Worksheet 2

1 Deformation tensor: examples

(1.a) Straining flow Take a velocity field with $u = x$, $v = -y$, and $w = 0$.

1. Calculate the deformation tensor $\boldsymbol{F}$.
2. Verify the Lagrangian equation of evolution for $\boldsymbol{F}$

$$\left.\frac{\partial \boldsymbol{F}}{\partial t}\right|_{\boldsymbol{\alpha}} = \nabla_{\boldsymbol{x}} \boldsymbol{v} \cdot \boldsymbol{F}. \tag{1}$$

(1.b) Take a velocity field with $u = 2x^{1/2}$, $v = 2y^{1/2}$, and $w = 2z^{1/2}$.

1. Do the same calculation.
2. Also verify the Eulerian equation of evolution for $\boldsymbol{F}$:

$$\left.\frac{\partial \boldsymbol{F}}{\partial t}\right|_{\boldsymbol{x}} + (\boldsymbol{v} \cdot \nabla_{\boldsymbol{x}})\boldsymbol{F} = \nabla_{\boldsymbol{x}} \boldsymbol{v} \cdot \boldsymbol{F}. \tag{2}$$

2 Evolution of an infinitesimal material line element

Take an infinitesimal material line element $\delta\boldsymbol{\ell}$, where it is the infinitesimal material element connecting $\boldsymbol{\ell}$ and $\boldsymbol{\ell} + \delta\boldsymbol{\ell}$. Show that its evolution equation follows the equation

$$\frac{D\delta\boldsymbol{\ell}}{Dt} = \nabla_{\boldsymbol{x}} \boldsymbol{v} \cdot \delta\boldsymbol{\ell}. \tag{3}$$

You will do this in the homework using (2). I want you to think about it from another perspective: use your physical intuition and argue using infinitesimal time.

Hint: local in time, the velocity at $\boldsymbol{\ell}$ is $\boldsymbol{v}$ and the velocity at $\boldsymbol{\ell} + \delta\boldsymbol{\ell}$ is $\boldsymbol{v} + \delta\boldsymbol{v}$. What is $\delta\boldsymbol{v}$?