Worksheet 1

1

Show that the following two statements are equivalent:

(1.a)

1. $f : B \to C$ is injective.
2. For any set A and any two functions $g, h : A \to B$ such that $f \circ g = f \circ h$, we must have $g = h$.

(1.b)

1. $f : A \to B$ is surjective.
2. For any set C and any two functions $g, h : B \to C$ such that $g \circ f = h \circ f$, we must have $g = h$.

2

Let $f : A \to B$ and $g : B \to C$ be two bijective functions. Show $g \circ f$ is bijective.

3

Let $f : A \to B$ and $C, D \subset A$. In lecture, you learned

$$f(C \cup D) = f(C) \cup f(D). \tag{1}$$

Show

$$f(C \cap D) \subset f(C) \cap f(D); \tag{2}$$
$$f^{-1}(C \cup D) = f^{-1}(C) \cup f^{-1}(D); \tag{3}$$
$$f^{-1}(C \cap D) = f^{-1}(C) \cap f^{-1}(D). \tag{4}$$

4

Let $f : X \to Y$ be a function between sets. Let $A, B \subset X$ be subsets.

(4.a) Show

$$f(A)/f(B) \subset f(A/B). \tag{5}$$

(4.b) Give an example function that shows the other direction is false.

5

Let $f : X \to Y$ be a function between sets and $U \subset A$ and $V \subset B$.

(5.a) Show that

$$f(f^{-1}(V)) \subset V \quad \text{and} \quad U \subset f^{-1}(f(U)). \tag{6}$$

(5.b) Give an example that shows the above relation cannot be equality.

6

If two sets A and B have the same cardinally, we write $A \sim B$. Show that $\sim$ is an equivalence relation on sets.

7

Decide for which natural numbers the inequality $2n > n^2$ is true. Prove your claim using induction.