Worksheet 12

1

[Exercise 5.2.4 of Lebl 2023] Prove the mean value theorem for integrals: If $f : [a, b] \to \mathbb{R}$ is continuous, then there exists a $c \in [a, b]$ such that

$$\int_a^b f = f(c)(b - a). \tag{1}$$

2

[Exercise 5.3.8 of Lebl 2023] Suppose $f : [a, b] \to \mathbb{R}$ is continuous and $\int_a^x f = \int_x^b f$ for all x in $[a, b]$. Show that $f(x) = 0$ for all $x \in [a, b]$.

3

[Exercise 5.3.9 of Lebl 2023] Suppose $f : [a, b] \to \mathbb{R}$ is continuous and $\int_a^x f = 0$ for all rational x in $[a, b]$. Show that $f(x) = 0$ for all $x \in [a, b]$.

4

[Exercise 6.1.8 of Lebl 2023] Let $\{f_n\}_{n=1}^{\infty}$, $\{g_n\}_{n=1}^{\infty}$, and $\{h_n\}_{n=1}^{\infty}$ be sequences of functions on $[a, b]$. Suppose $\{f_n\}_{n=1}^{\infty}$ and $\{h_n\}_{n=1}^{\infty}$ converge uniformly to some function $f : [a, b] \to \mathbb{R}$ and suppose $f_n(x) \le g_n(x) \le h_n(x)$ for all $x \in [a, b]$. Show that $\{g_n\}_{n=1}^{\infty}$ converges uniformly to f.

5

[Exercise 6.2.5 of Lebl 2023] Find an example of a sequence of continuous functions on $[0, 1]$ that converges pointwise to a continuous function on $[0, 1]$, but the convergence is not uniform.

References

Lebl, Jiri (July 11, 2023). *Basic Analysis I: Introduction to Real Analysis, Volume I*. version 6.0. URL: https://www.jirka.org/ra/realanal.pdf.