1   Helmholtz decomposition

The Helmholtz decomposition state that a velocity field $\boldsymbol{v}$ that decays fast enough at infinity, or is on a bounded domain, can be decomposed into a curl-free component and a divergence-free component (Petrascheck 2015)

$$\boldsymbol{v} = -\nabla\Phi + \nabla \times \boldsymbol{A}. \tag{1}$$

(1.a) Show that this is a generalization of the 2D situation where we have scalar potential and stream functions.

(1.b) How would you find these two potentials if you know the divergence and vorticity in the domain?

2   Baby steps in complex analysis.

We will learn a little bit of complex analysis. My reference is Stein and Shakarchi 2010.

References

Petrascheck, D. (Nov. 2015). "The Helmholtz Decomposition Revisited". In: *European Journal of Physics* 37.1, p. 015201. ISSN: 0143-0807. DOI: 10.1088/0143-0807/37/1/015201.

Stein, Elias M. and Rami Shakarchi (Apr. 22, 2010). *Complex Analysis*. Princeton University Press. 400 pp. ISBN: 978-1-4008-3115-9.