GFD Homework 2

Submit to NYU Classes by Nov. 3rd

1. Consider two functions

$$A(x) = \mathrm{Re}(\hat{A}\exp(i\frac{2n\pi x}{L}))$$
$$B(x) = \mathrm{Re}(\hat{B}\exp(i\frac{2n\pi x}{L})),$$

   where n is a (non zero) integer and $\mathrm{Re}(z) = \frac{1}{2}(z + z^*)$ is the real part of a complex number. We define the (zonal) average of a function f as

$$\overline{f} = \frac{1}{L}\int_0^L f(x)dx.$$

   Show that

$$\overline{AB} = \frac{1}{2}\mathrm{Re}(\hat{A}^*\hat{B}).$$

2. (Rossby waves and transport) Consider the linearized QG shallow water PV equation:

$$\frac{\partial}{\partial t}(\nabla^2\psi - \frac{\psi}{L_d^2}) + \beta\frac{\partial\psi}{\partial x} = 0.$$

   We consider Rossby waves as solution of this equation of the form

$$\psi(x, y, t) = \mathrm{Re}(\hat{\psi}\exp(i(kx + ly - \omega)).$$

   (a) Derive the dispersion relation for the Rossby wave, and compute the group velocity.

   (b) Show that the zonal mean potential vorticity flux $\overline{vq}$ associated with a Rossby wave vanishes, where $q = \nabla^2\psi - \frac{\psi}{L_d^2}$ is the potential vorticity. [You can use the results from question 1]

   (c) Show that the meridional momentum transport $\overline{uv}$ is in the opposite direction to the meridional group velocity c_{gy}.