GFD Homework 1

Submit to NYU Classes by Sept. 30th

1. (Coriolis force) Study the trajectory of a free particle of mass M released from a state of rest on a rotating, slopping rigid plane. The angular rotation rate is Ω and the angle formed by the plane with the horizontal is α. Fritcion and the centrifugal force are negligible. What is the maximum speed acquired by the particle, and what is its maximum downhill displacement?

2. (Riemann Invariants and non-linear waves) Consider the one dimensional shallow water equation on a plane:

$$\frac{du}{dt} = -g\frac{\partial h}{\partial x} \tag{1}$$

$$\frac{\partial h}{\partial t} + \frac{\partial (uh)}{\partial x} = 0 \tag{2}$$

   (a) Show that the governing equations for the two Riemann invariants $R^+ = u + 2\sqrt{gh}$ and $R^- = u - 2\sqrt{gh}$ can be written as

$$\frac{\partial R^+}{\partial t} + c^+\frac{\partial R^+}{\partial x} = 0$$

$$\frac{\partial R^-}{\partial t} + c^-\frac{\partial R^-}{\partial x} = 0.$$

   Determine the propogation speed c^+ and c^-2.

   (b) Consider a small amplitude perturbaton with $h(x,t) = H = h'(x,t)$ and $u(x,t) = u'(x,t)$, as in section 3.8.1. Compute the leading order expansion for the Riemann inavriants and propagation speed and compare to the linear solution obtained in section 3.8.1.

3. Goestrophic adjustment Consider the evolution of the linearized f-plane shallow water equations in an infinite domain. Suppose that initially

the surface is flat, the zonal velocity is zero and the meridional velocity is given by

$$\begin{aligned} v(x,y) &= v_0(1+\cos(\pi x/L)) \text{ for } -L < x < L \\ &= 0 \text{ otherwise.} \end{aligned} \tag{3}$$

(a) Find the equilibrium height and velocity fields at $t = \infty$.

(b) What are the initial and final kinetic and potential energies?

(c) How does the split between kinetic and potential energies in the final state depend on the width of the initial jet L? Contrast wide and narrow jets, and discuss what narrow and wide mean here.