Worksheet 5

1

Study the convergence of the series

(1.a)

$$\sum_{n\geq 1} \frac{n^5}{3^n}. \tag{1}$$

(1.b)

$$\sum_{n\geq 1} \frac{3^n}{n!}. \tag{2}$$

(1.c)

$$\sum_{n\geq 1} \cos(n). \tag{3}$$

(1.d)

$$\sum_{n\geq 1} \frac{1}{[n+(-1)^n]^2}. \tag{4}$$

(1.e)

$$\sum_{n\geq 1} \frac{n!}{n^n}. \tag{5}$$

(1.f)

$$\sum_{n\geq 2} \frac{1}{(\log_2 n)^{\log_2 n}}. \tag{6}$$

(1.g)

$$\sum_{n\geq 1} \frac{(-1)^n n!}{2^n}. \tag{7}$$

2

Let $\{a_n\}_{n\geq 1}$ be a decreasing sequence with $\lim_{n\to\infty} a_n = 0$. Let $\{b_n\}_{n\geq 1}$ be a sequence of real numbers such that $\{\sum_{k=1}^n b_k\}_{n\geq 1}$ is bounded. Then $\sum_{n\geq 1} a_n b_n$ converges.

3

(3.a) Give an example of a divergent series $\sum_{n\geq 1} a_n$ for which $\sum_{n\geq 1} a_n^2$ converges.

(3.b) Show that if $\sum_{n\geq 1} a_n$ is absolutely convergent, then the series $\sum_{n\geq 1} a_n^2$ also converges.

(3.c) Give an example of a convergent series $\sum_{n\geq 1} a_n$, for which $\sum_{n\geq 1} a_n^2$ diverges.

4

Let $\{a_n\}_{n\geq 1}$ be a decreasing sequence of non-negative numbers such that $\sum_{n\geq 1} a_n < \infty$. Show that

$$\lim_{n\to\infty} na_n = 0. \tag{8}$$