$$\begin{bmatrix} a_{11} & a_{12} & \cdots \\ a_{21} & a_{22} & \\ a_{31} & a_{32} & \\ & & \ddots \end{bmatrix} \rightarrow \left[\begin{array}{c|ccc} a_{11} & a_{12} & \cdots & \\ \hline 0 & \hat{a}_{22} & & \\ 0 & \hat{a}_{32} & & \\ 0 & \vdots & & \ddots \end{array}\right]$$

$$\hat{a}_{21} = a_{21} - a_{11}\frac{a_{21}}{a_{11}} = 0$$

$$\hat{a}_{22} = a_{22} - a_{12}\frac{a_{21}}{a_{11}} \Rightarrow |\hat{a}_{22}| \geq |a_{22}| - |a_{12}|\left|\frac{a_{21}}{a_{11}}\right|$$

$$\hat{a}_{j2} = a_{j2} - a_{12}\frac{a_{j1}}{a_{11}} \Rightarrow |\hat{a}_{j2}| \leq |a_{j2}| + |a_{12}|\left|\frac{a_{j1}}{a_{11}}\right|$$

$(j \geq 3)$

want $|\hat{a}_{22}| > \sum_{j\geq 3} |\hat{a}_{j2}|$

$$\sum_{j\geq 3} |\hat{a}_{j2}| \leq \sum_{j\geq 3} |a_{j2}| + \sum_{j\geq 3} |a_{12}|\left|\frac{a_{j1}}{a_{11}}\right|$$

$\wedge$?

$$|\hat{a}_{22}| \geq |a_{22}| - |a_{12}|\left|\frac{a_{21}}{a_{11}}\right|$$

Now $$\sum_{j\geq 3} |a_{j2}| + \sum_{j\geq 3} |a_{12}|\left|\frac{a_{j1}}{a_{11}}\right| + |a_{12}|\left|\frac{a_{21}}{a_{11}}\right|$$

$$= \sum_{j\geq 3} |a_{j2}| + |a_{12}|\underbrace{\left|\sum_{j\geq 2}\left|\frac{a_{j1}}{a_{11}}\right|\right|}_{<1}$$

$$\sum_{j\neq 2} |a_{j2}| < |a_{22}|$$