Worksheet 2

1

Let S be a non-empty bounded subset of $\mathbb{R}$.

(1.a) Prove that $\inf S \leq \sup S$.

(1.b) What can you say about S if $\inf S = \sup S$.

2

Let A be a non-empty subset of $\mathbb{R}$ which is bounded below and let

$$-A = \{-a : a \in A\}. \tag{1}$$

Prove that $\inf A = -\sup(-A)$.

3

Let S and T be two non-empty bounded subsets of $\mathbb{R}$ and let

(3.a) [1.1.4 of Lebl 2023] Prove that if $S \subset T$, then $\inf T \leq \inf S \leq \sup S \leq \sup T$.

(3.b) Prove that $\sup(S \cup T) = \max\{\sup(S), \sup(T)\}$.

4

Let $F \subset \mathbb{R}$ be a closed set bounded above. Prove that $\sup F \in F$.

5

Let's work with the vector space $\mathbb{R}^n$ and A a non-empty subset of $\mathbb{R}^n$. Prove that A is open if and only if it can be written as the union of a family of open balls of the form

$$B_r(x) = \{y \in \mathbb{R}^n : \|x - y\|_2 < r\}. \tag{2}$$

6

(6.a) Show that if a sequence $\{a_n\}_{n\in\mathbb{N}}$ of real numbers converges to a, then the sequence $\{|a_n|\}_{n\in\mathbb{N}}$ converges to $|a|$.

(6.b) Show (via an example) that the converse is not true.

References

Lebl, Jiri (July 11, 2023). *Basic Analysis I: Introduction to Real Analysis, Volume I*. version 6.0. URL: https://www.jirka.org/ra/realanal.pdf.