GFD Homework 2

Submit to NYU Classes by Dec. 4th

1. (Available Potential Energy in the Boussines approximation.) We onsider a Boussinesq system:

$$\frac{D\mathbf{u}}{Dt} = -\nabla P + B\mathbf{k} \quad (1)$$

$$\frac{DB}{Dt} = 0 \quad (2)$$

$$\nabla \cdot \mathbf{u} = 0 \quad (3)$$

where B is the buoyancy and $\mathbf{k}$ is the unit vector in the vertical direction.

   (a) Show that an internal energy can be defined as

   $$M(B, z) = (z_0 - z)B$$

   so that the totql energy - defined as sum of the internal energy and potential energy - is conserved.

   (b) Assume a small amplitude perturbation of the form

   $$\mathbf{u} = \epsilon\mathbf{u}'(x, y, z, t) \quad (4)$$

   $$B = N^2 z + \epsilon b'(x, y, z, t), \quad (5)$$

   where ϵ is small. Write the linearized equations of motion for a small pertuabtion and shows that a generic energy conservation here is for the sum of the kinetic energy and an Available Potentisal Energy (APE) that is proportional to b'^2.

   (c) Show that this APE corresponds to the difference in internal energy (as defined in part a) between the perturbed state state and a reference state where all parcels have been moved back to their level of neutral buoyance $z_{LNB} = \frac{B}{N^2}$.

(d) A more general definition of APE (not limited to small perturbations) is to define it as the difference between the internal energy of the current state and that of a reference state. The reference state is defined as the state that minizes the total internal energy after an adiabatic redistribution of the fluid parcel while maintaining the hydrostatic balance. Consider a buoyancy profile of the shape

$$B(z) = N^2(z - z_0)^2 \quad \text{for } z \geq 0,$$

with both $N^2 > 0$ and z_0. Find the reference profile and compute the corresponding Available Potential Energy.