21-8-23 (NA)

*) Root finding method :

Equation এর solution'ও বলা হয়।

$$f(a) = x - 1 = 0$$

$$\Rightarrow x = 1$$

x axis কে touch করে, y=0 ~~যেই~~ যেই মানে root বলে।

i.) Bisection Method

ii) Method of false Position

iii) Newton Raphson Method

iv) Fixed Position iteration Method

$$x^3 - 6x^2 + 11x - 6 = 0 \longrightarrow$$ exact solution নেই

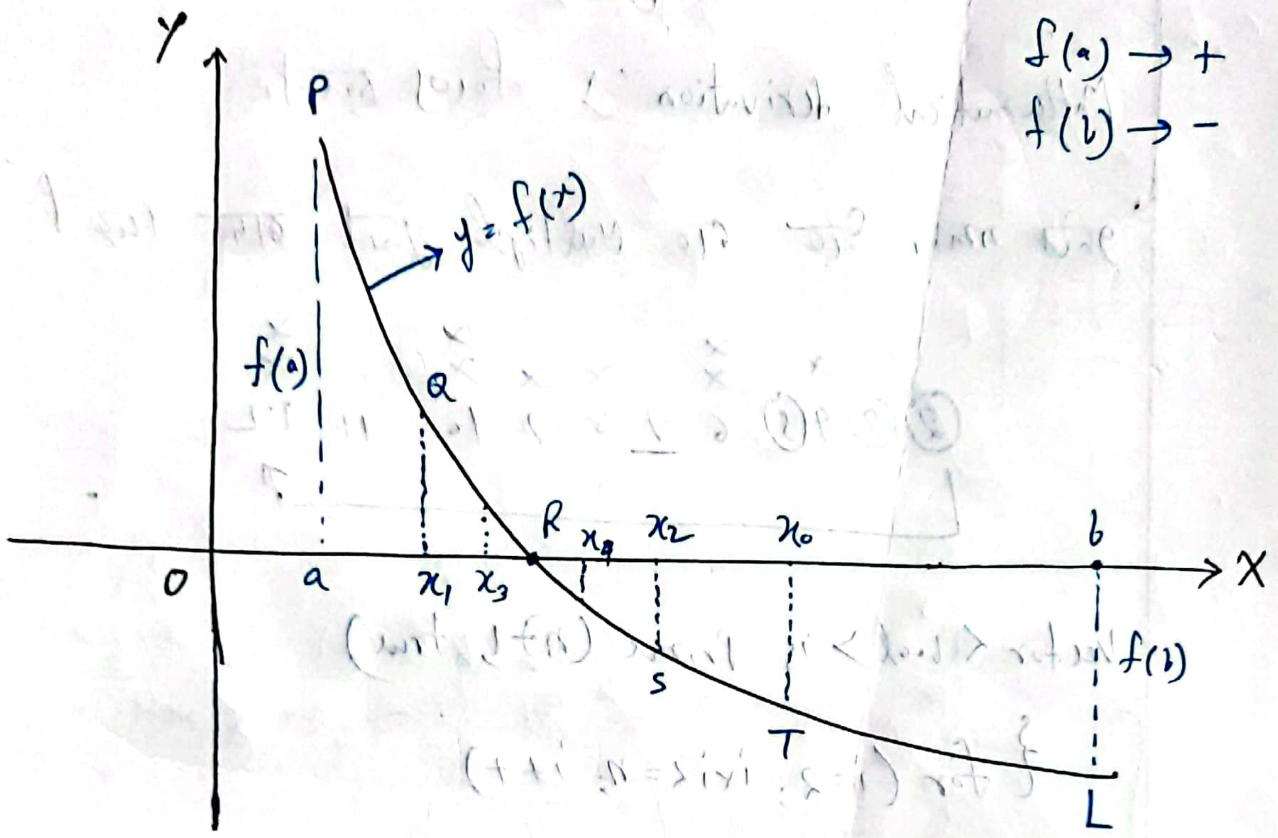
3টা root আছে।

i) Bisection Method : If a function $f(x)$ is continuous between a and b, and $f(a).f(b) < 0$ (or $f(a)$ and $f(b)$ are of opposite signs) then there exists

[Code গুলো দেখবো, MATLAB দিয়ে করবো]

[details এই হতে পড়তে হবে] [এর function এর নাম Zero function /root.]

<u>at least one root</u> between a and b.



1st ধাপ → root থাকার chance,

$$x_0 = \frac{a+b}{2}$$

Proem করবো, অথবা solution পেয়ে গেছি → i) $f(x_0) = 0$,

ii) $f(x_0) \neq 0$

2nd ধাপ: $x_1 = \frac{x_0 + a}{2}$ [এই example এ $f(a) \cdot f(x_0) < 0$ হবে তাই root টা বাম দিকে হবে]

[এভাবে repeatedly করে করে solution/root এর কাছাকাছি মান পাবো]

NA: Bisection method math

Q: Find the real root of the equation, $x^3-2x-5=0$.

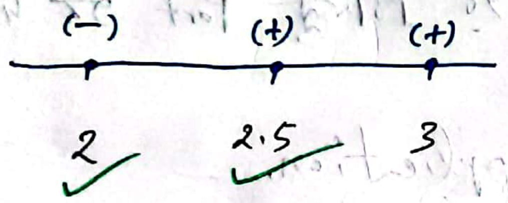
Soln: $f(x) = x^3-2x-5$

$f(0) = -5$

$f(1) = 1-2-5 = -6$

$f(2) = 8-4-5 = -1$

$f(3) = 27-6-5 = 16$



$x_1 = \frac{2+3}{2} = 2.5$

$f(2.5) = (2.5)^3 - 2(2.5) - 5$

$= 5.6250$

Ans: 2.09424

x	(−) a	(+) b	$x_i = \frac{a+b}{2}$	$f(x)$
1	2	3	2.5	5.6250 >0
2	2	2.5	2.25	1.8906 >0
3	2	2.25	2.125	0.3457 >0
4	2	2.125	2.0625	−0.3513 <0
5	2.0625	2.125	2.09375	−0.0089 <0
6	2.09375	2.125	2.10938	0.1668 >0
7	2.09375	2.10938	2.10156	0.07856 >0
8	2.09375	2.10156	2.09766	0.03471 >0
9	2.09375	2.09766	2.09570	0.01286 >0
10	2.09375	2.09570	2.09473	0.00195 >0
11	2.09375	2.09473	2.09424	
12				

NA (1-7-23)

$x^3-2x^2-4=0$

$f(x)=x^3-2x^2-4$

$f(a)=-4 \quad [a=2]$

$f(b)=5 \quad [b=3]$

$x_0=\frac{a+b}{2}=2.5$

$f(x_0)=-1.875$

$x_1=\frac{2.5+3}{2}=2.75$

$f(x_1)=1.171875$

$x_2=\frac{x_1+x_0}{2}$

$=\frac{2.75+2.5}{2}=2.625$

$f(x_2)=0.30640$

$x_3=\frac{x_0+x_2}{2}$

$=\frac{2.5+2.625}{2}=2.5625$

$f(x_3)=-0.306396$

$x_4=\frac{2.625+2.5625}{2}=2.59375$

$f(x_4)=-0.005523$

$x_5=\frac{2.625+2.59375}{2}=2.609375$

$f(x_5)=0.149135$

$x_6=\frac{2.59375+2.609375}{2}=2.6015625$

$f(x_6)=0.071451$

$x_7=\frac{2.59375+2.6015625}{2}$

$=2.597656$

$f(x_7)=0.032875$

$x_8=\frac{2.59765625+2.59375}{2}$

$=2.595703125$

$f(x_8)=0.013653$

$x_9=\frac{2.595703125+2.59375}{2}$

$=2.5947265625$

$f(x_9)=0.00405.$

$$x_{10} = \frac{2.59375 + 2.59472656525}{2} = 2.594238281 25$$

$f(x_{10}) = -0.0007334 \; (10^{-3})$ y এর value হল।

root: x এর যে মানের জন্য y এর মান 0 হবে।

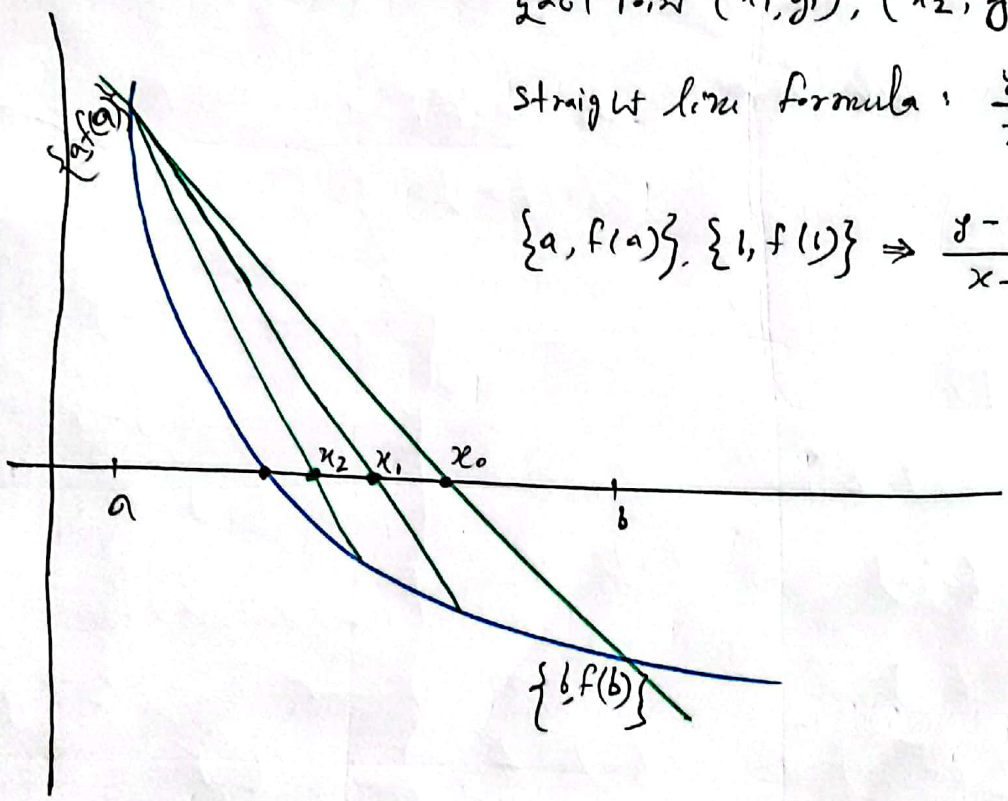
$\therefore$ root $= 2.5942382812 5$.

⑭ False Position Method: $f(a) \cdot f(b) < 0$.

দুইটা Point $(x_1, y_1), (x_2, y_2)$ হলে

Straight line formula: $\frac{y - y_1}{x - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$

$\{a, f(a)\}, \{b, f(b)\} \Rightarrow \frac{y - f(a)}{x - a} = \frac{f(a) - f(b)}{a - b}$



$$\Rightarrow \frac{0 - f(a)}{x - a} = \frac{f(a) - f(b)}{a - b} \Rightarrow x_0 - a = \left(\frac{f(a) - f(b)}{-f(a)(a - b)}\right)^{-1}$$

[x_0 এ/মিলন $y=0$ হয়] ([illegible] f.P. method (bisection method এর [illegible] meth [illegible])

$$\Rightarrow x_0 = a - \frac{f(a)(a-b)}{f(a)-f(b)}$$

$$= \frac{\cancel{af(a)} - af(b) \cancel{- af(a)} + bf(a)}{f(a)-f(b)}$$

$$= \boxed{\frac{bf(a)-af(b)}{f(a)-f(b)}}$$

[theory exm'এ a এবং b এর মান দেওয়া থাকবে]

NA (11-9-23)

$x^3 - 2x^2 - 4 = 0$ Using false-position method.

$f(x) = x^3 - 2x^2 - 4$ and $a = 2, b = 3$

$f(a) = -4$

$f(b) = 5$

$$x_0 = \frac{af(b) - bf(a)}{f(b) - f(a)} = 2.444444$$

$$f(x_0) = -1.34430727$$

$$x_1 = \frac{x_0 f(b) - b f(x_0)}{f(b) - f(x_0)} = 2.562162162$$

$$f(x_1) = -0.309588138$$

$$x_2 = \frac{x_1 f(b) - b f(x_1)}{f(b) - f(x_1)} = 2.587691337$$

$$f(x_2) = -0.0647332741$$

⋮

12^{th} approx^m, $x_{11} = \dfrac{x_{10} f(b) - b(f x_{10})}{f(b) - f(x_{10})} = 2.594313012$

Geometric interpretation, and with practice

H.W : Newton Raphson's Method (অবশ্যই Practice করতে হবে)

Ans : 0.75 . . . . (x_5 এর দিলে পেয়ে যাবে) (Sir দুই এর প্রশ্নই মিলাতে বলেছিলেন)

$x^3 + x^2 - 1 = 0$, $a = 0$, $b = 1$ Using false position method

$f(a) = -1$

$f(b) = 1$

$$x_0 = \frac{a f(b) - b f(a)}{f(b) - f(a)} = 0.5$$

$f(x_0) = -0.125$

$$x_1 = \frac{x_0 f(b) - b f(x_0)}{f(b) - f(x_0)}$$

$= 0.6923076923$

$f(x_1) = -0.1888939629$

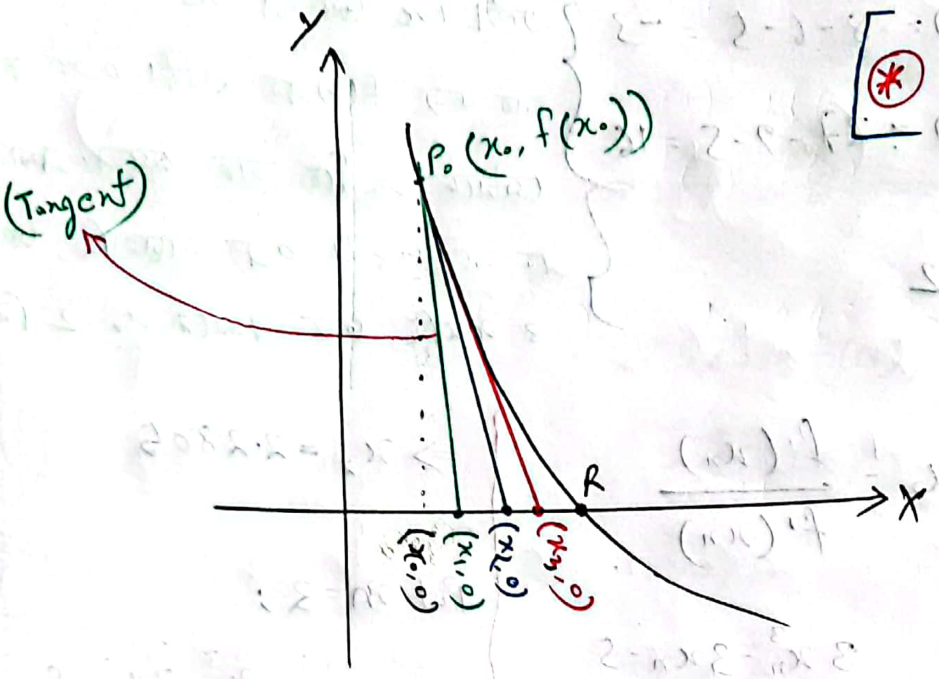
$$x_2 = \frac{x_1 f(b) - b f(x_1)}{f(b) - f(x_1)}$$

$= 0.7411940229$

$f(x_2) =$

Newton Raphson Method : $Y - Y_1 = \frac{dy}{dx}(x - x_1)$

$$\boxed{f'(x_n) \neq 0}$$



Equation of tangent : $Y - f(x_0) = \frac{dy}{dx}(x - x_0)$

$$\Rightarrow Y - f(x_0) = f'(x_0)(x - x_0)$$

$$\Rightarrow 0 - f(x_0) = f'(x_0)(x_1 - x_0)$$

$$\Rightarrow x_1 - x_0 = -\frac{f(x_0)}{f'(x_0)}$$

$$\Rightarrow x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$\Rightarrow \boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad f'(x_n) \neq 0}$$

$x^3 - 3x - 5 = 0$; use Newton Rapshon Method.

$\Rightarrow f(2) = 8 - 6 - 5 = -3$

$f(3) = 27 - 9 - 5 = 16$

$x_0 = 2$

একটা +ve আরেকটা -ve মান বের করলো। তারপর যার জন্য $f(x)$ এর মানটা 0 এর কাছাকাছি হবে সেটাকে x_0 ধরে কাজ করবো। এখানে -3 ও 16 এর মধ্যে -3 টা 0 এর কাছাকাছি, আর 2 এর জন্য -3 পাইছি, তাই এখানে $x_0 = 2$ নিয়ে .....

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{3x_n^3 - 3x_n - 5}{3x_n^2 - 3}$$

Put, $n = 0$;

$$x_1 = x_0 - \frac{x_0^3 - 3x_0 - 5}{3x_0^2 - 3} = 2 - \frac{-3}{9}$$

$$= 2.3333$$

Put, $n = 1$;

$$x_2 = x_1 - \frac{x_1^3 - 3x_1 - 5}{3x_1^2 - 3}$$

$$= (2.\dot{3}) - \frac{(2.\dot{3})^3 - 3(2.\dot{3}) - 5}{3(2.\dot{3})^2 - 3}$$

$$\Rightarrow x_2 = 2.2805$$

Put $n = 2$;

$$x_3 = x_2 - \frac{x_2^3 - 3x_2 - 5}{3x_2^2 - 3}$$

$$= 2.2805 - \frac{2.2805^3 - (3 \times 2.2805) - 5}{3(2.2805)^2 - 3}$$

$$= 2.2790$$

Put $n = 3$;

$$x_4 = x_3 - \frac{x_3^3 - 3x_3 - 5}{3x_3^2 - 3}$$

$$= 2.2790 - \frac{2.2790^3 - 3(2.2790) - 5}{3 \times (2.2790)^2 - 3}$$

$$= 2.2790$$

$\therefore$ Root $= 2.2790$.

NA (18-9-23) [Chapra → বই, writer]

Solution of Linear System:

$$\left.\begin{aligned} x+2y &= 1 \\ 2x-3y &= 0 \end{aligned}\right\}$$

[সমাধানের অনেক জানা method গুলো পড়ে আসবো।]

Gaussian Elemination Method: This is an elementary elemination method. It reduces the system of Linear equation to an equivalent upper triangular system which can be solved by back substitution.

$$\begin{bmatrix} 1 & * & * \\ 0 & 1 & * \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix}$$

upper triangular matrix, মূল কর্ণ বরাবর unit matrix

L_3 এর মান জানলাম, প্রথমে এই L_3 এর মান দিয়ে L_2 থেকে পাবো তারপর L_3 আর L_2 এর মান দিয়ে L_1 পাবো, তাই একে back substitution বলা হয়।

* Consider a system of linear equation with n unknowns.

$$\left.\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\ \cdots\cdots & \\ \cdots\cdots & \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n &= b_n \end{aligned}\right\} \cdots\cdots (1)$$

(first row, 2nd colum → a_{12})

একে আমরা $AX=B$ format এ লিখতে পারি।

$\cancel{a_{11}\ a_{12}}$

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \\ \cdots \\ \cdots \\ x_n \end{bmatrix} \quad B = \begin{bmatrix} b_1 \\ b_2 \\ \cdots \\ \cdots \\ b_n \end{bmatrix}$$

We have the augmented matrix of (1).

$$(A:B) = \left[\begin{array}{cccc|c} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & b_2 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} & b_n \end{array}\right]$$

Dividing first row by a_{11}, and then subtract this row multiplied by a_{21}, a_{31}, a_n from the

{first: a_{11} দিয়ে ভাগ করতে
Second: $a_{21} - (1 \times a_{21})$}

2nd, 3rd ... n^{th} rows respectively

$$= \left[\begin{array}{cccc|c} 1 & a_{12}' & \cdots & a_{1n}' & b_1' \\ 0 & a_{22}' & \cdots & a_{2n}' & b_2' \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & a_{n2}' & \cdots & a_{nn}' & b_n' \end{array}\right]$$

[3rd: a_{22}' কে 1 করতে হবে।]

finally we obtain,

$$\left[\begin{array}{ccccc|c} 1 & a_{12}' & a_{13}' & \cdots & a_{1n}' & l_1 \\ 0 & 1 & a_{23}'' & \cdots & a_{2n}'' & l_2 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1 & l_n \end{array}\right]$$

$$x_1 + a_{12}' x_2 + a_{13}' x_3 + \cdots + a_{1n}' x_n = l_1$$

[এভাবে equation আলাদাভাবে লেখা যায়]

$$\left.\begin{array}{l} x_1 + a'_{12}x_2 + a'_{13}x_3 + \cdots + a'_{1n}x_n = l_1 \\ 0 + x_2 + a''_{23}x_3 + \cdots + a''_{2n}x_n = l_2 \\ \cdots\cdots\cdots \\ \cdots\cdots\cdots \\ x_n = l_n \end{array}\right\}$$

Problem:
$$\left.\begin{array}{l} 2x+2y+z+2u = 7 \\ x-2y+0\cdot z-u = 2 \\ 3x-y-2z-u = 3 \\ x+0\cdot y+0\cdot z-2u = 0 \end{array}\right\}$$

$$A = \begin{bmatrix} 2 & 2 & 1 & 2 \\ 1 & -2 & 0 & -1 \\ 3 & -1 & -2 & -1 \\ 1 & 0 & 0 & -2 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \\ u \end{bmatrix} \quad B = \begin{bmatrix} 7 \\ 2 \\ 3 \\ 0 \end{bmatrix}$$

$$(A:B) = \left[\begin{array}{cccc|c} 2 & 2 & 1 & 2 & 7 \\ 1 & -2 & 0 & -1 & 2 \\ 3 & -1 & -2 & 1 & 3 \\ 1 & 0 & 0 & -2 & 0 \end{array}\right] = \left[\begin{array}{cccc|c} 1 & 1 & 1/2 & 1 & 7/2 \\ 0 & -3 & -1/2 & -2 & -3/2 \\ 0 & -4 & -7/2 & -4 & -15/2 \\ 0 & -1 & -1/2 & -3 & -7/2 \end{array}\right] \quad \begin{array}{l} [R_1' = R_1/2] \\ [R_2' = R_2 - R_1] \\ [R_3' = R_3 - 3R_1] \\ [R_4' = R_4 - R_1] \end{array}$$

$$= \left[\begin{array}{cccc|c} 1 & 1 & 1/2 & 1 & 7/2 \\ 0 & 1 & 1/6 & \frac{2}{3} & 1/2 \\ 0 & 0 & -17/6 & -4/3 & -11/2 \\ 0 & 0 & -1/3 & -7/3 & -3 \end{array}\right] \quad \begin{array}{l} R_2' = R_2/(-3) \\ R_3' = R_3 + 4R_2 \\ R_4' = R_4 + R_2 \end{array}$$

$$
= \left[\begin{array}{cccc|c} 1 & 1 & \frac{1}{2} & 1 & \frac{7}{2} \\ 0 & 1 & \frac{1}{6} & \frac{2}{3} & \frac{1}{2} \\ 0 & 0 & 1 & \frac{8}{17} & \frac{33}{17} \\ 0 & 0 & 0 & -\frac{37}{17} & \frac{90}{37} \end{array}\right]
\left\{\begin{array}{l} R_3' = R_3 / \left(-\frac{17}{6}\right) \\ R_4' = R_4 + \frac{1}{3} R_3 \\ \frac{37}{17} \text{ দিয়ে ভাগ দিয়ে} \end{array}\right\}
= \left[\begin{array}{cccc|c} 1 & 1 & \frac{1}{2} & 1 & \frac{7}{2} \\ 0 & 1 & \frac{1}{6} & \frac{2}{3} & \frac{1}{2} \\ 0 & 0 & 1 & \frac{8}{17} & \frac{33}{17} \\ 0 & 0 & 0 & 1 & \frac{90}{37} \end{array}\right]
$$

$$x + y + \tfrac{1}{2} z + u = \tfrac{7}{2}$$

$$y + \tfrac{1}{6} z + \tfrac{2}{3} u = \tfrac{1}{2}$$

$$z + \tfrac{8}{17} u = \tfrac{33}{17}$$

$$u = \tfrac{90}{37}$$

By back substitution:

$$u = \tfrac{90}{37}$$

$$z = \tfrac{33}{37}$$

$$y = -\tfrac{17}{37}$$

$$x = \tfrac{80}{37}$$

অন্যান্য method গুলো

Gauss → jordan

Gauss → seidel

[Programming এর concept যেখানে হবে → সেখানে iteration টা হবে]

* **Gauss jordan method :** Unit matrix বানাতে হবে

$$
[A:B] = \left[\begin{array}{cccc|c} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & b_2 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} & b_n \end{array}\right]
= \left[\begin{array}{cccc|c} 1 & a_{12}' & \cdots & a_{1n}' & b_1' \\ 0 & a_{22}' & \cdots & a_{2n}' & b_2' \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & a_{2n}' & \cdots & a_{nn}' & b_n' \end{array}\right]
$$

$$
= \left[\begin{array}{ccccc|c} 1 & 0 & 0 & \cdots & 0 & l_1 \\ 0 & 1 & 0 & \cdots & 0 & l_2 \\ 0 & 0 & 1 & \cdots & 0 & l_3 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1 & l_4 \end{array}\right]
\approx \left[\begin{array}{ccccc|c} x_1 & 0 & 0 & \cdots & 0 & l_1 \\ 0 & x_2 & 0 & \cdots & 0 & l_2 \\ 0 & 0 & x_3 & \cdots & 0 & l_3 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & x_n & l_n \end{array}\right]
$$

CamScanner

$x_n = l_n, \quad x_2 = l_2, \quad x_3 = l_3, \quad x_1 = l_1 \;;$ ⟶ এভাবে ans পেয়ে যাবো।

$$\left.\begin{aligned} x + 2y + 3z - 2s &= 6 \\ 2x - y - 2z - 3s &= 8 \\ 3x + 2y - z + 2s &= 4 \\ 2x - 3y + 2z + s &= -8 \end{aligned}\right\} \longrightarrow (1)$$

$$[A : B] = \left[\begin{array}{cccc|c} 1 & 2 & 3 & -2 & 6 \\ 2 & -1 & -2 & -3 & 8 \\ 3 & 2 & -1 & 2 & 4 \\ 2 & -3 & 2 & 1 & -8 \end{array}\right] = \left[\begin{array}{cccc|c} 1 & 2 & 3 & -2 & 6 \\ 0 & -5 & -8 & 1 & -4 \\ 0 & -4 & -10 & 8 & -14 \\ 0 & -7 & -1 & -5 & -20 \end{array}\right] \quad \begin{aligned} R_2' &= R_2 - 2R_1 \\ R_3' &= R_3 - 3R_1 \\ R_4' &= R_4 - 2R_1 \end{aligned}$$

$$= \left[\begin{array}{cccc|c} 1 & 2 & 3 & -2 & 6 \\ 0 & 1 & 8/5 & -1/5 & 4/5 \\ 0 & 0 & -\frac{18}{5} & \frac{36}{5} & \frac{36}{5} \\ 0 & 0 & \frac{36}{5} & \frac{18}{5} & -\frac{72}{5} \end{array}\right] \quad \begin{aligned} R_3' &= R_3 + 4R_2 \\ R_2' &= \underbrace{R_2/(-5)}_{\text{both}} \\ R_4' &= R_4 + 7R_2 \end{aligned} \quad \text{Sir} \quad \left[\begin{array}{cccc|c} 1 & 2 & 3 & -2 & 6 \\ 0 & 1 & 8/5 & -1/5 & 4/5 \\ 0 & -4 & -10 & 8 & -14 \\ 0 & -7 & -4 & 5 & 20 \end{array}\right]$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & -1/5 & -8/5 & 22/5 \\ 0 & 1 & 8/5 & -1/5 & 4/5 \\ 0 & 0 & -18/5 & 36/5 & -54/5 \\ 0 & 0 & -36/5 & 18/5 & -72/5 \end{array}\right] \left\{\begin{aligned} r_1' &= r_1 - 2r_2 \\ r_3' &= r_3 + 4r_2 \\ r_4' &= r_4 + 7r_2 \end{aligned}\right. \approx \left[\begin{array}{cccc|c} 1 & 0 & -1/5 & -8/5 & 22/5 \\ 0 & 1 & 8/5 & -1/5 & 4/5 \\ 0 & 0 & 1 & -2 & 3 \\ 0 & 0 & 36/5 & 18/5 & 72/5 \end{array}\right] \quad r_3' = r_3/\left(-\frac{18}{5}\right)$$

$$\approx \left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & 5 \\ 0 & 1 & 0 & 3 & -4 \\ 0 & 0 & 1 & -2 & 3 \\ 0 & 0 & 0 & 18 & -36 \end{array}\right] \quad \begin{aligned} r_1' &= r_1 + 1/5\, r_3 \\ r_2' &= r_2 - 8/5\, r_3 \\ r_4' &= r_4 - 36/5\, r_3 \end{aligned} \approx \left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & 5 \\ 0 & 1 & 0 & 3 & -4 \\ 0 & 0 & 1 & -2 & 3 \\ 0 & 0 & 0 & 1 & -2 \end{array}\right] \quad r_4' = r_4/18$$

$$
= \left[\begin{array}{cccc|c}
1 & 0 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 & 2 \\
0 & 0 & 1 & 0 & -1 \\
0 & 0 & 0 & 1 & -2
\end{array}\right]
\qquad
\begin{aligned}
r_1' &= r_1 + 2r_4 \\
r_2' &= r_2 - 3r_4 \\
r_3' &= r_3 + 2r_4
\end{aligned}
$$

* Practice করা লাগবে *

$1 \cdot x + 0 \cdot y + 0 \cdot z + 0 \cdot s = 1$; $\quad x = 1$

$0 \cdot x + 1 \cdot y + 0 \cdot z + 0 \cdot s = 2$; $\quad y = 2$

$0 \cdot x + 0 \cdot y + 1 \cdot z + 0 \cdot s = -1$; $\quad z = -1$

$0 \cdot x + 0 \cdot y + 0 \cdot z + 1 \cdot s = -2$; $\quad s = -2$

NA : (25-9-23)

Iterative Method :

i) Gauss Jacobi Iterative Method

ii) Gauss Seidel Iterative "

Problem:

$10x - 2y + z = 2 \Rightarrow x = (2 + 2y - z)/10$ (x কে তো দিলাম yz এর func. পেলাম)

$-3x + 11y + 2z = 5 \Rightarrow y = (5 + 3x - 2z)/11$

$x - y + 5z = 1 \Rightarrow z = (1 - x + y)/5$

(i) $x = g_1(y, z)$ (ii) $y = g_2(x, z)$ (iii) $z = g_3(x, y)$

এই system যখন diagonally dominated হবে তখনই এই method use করতে পারবো. না থাকলে কাজ করবে না। (এটা একটা limitation)

1st এ x বড়, & 2nd এ y বড়, 3rd এ z বড় → diagonally dominated.

Concept:

Jacobi:

$$x_i = g_1(y_{i-1}, z_{i-1})$$
$$y_i = g_2(x_{i-1}, z_{i-1})$$
$$z_i = g_3(x_{i-1}, y_{i-1})$$

প্রত্যেকবার আগের iteration এর value দিয়ে calculate করে যাবো।

Seidel:

$$x_i = g_1(y_{i-1}, z_{i-1})$$
$$y_i = g_2(x_i, z_{i-1})$$
$$z_i = g_3(x_i, y_i)$$

* fast বেশি
* accurate বেশি

এখানে পাওয়ার পর update হয়ে গেলে যেটা আছে সেটা দিয়েই পরেরটা বের করবে।

Jacobi

i	x	y	z
0	0	0	0
1	0.2	0.4545	0.2
2	0.2709	0.4727	0.2509
3	0.27295	0.4828	0.2402
4	0.27254	0.4843	0.24217
5	0.27259	0.48475	0.2424

$x = (2+0\mp 0)/10 = .2$

$x = (2+2\times .4545 - 0.2)/10$

[Programming করে এটা solve করতে হবে]

[অ্যালগরিদমটাও করলে ভালো]

[কিছু কিছু Problem Practise করবো]

Seidel

i	x	y	z
0	0	0	0
1	0.2	0.5090	0.2618
2	0.2736	0.4821	0.2913
3	0.2729	0.4899	0.2425
4	0.2727	0.4898	0.2424
5	0.2727	0.4898	0.2424

$x = (2+0\mp 0)/10$

$x = (2+2\times .4821 - .2913)/10$

$y = (5+3\times .2729 - 2\times .2913)/11$

$z = (1 - .2729 + 0.4899)/5$

NA : (9-10-23) [Approximate / Predict value]

Interpolation: আমাদেরকে কিছু ডেটা দেয়া থাকে একটা Prediction করি।

Using Newton's formula for interpolation, estimate the population for the year 1905.

	1905			1925	
Year	1981	1901	1911	1921	1931
Population	98,752	132285	168076	195690	246050

forward ← যেকোনো একটা → Backward

[formula এর background জানাই লাগবে নিবো।] [মোট 2টা method আছে]

এখন 1905 এর Population কিভাবে বের করবো? → Netwon's formula: backward formula, forward formula, যেকোন ভালো দেখে নিতে হবে কোনটা use করবো। এই ক্ষেত্রে netwons forward formula use করতে হবে, Range → 1981 → 1931, এই range শুরুতেই আছে forward এ পাওয়া যাবে, 1920 চাইলে হয়তো backward এ চাইতাম। এখন 1905 এর জন্য netwons forward table তৈরি করতে হবে (Netwons forward difference table).

[এখানে root finding method, 2nd উদাহ, Solution করবো]

Year (x)	Population (y)	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1891 x_0	98752 y_0	33533 Δy_0	2258 $\Delta^2 y_0$	-10935 $\Delta^3 y_0$	41358 $\Delta^4 y_0$
1901 x_1	132285 y_1	35791 Δy_1	-8177 $\Delta^2 y_1$	30923 $\Delta^3 y_1$	
1911 x_2	168076 y_2	27614 Δy_2	22746 $\Delta^2 y_2$		
1921 x_3	195690 y_3	50360 Δy_3			
1931 x_4	246050 y_4				

(নিউটন - র) ← forward

By Netwon forward formula, we have,

$$y(x) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_0 + \dots$$

$u = \frac{x - x_0}{h}$; here → $x = 1905$ [যাহারা 1905 হওয়ার জন্য]

$x_0 = 1891$ [1st value টা]

$h = 10$ [1901 − 1891] [value গুলোর difference]

$$\cancel{y(x)}\ y(1905) = 98752 + (1.4)(33533) + \frac{(1.4)(1.4-1)}{2!}(2258) + \frac{(1.4)(1.4-1)(1.4-2)}{3!} \times (-10935) + \frac{(1.4)(1.4-1)(1.4-2)(1.4-3)}{4!}(41358)$$

$= 147,841.$ (Approx.)

Year (x)	Population (y)	∇y	$\nabla^2 y$	∇y^3	∇y^4
1891	98752				
1901	132285	33533			
1911	168076	35791	2258		
1921	195690	27614	-8177	-10435	
1931	246050	50360	22746	30923	41358

By Newtons Backword formula, we have,

$$y(x) = y_n + u\nabla y_n + \frac{u(u-1)}{2!}\nabla^2 y_n + \frac{u(u-1)(u-2)}{3!} y_n + \frac{u(u-1)(u-2)(u-3)}{4!}\nabla y^4_n + \dots$$

$u = \frac{x - x_n}{h}$, here $x = 1925$

$x_n = \cancel{1891}\ 1931$

$= -0.6$

$h = 10$

$$\therefore y(1925) = 246050 + (-0.6)\,50360 + \frac{(-0.6)(-0.6-1)}{2!}(22746) +$$

$$\frac{(-0.6)(-0.6-1)(-0.6-2)}{3!}(30923) + \cancel{(-0.6)(-0.6-1)(-0.6-2)}$$

$$\frac{(-0.6)(-0.6-1)(-0.6-2)(-0.6-3)}{4!}(41358).$$

$= 229372$ (Approx)

Lagrange's Interpolation formula:

Let $y = f(x)$ be a polynomial of nth degree, which takes the value of $f(x_0), f(x_1), \ldots f(x_n)$ for any value $x_0, x_1, \ldots x_n$ of the argument x. The polynomial may be written as:

$$f(x) = a_0 (x-x_1)(x-x_2) \ldots (x-x_n) + a_1 (x-x_0)(x-x_2) \ldots (x-x_n)$$

$$+ a_2 (x-x_0)(x-x_1)(x-x_3) \ldots (x-x_n) + \ldots +$$

$$a_n (x-x_0)(x-x_1) \ldots (x-x_{n-1}) \quad \ldots (1)$$

[where a's are constant]

$$f(x_0) = a_0 (x_0 - x_1)(x_0 - x_2) \ldots (x_0 - x_1)$$

$$a_0 = \frac{f(x_0)}{(x_0 - x_1)(x_0 - x_2) \ldots (x - x_1)}$$

$$a_1 = \frac{f(x_1)}{(x_1 - x_0)(x_1 - x_2) \ldots (x_1 - x_n)}$$

Putting all values in eq^n (1),

$$f(x) = \frac{(x-x_1)(x-x_2) \ldots (x-x_n)}{(x_0-x_1)(x_0-x_2) \ldots (x_0-x_n)} f(x_0) + \frac{(x-x_0)(x-x_2) \ldots (x-x_n)}{(x_1-x_0)(x_1-x_2) \ldots (x_1-x_n)} f(x_1)$$

$$+ \ldots + \frac{(x-x_0)(x-x_1) \ldots (x-x_{n-1})}{(x_n-x_0)(x_n-x_1) \ldots (x_n-x_{n-1})} f(x_n).$$

Math :

x	0	1	3	4
$f(x)$	-12	0	12	24

formula তে বসিয়ে পাই →

$$= \frac{(x-1)(x-3)(x-4)}{(-1)(-3)(-4)} \times (-12) + \frac{x(x-3)(x-4)}{(1)(-2)(-3)} \times 0 + \frac{x(x-1)(x-4)}{(3)(2)(-1)} \times 12$$

$$+ \frac{x(x-1)(x-3)}{(4)(3)(1)} \times 24$$

$$= (x-1)(x-3)(x-4) + 0 - 2x(x-1)(x-4) + 2x(x-1)(x-3)$$

$$= x^3 - 8x^2 + 19x - 12 - 2x^3 + 10x^2 - 8x + 2x^3 - 8x^2 + 6x$$

$$= x^3 - 6x^2 + 17x - 12 \quad \{\text{required form of } f(x)\}$$

H.W →

x	321.0	322.8	324.2	325.0
$\log_{10} x$	2.50651	2.50893	2.51081	2.51188

Compute the value of $\log_{10}^{323.5}$ using Lagrange's interpolation formula

[Numerical differential → next class.]

NA (11-10-23): [writer : Shyree ...]

1. Euler Method
2. Runge Kutta Method

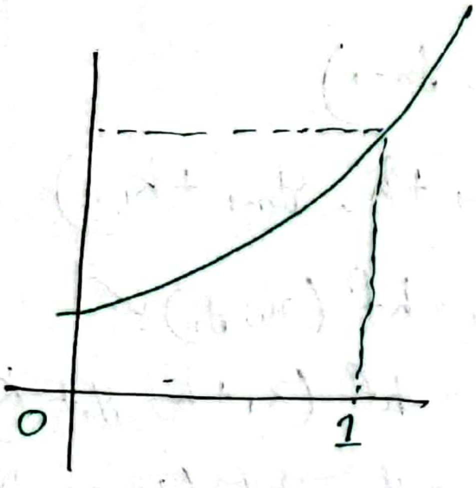
Ordinary Differential Equation:

$$\frac{dy}{dx} = x+1$$

$$\Rightarrow dy = (x+1)\,dx$$

$$\Rightarrow \int dy = \int (x+1)\,dx$$

$$\Rightarrow y = \frac{1}{2}x^2 + x + c \longrightarrow y = \frac{1}{2}x^2 + x$$



x	y
0	0
0.1	–
.	
.	
1	

At $x=0$, $y(0) = 0 + c$

$$\Rightarrow c = 0$$

Euler Method: $\frac{dy}{dx} = f(x,y)$, $y(x_0) = y_0, h$ দেওয়া given.

$$y_n = y_{n-1} + hf(x_{n-1}, y_{n-1})$$

Example: Solve $\frac{dy}{dx} = x+y$, $y(0) = 1$ for $\underbrace{0 \le x \le 1}_{\text{range}}$ with $\underbrace{h = 0.1}_{\text{difference}}$

$\Rightarrow$

x	y
0	1
.1	1.1
.2	1.22
.3	.
.4	.
.5	.
.	.
.	.
1	

$$y_1 = 1 + 0.1(0+1) = 1.1$$

Runge-kutta Method: $\frac{dy}{dx} = f(x,y)$, $y(x_0) = y_0$, h

সবসময় কিছু constant আসে বের করা লাগে, at least দুইটা। যত বেশি করবো তত accuracy বাড়বে। k_1, k_2

$$k_{1,n} = hf(x_{n-1}, y_{n-1})$$

$$k_{2,n} = hf(x_{n-1} + h, y_{n-1} + k_{1,n})$$

{k বের করার formula টা কেমন হবে তা শিখবো}

Generalize করলে,

$$\begin{cases} k_1 = hf(x_1, y_1) \\ k_2 = hf(x_1 + h, y_1 + k_1) \\ y_n = y_{n-1} + \frac{1}{2}(k_1 + k_2) \end{cases}$$

Solve: $\frac{dy}{dx} = \frac{x^2 + y^2}{10}$, $y(0) = 1$, for $0 \le x \le 0.4$, $h = 0.1$

~~Given~~, $\frac{dy}{dx} = \frac{1}{10}(x^2 + y^2) = f(x,y)$, $x_0 = 0$, $y_0 = 1$, $h = 0.1 \to$ প্রথমে Given

$$k_1 = 0.1\left(\frac{0+1}{10}\right) = 0.01$$

$$k_2 = 0.1\left(\frac{0.1^2 + 1.01^2}{10}\right) = 0.10301$$

$y_n = \text{-----}$ formula টা লিখবো

$$y_1 = y_0 + \frac{1}{2}(k_1 + k_2) = 1 + \frac{1}{2}(0.01 + 0.1030) = 1.01615$$

এভাবে, k_1, k_2 change হবে, এভাবে শেষ পর্যন্ত বের করবো, মোট 5 টা, y_4 পর্যন্ত।

y_0 দেয়া আছে, y_1 y_2 y_3 y_4 বের করবো।

NA (6-11-23): [Introductory chapter, chapter-1 গুলো পড়বো]

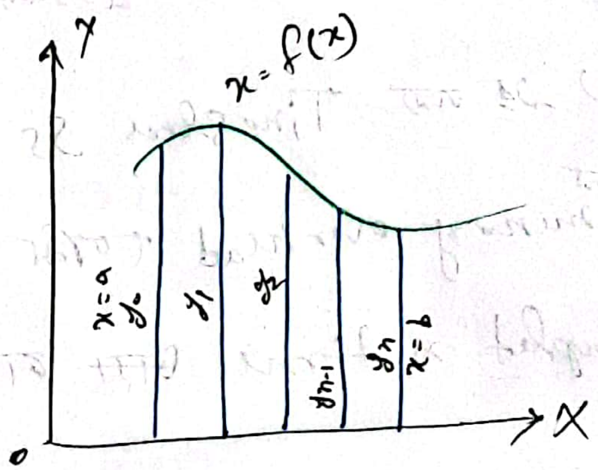
(এর উপর assignment, chapter এর উপর। পরে question গুলো দেওয়া হবে, এর solution করে group wise দেড় পেজ করে জমা করো। next sat এ অনলাইন exam. 12 টায়। System of linear equation: next মঙ্গলবার Assignment টা আজই অফিসে দিতে হবে।)

Numerical Integration:

formula দেখো। দিয়ে Math করো:

① Trapezoidal rule for Numerical Integration:

$$\int_a^b f(x)\,dx = \frac{h}{2}\left[y_0 + 2(y_1 + y_2 + \dots y_{n-1}) + y_n\right]$$



এর জন্য জোড় হতে হবে যে কোন সংখ্যক নিতে

② Simson's 1/3 rule:

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[y_0 + 4(y_1 + y_3 + \dots + y_{n+1}) + 2(y_2 + y_4 + \dots y_{n-2}) + y_n\right]$$

③ Simson's 3/8 rule: (3 এর গুণিতক সংখ্যক নিতে হবে প্রয়োজন)

$$\int_a^b f(x)\,dx = \frac{3h}{8}\left[y_0 + 3(y_1 + y_2 + y_4 + y_5 + \dots + y_{n-2} + y_{n-1}) + 2(y_3 + y_6 + y_9 + \dots y_{n-3}) + y_n\right]$$

* **Math:** $\int_0^6 \frac{dx}{1+x^2}$ using ~~simpsons~~ those $\frac{1}{3}$ formula:

$h = \frac{6-0}{6} = 1$

	y_0	y_1	y_2	y_3	y_4	y_5	y_6
x	0	1	2	3	4	5	6
$y = \frac{1}{1+x^2}$	1	0.5	0.2	0.1	0.0588	0.0384	0.02702

*) $\int_0^6 f(x)\,dx = \frac{h}{2}\left[y_0 + \ldots \right.$

$= \frac{1}{2}\left[1 + 2(0.5 + 0.2 + 0.1 + 0.05 + 0.03) + 0.027\right] = 1.4107985$

*) Simson's $\frac{1}{3}$ formula: $\frac{1}{3}\left[1 + 4(0.5 + 0.1 + 0.0384) + 2(0.2 + 0.58) + 0.027\right]$

$= 1.36617$

*) Simpson's $\frac{3}{8}$ formula: . . . .

6 এখন এইটা যায় জোড়, আবার ৩ এর গুণিতক ও, so এটা দিয়ে ৩ টা formula ই use করা যাবে আবার 12 দিয়েও হত, ৭ দিলে হবেনা যাবে না।

ই formula এর background study নিজেরা তোমরা করে study করবে।

NA LaB 1st class:

*) mmm jacobi Practise করবো।

Gaussian Elimination:

$\boxed{\text{find} \rightarrow \text{Problem solve}}$

$$-3x_1 + 2x_2 - x_3 = -1$$
$$6x_1 - 6x_2 + 7x_3 = -7$$
$$3x_1 - 4x_2 + 4x_3 = -6$$

$$\begin{pmatrix} -3 & 2 & -1 \\ 6 & -6 & 7 \\ 3 & -4 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -1 \\ -7 \\ -6 \end{pmatrix}$$

$$\boxed{x = A^{-1} b}$$

i) Augmented matrix তৈরি:

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{pmatrix}$$

1st Pivot

$$\left(\begin{array}{ccc|c} -3 & 2 & -1 & -1 \\ 6 & -6 & 7 & -7 \\ 3 & -4 & 4 & -6 \end{array}\right)$$

$r_2 = 2r_1 + r_2$

$r_3 = r_3 + r_1$

$r_2 = r_2 - (\text{ratio} * r_1)$

$$\frac{r_2 \text{ (current row)}}{r_1 \text{ (Pivot)}}$$

2nd Pivot

ii) Forward Elemination:

$$\left(\begin{array}{ccc|c} -3 & 2 & -1 & -1 \\ 0 & -2 & 5 & -9 \\ 0 & -2 & 3 & -7 \end{array}\right)$$

$r_3' = r_3 - r_2$

$\rightarrow$ upper triangular matrix বানাও

$$\left(\begin{array}{ccc|c} -3 & 2 & -1 & -1 \\ 0 & -2 & 5 & -9 \\ 0 & 0 & -2 & 2 \end{array}\right)$$

iii) Backward Substitution:

$$x_3 = \frac{2}{-2} = -1$$

$$x_2 = \frac{-9 - 5x_3}{-2} = \frac{-9+5}{-2} = 2$$

$$x_1 = \frac{-1 + x_3 - 2x_2}{-3} = \frac{-1-1-4}{-3} = 2$$