Computing Theory – Homework #1

Due: April 5, 2016 13:00

Take care of the readability of your solutions, from which you may lose 10 points.

1. **[25 Points]** For every prime number p, prove that $\sqrt{p}$ is irrational.

2. **[25 Points]** Prove by induction that $n^4 - 4n^2$ is divisible by 3, for all integers $n \geq 1$.

3. **[25 Points]** Design automata (NFA / DFA) to accept the following languages:

   a. A = {w ∈ {0, 1}* : w has a 1 in the third position from the right}.

   b. B = {w ∈ {0, 1}* : w contains at least two 0s}

   c. C = {w ∈ {0, 1}* : the length of w is divisible by three}

   d. D = {w ∈ {0, 1}* : w contains exactly two 0s and at least two 1s}.

4. **[25 Points]** Give regular expressions describing the following languages:

   a. A = { w ∈ {0, 1}* : w contains at least three 1s}.

   b. B = { w ∈ {0, 1}* : w contains at least two 1s and at most one 0},

   c. C = { w ∈ {0, 1}* : w contains an even number of 0s and exactly two 1s}.

   d. D = { w ∈ {0, 1}* : w contains an even number of 0s and each 0 is followed by at least one}

Best luck!

The course book: *Introduction to the theory of computation, 2nd Ed., Massachusetts Institute of Technology, by Micheal Sipser.*