数据结构B

基于状压的线性 RMQ 算法

严格 $\mathcal{O}(N)$ 预处理, $\mathcal{O}(1)$ 查询。

```
1
    template<class T, class Cmp = less<T>> struct RMQ {
 2
         const Cmp cmp = Cmp();
 3
        static constexpr unsigned B = 64;
 4
        using u64 = unsigned long long;
 5
        int n;
 6
        vector<vector<T>> a;
 7
        vector<T> pre, suf, ini;
 8
        vector<u64> stk;
 9
         RMQ() \{ \}
         RMQ(const vector<T> &v) {
10
11
             init(v);
12
13
        void init(const vector<T> &v) {
14
             n = v.size();
             pre = suf = ini = v;
15
             stk.resize(n);
16
17
             if (!n) {
18
                 return:
19
             }
20
             const int M = (n - 1) / B + 1;
21
             const int lg = \underline{\hspace{0.1cm}} lg(M);
             a.assign(lg + 1, vector<T>(M));
22
23
             for (int i = 0; i < M; i++) {
24
                 a[0][i] = v[i * B];
25
                 for (int j = 1; j < B && i * B + j < n; j++) {
26
                     a[0][i] = min(a[0][i], v[i * B + j], cmp);
27
28
29
             for (int i = 1; i < n; i++) {
30
                 if (i % B) {
                     pre[i] = min(pre[i], pre[i - 1], cmp);
31
32
                 }
33
34
             for (int i = n - 2; i >= 0; i--) {
35
                 if (i % B != B - 1) {
                     suf[i] = min(suf[i], suf[i + 1], cmp);
36
37
38
             for (int j = 0; j < lg; j++) {
39
                 for (int i = 0; i + (2 << j) <= M; i++) {
40
41
                     a[j + 1][i] = min(a[j][i], a[j][i + (1 << j)], cmp);
42
                 }
43
44
             for (int i = 0; i < M; i++) {
45
                 const int l = i * B;
46
                 const int r = min(1U * n, 1 + B);
```

```
47
                  u64 s = 0;
48
                  for (int j = 1; j < r; j++) {
49
                      while (s \&\& cmp(v[j], v[\_]g(s) + 1])) {
50
                           s \land = 1ULL << __lg(s);
51
52
                      s = 1ULL << (j - 1);
53
                      stk[j] = s;
54
                  }
             }
55
56
         }
         T operator()(int 1, int r) {
57
             if (1 / B != (r - 1) / B) {
58
59
                  T ans = min(suf[1], pre[r - 1], cmp);
60
                  1 = 1 / B + 1;
                  r = r / B;
61
                  if (1 < r) {
62
                      int k = \underline{\hspace{1cm}} \lg(r - 1);
63
64
                      ans = min({ans, a[k][1], a[k][r - (1 << k)]}, cmp);
65
                  }
66
                  return ans;
             } else {
67
                  int x = B * (1 / B);
68
69
                  return \ ini[\_builtin\_ctz]](stk[r - 1] >> (l - x)) + l];
70
             }
71
         }
72
    };
```

珂朵莉树 (OD Tree)

区间赋值的数据结构都可以骗分,在数据随机的情况下,复杂度可以保证,时间复杂度: $\mathcal{O}(N\log\log N)$ 。

```
1
    struct ODT {
 2
         struct node {
 3
             int 1, r;
 4
             mutable LL v;
 5
             node(int 1, int r = -1, LL v = 0) : 1(1), r(r), v(v) {}
             bool operator<(const node &o) const {</pre>
 6
 7
                  return 1 < 0.1;
             }
 8
 9
         };
10
         set<node> s;
11
         ODT() {
12
              s.clear();
13
         }
         auto split(int pos) {
14
15
             auto it = s.lower_bound(node(pos));
16
             if (it != s.end() && it->1 == pos) return it;
17
             it--;
             int l = it \rightarrow l, r = it \rightarrow r;
18
19
             LL v = it -> v;
20
             s.erase(it);
21
             s.insert(node(l, pos - 1, v));
```

```
return s.insert(node(pos, r, v)).first;
22
23
        }
24
        void assign(int 1, int r, LL x) {
25
             auto itr = split(r + 1), itl = split(l);
26
             s.erase(itl, itr);
27
             s.insert(node(1, r, x));
28
29
        void add(int 1, int r, LL x) {
             auto itr = split(r + 1), itl = split(l);
30
31
             for (auto it = itl; it != itr; it++) {
32
                 it->v += x;
33
             }
34
35
        LL kth(int 1, int r, int k) {
             vector<pair<LL, int>> a;
36
             auto itr = split(r + 1), itl = split(l);
37
             for (auto it = itl; it != itr; it++) {
38
39
                 a.push_back(pair<LL, int>(it->v, it->r - it->1 + 1);
40
             }
             sort(a.begin(), a.end());
41
42
             for (auto [val, len] : a) {
43
                 k \rightarrow 1en;
44
                 if (k <= 0) return val;
45
             }
46
47
        LL power(LL a, int b, int mod) {
48
             a \% = mod;
49
            LL res = 1;
50
             for (; b; b /= 2, a = a * a % mod) {
51
                 if (b % 2) {
52
                     res = res * a % mod;
53
54
             }
55
             return res;
56
57
        LL powersum(int 1, int r, int x, int mod) {
             auto itr = split(r + 1), itl = split(l);
59
            LL ans = 0;
60
             for (auto it = itl; it != itr; it++) {
                 ans = (ans + power(it->v, x, mod) * (it->r - it->l + 1) % mod) % mod;
             }
63
             return ans;
64
        }
65
    };
```

pbds 扩展库实现平衡二叉树

记得加上相应的头文件,同时需要注意定义时的参数,一般只需要修改第三个参数:即定义的是大根堆还是小根堆。

附常见成员函数:

```
1empty() / size()2insert(x) // 插入元素x3erase(x) // 删除元素/迭代器x4order_of_key(x) // 返回元素x的排名5find_by_order(x) // 返回排名为x的元素迭代器6lower_bound(x) / upper_bound(x) // 返回迭代器7join(Tree) // 将Tree树的全部元素并入当前的树8split(x, Tree) // 将大于x的元素放入Tree树
```

```
1
    #include <ext/pb_ds/assoc_container.hpp>
 2
    using namespace __gnu_pbds;
 3
    using V = pair<int, int>;
 4
    tree<V, null_type, less<V>, rb_tree_tag, tree_order_statistics_node_update> ver;
 5
    map<int, int> dic;
 6
 7
    int n; cin >> n;
 8
    for (int i = 1, op, x; i \le n; i++) {
9
        cin >> op >> x;
        if (op == 1) { // 插入一个元素x, 允许重复
10
11
            ver.insert({x, ++dic[x]});
12
        } else if (op == 2) { // 删除元素x, 若有重复, 则任意删除一个
13
            ver.erase({x, dic[x]--});
14
        } else if (op == 3) { // 查询元素x的排名(排名定义为比当前数小的数的个数+1)
15
            cout << ver.order_of_key(\{x, 1\}) + 1 << end];
16
        } else if (op == 4) { // 查询排名为x的元素
            cout << ver.find_by_order(--x)->first << endl;</pre>
17
        } else if (op == 5) { // 查询元素x的前驱
18
19
            int idx = ver.order_of_key({x, 1}) - 1; // 无论x存不存在, idx都代表x的位置, 需要-1
20
            cout << ver.find_by_order(idx)->first << endl;</pre>
21
        } else if (op == 6) { // 查询元素x的后继
22
            int idx = ver.order_of_key( {x, dic[x]}); // 如果x不存在,那么idx就是x的后继
23
            if (ver.find({x, 1}) != ver.end()) idx++; // 如果x存在,那么idx是x的位置,需要+1
24
            cout << ver.find_by_order(idx)->first << endl;</pre>
25
        }
26
    }
```

vector 模拟实现平衡二叉树

```
1
    #define ALL(x) x.begin(), x.end()
 2
    #define pre lower_bound
 3
    #define suf upper_bound
 4
    int n; cin >> n;
 5
    vector<int> ver;
 6
    for (int i = 1, op, x; i \le n; i++) {
        cin >> op >> x;
 7
 8
        if (op == 1) ver.insert(pre(ALL(ver), x), x);
9
        if (op == 2) ver.erase(pre(ALL(ver), x));
10
        if (op == 3) cout << pre(ALL(ver), x) - ver.begin() + 1 <math><< end1;
        if (op == 4) cout << ver[x - 1] << end];
11
12
        if (op == 5) cout << ver[pre(ALL(ver), x) - ver.begin() - 1] <math><< end1;
13
        if (op == 6) cout << ver[suf(ALL(ver), x) - ver.begin()] << endl;</pre>
```

```
14 }
```

取模类

集成了常见的取模四则运算,运算速度与手动取模相差无几,效率极高。

```
1
    using i64 = long long;
 2
 3
    template<class T> constexpr T mypow(T n, i64 k) {
 4
        T r = 1;
 5
        for (; k; k \neq 2, n *= n) {
            if (k % 2) {
 6
                 r *= n;
 8
            }
9
10
        return r;
11
12
13
    template<class T> constexpr T power(int n) {
14
        return mypow(T(2), n);
15
    }
16
17
    template<const int &MOD> struct Zmod {
18
        int x;
19
        Zmod(signed x = 0) : x(norm(x % MOD)) {}
        Zmod(i64 x) : x(norm(x % MOD)) {}
20
21
22
        constexpr int norm(int x) const noexcept {
23
            if (x < 0) [[unlikely]] {</pre>
24
                X += MOD;
25
            if (x \ge MOD) [[unlikely]] {
26
27
                 X -= MOD;
28
29
            return x;
30
        }
        explicit operator int() const {
31
32
             return x;
33
        constexpr int val() const {
34
35
             return x;
36
37
        constexpr Zmod operator-() const {
            Zmod\ val = norm(MOD - x);
38
39
             return val;
40
        constexpr Zmod inv() const {
41
            assert(x != 0);
42
             return mypow(*this, MOD - 2);
43
44
        friend constexpr auto &operator>>(istream &in, Zmod &j) {
45
46
            int v;
```

```
47
            in >> v;
48
             j = Zmod(v);
49
             return in;
50
        friend constexpr auto &operator<<(ostream &o, const Zmod &j) {</pre>
51
52
             return o << j.val();</pre>
53
        constexpr Zmod &operator++() {
54
55
            x = norm(x + 1);
56
             return *this;
57
58
        constexpr Zmod &operator--() {
59
            x = norm(x - 1);
60
             return *this;
61
        constexpr Zmod operator++(signed) {
62
63
            Zmod res = *this;
64
            ++*this;
65
            return res;
66
        constexpr Zmod operator--(signed) {
67
            Zmod res = *this;
68
69
             --*this;
70
             return res:
71
        constexpr Zmod &operator+=(const Zmod &i) {
72
73
            x = norm(x + i.x);
74
             return *this;
75
        constexpr Zmod &operator-=(const Zmod &i) {
76
77
            x = norm(x - i.x);
78
             return *this;
79
80
        constexpr Zmod &operator*=(const Zmod &i) {
81
            x = i64(x) * i.x % MOD;
82
            return *this;
83
        constexpr Zmod &operator/=(const Zmod &i) {
84
85
             return *this *= i.inv();
86
        constexpr Zmod &operator%=(const int &i) {
87
88
             return x %= i, *this;
89
90
        friend constexpr Zmod operator+(const Zmod i, const Zmod j) {
             return Zmod(i) += j;
91
92
93
        friend constexpr Zmod operator-(const Zmod i, const Zmod j) {
94
             return Zmod(i) -= j;
95
96
        friend constexpr Zmod operator*(const Zmod i, const Zmod j) {
97
             return Zmod(i) *= j;
98
        }
99
        friend constexpr Zmod operator/(const Zmod i, const Zmod j) {
```

```
100
             return Zmod(i) /= j;
101
102
         friend constexpr Zmod operator%(const Zmod i, const int j) {
103
              return Zmod(i) %= j;
104
105
         friend constexpr bool operator==(const Zmod i, const Zmod j) {
106
              return i.val() == j.val();
107
         }
         friend constexpr bool operator!=(const Zmod i, const Zmod j) {
108
109
              return i.val() != j.val();
110
         friend constexpr bool operator<(const Zmod i, const Zmod j) {</pre>
111
112
              return i.val() < j.val();</pre>
113
         }
         friend constexpr bool operator>(const Zmod i, const Zmod j) {
114
115
              return i.val() > j.val();
116
         }
117
     };
118
     int MOD[] = \{998244353, 1000000007\};
119
120
     using Z = Zmod<MOD[1]>;
```

分数运算类

定义了分数的四则运算,如果需要处理浮点数,那么需要将函数中的 gcd 运算替换为 fgcd 。

```
template<class T> struct Frac {
 1
 2
        T x, y;
 3
        Frac() : Frac(0, 1) {}
 4
        Frac(T x_) : Frac(x_, 1) \{\}
 5
        Frac(T x_{,} T y_{,}) : x(x_{,}), y(y_{,}) {
 6
            if (y < 0) {
 7
                 y = -y;
 8
                 x = -x;
 9
             }
10
        }
11
        constexpr double val() const {
12
13
             return 1. * x / y;
14
        }
15
        constexpr Frac norm() const { // 调整符号、转化为最简形式
16
            T p = gcd(x, y);
17
             return \{x / p, y / p\};
18
19
         friend constexpr auto &operator<<(ostream &o, const Frac &j) {
20
            T p = gcd(j.x, j.y);
21
            if (j.y == p) {
                 return o << j.x / p;
22
23
            } else {
24
                 return o << j.x / p << "/" << j.y / p;
25
             }
        }
26
```

```
constexpr Frac &operator/=(const Frac &i) {
27
28
            x *= i.y;
29
            y *= i.x;
30
            if (y < 0) {
31
                x = -x;
32
                y = -y;
33
            }
34
            return *this;
35
        constexpr Frac & operator+=(const Frac &i) { return x = x * i.y + y * i.x, y *= i.y,
36
        constexpr Frac & operator = (const Frac &i) { return x = x * i.y - y * i.x, y *= i.y,
37
    *this: }
38
        constexpr Frac &operator*=(const Frac &i) { return x *= i.x, y *= i.y, *this; }
39
        friend constexpr Frac operator+(const Frac i, const Frac j) { return i += j; }
        friend constexpr Frac operator-(const Frac i, const Frac j) { return i -= j; }
40
        friend constexpr Frac operator*(const Frac i, const Frac j) { return i *= j; }
41
        friend constexpr Frac operator/(const Frac i, const Frac j) { return i /= j; }
42
43
        friend constexpr Frac operator-(const Frac i) { return Frac(-i.x, i.y); }
        friend constexpr bool operator<(const Frac i, const Frac j) { return i.x * j.y <</pre>
44
    i.y * j.x; }
        friend constexpr bool operator>(const Frac i, const Frac j) { return i.x * j.y >
45
    i.y * j.x; }
        friend constexpr bool operator==(const Frac i, const Frac j) { return i.x * j.y ==
46
    i.y * j.x; }
        friend constexpr bool operator!=(const Frac i, const Frac j) { return i.x * j.y !=
47
    i.y * j.x; }
48
    };
```

大整数类(高精度计算)

```
const int base = 1000000000;
    const int base_digits = 9; // 分解为九个数位一个数字
 2
 3
    struct bigint {
        vector<int> a;
 4
 5
        int sign;
 6
 7
        bigint() : sign(1) {}
        bigint operator-() const {
 8
            bigint res = *this;
 9
10
            res.sign = -sign;
            return res;
11
12
        bigint(long long v) {
13
            *this = v;
14
15
        bigint(const string &s) {
16
17
            read(s);
18
        void operator=(const bigint &v) {
19
20
            sign = v.sign;
21
            a = v.a;
```

```
22
23
        void operator=(long long v) {
24
            a.clear();
25
            sign = 1;
            if (v < 0) sign = -1, v = -v;
26
            for (; v > 0; v = v / base) {
27
                a.push_back(v % base);
28
29
            }
30
        }
31
32
        // 基础加减乘除
33
        bigint operator+(const bigint &v) const {
            if (sign == v.sign) {
34
35
                bigint res = v;
                 for (int i = 0, carry = 0; i < (int)max(a.size(), v.a.size()) || carry;</pre>
36
    ++i) {
                     if (i == (int)res.a.size()) {
37
38
                         res.a.push_back(0);
39
                     res.a[i] += carry + (i < (int)a.size() ? a[i] : 0);
40
41
                     carry = res.a[i] >= base;
42
                     if (carry) {
43
                         res.a[i] -= base;
44
                     }
45
                 }
46
                 return res;
47
            }
48
            return *this - (-v);
49
        bigint operator-(const bigint &v) const {
50
51
            if (sign == v.sign) {
52
                if (abs() >= v.abs()) {
                     bigint res = *this;
53
54
                     for (int i = 0, carry = 0; i < (int)v.a.size() || carry; ++i) {
55
                         res.a[i] -= carry + (i < (int)v.a.size() ? v.a[i] : 0);
56
                         carry = res.a[i] < 0;
                         if (carry) {
57
                             res.a[i] += base;
58
59
                         }
60
                     res.trim();
61
62
                     return res;
63
64
                 return -(v - *this);
65
66
            return *this + (-v);
67
        void operator*=(int v) {
68
69
            check(v);
70
            for (int i = 0, carry = 0; i < (int)a.size() || carry; ++i) {
71
                 if (i == (int)a.size()) {
72
                     a.push_back(0);
73
                }
```

```
74
                  long long cur = a[i] * (long long)v + carry;
 75
                  carry = (int)(cur / base);
 76
                  a[i] = (int)(cur \% base);
 77
              }
 78
              trim();
 79
         void operator/=(int v) {
 80
 81
              check(v);
              for (int i = (int)a.size() - 1, rem = 0; i >= 0; --i) {
 82
 83
                  long long cur = a[i] + rem * (long long)base;
                  a[i] = (int)(cur / v);
 84
                  rem = (int)(cur % v);
 85
 86
              }
 87
              trim();
 88
         int operator%(int v) const {
 89
 90
              if (v < 0) {
 91
                  V = -V;
 92
              }
              int m = 0;
 93
 94
              for (int i = a.size() - 1; i >= 0; --i) {
 95
                  m = (a[i] + m * (long long)base) % v;
 96
 97
              return m * sign;
 98
 99
100
         void operator+=(const bigint &v) {
101
              *this = *this + v;
102
103
         void operator-=(const bigint &v) {
104
              *this = *this - v;
105
106
         bigint operator*(int v) const {
107
              bigint res = *this;
108
              res *= v;
109
              return res;
110
         bigint operator/(int v) const {
111
112
              bigint res = *this;
113
              res /= v;
114
              return res;
115
116
         void operator%=(const int &v) {
117
              *this = *this % v;
118
         }
119
120
         bool operator<(const bigint &v) const {</pre>
121
              if (sign != v.sign) return sign < v.sign;</pre>
122
              if (a.size() != v.a.size()) return a.size() * sign < v.a.size() * v.sign;</pre>
123
              for (int i = a.size() - 1; i >= 0; i--)
124
                  if (a[i] != v.a[i]) return a[i] * sign < v.a[i] * sign;
125
              return false;
126
         }
```

```
127
         bool operator>(const bigint &v) const {
128
              return v < *this;
129
130
         bool operator<=(const bigint &v) const {</pre>
131
              return !(v < *this);</pre>
132
         }
133
         bool operator>=(const bigint &v) const {
              return !(*this < v);</pre>
134
135
         }
136
         bool operator==(const bigint &v) const {
137
              return !(*this < v) && !(v < *this);
138
         }
         bool operator!=(const bigint &v) const {
139
140
              return *this < v || v < *this;
141
         }
142
         bigint abs() const {
143
144
             bigint res = *this;
145
              res.sign *= res.sign;
146
              return res:
147
148
         void check(int v) { // 检查输入的是否为负数
149
             if (v < 0) {
150
                  sign = -sign;
151
                  V = -V;
152
             }
153
         }
154
         void trim() { // 去除前导零
155
             while (!a.empty() && !a.back()) a.pop_back();
156
             if (a.empty()) sign = 1;
157
         }
158
         bool iszero() const { // 判断是否等于零
159
              return a.empty() \mid \mid (a.size() == 1 && !a[0]);
160
         friend bigint qcd(const bigint &a, const bigint &b) {
161
162
              return b.isZero() ? a : gcd(b, a % b);
163
         friend bigint lcm(const bigint &a, const bigint &b) {
164
165
              return a / \gcd(a, b) * b;
166
         void read(const string &s) {
167
168
             sign = 1;
169
              a.clear();
170
             int pos = 0;
             while (pos < (int)s.size() && (s[pos] == '-' || s[pos] == '+')) {
171
172
                  if (s[pos] == '-') sign = -sign;
173
                 ++pos;
174
175
              for (int i = s.size() - 1; i >= pos; i -= base_digits) {
176
                  int x = 0;
177
                  for (int j = max(pos, i - base\_digits + 1); j <= i; j++) x = x * 10 + s[j]
     - '0';
178
                  a.push_back(x);
```

```
179
              }
180
              trim();
181
182
          friend istream &operator>>(istream &stream, bigint &v) {
183
              strina s:
184
              stream >> s;
185
              v.read(s);
186
              return stream;
187
188
          friend ostream &operator<<(ostream &stream, const bigint &v) {
              if (v.sign == -1) stream << '-';
189
190
              stream << (v.a.empty() ? 0 : v.a.back());</pre>
191
              for (int i = (int)v.a.size() - 2; i >= 0; --i)
192
                  stream << setw(base_digits) << setfill('0') << v.a[i];</pre>
193
              return stream;
194
         }
195
196
         /* 大整数乘除大整数部分 */
197
          typedef vector<long long> vll;
          bigint operator*(const bigint &v) const { // 大整数乘大整数
198
199
              vector<int> a6 = convert_base(this->a, base_digits, 6);
              vector<int> b6 = convert_base(v.a, base_digits, 6);
200
201
              v11 a(a6.begin(), a6.end());
202
              vll b(b6.begin(), b6.end());
              while (a.size() < b.size()) a.push_back(0);</pre>
              while (b.size() < a.size()) b.push_back(0);</pre>
204
205
              while (a.size() & (a.size() - 1)) a.push_back(0), b.push_back(0);
206
              vll c = karatsubaMultiply(a, b);
207
              bigint res;
              res.sign = sign * v.sign;
208
209
              for (int i = 0, carry = 0; i < (int)c.size(); i++) {
210
                  long long cur = c[i] + carry;
                  res.a.push_back((int)(cur % 1000000));
211
212
                  carry = (int)(cur / 1000000);
213
214
              res.a = convert_base(res.a, 6, base_digits);
215
              res.trim();
216
              return res;
217
218
         friend pair<br/>bigint, bigint> divmod(const bigint &a1,
                                              const bigint &b1) { // 大整数除大整数,同时返回答案
219
     与余数
220
              int norm = base / (b1.a.back() + 1);
              bigint a = a1.abs() * norm;
221
              bigint b = b1.abs() * norm;
222
223
              bigint q, r;
224
              q.a.resize(a.a.size());
225
              for (int i = a.a.size() - 1; i >= 0; i--) {
226
                  r *= base;
227
                  r += a.a[i];
228
                  int s1 = r.a.size() \leftarrow b.a.size() ? 0 : r.a[b.a.size()];
                  int s2 = r.a.size() \leftarrow b.a.size() - 1 ? 0 : r.a[b.a.size() - 1];
229
230
                  int d = ((long long)base * s1 + s2) / b.a.back();
```

```
231
                  r -= b * d;
232
                  while (r < 0) r += b, --d;
233
                  q.a[i] = d;
234
             }
              q.sign = a1.sign * b1.sign;
235
236
              r.sign = a1.sign;
237
              q.trim();
238
              r.trim();
239
              return make_pair(q, r / norm);
240
241
         static vector<int> convert_base(const vector<int> &a, int old_digits, int
     new_digits) {
242
             vector<long long> p(max(old_digits, new_digits) + 1);
243
              p[0] = 1;
244
              for (int i = 1; i < (int)p.size(); i++) p[i] = p[i - 1] * 10;
245
             vector<int> res;
             long long cur = 0;
246
247
             int cur_digits = 0;
248
              for (int i = 0; i < (int)a.size(); i++) {
                  cur += a[i] * p[cur_digits];
249
250
                  cur_digits += old_digits;
                  while (cur_digits >= new_digits) {
251
252
                      res.push_back((int)(cur % p[new_digits]));
253
                      cur /= p[new_digits];
254
                      cur_digits -= new_digits;
255
                  }
256
             }
257
              res.push_back((int)cur);
258
              while (!res.empty() && !res.back()) res.pop_back();
259
              return res;
260
261
         static vll karatsubaMultiply(const vll &a, const vll &b) {
              int n = a.size();
262
263
             vll res(n + n);
264
              if (n <= 32) {
265
                  for (int i = 0; i < n; i++) {
                      for (int j = 0; j < n; j++) {
266
267
                          res[i + j] += a[i] * b[j];
268
269
                  }
270
                  return res;
271
             }
272
273
             int k = n \gg 1;
              vll a1(a.begin(), a.begin() + k);
274
275
             v11 a2(a.begin() + k, a.end());
276
             vll b1(b.begin(), b.begin() + k);
277
             v11 b2(b.begin() + k, b.end());
278
279
             vll a1b1 = karatsubaMultiply(a1, b1);
280
              vll a2b2 = karatsubaMultiply(a2, b2);
281
282
              for (int i = 0; i < k; i++) a2[i] += a1[i];
```

```
283
             for (int i = 0; i < k; i++) b2[i] += b1[i];
284
285
             vll r = karatsubaMultiply(a2, b2);
286
             for (int i = 0; i < (int)a1b1.size(); i++) r[i] -= a1b1[i];
             for (int i = 0; i < (int)a2b2.size(); i++) r[i] -= a2b2[i];
287
288
289
             for (int i = 0; i < (int)r.size(); i++) res[i + k] += r[i];
             for (int i = 0; i < (int)a1b1.size(); i++) res[i] += a1b1[i];
290
             for (int i = 0; i < (int)a2b2.size(); i++) res[i + n] += a2b2[i];
291
292
             return res;
293
         }
294
295
         void operator*=(const bigint &v) {
296
             *this = *this * v;
297
         bigint operator/(const bigint &v) const {
298
299
             return divmod(*this, v).first;
300
301
         void operator/=(const bigint &v) {
             *this = *this / v:
302
303
304
         bigint operator%(const bigint &v) const {
305
             return divmod(*this, v).second;
306
         }
307
         void operator%=(const bigint &v) {
             *this = *this % v;
308
309
         }
310
     };
```

常见结论

题意:(区间移位问题)要求将整个序列左移/右移若干个位置,例如,原序列为 $A=(a_1,a_2,\ldots,a_n)$,右移 x 位后变为 $A=(a_{x+1},a_{x+2},\ldots,a_n,a_1,a_2,\ldots,a_x)$ 。

区间的端点只是一个数字,即使被改变了,通过一定的转换也能够还原,所以我们可以 $\mathcal{O}(1)$ 解决这一问题。为了方便计算,我们规定下标从 0 开始,即整个线段的区间为 [0,n) ,随后,使用一个偏移量 shift 记录。使用 shift = (shift + x) % n; 更新偏移量;此后的区间查询/修改前,再将坐标偏移回去即可,下方代码使用区间修改作为示例。

```
cin >> 1 >> r >> x;
2
    1--; // 坐标修改为 0 开始
    r--;
3
   1 = (1 + shift) % n; // 偏移
    r = (r + shift) % n;
6
   if (1 > r) { // 区间分离则分别操作
7
        segt.modify(1, n - 1, x);
8
        segt.modify(0, r, x);
9
    } else {
10
        segt.modify(1, r, x);
    }
11
```

常见例题

题意: (带修莫队-维护队列) 要求能够处理以下操作:

- 'Q' 1 r: 询问区间 [l, r] 有几个颜色;
- 'R' idx w: 将下标 idx 的颜色修改为 w。

输入格式为: 第一行 n 和 q ($1 \le n, q \le 133333$) 分别代表区间长度和操作数量;第二行 n 个整数 $a_1, a_2 \dots, a_n$ ($1 \le a_i \le 10^6$) 代表初始颜色;随后 q 行为具体操作。

```
const int N = 1e6 + 7;
 2
    signed main() {
 3
        int n, q;
 4
        cin >> n >> q;
 5
        vector<int> w(n + 1);
 6
        for (int i = 1; i <= n; i++) {
 7
            cin >> w[i];
 8
        }
 9
10
        vector<array<int, 4>> query = {{}}}; // {左区间, 右区间, 累计修改次数, 下标}
11
        vector<array<int, 2>> modify = {{}}; // {修改的值, 修改的元素下标}
12
        for (int i = 1; i \le q; i++) {
            char op;
13
14
            cin >> op;
15
            if (op == 'Q') {
                int 1, r;
16
17
                cin >> 1 >> r;
                query.push_back({1, r, (int)modify.size() - 1, (int)query.size()});
18
19
            } else {
20
                int idx, w;
21
                cin >> idx >> w;
22
                modify.push_back({w, idx});
23
            }
24
        }
25
26
        int Knum = 2154; // 计算块长
27
        vector<int> K(n + 1);
        for (int i = 1; i <= n; i++) { // 固定块长
28
29
            K[i] = (i - 1) / Knum + 1;
30
        sort(query.begin() + 1, query.end(), [&](auto x, auto y) {
31
32
            if (K[x[0]] != K[y[0]]) return x[0] < y[0];
33
            if (K[x[1]] != K[y[1]]) return x[1] < y[1];
34
            return x[3] < y[3];
35
        });
36
37
        int l = 1, r = 0, val = 0;
        int t = 0; // 累计修改次数
38
39
        vector<int> ans(query.size()), cnt(N);
40
        for (int i = 1; i < query.size(); i++) {
41
            auto [q], qr, qt, id] = query[i];
42
            auto add = [\&](int x) -> void {
43
                if (cnt[x] == 0) ++ val;
```

```
44
                ++ cnt[x];
45
            };
46
            auto del = [\&](int x) -> void {
47
                -- cnt[x];
48
                if (cnt[x] == 0) -- val;
49
            };
50
            auto time = [\&] (int x, int 1, int r) -> void {
51
                if (l \le modify[x][1] && modify[x][1] \le r) { //当修改的位置在询问期间内部时才会
    改变num的值
52
                    del(w[modify[x][1]]);
53
                    add(modify[x][0]);
54
                }
55
                swap(w[modify[x][1]], modify[x][0]); //直接交换修改数组的值与原始值,减少额外的数
    组开销,且方便复原
56
            };
57
            while (1 > q1) add(w[--1]);
58
            while (r < qr) add(w[++r]);
59
            while (1 < q1) del(w[1++]);
60
            while (r > qr) del(w[r--]);
61
            while (t < qt) time(++t, ql, qr);</pre>
            while (t > qt) time(t--, ql, qr);
63
            ans[id] = val;
64
        }
65
        for (int i = 1; i < ans.size(); i++) {
            cout << ans[i] << endl;</pre>
67
        }
68
   }
```

/END/