## 1

Let $A$ be a non-empty bounded subset of $\mathbb{R}$ and suppose $\sup A \notin A$. Show that there exists an increasing sequence of points $\{a_n\}_{n\geq 1}$ in $A$ such that $\lim_{n\to\infty} a_n = \sup A$.

## 2

**(2.a)** [Proposition 2.2.7 of Lebl 2023] Show that if a sequence $\{a_n\}_{n\in\mathbb{N}}$ of real numbers converges to $a$, then the sequence $\{|a_n|\}_{n\in\mathbb{N}}$ converges to $|a|$.

**(2.b)** Show (via an example) that the converse is not true.

## 3

Let $\{a_n\}_{n\geq 1}$ be a sequence of positive numbers. Show that

$$\lim_{n\to\infty} \frac{1}{a_n} = \infty \iff \lim_{n\to\infty} a_n = 0. \tag{1}$$

## 4

Show that the sequence

$$\lim_{n\to\infty} \left(\sqrt{n^2+n} - n\right) = \frac{1}{2}. \tag{2}$$

## 5

[Exercise 2.2.12 of ibid.] Let $\{a_n\}_{n\geq 1}$ and $\{b_n\}_{n\geq 1}$ be two sequences of real numbers such that $\{a_n\}_{n\geq 1}$ is bounded and $\{b_n\}_{n\geq 1}$ converges to 0. Show that the sequence $\{a_n b_n\}_{n\geq 1}$ converges to 0.

## 6

Let $\{a_n\}_{n\geq 1}$ be a convergent sequence of real numbers and let $a \in \mathbb{R}$ such that $\lim_{n\to\infty} a_n > a$. Show that there exists $n_0 \in \mathbb{N}$ such that $a_n > a$ for all $n > n_0$.

## References

Lebl, Jiri (July 11, 2023). *Basic Analysis I: Introduction to Real Analysis, Volume I*. version 6.0. URL: https://www.jirka.org/ra/realanal.pdf.