22.08.23

Algorithm

Seive Prime

Pseudo Code:-

```
vector<bool> isPrime(n+1, true);

for (i=2; i*i<=n; i++)  → √n
{     if(isPrime[i]){
    for(int j = i*i; j<=n; j+=i)
  {
        isPrime[j] = false;

  }
 }
}
```

Complexity:

$n/2 + n/3 + n/4 + \dots$

$= n(1/2 + 1/3 + 1/4 + \dots)$

$= n \log n$

$n \rightarrow$ maximum number $\quad P \rightarrow$ any prime No.

inner loop $\rightarrow n/p$

total loop $\rightarrow \sum\limits_{p \text{ isprime} <= n} n/p$

$$= n \sum 1/p$$

* To find prime numbers in n numbers.

$$n/\ln(n) \Rightarrow \text{Approximate}$$

* k th prime number $= k\ln(k)$

* $$n \sum 1/p \approx \sum_{k=1}^{n/\ln(n)} \frac{1}{k\ln(k)}$$

$$\approx \frac{1}{2} + \sum_{k=2}^{n/\ln(n)} \frac{1}{k\ln(k)}$$

$$\sum_{k=2}^{n/\ln(n)} \frac{1}{k\ln(k)} = \int_2^{n/\ln(n)} \frac{1}{k\ln(k)}\, dk$$

$$= \left[\ln\ln(k)\right]_2^{n/\ln(n)}$$

$$= \ln\ln\left(\frac{n}{\ln(n)}\right) - \ln\ln\left(\frac{n}{\ln(2)}\right)$$

$$= \ln\left[\frac{\ln\left(\frac{n}{\ln(n)}\right)}{\ln\left(\frac{n}{\ln(2)}\right)}\right]$$

$$= \ln\left[\frac{\ln(n) - \ln\ln(n)}{\ln(n) - \ln\ln(2)}\right]$$

$$= \ln\cancel{\ln(n)} - \ln\ln\ln(n)$$

$$- \ln\cancel{\ln(n)} + \ln\ln\ln(2)$$

$$= \ln\ln\ln(2) - \ln\ln\ln(n)$$

$$\approx \ln\ln(n)$$

~~2 Complexity~~.

Total complexity

$n\ln\ln(n)$

Segmented Sieve

$0 \longleftrightarrow \sqrt{R}$

$R - L \text{ (maximum)} \longrightarrow 10^5$

arr[0] = L

arr[i] = L+i

Steps

(i) $\sqrt{R} \longrightarrow$ prime generate

(ii) int p; for(j = p×p; j<=R; j+=p)

* L এর পর 1st যে No. P দ্বারা ভাগ যায় $= \dfrac{n+p-1}{p} \times p$

$= \dfrac{L+P-1}{P} \times P$

H.W. Time Complexity.

Complexity of sieve prime

without if condition $\Rightarrow$

$$\frac{n}{2} + \frac{n}{3} + \frac{n}{4} + \frac{n}{5} + \cdots$$

$$= n\left(1/2 + 1/3 + 1/4 + 1/5 + \cdots\right)$$

$$= n\log(n).$$

But when we use if (isprime[i]) condition then only prime numbers will be counted.

$$\frac{n}{2} + \frac{n}{3} + \frac{n}{5} + \frac{n}{7} + \cdots$$

$$= n\left(n/2 + n/3 + n/5 + n/7 + \cdots\right)$$

$$= n\log\log(n).$$

Number of divisor (NOD/σ)

NOD(12) = 6 (1, 2, 3, 4, 6, 12)

NOD (7) = 2

```
for(int i=1; i<=sqrt(N); i++)
{    if(N % i == 0)
         count += 2;
     if(i*i == N)
         count--;
}
```

Complexity = $O(\sqrt{n})$.

$12 = \boxed{2^2 \times 3}$

Divisor = 1, 2, 3, 4, 6, 12

$1 = 2^0 \times 3^0$

$2 = 2^1 \times 3^0$

$3 = 2^0 \times 3^1$

$(2+1) \times (1+1) = 6$

NOD

$$D = P_1^{a_1} \times P_2^{a_2} \times P_3^{a_3} \times \cdots \times P_n^{a_n}$$

$$Div(D) = P_1^{b_1} \times P_2^{b_2} \times P_3^{b_3} \times \cdots \times P_n^{b_n}$$

$$0 \leq b_i <= a_i$$

$$NOD(D) = (a_1+1) * (a_2+1) * (a_3+1) * \cdots$$

$$\cdots * (a_n+1)$$

<u>Pseudo code</u>

```
int sq = sqrt (n);
  int nod = 1;
                              primes[i]<=sq;
for(i=0; i<primes.size() && i<=sq;
        i++)
{
        int p=0;
   while(n % prime [i])
     { p++;
       n/ = prime[i];
     }
   nod *= (p+1);
```

NOD(prime number) = 2

NOD(Prime^2) = 3

NOD (prime1 x prime2) = 4

Time complexity = $O(\sqrt[3]{n})$

```
for (i=0; i< pr.size(); i++)
{ if (pr[i]*pr[i]*pr[i] >= n)
    break;

  p=0
  while (n % pr[i]==0)
  {  p++;
     n/= pr[i];
  }
  nod *= (p+1);
}
cout << nod << endl;
```

Hydronix

GCD

gcd(a, b%a)

Extended Euclid GCD

$$\boxed{ax + by = gcd(a,b)} \rightarrow \text{Bezart's identity.}$$

$$GCD(240, 46) = GCD(46, 240 \% 46)$$

$$\vdots$$

$$= GCD(2, 0)$$

$r_0 = a \qquad r_1 = b \qquad q_0 = \frac{r_1}{r_0}$

$$q_i = \left\lfloor \frac{r_{i-2}}{r_{i-1}} \right\rfloor \qquad r = 240 - 2 \times 46$$

$r_0 = b$

$r_1 = a$

a	b	q	r
			$r_0 = 240$
			$r_1 = 46$
$a = 10$, $b = 46$		$q = \frac{r_0}{r_1} = 5$	$r = 10$ $(240 - 5 * 46)$
10	6	$q = 4$	$r = 6$
$b > a$		$q = 1$	$r = 4$
6	4	$q = 1$	$r = 2$
4	2	$q = 2$	$r = 0$

$= GCD = 2$

$$r_2 = r_0 - q * r_1$$

$$r_i = r_{i-2} - q_i * r_{i-1}$$

For finding Bezout's identity x & y :⇒

$$ax + by = \gcd(a,b)$$

$$r_i = ax_i + by_i$$

$$\gcd(a,b) = ax + by$$

$$\Rightarrow 2 = \cancel{6x + 9y}\ 4x + 6y$$

Initially, $(x_0, y_0) = \cancel{(0,1)}\ (1,0)$

$(x_1, y_1) = \cancel{(1,0)}\ (0,1)$

$$r_i = r_{i-2} - q_i * r_{i-1}$$

$$r_i = ax_i + by_i$$

$$\therefore r_{i-2} = ax_{i-2} + by_{i-2}$$

$$r_i = ax_{i-2} + by_{i-2} - q_i * r_{i-1}$$

$$r_i = ax_{i-2} + by_{i-2} - q_i * (ax_{i-1} + by_{i-1})$$

$$= ax_{i-2} + by_{i-2} - q_i a x_{i-1} - q_i b y_{i-1}$$

$$r_i = a(x_{i-2} - q_i x_{i-1}) + b(y_{i-2} - q_i y_{i-1})$$

$$x_i = x_{i-2} - q_i x_{i-1}$$

$$y_i = y_{i-2} - q_i y_{i-1}$$

$1 - 5\times 0$
$0 - 5\times 1$
$0 - 4\times 1$
$1 - 4\times -5$
$1 - 1\times -4$
$-5 - 1\times 21$
$-4\times 1\times$
$-4 - 1\times 4$
$20 - 1\times -25$
$4 - 2\times -8$
$16+9$
$-25 - 2\times 45$
90
25
115
$1+5\times 4$
$1+4\times$

q	r	x	y
	240	1	0
	46	0	1
5	10	1	-5
4	6	-4	$\cancel{20}$ 21
1	4	$\cancel{4}$ 5	$-2\cancel{5}6$
1	2	$\cancel{-8}$ -9	$4\cancel{7}$ 47
2	0	$\cancel{20}$ 23	$\cancel{-115}$

∴ equation —

~~2 = 240×-9 + 46×47~~

$$2 = 240 \times \underbrace{-9}_{x} + 46 \times \underbrace{47}_{y}$$

gcd(240, 46)

GCD(b, a%b)

H.W.

x ও y এর multiple value

Big mode

Modular Multiplicative Inverse (MMI)

$\gcd(3,5) = 1$

$3x + 5y = 1$

$3u \equiv 1 \bmod 5$

$-2 \% 5 = -2$

$2 \% 5 = 2 + 3$

$3(2) + 5$

$AX \equiv 1 \pmod{M} \Rightarrow X = A^{-1} \pmod{M}$

co-prime

$\gcd(A, M) = 1$

$AX \equiv 1 \pmod{M}$

$\Rightarrow (AX - 1) \equiv 0 \pmod{M}$

If $\gcd(A, M) = D$, $D > 1$

$A \% D = 0 \quad M \% D = 0$

$\Rightarrow AX \% D = 0$

Euler's Theorem:

Fermat's Little Theorem:

Euler's Theorem

a, m co-prime

$$a^{\phi(m)} \equiv 1 \pmod{m}$$

ϕ = eulers tossen function

$\phi(m)$

$\phi(6) = 2$

$\phi(7) = 6$

Co ~~prime~~

6

ϕ

1, 2, 3, 4, 5

(1, 5) এর সাথে 6

এর gcd 1

$\phi(6) = 2$

Proof:

$$A = \{b_1, b_2, b_3, \dots\dots, b_{\phi(n)}\}$$

$b_i \to n \to$ coprime

$$B = \{ab_1, ab_2, ab_3, \dots\dots, ab_{\phi(n)}\}$$

Lemma $\to A = B$.

$$\{b_1 \times b_2 \times b_3 \times \dots \times b_{\phi(n)}\} \equiv \{ab_1 \times ab_2 \times ab_3 \times \dots \times ab_{\phi(n)}\} \pmod{n}$$

$$= a^{\phi(n)} \times b_1 \times b_2 \times \dots b_{\phi(n)}$$

$$\Rightarrow \cancel{a^{\phi(n)}}\ 1 = a^{\phi(n)} \pmod{n}$$

$$\Rightarrow a^{\phi(n)} \equiv 1 \pmod{n}$$

Fermat's Little Theorem:

$a, n \rightarrow$ Coprime $\quad n \rightarrow$ prime

$\cancel{a^{n-1} \equiv 1 \pmod n}$

$$a^{n-1} \equiv 1 \pmod n$$

$\phi(n) = n-1 \qquad n \rightarrow$ prime

$$\Rightarrow a^{\phi(n)} \equiv 1 \pmod n$$

$$AX \equiv 1 \pmod m$$

$$X \equiv A^{-1} \pmod m$$

$$A^{-1} \equiv 1 \pmod m$$

$A^{m-1} \equiv 1 \qquad m \rightarrow$ prime

$A, m \rightarrow$ Coprime

$$\Rightarrow \frac{A^{m-1}}{A} \equiv \frac{1}{A}$$

$$\Rightarrow A^{m-2} \equiv A^{-1} \pmod m$$

$$A^{\phi(m)} \equiv 1 \rightarrow A^{\phi(m)-1} \equiv A^{-1}$$

$$Ax + By = GCD(A, B)$$

$$\Rightarrow Ax + My \equiv 1$$

04.10.23

Linear Diophantine Equation

$$Ax + By = C$$

$0 \cdot x + 0 \cdot y = C$ if $C \neq 0$, trivial case

$$2x + 3y = 1$$

$Ax \equiv C \pmod{B}$ or $By \equiv C \pmod{A}$

$$\frac{Ax}{B} + \frac{By}{B} = \frac{C}{B}$$

P   O   Z

$$P \equiv Z$$

$$Ax \equiv C \pmod{B}$$

1st step:

$$Ax_g + By_g = \gcd(A, B)$$

Hydronix

$$\Rightarrow A\underbrace{\boxed{x_g \frac{c}{\gcd(A,B)}}}_{x} + B\underbrace{\boxed{y_g \cdot \frac{c}{\gcd(A,B)}}}_{y} = c$$

$$Ax + By = c$$

$$Ax + By + D - D = c$$

$$\Rightarrow Ax_0 + By_0 + \frac{A \cdot B}{g} - \frac{A \cdot B}{g} = c$$

$$\Rightarrow Ax_0 + \frac{AB}{g} + By_0 - \frac{BA}{g} = c$$

$$\Rightarrow A\left(x_0 + \frac{B}{g}\right) + B\left(y_0 - \frac{A}{g}\right) = c$$

$$\Rightarrow A\underbrace{\left(x_0 + \frac{KB}{g}\right)}_{x} + B\underbrace{\left(y_0 - \frac{KA}{g}\right)}_{y} = c$$

H.W.

Particular range এ x ও y এর কয়টা value আছে বের কর।

DP → Dynamic Programming.

recursive call

LIS ⇒ Longest I substitutes.

OS TT → 14-11-23 (11.00AM)
G-1

Algo TT → ~~13~~ 20-11-23 (11.00AM)
G-1

* Matrix chain Multiplication
* KMP (String ~~Mat~~ Matching Algo)
* Radix, Bucket, Counting Sort
(Step. implementation, scenerio)

Safayet planet
YT channel

dP

0-1 knapsack / Coin change:

Target 22

Array = {2, 5, 9, 13, 15}.

15 + 5 + 2
13 + 9

Pseudo Code:-

~~function~~(index, remaining_weight)
{ if (r-w < 0)
return ∞;

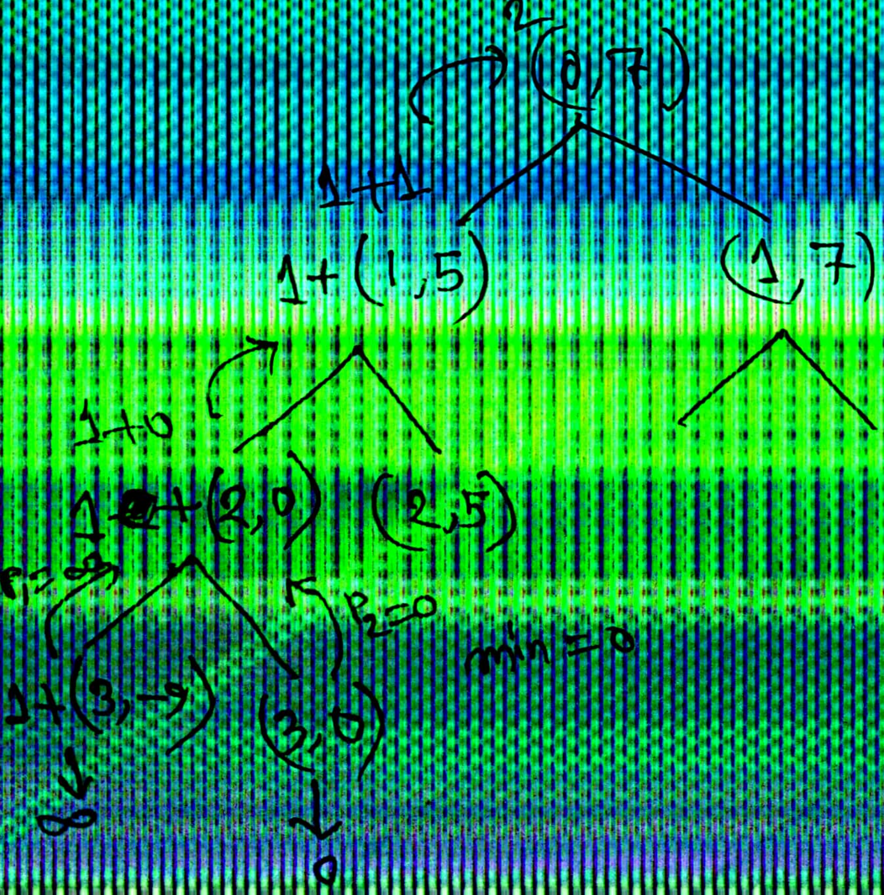
if ~~(r-w index == length array)~~ ~~if~~ r w == 0
{ ~~if~~

```
f(index, length)
{ if (rw == 0)
      return 0;
  }
  result1 = 1 + f(index+1,
                  rw - Array[index]);
  result2 = f(index+1, rw);
  return min(result1, result2);
```

~~I~~ Recursive Tree:-

{2, 5, 9}

Target = 7



~~DP~~ | Using DP

```
dp[index][rw] = min(r1, r2);
    return dp[i][rw];

r1 = 1 + f(index , rw - Array[index]);
r2 = f(index+1, rw);
```

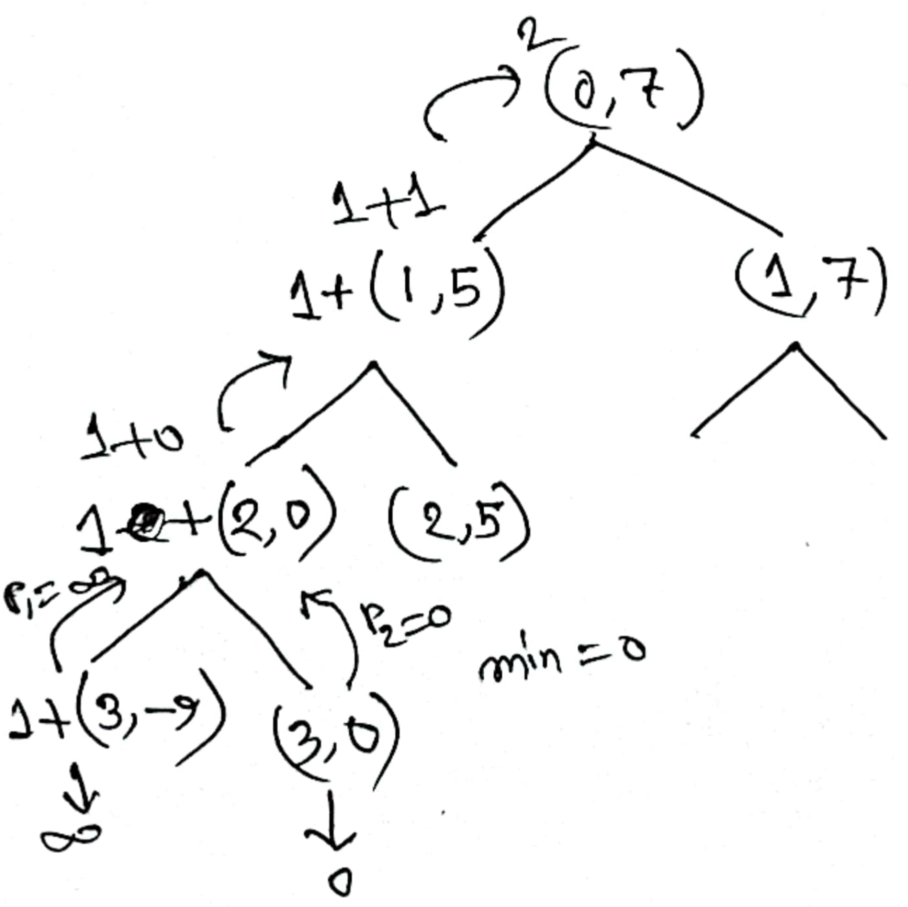
```
if( index == length)
{ if (rw == 0)
        return 0;
   else
        return ∞;
}

result1 = 1 + function(index++,
                    rw - Array[index]);

result2 = f(index+1, rw);

return min(result1, result2);
```

Recursive Tree :-



{2, 5, 9}
Target = 7

Formal Proof:

Automata theory:

Regular Language → Regular Expression
→ Finite Automata
→ Deterministi
(DFA)
→ Non-Determ
(NFA)

Context free language
→ Push down Automata
→ Chomsky Normal Form

Turing Machine

TOC

19.09.23

Formal Proof:

If Hypothesis then conclusion.

Deductive Proof

If x is ~~the~~ the sum of the square of four positive integers. then

$2^x \geq x^2$

1. $x = a^2 + b^2 + c^2 + d^2$
2. $a \geq 1, \; b \geq 1, \; c \geq 1, \; d \geq 1$
3. $a^2 \geq 1, \; b^2 \geq 1, \; c^2 \geq 1, \; d^2 \geq 1$
4. $x \geq 4$
5. $2^x \geq x^2$

Inductive Proof

Properties $\rightarrow$ (i) ~~Basic~~ Basis

(ii) Inductive Step.

For all $n \geq 0$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} = S(n)$$

If $S(n)$ then $S(n+1)$.

$$S(n+1) = \sum_{i=1}^{n+1} i^2 = \frac{(n+1)(n+2)(2n+3)}{6}$$

$$\left(\sum_{i=1}^{n} i^2\right)^2 + (n+1)^2$$

H.W.

$$= \left\{\frac{n(n+1)(2n+1)}{6}\right\}^2$$

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Proof 26.09.2023

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

Basis $n=0$

$$= S(n) \quad \text{—— (i)}$$

হলে, ans $=0$

Inductive Steps:

if $S(n)$ then $S(n+1)$

$$S(n+1) = \sum_{i=1}^{n+1} i^2 = \frac{(n+1)(n+2)(2n+3)}{6}$$

$$= \frac{2n^3+9n^2+13n+6}{6} \quad \text{—— (ii)}$$

$\cancel{(i) \times (ii) \Rightarrow}$

$$\sum_{i=1}^{n} i^2 = \frac{2n^3+3n^2+n}{6} \quad \text{—— (i)}$$

$$\cancel{\sum_{i=1}^{n} i^2 + \sum_{i=1}^{n+1} i^2}$$

$$\sum_{i=1}^{n} i^2 + (n+1)^2 = \sum_{i=1}^{n+1} i^2 \quad \cancel{= 2n^3+9n^2+1}$$

$$= \frac{2n^3+9n^2+13n+6}{6}$$

$$L.H.S. = \frac{2n^3+3n^2+n}{6} + (n+1)^2$$

$$= \frac{2n^3+3n^2+n+(n+1)^2.6}{6}$$

$$= \frac{2n^3+3n^2+n+6n^2+12n+6}{6}$$

$$= \frac{2n^3+9n^2+13n+6}{6}$$

$= R.H.S.$

if $S(n)$, then $S(n+1)$ (Proved)

If $x \geq 4$ then $2^x \geq x^2 \rightarrow$ true

Inductive step: if $S(n)$ then $S(n+1)$

$2^{(n+1)} \geq (n+1)^2$

$\Rightarrow 2^n \cdot 2^1 \geq n^2 + 2n + 1$

$\Rightarrow n^2 \cdot 2 \geq n^2 + 2n + 1$

$2^n \geq n^2$
Basis
$2^n = n^2$

$\Rightarrow n^2 \geq 2n + 1$

$\Rightarrow n \geq 2 + \frac{1}{n}$

for $n = 4$

$4 \geq 2 + \frac{1}{4}$ true

$5 \geq 2 + \frac{1}{5}$ true

(Proved)

* All prime numbers are odd

Automata

Abstract design of a machine

Deterministic Finite Automata

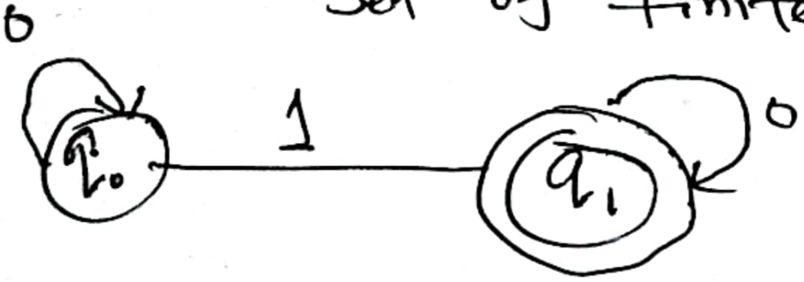
$Q \rightarrow$ set of states

$\Sigma \rightarrow$ ~~ste~~ set of alphabet

δ: $Q \times \Sigma \rightarrow$ Transition functions

$q_0 \rightarrow$ start state

$F \subseteq Q \rightarrow$ Accept state / Finish state
set of finite final state.



$Q = \{q_0, q_1\}$

$\Sigma = \{0, 1\}$

$\delta : \{(q_0, 0) \rightarrow q_0, (q_0, 1) \rightarrow q_1,$

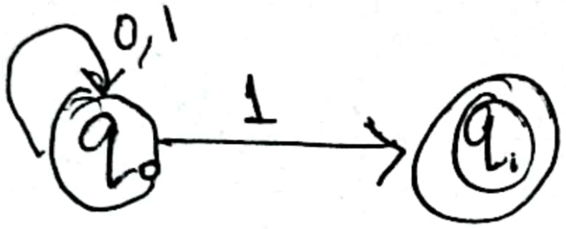
$(q_1, 0) \rightarrow q_1, (q_1, 1) \rightarrow q_0\}$

$F = \{q_1\}$

$0^* 1 0^*$

DFA

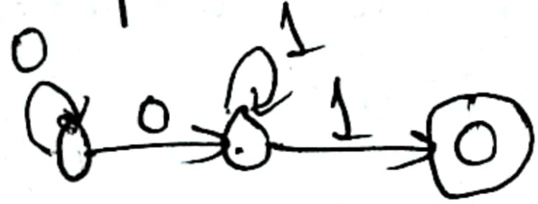
$L(M) = \{w \mid w \text{ ends with '1'}\}$



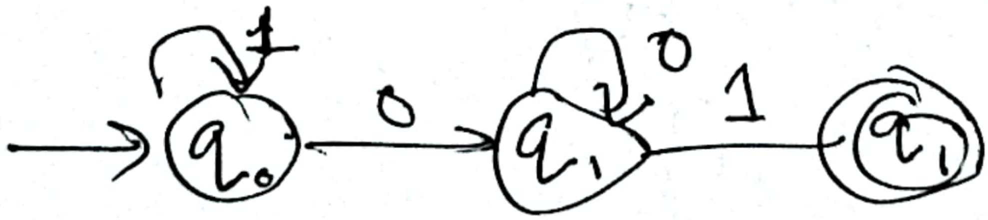
0 0 0 1

1 1

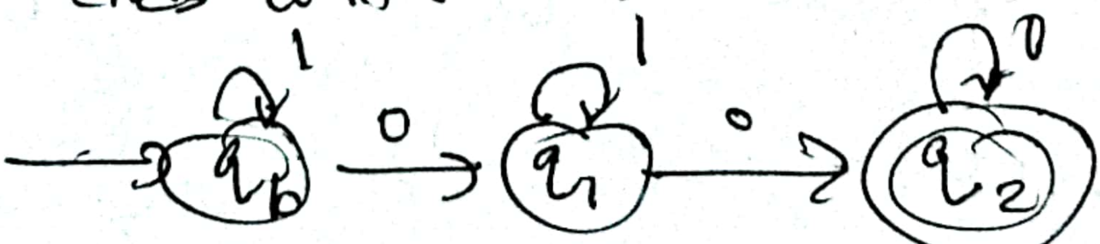
1



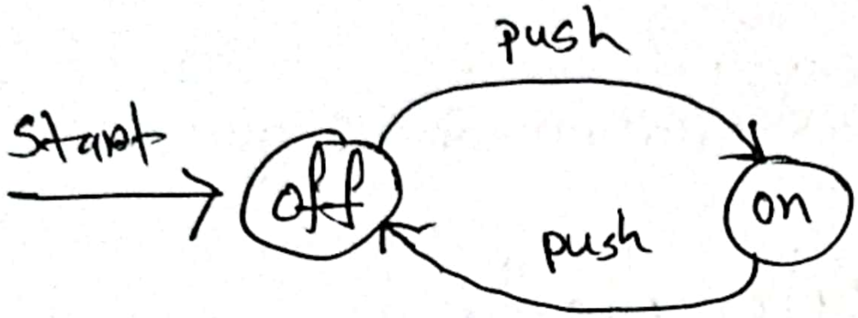
1 1 1 1 0 1

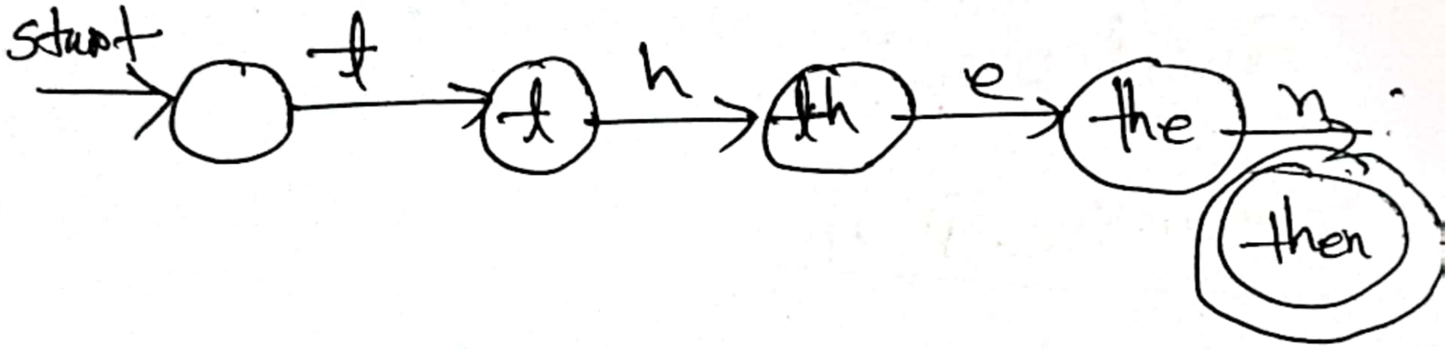


ends with '00'



The simplest nontrivial finite automation is an on/off switch.





The start state corresponds to the empty string, and each state has a transition on the next letter of then to the state that corresponds to the next-larger prefix.

NFA:

$$\delta = Q \times \Sigma \longrightarrow Q$$

$$Q = \{q_0, q_1\}$$

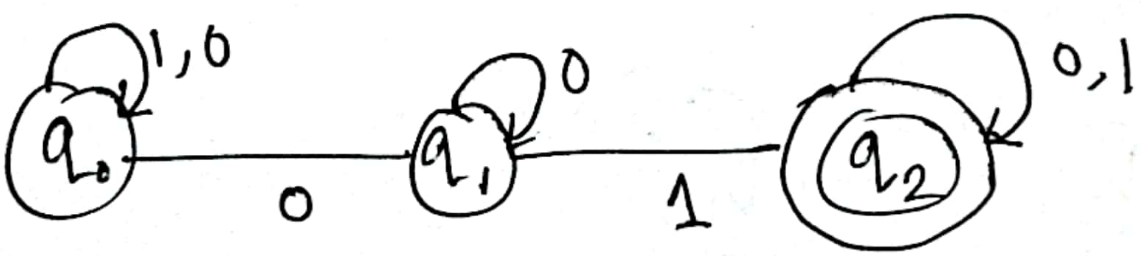
$$\Sigma = \{0, 1\}.$$

$(q_0, 0)$, $(q_0, 1)$

$(q_0, 0) \rightarrow \{q_1, q_2, \ldots\}$

↳ multiple path (NFA)

An NFA that contains "01"



	0	1
→ q_0	$q_1 q_1$	q_0
q_1	q_1	q_2
q_2	q_2	q_2

NFA state Table

Before NLP

Regular Expression:

07.11.23

Context Free Language:

- Push down automata
- Powerful than Regular Language
- Terminal symbol
- Non Terminal symbol

$G = \{(S, A), (a, b), (S \rightarrow aAb, A \rightarrow aAb | \epsilon$

$S \rightarrow aAb$

$S \rightarrow aaAbb$

$S \rightarrow aabb$

Push Down Automata: (PDA)

$P = \{Q, \Sigma, \Gamma, \delta, q_0, Z_0, F\}$

$Q \rightarrow$ set of states

$\Sigma \rightarrow$ Set of input symbols

~~TG~~ $\Gamma \rightarrow$ Set of ~~stat~~ stack alphabet

$\delta \rightarrow$ Transition function

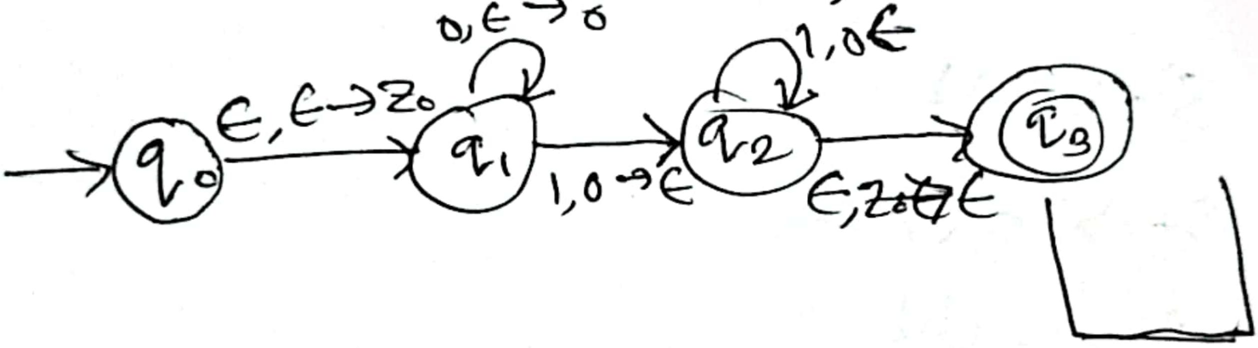
$q_0 \rightarrow$ start state

$Z_0 \rightarrow$ start stack state

$F \rightarrow$ set of Final states

PDA = FSM + stack

* $L = \{0^n 1^n \mid n \geq 0\}$.



stack

$a, b \rightarrow c$

a b → c

a → Input

b → popped from stack

c → push into stack

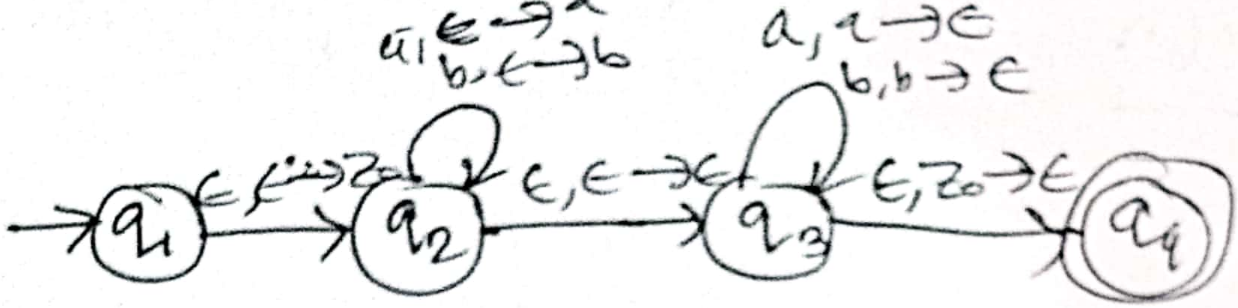
$\Gamma = \Sigma \cup Z_0 = \{0, 1\} \cup Z_0$

$= 0, 1, Z_0$

$L = \{ ww^R \mid w = (a+b)^+ \}$

NO → W

W^R → ON



Pumping Lemma: For regular language.

$a^n b^n$

If A is a regular language, then A has a pumping length "p" such that any string "s" where $|s| \geq p$ may be divided into 3 parts $s = xyz$, such that the following condition is true.

FSM X

1. $xy^iz \in A$ where $i \geq 0$
2. $|y| > 0$
3. $|xy| \leq p$ any $y \leq p$

Ex

$P = 7$

$$\underbrace{aa}_{x}\overbrace{aaaaa}^{y}\underbrace{bbbbbbb}_{z}$$

case-1: Y is in "a" part.

if $i = 2$,

$xy^{i}z =$

$$\underbrace{aaa}_{x}\underbrace{aaaaaaaa}_{y^2}\underbrace{bbbbbbb}_{z}$$

$a = 11 \quad b = 7$ doesn't match $a^n b^n$

Case: 2 → Y is in "b" part

$$S = \underbrace{aaaaaaa}_{z}\underbrace{bbbb}_{Y}\underbrace{bbb}_{x}$$

Case: 3 ⇒ y is in "a" and "b" part

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3.5

Simplification of CFG:

Ambiguity

S → NP VP

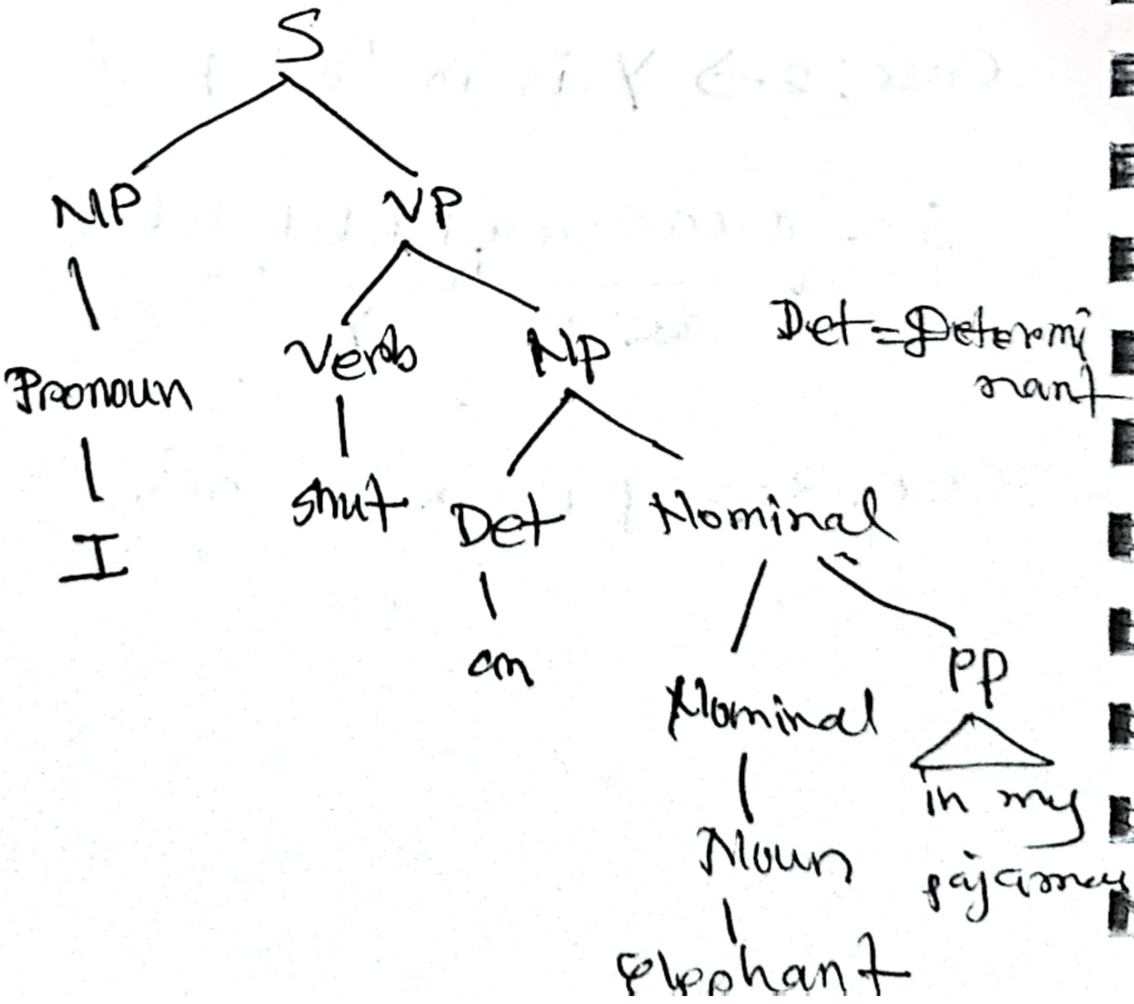
NP → noun / pronoun | Det Nominal

VP → Verb NP

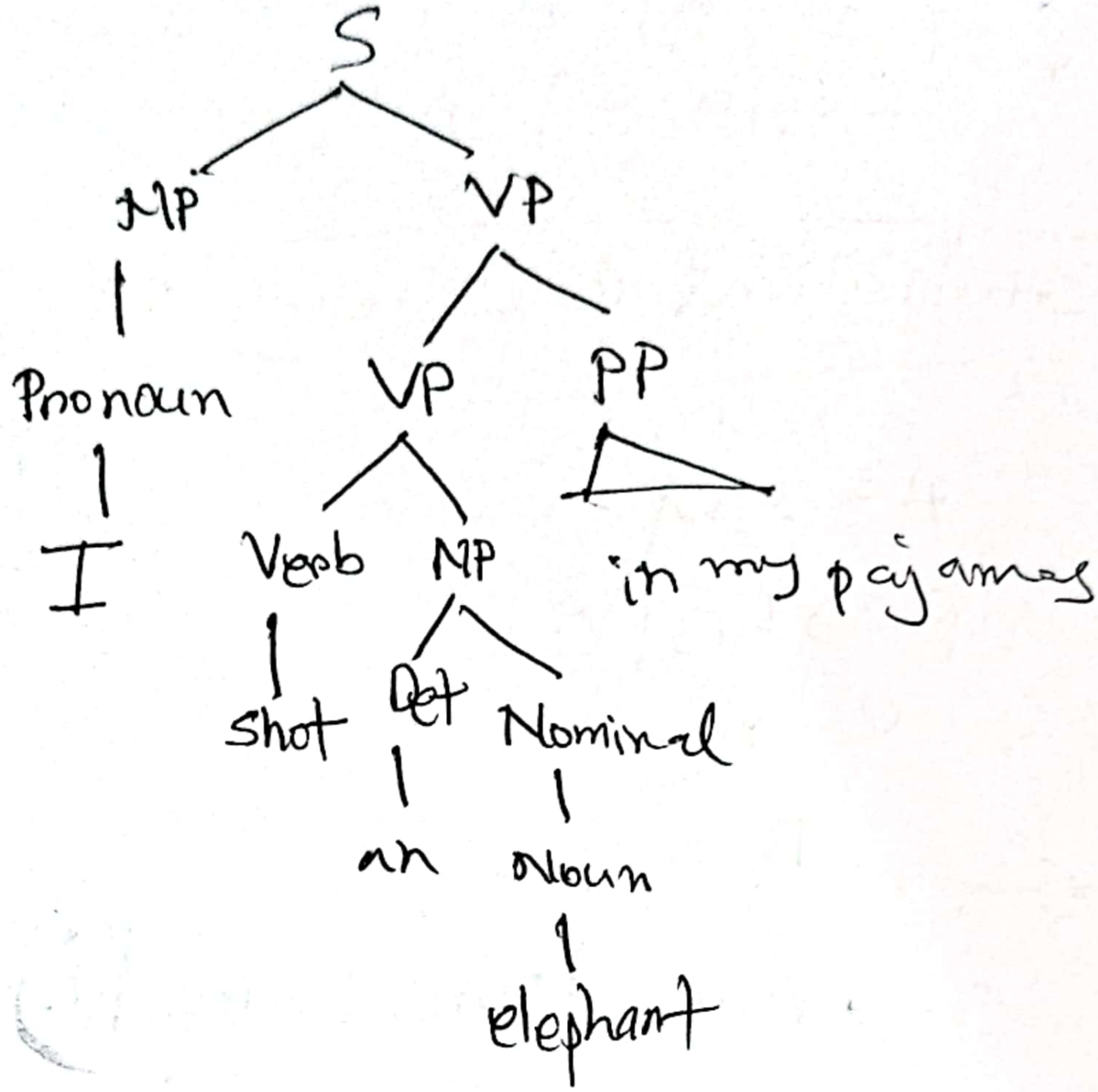
S → Noun verb NP

S → noun verb Det Nominal.

parse tree –



Det = Determinant



(i) Remove Null Production } to simplify
(ii) " Unit "

$S \rightarrow ABAc, \quad A \rightarrow aA \mid \epsilon, \quad B \rightarrow bB \mid \epsilon$

$\cancel{C \rightarrow c} \quad C \rightarrow c$

$A \rightarrow \epsilon$

Removing ~~A~~ $A \to \epsilon$

$S \to ABAC \mid BAC \mid ABC \mid BC$

* Statistical Consistency Parsing

Not important

$A \to aA$

$B \to bB \mid \epsilon$

$C \to c$

$B \to \epsilon$

~~S →~~

$S \to ABAC \mid BAC \mid ABC \mid BC \mid AAC$
$\mid AC \mid C$

$A \to aA$

$B \to bB$

$C \to c$

Removing of Unit Production

$S \rightarrow XY$

$X \rightarrow a$, $Y \rightarrow Z | b$, $Z \rightarrow M$,

$M \rightarrow N$, $N \rightarrow a$

24.08.23

Numerical Analysis:

For finding approximation

Convert algorithm to program

Management Information System:

Root finding Methods:

1. Bisection Method.
2. Method of false position.
3. Newton Raphson Method.
4. Fixed Position iteration method.

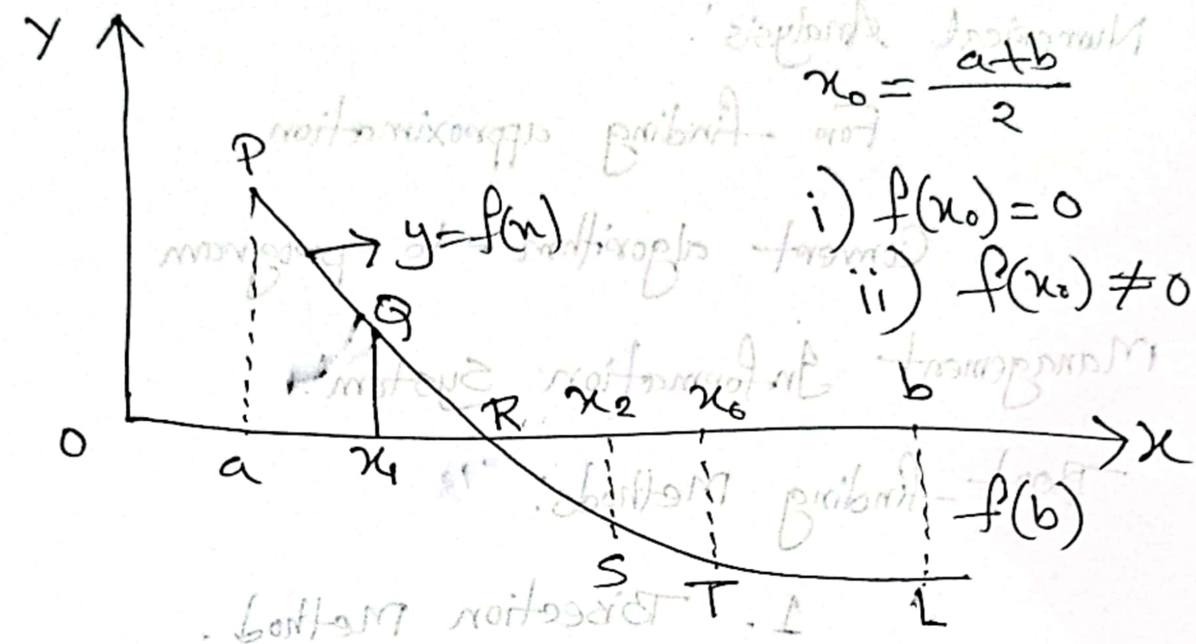
Bisection Method:

If a function $f(x)$ is continuous between a and b, and $f(a).f(b)<0$ (or $f(a)$ and $f(b)$ are of opposite signs) then their exists at least one root between a and b.

$$x_0 = \frac{a+b}{2}$$

i) $f(x_0) = 0$

ii) $f(x_0) \neq 0$



$f(a) \cdot f(b) < 0$ or $f(x_0) \cdot f(b) < 0$

iteration occur

$$x_1 = \frac{x_0 + a}{2}$$

$\cancel{f(x_1}$ $f(x_1) = 0$

root → zero function.

04-03-23

H.W. Calculator use
7 days

$x^3 - 2x^2 - 4 = 0$

$f(x) = x^3 - 2x^2 - 4$

$f(a) = f(2) = -4$

$f(b) = f(3) = 5$

$x_0 = \frac{a+b}{2} = \frac{2+3}{2} = \frac{5}{2} = 2.5$

$f(x_0) = -0.875$

$x_1 = \frac{2.5+3}{2} = 2.75$

$f(x_1) = 1.671875$

$x_2 = \frac{x_1 + x_0}{2} = \frac{2.5 + 2.75}{2} = 2.625$

$f(x_2) = 0.30640$

$x_3 = \frac{x_0 + x_2}{2} = \frac{2.5 + 2.625}{2}$

$= 2.5625$

$f(x_3) = -0.306396$

$$x_4 = \frac{x_3 + x_2}{2} = \frac{2.5625 + 2.625}{2}$$

$$= \cancel{2.60625}\ 2.59375$$

$$f(x_4) = -0.005523$$

$$x_5 = \frac{x_2 + x_4}{2} = \frac{\cancel{0.3}\ 2.625 + \cancel{2.60625}\ 2.59375}{2}$$

$$= 2.609375$$

$$f(x_5) = 0.149135$$

$$x_6 = \frac{x_4 + x_5}{2} = \frac{2.59375 + 2.609375}{2}$$

$$= 2.6015625$$

$$f(x_6) = 0.071451$$

$$x_7 = \frac{\cancel{x_5}\ x_4 + x_6}{2} = \frac{\cancel{0.2.609375 + 0.}\ 2.59375\ 2.6015625}{2}$$

$$= 2.5976525$$

$$f(x_7) = 0.032875$$

$$x_8 = \frac{x_7 + x_4}{2} = \frac{2.5976 5625 + 2.59375}{2}$$

$$= 2.595703125$$

$$f(\cancel{x}_8) = 0.013653$$

$$x_9 = \frac{\cancel{8}\, x_8 + x_4}{2} = \frac{2.595703125 + 2.5937}{2}$$

$$= 2.594726 5625$$

$$f(x_9) = 0.0040595$$

$$x_{10} = \frac{x_4 + x_9}{2} = \frac{2.59375 + 2.5947265}{2}$$

$$= 2.59423828125$$

$$f(x_{10}) = -0.0007334$$

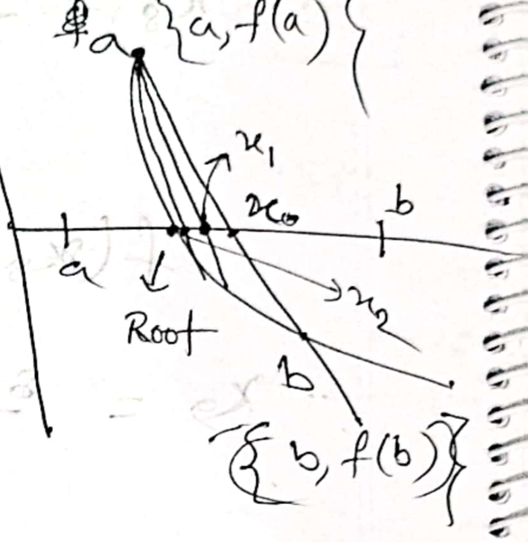
H.W. নিজে Q বানিয়ে solve কর
Bisection Q

False ~~Pot~~ Position Method

root is between x_0 and a

then → x_1 and a

then → x_2 and a



(x_1, y_1) (x_2, y_2)

eqn → $\dfrac{y-y_1}{x-x_1} = \dfrac{y_1-y_2}{x_1-x_2}$

$$\Rightarrow \frac{y-f(a)}{x-a} = \frac{f(a)-f(b)}{a-b}$$

$$\Rightarrow (x-a)(f(a)-f(b)) = (a-b)(y-f(a))$$

$$\Rightarrow (x-a) = \frac{(a-b)(y-f(a))}{f(a)-f(b)}$$

$$\Rightarrow x = a + \frac{(a-b)(y-f(a))}{f(a)-f(b)}$$

when, $x = x_0$

$$\therefore x_0 = a + \frac{(a-b)\{-f(a)\}}{f(a)-f(b)}$$

$$= a \cancel{b} - \frac{f(a)\{(a-b)\}}{f(a)-f(b)}$$

$$x_0 = \frac{a\{f(a)-f(b)\} - f(a)\{(a-b)\}}{f(a)-f(b)}$$

$$= \frac{\cancel{af(a)} - af(b) \cancel{-af(a)} + b(f(a))}{f(a)-f(b)}$$

$$= \frac{bf(a) - af(b)}{f(a)-f(b)}$$

Writer— Chapra
Book

Numerical Analysis

Solution of Linear System:

Gaussian Elemination Method:

This is an elementary elemination method. It reduces the system of linear equation to an equivalent upper triangular which can be solved by back ~~sush~~ substitution.

Consider a system of a linear equation with n unknowns:

$$
\left.
\begin{aligned}
a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\
a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\
\cdots \quad \cdots \quad \cdots & \\
\cdots \quad \cdots \quad \cdots & \\
a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n &= b_n
\end{aligned}
\right\} \quad (1)
$$

$$AX = B$$

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

$$\cancel{x} = x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \qquad B = \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix}$$

We have ~~the~~ augmented matrix of 1

$$(A : B) = \left[\begin{array}{cccc|c} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & b_2 \\ \cdots & \cdots & \cdots & \cdots & \vdots \\ \cdots & \cdots & \cdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} & b_n \end{array}\right]$$

~~For example:~~

$$= \left[\begin{array}{cccc|c} 1 & a_{12}' & \cdots & a_{1n}' & b_1' \\ 0 & a_{22}' & \cdots & a_{2n}' & b_2' \\ \cdots & \cdots & \cdots & \cdots & \vdots \\ 0 & a_{n2}' & \cdots & a_{nn}' & b_n' \end{array}\right] \begin{array}{l} R_1 \to R_1/a_{11} \\ R_2 \to R_2 - a_{21}R_1 \\ \\ R_n \to \end{array}$$

$$= \left[\begin{array}{ccccc|c} 1 & a_{12}' & a_{13}' & \cdots & a_{1n}' & b_1' \\ 0 & 1 & a_{23}'' & \cdots & a_{2n}'' & b_2'' \\ - & - & - & - & - & \vdots \\ 0 & 0 & 0 & \cdots & 1 & b_n \end{array}\right] \left[\begin{array}{c} l_1 \\ l_2 \\ \vdots \\ l_n \end{array}\right]$$

We obtain →

$$x_1 + a_{12}'x_2 + a_{13}'x_3 + \cdots a_{1n}'x_n = l_1 \quad \text{--- (i)}$$

$$x_2 + a_{23}''x_3 + \cdots a_{2n}''x_n = l_2 \quad \text{--- (ii)}$$

$$x_n = l_n \quad \text{--- (iii)}$$

$$\frac{-15}{2} + 2\,\frac{-15+4}{2} \qquad -17/$$

$$-2 - \frac{3}{2} \qquad -1-$$

$$-4 + \frac{2\times 4}{3} \qquad \frac{-4-3}{2}, \; -7/2$$

$$\frac{-12+8}{3} \qquad \frac{-4}{3} \qquad 3 - \frac{21}{2}$$

$$\frac{6-21}{2}$$

$$-7/2 + 7/6\,3$$

$$-7/2 + 2/3$$

$$\frac{-21+4}{6} \qquad -17/$$

$2-\frac{7}{2}$

$\frac{4-7}{2}$

$-\frac{3}{2}$

Q]
$$\left.\begin{aligned} 2x+2y+z+2u &= 7 \\ x-2y+0z-u &= 2 \\ 3x-y-2z-u &= 3 \\ x+0y+0z-2u &= 0 \end{aligned}\right\}$$

Augmented matrix ——

$$\left[\begin{array}{cccc|c} 2 & 2 & 1 & 2 & 7 \\ 1 & -2 & 0 & -1 & 2 \\ 3 & -1 & -2 & -1 & 3 \\ 1 & 0 & 0 & -2 & 0 \end{array}\right]$$

$$= \left[\begin{array}{cccc|c} 1 & 1 & 1/2 & 1 & 7/2 \\ 1 & -2 & 0 & -1 & 2 \\ 3 & -1 & -2 & -1 & 3 \\ 1 & 0 & 0 & -2 & 0 \end{array}\right] \quad R_1 \leftarrow R_1/2$$

~~$R_2 \leftarrow R_2 - R_1$~~

$$= \left[\begin{array}{cccc|c} 1 & 1 & 1/2 & 1 & 7/2 \\ 0 & -3 & -1/2 & -2 & -3/2 \\ 3 & -1 & -2 & -1 & 3 \\ 1 & 0 & 0 & -2 & 0 \end{array}\right] \quad R_2 \leftarrow R_2 - R_1$$

$-3/2 \times \frac{1}{-3}$

$-2/-3$

$1/6 \cdot 4/2$

$$= \left[\begin{array}{cccc|c} 1 & 1 & 1/2 & 1 & 7/2 \\ 0 & -3 & -1/2 & -2 & -3/2 \\ 0 & -4 & -7/2 & -4 & -15/2 \\ 1 & 0 & 0 & -2 & 0 \end{array}\right] \quad R_3 \leftarrow R_3 - 3R_1$$

$$= \left[\begin{array}{cccc|c} 1 & 1 & 1/2 & 1 & 7/2 \\ 0 & 1 & 1/6 & 2/3 & \frac{1}{2} \\ \cancel{3} & \cancel{-4} & \cancel{-7/2} & \cancel{-4} & 3 \\ 1 & 0 & 0 & -2 & 0 \end{array}\right] \quad \begin{array}{l} R_2/-3 \\ R_3 \leftarrow R \end{array}$$

$$= \left[\begin{array}{cccc|c} 1 & 1 & 1/2 & 1 & 7/2 \\ 0 & 1 & 1/6 & 2/3 & 1/2 \\ 0 & -4 & -7/2 & -4 & -15/2 \\ 0 & -1 & -1/2 & 2 & -7/2 \end{array}\right] \quad \begin{array}{l} R_2 \leftarrow R_2/-3 \\ R_3 \leftarrow R_3 - 3R_1 \\ R_4 \leftarrow R_4 - R_1 \end{array}$$

$$= \left[\begin{array}{cccc|c} 1 & 1 & 1/2 & 1 & 7/2 \\ 0 & 1 & 1/6 & 2/3 & 1/2 \\ 0 & 0 & -17/6 & -4/3 & -11/2 \\ 0 & -1 & -1/3 & -7/3 & -3 \end{array}\right] \quad \begin{array}{l} R_3 \leftarrow R_3 + 4R_2 \\ R_4 \leftarrow R_4 + R_2 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1/2 & 1 & 7/2 \\ 0 & 1 & 1/6 & 2/3 & 1/2 \\ 0 & 0 & 1 & 8/17 & 33/17 \\ 0 & 0 & 0 & 1 & 40/37 \end{array}\right]$$

$$x + y + 1/2 z + u = 7/2$$

$$y + 1/6 z + 2/3 u = 1/2$$

$$z + 8/17 u = 33/17$$

$$u = 40/37$$

$z = 53/37$ $\quad y = -17/37$ $\quad x = 80/37$

Gauss Jordan Method

$$[A:B] = \left[\begin{array}{cccc|c} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & b_2 \\ - & - & - & - & \\ a_{n1} & a_{n2} & \cdots & a_{nn} & b_n \end{array}\right]$$

$$= \left[\begin{array}{cccc|c} 1 & a_{12}' & \cdots & a_{1n}' & b_1' \\ 0 & a_{22}' & \cdots & a_{2n}' & b_2' \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & a_{2n}' & \cdots & a_{nn}' & b_n' \end{array}\right]$$

$$= \left[\begin{array}{ccccc|c} x_1 & 0 & 0 & \cdots & 0 & l_1 \\ 0 & x_2 & 0 & \cdots & 0 & l_2 \\ 0 & 0 & x_3 & \cdots & 0 & l_3 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & x_n & l_n \end{array}\right]$$

$\therefore x_1 = l_1 \quad x_2 = l_2$

$x_3 = l_3 \quad \cdots \quad x_n = l_n$

Q2

$$\left.\begin{array}{l} x + 2y + 3z - 2s = 6 \\ 2x - y - 2z - 3s = 8 \\ 3x + 2y - z + 2s = 4 \\ 2x - 3y + 2z + s = -8 \end{array}\right\}$$

$4 - 18 \quad 8 - 12$

-14

$2 - 6$

$1 + 4$

$8 - 12$

Soln

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & -2 & 6 \\ 2 & -1 & -2 & -3 & 8 \\ 3 & 2 & -1 & 2 & 4 \\ 2 & -3 & 2 & 1 & -8 \end{array}\right]$$

$$= \left[\begin{array}{cccc|c} 1 & 2 & 3 & -2 & 6 \\ 0 & -5 & -8 & 1 & -4 \\ 0 & -4 & -10 & 8 & -14 \\ 0 & -7 & -4 & 5 & -20 \end{array}\right] \quad \begin{array}{l} R_2 = R_2 - 2R_1 \\ R_3 = R_3 - 3R_1 \\ R_4 = R_4 - 2R_1 \end{array}$$

$$= \left[\begin{array}{cccc|c} 1 & 2 & 3 & -2 & 6 \\ 0 & -5 & -8 & 1 & -4 \\ 0 & 0 & -18 & 36 & -54 \\ 0 & \cancel{-20}\,0 & 36 & 18 & -72 \end{array}\right] \quad \begin{array}{l} R_3 = 5R_3 - 4R_2 \\ R_4 = \cancel{4R_4 + 7R_2} \\ \quad 5R_4 - 7R_2 \end{array}$$

$$= \left[\begin{array}{cccc|c} 1 & 2 & 3 & -2 & 6 \\ 0 & -5 & -8 & 1 & -4 \\ 0 & 0 & -18 & 36 & -54 \\ 0 & 0 & 0 & 90 & -180 \end{array}\right] \quad R_4 = R_4 + 2R_3$$

$3 - \frac{2\times 8}{5}$ $\quad -2 + \frac{2}{5}$ $\quad \frac{-8}{5}$ $\quad 6 - \frac{4\times 2}{5}$ $\quad -\frac{1}{5} + \frac{8}{5} \times \frac{1}{5}$

$\frac{15-16}{5}$ $\quad \frac{-10+2}{5}$ $\quad \frac{30-8}{5}$ $\quad \frac{22}{5}$ $\quad \frac{-5+8}{25}$ $\quad \frac{-3}{}$

$$= \left[\begin{array}{cccc|c} 1 & 2 & 3 & -2 & 6 \\ 0 & 1 & \frac{8}{5} & \frac{-1}{5} & \frac{4}{5} \\ 0 & 0 & 1 & -2 & 3 \\ 0 & 0 & 0 & 1 & -2 \end{array}\right] \quad \begin{array}{l} R_2 = R_2/-5 \\ R_3 = R_3/-18 \\ R_4 = R_4/90 \end{array}$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & \frac{-1}{5} & \frac{-8}{5} & \cancel{-8}\ \frac{22}{5} \\ 0 & 1 & 0 & \frac{-3}{\cancel{25}} & -4 \\ & & & & \\ & & & & \end{array}\right] \quad \begin{array}{l} R_1 = R_1 - 2R_2 \\ R_2 = R_2 - \frac{8}{5}R_3 \end{array}$$

$-\frac{1}{5} + \frac{8}{5} \times 2$

$\frac{-1+16}{5}$

$\frac{15}{5}$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -2 \end{array}\right]$$

$x = 1 \qquad y = 2$

$z = -1 \qquad s = -2$

25.09.23

Gauss Jacobi Iterative Method

Gauss Seidel Iterative Method

$10x - 2y + z = 2 \Rightarrow x = (2y + 2 - z)/10$

$-3x + 11y + 2z = 5 \Rightarrow y = (5 + 3x - 2z)/11$

$x - y + 5 = 1 \Rightarrow z = (1 - x + y)/5$

$x = g_1(y, z) \qquad y = g_2(x, z) \qquad z = g_3(x, y)$

$$\left.\begin{aligned} x_i &= g_1(y_{i-1}, z_{i-1}) \\ y_1 &= g_2(x_{i-1}, z_{i-1}) \\ z_i &= g_3(x_{i-1}, y_{i-1}) \end{aligned}\right\} \text{Jacobi}$$

$$\left.\begin{aligned} x_i &= g_1(y_{i-1}, z_{i-1}) \\ y_i &= g_2(x_i, z_{i-1}) \\ z_i &= g_3(x_i, y_i) \end{aligned}\right\} \text{Seidel.}$$

Jacobi

i	x	y	z
0	0	0	0
1	0.2	0.9545 ~~0.5090~~	~~0.2618~~ 0.2
~~2~~	~~0.2756~~	~~0.482~~	~~0.2413~~
~~3~~	~~0.27227~~	~~0.4858~~	~~0.24128~~
~~4~~ 2	0.2709	0.4727	0.2509
~~5~~ 3	0.2695	0.4828	0.2403
4	0.2725	0.4844	0.24267
5	0.2726	0.4847	0.2423

Seidel ⇒

i	x	y	z
0	0	0	0
1	0.2	0.5090	0.2618
2	0.2756	0.4821	0.2413
3	0.2723	0.4849	0.2425
4	0.2727	0.4848	0.2424
5	0.2727	0.4848	0.2424

Interpolation:

Using Newton's formula for interpolation, estimate the population for the year 1905.

Year	1891	1901	1911	1921	1931
Population	98752	132285	168076	195690	246050

Year(x)	Population(y)	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
~~1981~~ 1891	98752	33533	2258	-16435	41358
1901	132285	35791	-8177	30923	
1911	168076	27614	-446		
1921	195696	50366			
1931	246050				

By Newton's foreward formula, we have

$$y(x) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y$$

$$u = \frac{x - x_0}{h}$$

$x \rightarrow$ The ~~b~~ value which we need to find.

$x_0 \rightarrow$ first value, here 1891

$h \rightarrow$ range

$y(1905) = 98752 +$

Backward formula

Year(x)	Population(y)	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
1891	98752				
1901	132285	33533			
1911	168076	~~33533~~ 35791	2258		
1921	195696	~~35791~~ 27614	-8177	-10435	
1931	246050	~~27~~ 50360	22746	30923	41358

By Newton's Backward formula we have,

~~$y(1925) =$~~

$$u = \frac{1925-1931}{10} = -0.6$$

$$y(x) = y_n + u\nabla y_n + \frac{u(u-1)}{2!}\nabla^2 y_n + \frac{u(u-1)(u-2)}{3!}\nabla^3 y_n + \frac{u(u-1)(u-2)(u-3)}{4!}\nabla^4 y_n$$

$y(1925) =$ $\quad u = \frac{x-x_n}{h}$

$$y(1925) = 246050 + (-0.6)\cdot 50360 + 10918.08 - 12863.968 + 15484.4352$$

$= \cancel{229372}$

$= 229372.54$

Lagrange's Interpolation formula

Let $y = f(x)$ be a polynomial of nth degree, which takes the value of $f(x_0), f(x_1), \dots f(x_n)$ for any value $x_0, x_1, \dots x_n$ of the argument x. The polynomial may be written as:

$$f(x) = a_0(x-x_1)(x-x_2)\dots(x-x_n) + a_1(x-x_0)(x-x_2)\dots(x-x_n) + a_2(x-x_0)(x-x_1)\dots(x-x_n) + \dots$$

$$- \cdots + a_n (x-x_0)(x-x_1)(x-x_2) \cdots (x-x_{n-1}) \quad \text{--- (i)}$$

where a's are constant.

$$f(x_0) = a_0 (x_0 - x_1)(x_0 - x_2) \cdots (x_0 - x_n)$$

$$\Rightarrow a_0 = \frac{f(x_0)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)}$$

$$a_1 = \frac{f(x_1)}{(x_1-x_0)(x_1-2)\cdots(x_1-x_n)}$$

Putting all values in equation (1)

$$\cancel{f(x) = f(x_0) + f(x_1) + f(x_2) + \cdots + f(x_n)}$$

$$f(x) = \frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)} f(x_0) +$$

$$\frac{(x-x_0)(x-x_2)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)\cdots(x_1-x_n)} f(x_1) + \cdots$$

$$+ \frac{(x-x_0)(x-x_1)\cdots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\cdots(x_n-x_{n-1})} f(x_n)$$

x	0	1	2	3	4
$f(x)$	-12	0	12	24	

x	0	1	3	4
$f(x)$	-12	0	12	24

$$= \frac{(x-1)(x-3)(x-4)}{(-1)(-3)(-4)} \times (-12) + \frac{x(x-3)(x-4)}{(1)(-2)(-3)} \times 0$$

$$+ \frac{x(x-1)(x-4)}{(3)(2)(-1)} \times 12 + \frac{x(x-1)(x-3)}{(4)(3)(1)} \times 24$$

$$= (x-1)(x-3)(x-4) + 0 - 2x(x-1)(x-4) + 2x(x-1)(x-3)$$

H.W

x	321.0	322.8	324.2	325.0
$\log_{10} x$	2.50651	2.50893	2.51081	2.51188

Compute the value of $\log_{10} 323.5$ using Lagrange's interpolation formula.

swasthi book

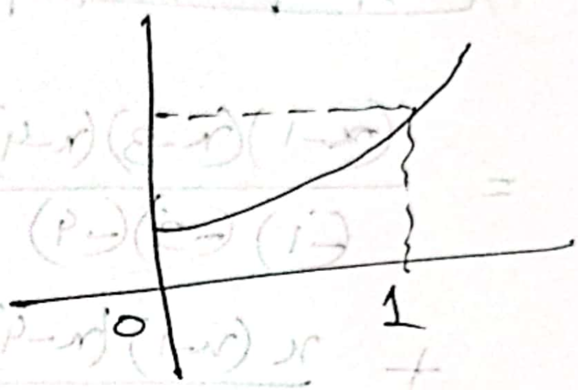
Ordinary Differential Equation :

$$\frac{dy}{dx} = x+1$$

$$\Rightarrow dy = (x+1)\,dx$$

$$\Rightarrow \int dy = \int (x+1)\,dx$$

$$\Rightarrow y = \frac{1}{2}x^2 + x + c$$



x	y
0	0
0.1	0
⋮	
1	

1. Euler Method
2. Runge Kutta metho

Euler Method :

$$\frac{dy}{dx} = f(x,y),\quad y(x_0) = y_0,\ h$$

$$y_n = y_{n-1} + h f(x_{n-1}, y_{n-1}).$$

Ex]
Solve $\frac{dy}{dx} = x+y$, $y(0) = 1$, for $0 \le x \le 1$ with

$h = 0.1$

x	0	0.1	0.2	0.3	0.4	0.5	...	1
y	1	1.2	1.22				---	

$Y_1 = 1.2$

$Y_2 =$

Runge kutta method.

$\frac{dy}{dx} = f(x,y),\ y(x_0) = y_0,\ h =$

$k_{1,n} = hf(x_{n-1}, y_{n-1})$

$k_{2,n} = hf(x_{n-1}+h, y_{n-1}+k_{1,n})$

$k_1 = hf(x_1, y_1)$

$k_2 = hf(x_1+h, y_1+k_1)$

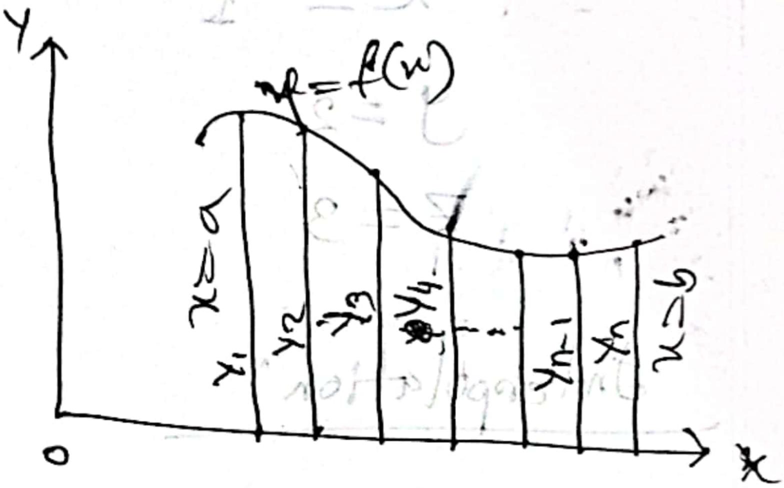
$y_n = y_{n-1} + \frac{1}{2}(k_1+k_2)$

Numerical Integration :

Formula :-

1. Trapezoidal Rule for Numerical Integration.

$$\int_a^b f(x)dx = \frac{h}{2}\left[y_0 + 2(y_1+y_2+\dots+y_{n-1}) + y_n\right]$$



2. SimSon's ~~1/3 1/3 1/3 Rule~~ 1/3 Rule.

$$\int_a^b f(x)dx = \frac{h}{3}\left[y_0 + 4(y_1+y_3+\dots+y_{n-1}) + 2(y_2+y_4+\dots+y_{n-2}) + y_n\right]$$

3. Simson's 3/8 Rule

$\underline{\underline{0}}$

$$\int_a^b f(x)\,dx = \frac{3h}{8}\Big[y_0 + 3(y_1 + y_2 + y_4 + y_5 + \cdots + y_{n-2} + y_{n-1}) + 2(y_3 + y_6 + y_9 + \cdots + y_{n-3}) + y_n\Big]$$

Q. 1 $\displaystyle\int_0^6 \frac{dx}{1+x^2}$ $h = \frac{6-0}{6} = 1$

x	0	1	2	3	4	5	6
$y = \frac{1}{1+x^2}$	1	0.5	0.2	0.1			