Credit Valuation Adjustment (CVA) of an Interest Rate Swap (IRS)

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Abstract

The aim of this project is to demonstrate techniques used to calculate a Credit Valuation Adjustment (CVA) for a fixed income instrument, an interest rate swap (IRS). An hypothetical scenario has been assumed where two different counterparties enter into an IRS written over 6 month LIBOR at T_0 for 5 years maturity and then credit value adjustments (CVA) are calculated to reflect the credit risk on the horizon until the end of fixed income instrument. The counterparty A receives the fixed leg and pays the floating leg where as counterparty B (Deutsche Bank) pays the fixed leg and receives the floating leg of the IRS. The mark to market (MTM) of the IRS has been simulated by using Heat-Jarrow-Morton framework model (HJM) over the recent data obtained from the Bank of England's (BOE) statistics on bank liability curve. Then, the yield curve structure has been simulated for different scenarios in order to obtain MTM values on future horizons. This process then followed by generating an expected exposure profile for the underlying fixed income instrument where the fair price of credit risk taken by counterparty A has been calculated using below formula set;

$$\mathrm{CVA} = \int_0^T (1 - R) * \mathrm{EE}_t * \mathrm{DF}_t * \mathrm{dPD}_t$$

where;

$$\mathrm{EE}_t = E^Q[\max(\mathrm{MTM}_t,\ 0]$$

$$\mathrm{dPD}_t = \Delta\mathrm{PD}_t = \mathrm{PD}_{(t_{i-1},t_i)} = P_{(0,\,t_{i-1})} - P_{(0,\,t_i)}$$

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Counterparty Credit Risk

Counterparty Credit Risk is the risk that, the counterparty to a financial contract will default prior to the expiration of the contract and will not make all the payments required by the contract. Therefore, a bank (dealer) (or the related counterparty who is facing against this credit risk) calculates a quantity known as credit value adjustment (CVA) . This adjustment is shaped by the expected loss because of the possibility of a default by the counterparty and therefore related instrument's value is adjusted (in this case reduced) with the same magnitude.

Counterparty risk is typically defined as arising from two broad classes of financial products: Over-the- counter (OTC) derivatives and securities financial transactions (SFT). The former category is the more significant due to the size and diversity of the OTC derivatives market and the fact that a significant amount of risk is not collateralized.

Only the contracts privately negotiated between counterparties over-the-counter (OTC) derivatives and security financing transactions (SFT) — are subject to counterparty risk. There are two ways in which OTC derivatives transactions are cleared. Either through central clearing parties or through bilateral agreements. Exchange-traded derivatives and derivatives cleared by the central clearing parties are not affected by counterparty risk, because the exchange guarantees the cash flows promised by the derivative to the counterparties. On the other hand, bilaterally cleared derivatives entail credit risk.

Counterparty risk is similar to other forms of credit risk in that the cause of economic loss is obligor's default. There are however two features that set counterparty risk apart from more traditional forms of credit risk:

The value of the contract in the future is uncertain - in most cases significantly so. The MTM value of a derivative at a potential default date will be the net value of all future cash flows required under that contract. This future value can be positive or negative and its typically highly uncertain as seen from today.

Since the value of the contract can be positive or negative , counterparty risk is typically bilateral. This means, each counterparty in a derivatives transaction has a risk to the other.

CVA

Dealer calculates the credit value adjustment (CVA) for each counterparty with which it has bilaterally cleared over the counter derivative. This is an estimation of a loss function from a default by the counterparty. Eventually, CVA reduces the value of the derivative instrument, ie. an increase (or decrease) in the total CVA during a period leads to a decrease (or increase) in profits for the period.

There are important components that define counterparty risk and related metrics.

Mark-to-market and replacement cost: Mark-to-market (MTM) is the starting point for analysis of of counterparty risk and related aspects. Current MTM does not constitute an immediate liability by one party to the other but rather is the present value of all the payments that a party is expecting to receive, less those it is obliged to make. These payments may be scheduled to occur may years in the future and may have values that are strongly dependent on market variables. MTM will be positive or negative, depending on the magnitude of the remaining payments and current market rates. The MTM with respect to a particular counterparty defines the net value of all positions and is therefore directly related to what could potentially be lost today in the event of a default.

However, other aspects such as the ability to net transactions in default and possibility to adjust positions with collateral amounts are also important. In addition, contractual features of transactions, such as close-out netting and termination features refer to replacement costs.

Credit exposure: Credit exposure, or simply exposure, defines the loss in the event of a counterparty defaulting. Exposure is characterized by the fact that a positive value of a portfolio corresponds to a claim on a defaulted counterparty , whereas in the event of negative value , a party is still obliged to honour their contractual payments (at least to the extent that they exceed those of defaulted counterparty.) Exposure is also a time-sensitive measure, since a counterparty can default at any time in the future and one must consider the impact of such an event many years from now.

Default Probability and credit spreads: While assessing counterparty risk, we should consider the credit quality of a counterparty over the entire lifetime of the relevant transaction. Such time horizons can be extremely long. Credit migrations or discrete changes in credit quality such as due to ratings changes, are crucial since they influence the term structure of default probability. They should also be considered, since they may cause issues even when a counterparty is not yet in default. The term structure of default is very important to consider.

Recovery and loss given default: Recovery rate represents the percentage of the outstanding claim recovered when a counterparty defaults. The alternative variable to recovery is loss given default (LGD), which in percentage term sis 1005 minus the recovery rate. While credit exposure is measured independently but LGD is relevant in the quantification of the CVA.

Quantification

This section describes the general framework for calculating the potential future exposure on the OTC derivative product in our case IRS.

The default risk is needed to be correctly quantified and ensured that non-defaulting counterparty is being correctly compensated for the credit risk that they take. This is achieved via CVA, which has been increasingly used in the recent years as a means of assigning an economic value on the counterparty risk.

Modelling Credit Exposure

This framework is necessary for any banking counterparty to compare exposure against limits to price and hedge counterparty credit risk and to calculate economic and regulatory capital.This exposure framework is such an important point so that it allows the financial institution to calculate the entire exposure distribution at any future given date. Future market scenarios have been generated for a fixed set of simulation dates for the underlying instrument.

Instrument Valuation

For each simulation and realization of the underlying risk factor (change in yield curve structure), instrument valuation has been performed for each trade. For each simulation and realization of the underlying risk factor, counterparty - level exposure is obtained according to the formula.

Hence, the outcome of this above process is a set of realizations of counterparty - level exposure at each simulation.

Calculating Default Probabilities

The Default Probabilities are taken and implied from the CDS spreads using the following formula:

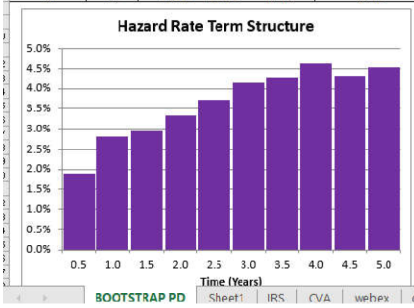
$$P(T_N) = \frac{\sum_{n=1}^{N-1} D(0,T_n)\left[\mathrm{L}P(T_{n-1}) - (L + \Delta t_n S_N) P(T_n)\right]}{D(0,T_n)(L + \Delta t_n S_N)} + \frac{P(T_{n-1})L}{(L + \Delta t_n S_N)}$$

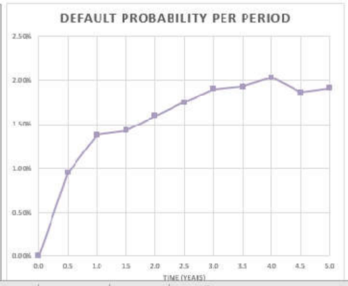
P is the survival probability, L=1-RR is the expected loss calculated from the recovery rate RR, Δ_t is the payment frequency, S is the CDS spread and D is the discount factor.

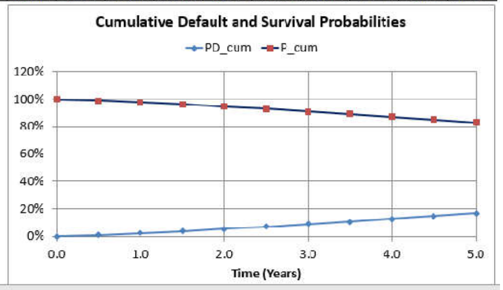
The counterparty B is " Deutsche Bank" and its credit spread are taken from the Markit on date 26 June 2017. The mid levels which are not observable at the market were simply interpolated linearly. Full calculations can be found in the below spreadsheet.

PROBABILITY OF DEFAULT	
Recovery Rate	40%
Counterparty B	Deutsche Bank

TIME (in Years)	Time Step dt	MARKET Level CDS	Discount Factors	IMPLIED HAZARD RATES Lambda	PERIOD DEFAULT PROB P(t-1)-P(t) PD	CUM DEFAULT PROB 1-P(t) PD_cum	CUM SURVIVAL PROB P(t) P_cum	1st term	2nd term	3rd term	4th term	5th term	6th term	7th term	8th term	9th term	sum	quotient	first term	last term
0.0			1.0000		0.00%	0.00%	100.00%													
0.5	0.5	114.40	0.9958	1.90%	0.94%	0.94%	99.06%													
1.0	0.5	141.76	0.9910	2.81%	1.38%	2.33%	97.67%	-0.00135									-0.00135	0.60150	-0.00224	0.97899
1.5	0.5	153.56	0.9857	2.94%	1.43%	3.75%	96.25%	-0.00193	0.00078								-0.00115	0.59899	-0.00193	0.96441
2.0	0.5	165.36	0.9801	3.34%	1.60%	5.35%	94.65%	-0.00251	0.00021	0.00059							-0.00171	0.59617	-0.00287	0.94040
2.5	0.5	176.96	0.9741	3.73%	1.75%	7.10%	92.90%	-0.00309	-0.00035	0.00004	0.00117						-0.00222	0.59308	-0.00375	0.93277
3.0	0.5	188.56	0.9678	4.14%	1.90%	9.00%	91.00%	-0.00366	-0.00092	-0.00051	0.00064	0.00170					-0.00275	0.58932	-0.00466	0.91465
3.5	0.5	197.94	0.9612	4.28%	1.93%	10.93%	89.07%	-0.00412	-0.00137	-0.00095	0.00020	0.00127	0.00233				-0.00263	0.58625	-0.00449	0.89523
4.0	0.5	207.32	0.9542	4.62%	2.03%	12.96%	87.04%	-0.00458	-0.00182	-0.00140	-0.00023	0.00085	0.00192	0.00223			-0.00304	0.58244	-0.00521	0.87561
4.5	0.5	212.85	0.9469	4.33%	1.87%	14.83%	85.17%	-0.00486	-0.00209	-0.00166	-0.00049	0.00060	0.00168	0.00200	0.00281		-0.00202	0.57822	-0.00349	0.85523
5.0	0.5	218.38	0.9392	4.54%	1.91%	15.74%	83.26%	-0.00513	-0.00236	-0.00192	-0.00075	0.00035	0.00143	0.00176	0.00258	0.00130	-0.00224	0.57377	-0.00391	0.83651







BOOTSTRAP PD | Sheet1 | IRS | CVA | webex | data | pv_input | Sheet3

Discount Factors & Forward Rates

In order to calculate the future price of the underlying derivative instrument forward rates are generated. This process is described by the following steps.

The historical spot value (Bank liability curve) data set is taken from the BOE website dating from 03 March 2014 until 31 May 2017

http://www.bankofengland.co.uk/statistics/Pages/yieldcurve/archive.aspx

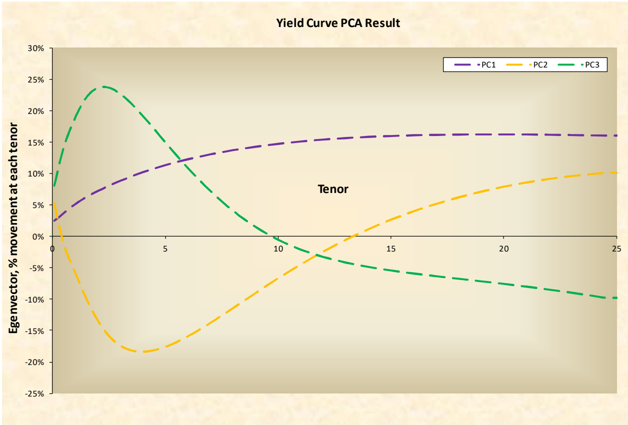
Those items then pasted into "Data" spreadsheet on the excel file named "HJM-Model-PCA-new" from there we can visualize historic forward curves and historic prices of the interest rates for different tenors.

Then day- to-day changes for each tenor are calculated. This produces a set of independent random variables that can be used to get principal components (PC) and conduct principal component analysis (PCA). The largest three eigenvalues corresponding eigenvectors are taken because the differences between each data is taken and covariance matrix is formed. From this covariance matrix a Vectorized eigenvector matrix is obtained.

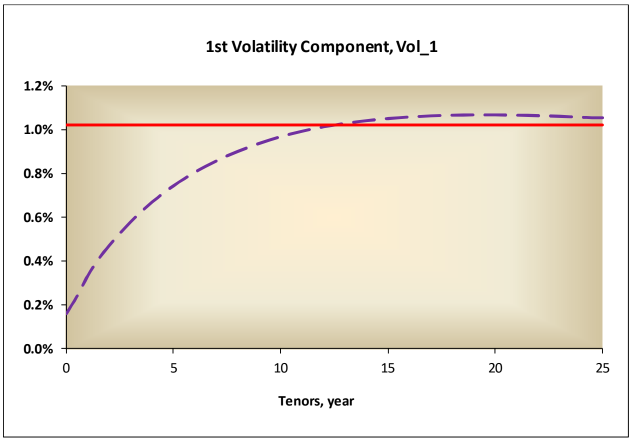
Then I sorted the eigenvalues from largest to smallest value, and first three largest eigenvalues is assumed to describe the forward evolution of the yield curve. The results are summarized below:

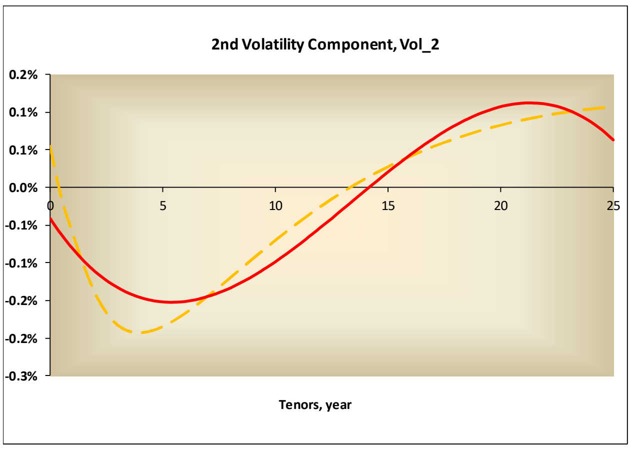
-	Tenor	Eigenvalue	Cum. R^2
3 rd largest	3 year	0.0000600261197280793	0.9946
2 nd largest	25 year	0.000109811659833738	0.9813
1 st largest	20.5 year	0.00433078474341413	0.9571

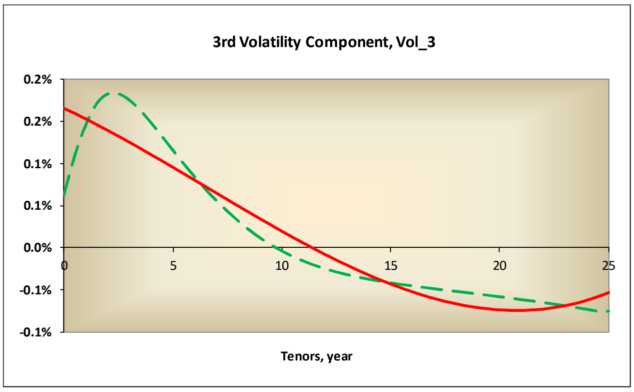
Below figure shows the resulting eigenvectors of the principal components. The largest component PC1 is attributed to control the parallel shifts in the curve, the second largest PC2 controls the stepping/flattening (skewness) and third largest PC3 controls the bending of the curve at specific maturity points (convexity).



After getting the PCA results, the relevant three largest eigenvectors are used to obtain the volatility functions for the HJM Model.







The drift function $\mu(t)$ is obtained by integrating over the principal components and assuming that volatility is a function of time.

The calibrated volatility and drift functions are then entered into the HJM model and forward LIBOR rates are then generated.

Next, the discount factors are then derived from the HJM forward LIBOR simulations.

Forward Rates:

$$L_1 = S_1$$

$$L_i = \frac{S_i\,T_i - S_{i-1}\,T_{i-1}}{T_i - T_{i-1}}$$

Discount Factors:

$$\mathrm{DF}_i = \exp(-S_i\,T_i)$$

Both, discount factors and Forward rates can be found at the "data" worksheet of the CVA_Component_E.Ozpeynirci excel file.

I have computed two different results from this point onwards. The first one is the above mentioned scenario, where first discount factors and then the forward rates have been calculated using the HJM output. The results and the findings are at the “data” worksheet.

The second approach is from the excel worksheet provided with the last webex session on 11 July 2017. again the same HJM output has been used to reach those results and they can be found at the “webex” worksheet of the excel file.

Forward L(T, T+)	L_6M today											
t=T_i	fix, not simulated	**5.5265%**	**6.1627%**	**6.8397%**	**8.3019%**	**7.9555%**	**8.7549%**	**7.5657%**	**7.3051%**	**7.3409%**	**~~7.7066%~~**	
Z(0, T+) implied	1	0.9727	0.9432	0.9115	0.8745	0.8404	0.8044	0.7745	0.7467	0.7198	~~0.6926~~	
Z(0, T+) curve-wise	1	0.9727	0.9415	0.9054	0.8568	0.8369	0.7911	0.8017	0.7856	0.7688	~~0.7688~~	
												DF for MtM
t=0	fix, not simulated	3.75%	4.57%	5.11%	5.47%	5.80%	6.15%	6.53%	6.91%	7.31%	7.71%	
t=0.5Y	...	4.26%	4.80%	5.18%	5.50%	5.82%	6.18%	6.55%	6.94%	7.34%		0.768836
t=1Y		4.81%	5.20%	5.52%	5.83%	6.16%	6.53%	6.91%	7.31%			0.785605
t=1.5Y		5.11%	5.52%	5.90%	6.29%	6.70%	7.13%	7.57%				0.801677
t=2Y		6.91%	7.26%	7.61%	7.97%	8.36%	8.75%					0.791117
t=2.5Y		6.32%	6.71%	7.11%	7.52%	7.96%						0.836902
t=3Y		7.17%	7.53%	7.91%	8.30%							0.856811
t=3.5Y		6.42%	6.61%	6.84%								0.905412
t=4Y		5.90%	6.16%									0.941480
t=4.5Y		5.53%										0.972746
t=5Y		6.32%										0.968873

Credit Valuation Adjustment (CVA)

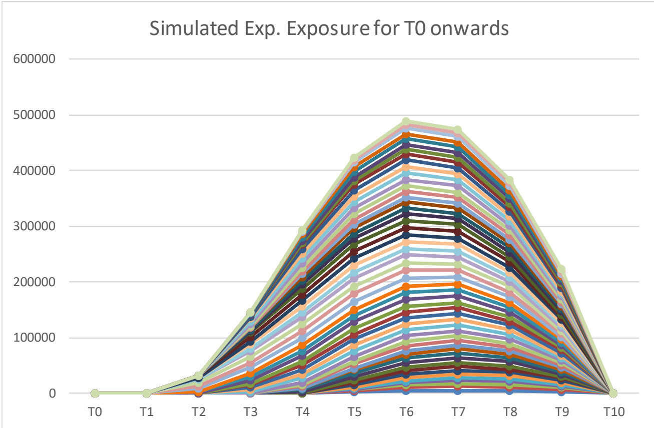
For the CVA to be calculated first MTM value of the IRS is calculated using the below formula for the both approaches:

$$\mathrm{V}(\mathrm{T}_i = 0) = \sum_{i=1}^{11} N * D(0,\ T_i) * (L_i - K)$$
$$\mathrm{V}(\mathrm{T}_i = 0.5) = \sum_{i=2}^{11} N * D(0.5,\ T_i) * (L_i - K)$$
$$\mathrm{V}(\mathrm{T}_i = 1) = \sum_{i=3}^{11} N * D(1.0,\ T_i) * (L_i - K)$$
....
$$\mathrm{V}(\mathrm{T}_i = 5.0) = \sum_{i=11}^{11} N * D(5.0,\ T_i) * (L_i - K)$$

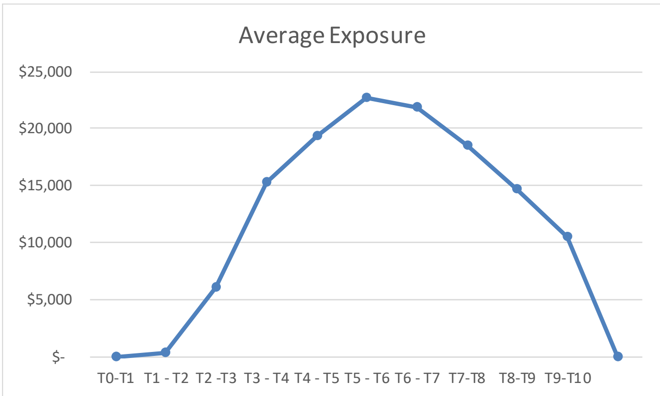
where V is the value of the IRS, N is the notional amount assumed 1 million,
$L_i = L(t;\ T_{i-1},\ T_i)$ is the effective LIBOR rate for the relevant period and D is the discount factor.

Swap at par gives a strike level of **6.77023%** to have a zero initial cashflow at the time of entering the swap.

Finally the exposure for each tenor is calculated from the positive part of the MTM simulations.
$E_i = \max(V_i, 0)$



And an average expected exposure is calculated from the exposure profile.

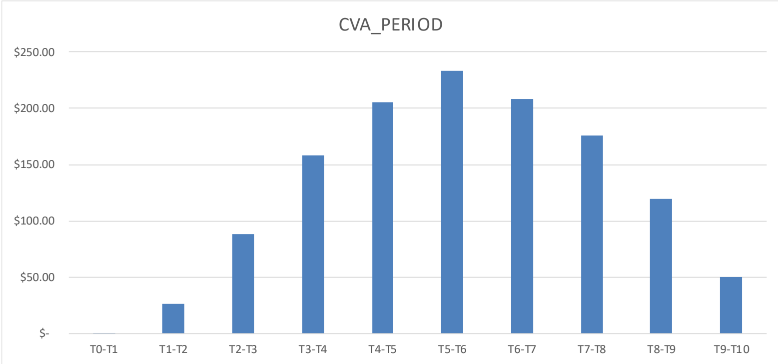


Finally CVA for each period has been calculated after each consecutive periods have been averaged between themselves by using the below formula.

$$\mathrm{CVA} = \int_0^T (1 - R) * \mathrm{EE}_t * \mathrm{DF}_t * \mathrm{dPD}_t$$

CVA Computation										
	T0-T1	T1-T2	T2-T3	T3-T4	T4-T5	T5-T6	T6-T7	T7-T8	T8-T9	T9-T10
Expected EXPOSURE	$ 145.28	$ 3,208.48	$ 10,716.92	$ 17,335.81	$ 21,042.08	$ 22,269.63	$ 20,155.07	$ 16,566.65	$12,589.23	$ 5,268.03
DF	0.9812	0.9785	0.9591	0.9518	0.9286	0.9164	0.8957	0.8681	0.8497	0.8252
PD	0.94%	1.38%	1.43%	1.60%	1.75%	1.90%	1.93%	2.03%	1.87%	1.91%
(1-R)	60%	60%	60%	60%	60%	60%	60%	60%	60%	60%
CVA_PERIODO	$ 0.81	$ 26.01	$ 87.98	$ 157.96	$ 205.19	$ 233.01	$ 208.61	$ 175.50	$ 119.78	$ 49.90
CVA	**$ 1,264.75**									

Below graph illustrates the CVA calculated per period.



By using the spreadsheet provided at the webex the CVA calculation resulted the following;

CVA Computation										
	T0-T1	T1-T2	T2-T3	T3-T4	T4-T5	T5-T6	T6-T7	T7-T8	T8-T9	T9-T10
Expected EXPOSURE	$ 24,844.20	$ 30,214.89	$ 32,561.98	$ 29,817.10	$ 24,264.44	$ 17,369.85	$11,157.01	$ 8,255.07	$ 5,798.95	$ 2,252.42
DF	0.9727	0.9432	0.9115	0.8745	0.8404	0.8044	0.7745	0.7467	0.7198	0.6926
PD	0.94%	1.38%	1.43%	1.60%	1.75%	1.90%	1.93%	2.03%	1.87%	1.91%
(1-R)	60%	60%	60%	60%	60%	60%	60%	60%	60%	60%
CVA_PERIODO	$ 136.93	$ 236.12	$ 254.06	$ 249.62	$ 214.12	$ 159.52	$ 99.86	$ 75.23	$ 46.74	$ 17.91
CVA	**$ 1,490.10**									

Conclusion

With respect to the calculations above the price of the IRS need to be reduced by the CVA amount, in order to be compensated by taking credit default risk of the counterparty. This would facilitate charging a CVA to a client while adjusting the price of the swap.