

Solutions to Chapter 8, Susanna Epp Discrete Math

5th Edition

<https://github.com/spamegg1>

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1 Exercise Set 8.1

1.1 Exercise 1

As in Example 8.1.2, the **congruence modulo 2** relation E is defined from \mathbb{Z} to \mathbb{Z} as follows: For every ordered pair $(m, n) \in \mathbb{Z} \times \mathbb{Z}$, $m E n \iff m - n$ is even.

1.1.1 (a)

Is $0 E 0$? Is $5 E 2$? Is $(6, 6) \in E$? Is $(21, 7) \in E$?

Proof. $0 E 0$ because $0 - 0 = 0 = 2 \cdot 0$, so $2 \mid (0 - 0)$. $5 \not E 2$ because $5 - 2 = 3$ and $3 \neq 2k$ for any integer k , so $2 \nmid (5 - 2)$. $(6, 6) \in E$ because $6 - 6 = 0 = 2 \cdot 0$, so $2 \mid (6 - 6)$. $(-1, 7) \in E$ because $-1 - 7 = -8 = 2 \cdot (-4)$, so $2 \mid (-1 - 7)$. \square

1.1.2 (b)

Prove that for any even integer n , $n E 0$.

Proof. Assume n is even. By definition of even, $n = 2k$ for some integer k . Then $n - 0 = 2k - 0 = 2k$ is also even. Therefore by definition of E , $n E 0$. \square

1.2 Exercise 2

Prove that for all integers m and n , $m - n$ is even if, and only if, both m and n are even or both m and n are odd.

Proof. \implies : Assume $m - n$ is even. [We want to prove that both m and n are even or both m and n are odd.] By definition of even, $m - n = 2k$ for some integer k . There are 4 cases:

Case 1: both m and n are even: Nothing to prove.

Case 2: both m and n are odd: Nothing to prove.

Case 3: m is even, n is odd: By definitions of even and odd, $m = 2k, n = 2l + 1$ for some integers k, l . So $m - n = 2k - 2l - 1 = 2(k - l - 1) + 1$ where $k - l - 1$ is an integer. So by definition of odd, $m - n$ is odd, a contradiction. So this case is impossible.

Case 4: m is odd, n is even: By definitions of even and odd, $m = 2k + 1, n = 2l$ for some integers k, l . So $m - n = 2k + 1 - 2l = 2(k - l) + 1$ where $k - l$ is an integer. So by definition of odd, $m - n$ is odd, a contradiction. So this case is impossible.

\Leftarrow : Assume both m and n are even or both m and n are odd. [*We want to prove that $m - n$ is even.*] There are 2 cases:

Case 1: both m and n are even: By definition of even, $m = 2k, n = 2l$ for some integers k, l . Then $m - n = 2k - 2l = 2(k - l)$ where $k - l$ is an integer. So by definition, $m - n$ is even.

Case 2: both m and n are odd: By definition of even, $m = 2k + 1, n = 2l + 1$ for some integers k, l . Then $m - n = 2k + 1 - 2l - 1 = 2(k - l)$ where $k - l$ is an integer. So by definition, $m - n$ is even. \square

1.3 Exercise 3

The congruence modulo 3 relation, T , is defined from \mathbb{Z} to \mathbb{Z} as follows: For all integers m and n , $m T n \iff 3 \mid (m - n)$.

1.3.1 (a)

Is $10 T 1$? Is $1 T 10$? Is $(2, 2) \in T$? Is $(8, 1) \in T$?

Proof. $10 T 1$ because $10 - 1 = 9 = 3 \cdot 3$, and so $3 \mid (10 - 1)$.

$1 T 10$ because $1 - 10 = -9 = 3 \cdot (-3)$, and so $3 \mid (1 - 10)$.

$2 T 2$ because $2 - 2 = 0 = 3 \cdot 0$, and so $3 \mid (2 - 2)$.

$8 \not T 1$ because $8 - 1 = 7 \neq 3k$, for any integer k . So $3 \nmid (8 - 1)$. \square

1.3.2 (b)

List five integers n such that $n T 0$.

Proof. One possible answer: 3, 6, 9, -3, -6 \square

1.3.3 (c)

List five integers n such that $n T 1$.

Proof. One possible answer: 4, 7, 10, -2, -5 \square

1.3.4 (d)

List five integers n such that $n T 2$.

Proof. One possible answer: 5, 8, 11, -1, -4 \square

1.3.5 (e)

Make and prove a conjecture about which integers are related by T to 0, which integers are related by T to 1, and which integers are related by T to 2.

All integers of the form $3k + 1$, for some integer k , are related by T to 1.

Proof. All integers of the form $3k$, for some integer k , are related by T to 0.

All integers of the form $3k + 1$, for some integer k , are related by T to 1.

All integers of the form $3k + 2$, for some integer k , are related by T to 2. □

1.4 Exercise 4

Define a relation P on \mathbb{Z} as follows: For every ordered pair $(m, n) \in \mathbb{Z} \times \mathbb{Z}$, $m P n \iff m$ and n have a common prime factor.

1.4.1 (a)

Is $15 P 25$?

Proof. Yes, because 15 and 25 are both divisible by 5, which is prime. □

1.4.2 (b)

Is $22 P 27$?

Proof. No, because 22 and 27 have no common prime factor. □

1.4.3 (c)

Is $0 P 5$?

Proof. Yes, because 0 and 5 are both divisible by 5, which is prime. □

1.4.4 (d)

Is $8 P 8$?

Proof. Yes, because 8 and 8 are both divisible by 2, which is prime. □

1.5 Exercise 5

Let $X = \{a, b, c\}$. Recall that $\mathcal{P}(X)$ is the power set of X . Define a relation \mathbf{S} on $\mathcal{P}(X)$ as follows: For all sets A and B in $\mathcal{P}(X)$, $A \mathbf{S} B \iff A$ has the same number of elements as B .

1.5.1 (a)

Is $\{a, b\} \mathbf{S} \{b, c\}$?

Proof. Yes, because both $\{a, b\}$ and $\{b, c\}$ have two elements. □

1.5.2 (b)

Is $\{a\} \mathbf{S} \{a, b\}$?

Proof. No, one has 1 element, the other has 2 elements. □

1.5.3 (c)

Is $\{c\} \mathbf{S} \{b\}$?

Proof. Yes, because both $\{c\}$ and $\{b\}$ have one element. □

1.6 Exercise 6

Let $X = \{a, b, c\}$. Define a relation \mathbf{J} on $\mathcal{P}(X)$ as follows: For all sets A and B in $\mathcal{P}(X)$, $A \mathbf{J} B \iff A \cap B \neq \emptyset$.

1.6.1 (a)

Is $\{a\} \mathbf{J} \{c\}$?

Proof. No, because $\{a\} \cap \{c\} = \emptyset$. □

1.6.2 (b)

Is $\{a, b\} \mathbf{J} \{b, c\}$?

Proof. Yes, because $\{a, b\} \cap \{b, c\} = \{b\} \neq \emptyset$. □

1.6.3 (c)

Is $\{a, b\} \mathbf{J} \{a, b, c\}$?

Proof. Yes, because $\{a, b\} \cap \{a, b, c\} = \{a, b\} \neq \emptyset$. □

1.7 Exercise 7

Define a relation R on \mathbb{Z} as follows: For all integers m and n , $m R n \iff 5 \mid (m^2 - n^2)$.

1.7.1 (a)

Is $1 R (-9)$?

Proof. Yes. $1 R (-9) \iff 5 \mid (1^2 - (-9)^2)$. But $1^2 - (-9)^2 = 1 - 81 = -80$, and $5 \mid (-80)$ because $-80 = 5 \cdot (-16)$. \square

1.7.2 (b)

Is $2 R 13$?

Proof. Yes, $2^2 - (13)^2 = 4 - 169 = -165 = 5 \cdot (-33)$. So $5 \mid 2^2 - (13)^2$. \square

1.7.3 (c)

Is $2 R (-8)$?

Proof. Yes, $2^2 - (-8)^2 = 4 - 64 = -60 = 5 \cdot (-12)$. So $5 \mid 2^2 - (-8)^2$. \square

1.7.4 (d)

Is $(-8) R 2$?

Proof. Yes, $(-8)^2 - 2^2 = 64 - 4 = 60 = 5 \cdot 12$. So $5 \mid (-8)^2 - 2^2$. \square

1.8 Exercise 8

Let A be the set of all strings of a 's and b 's of length 4. Define a relation R on A as follows: For every $s, t \in A$, $s R t \iff s$ has the same first two characters as t .

1.8.1 (a)

Is $abaa R abba$?

Proof. Yes, because both $abaa$ and $abba$ have the same first two characters ab . \square

1.8.2 (b)

Is $aabb R bbaa$?

Proof. No, because the first two characters of $aabb$ are different from the first two characters of $bbaa$. \square

1.8.3 (c)

Is $aaaa R aaab$?

Proof. Yes, because both $aaaa$ and $aaab$ have the same first two characters aa . \square

1.8.4 (d)

Is $baaa R abaa$?

Proof. No, because the first two characters of $baaa$ are different from the first two characters of $abaa$. \square

1.9 Exercise 9

Let A be the set of all strings of 0's, 1's, and 2's of length 4. Define a relation R on A as follows: For every $s, t \in A$, $s R t \iff$ the sum of the characters in s equals the sum of the characters in t .

1.9.1 (a)

Is $0121 R 2200$?

Proof. Yes, because the sum of the characters in 0121 is 4 and the sum of the characters in 2200 is also 4. \square

1.9.2 (b)

Is $1011 R 2101$?

Proof. No, because the sum of the characters in 1011 is 3, whereas the sum of the characters in 2101 is 4. \square

1.9.3 (c)

Is $2212 R 2121$?

Proof. No, because the sum of the characters in 2212 is 7, whereas the sum of the characters in 2121 is 6. \square

1.9.4 (d)

Is $1220 R 2111$?

Proof. Yes, because the sum of the characters in 1220 is 5 and the sum of the characters in 2111 is also 5. \square

1.10 Exercise 10

Let $A = \{3, 4, 5\}$ and $B = \{4, 5, 6\}$ and let R be the “less than” relation. That is, for every ordered pair $(x, y) \in A \times B$, $x R y \iff x < y$. State explicitly which ordered pairs are in R and R^{-1} .

Proof. $R = \{(3, 4), (3, 5), (3, 6), (4, 5), (4, 6), (5, 6)\}$

$R^{-1} = \{(4, 3), (5, 3), (6, 3), (5, 4), (6, 4), (6, 5)\}$

□

1.11 Exercise 11

Let $A = \{3, 4, 5\}$ and $B = \{4, 5, 6\}$ and let S be the “divides” relation. That is, for every ordered pair $(x, y) \in A \times B$, $x S y \iff x \mid y$. State explicitly which ordered pairs are in S and S^{-1} .

Proof. $S = \{(3, 6), (4, 4), (5, 5)\}$, $S^{-1} = \{(6, 3), (4, 4), (5, 5)\}$

□

1.12 Exercise 12

1.12.1 (a)

Suppose a function $F : X \rightarrow Y$ is one-to-one but not onto. Is F^{-1} (the inverse relation for F) a function? Explain your answer.

Proof. No. If $F : X \rightarrow Y$ is not onto, then F fails to be defined on all of Y . In other words, there is an element y in Y such that $(y, x) \notin F^{-1}$ for any $x \in X$. Consequently, F^{-1} does not satisfy property (1) of the definition of function. □

1.12.2 (b)

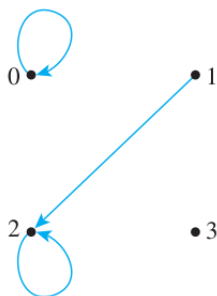
Suppose a function $F : X \rightarrow Y$ is onto but not one-to-one. Is F^{-1} (the inverse relation for F) a function? Explain your answer.

Proof. No. If $F : X \rightarrow Y$ is not one-to-one, then F for some y in Y , there will be multiple potential values for $F^{-1}(y)$. In other words, there is an element y in Y and elements $x_1, x_2 \in X$ such that $(y, x_1) \in F^{-1}$ and $(y, x_2) \in F^{-1}$. Consequently, F^{-1} does not satisfy property (2) of the definition of function. □

Draw the directed graphs of the relations defined in 13 – 18.

1.13 Exercise 13

Define a relation R on $A = \{0, 1, 2, 3\}$ by $R = \{(0, 0), (1, 2), (2, 2)\}$.

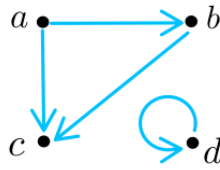


Proof.

□

1.14 Exercise 14

Define a relation S on $B = \{a, b, c, d\}$ by $S = \{(a, b), (a, c), (b, c), (d, d)\}$.

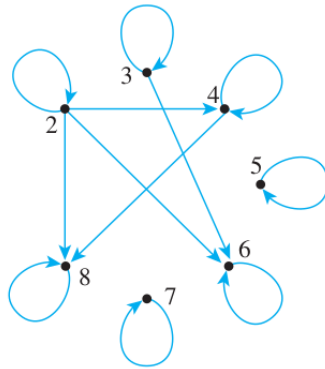


Proof.

□

1.15 Exercise 15

Let $A = \{2, 3, 4, 5, 6, 7, 8\}$ and define a relation R on A as follows: For every $x, y \in A$, $x R y \iff x \mid y$.

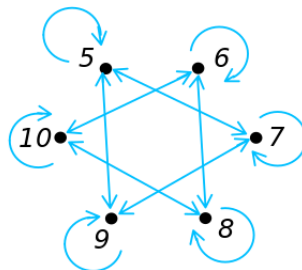


Proof.

□

1.16 Exercise 16

Let $A = \{5, 6, 7, 8, 9, 10\}$ and define a relation S on A as follows: For every $x, y \in A$, $x S y \iff 2 \mid (x - y)$.



Proof.

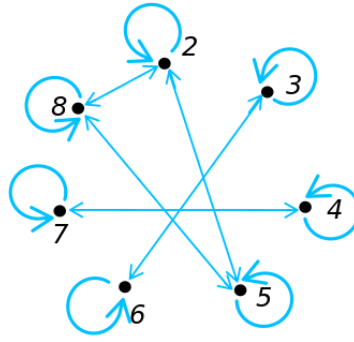
□

1.17 Exercise 17

Let $A = \{2, 3, 4, 5, 6, 7, 8\}$ and define a relation T on A as follows: For every $x, y \in A$, $x T y \iff 3 \mid (x - y)$.

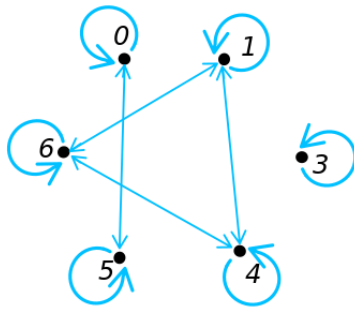
Proof.

□



1.18 Exercise 18

Let $A = \{0, 1, 3, 4, 5, 6\}$ and define a relation V on A as follows: For every $x, y \in A$, $x V y \iff 5 \mid (x^2 - y^2)$.



Proof.

□

1.19 Exercise 19

Let $A = \{2, 4\}$ and $B = \{6, 8, 10\}$ and define relations R and S from A to B as follows: For every $(x, y) \in A \times B$, $x R y \iff x \mid y$ and $x S y \iff y - 4 = x$. State explicitly which ordered pairs are in $A \times B$, R , S , $R \cup S$, and $R \cap S$.

Proof. $A \times B = \{(2, 6), (2, 8), (2, 10), (4, 6), (4, 8), (4, 10)\}$

$R = \{(2, 6), (2, 8), (2, 10), (4, 8)\}$, $S = \{(2, 6), (4, 8)\}$, $R \cup S = R$, $R \cap S = S$

□

1.20 Exercise 20

Let $A = \{-1, 1, 2, 4\}$ and $B = \{1, 2\}$ and define relations R and S from A to B as follows: For every $(x, y) \in A \times B$, $x R y \iff |x| \mid |y|$ and $x S y \iff x - y$ is even. State explicitly which ordered pairs are in $A \times B$, R , S , $R \cup S$, and $R \cap S$.

Proof. $A \times B = \{(-1, 1), (-1, 2), (1, 1), (1, 2), (2, 1), (2, 2), (4, 1), (4, 2)\}$

$R = \{(-1, 1), (1, 1), (2, 2)\}$, $S = \{(-1, 1), (1, 1), (2, 2), (4, 2)\}$, $R \cup S = S$, $R \cap S = R$

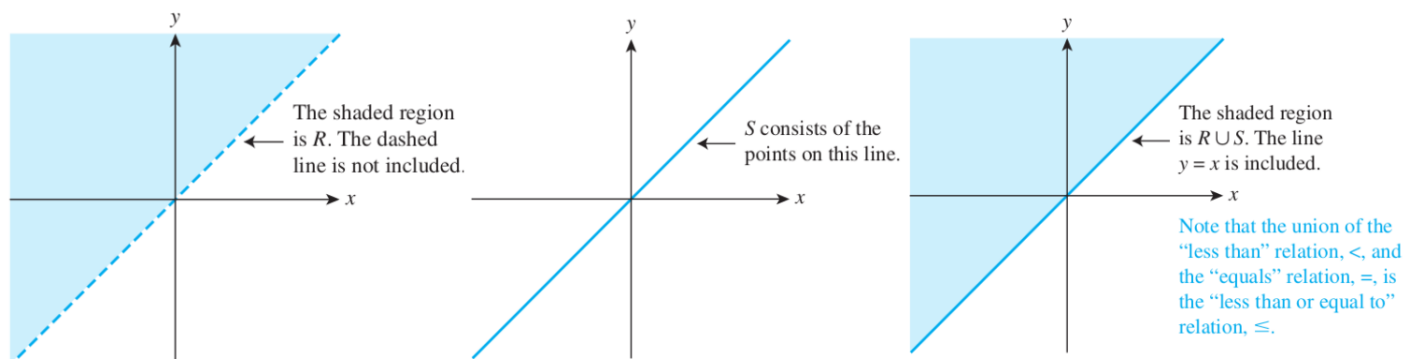
□

1.21 Exercise 21

Define relations R and S on \mathbb{R} as follows: $R = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid x < y\}$ and

$S = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid x = y\}$. That is, R is the “less than” relation and S is the “equals” relation on \mathbb{R} . Graph $R, S, R \cup S$, and $R \cap S$ in the Cartesian plane.

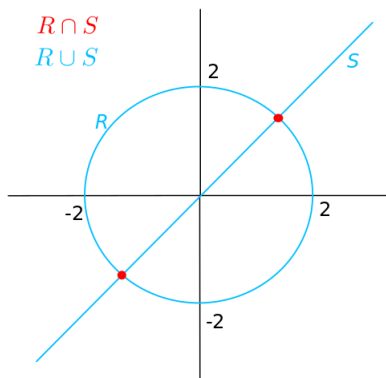
Proof. The graph of the intersection of R and S is obtained by finding the set of all points common to both graphs. But there are no points for which both $x < y$ and $x = y$. Hence $R \cap S = \emptyset$ and the graph consists of no points at all.



□

1.22 Exercise 22

Define relations R and S on \mathbb{R} as follows: $R = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid x^2 + y^2 = 4\}$ and $S = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid x = y\}$. Graph $R, S, R \cup S$, and $R \cap S$ in the Cartesian plane.

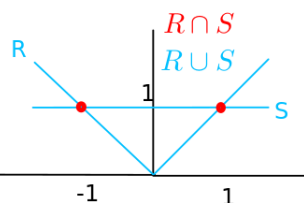


Proof.

□

1.23 Exercise 23

Define relations R and S on \mathbb{R} as follows: $R = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid y = |x|\}$ and $S = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid y = 1\}$. Graph $R, S, R \cup S$, and $R \cap S$ in the Cartesian plane.



Proof.

□

1.24 Exercise 24

In Example 8.1.7 consider the query `SELECT Patient_ID#, Name FROM S WHERE Primary_Diagnosis = X`. The response to the query is the projection onto the first two coordinates of the intersection of the database with the set $A_1 \times A_2 \times A_3 \times \{X\}$.

1.24.1 (a)

Find the result of the query `SELECT Patient_ID#, Name FROM S WHERE Primary_Diagnosis = pneumonia`.

Proof. 574329 Tak Kurosawa, 011985 John Schmidt □

1.24.2 (b)

Find the result of the query `SELECT Patient_ID#, Name FROM S WHERE Primary_Diagnosis = appendicitis`.

Proof. 466581 Mary Lazars, 778400 Jamal Baskers □

2 Exercise Set 8.2

2.1 Exercise 1

2.1.1 (a)

Proof. □

2.1.2 (b)

Proof. □

2.1.3 (c)

Proof. □

2.1.4 (d)

Proof. □

2.2 Exercise 2

2.2.1 (a)

Proof. □

2.2.2 (b)

Proof. □

2.2.3 (c)

Proof.



2.2.4 (d)

Proof.



2.3 Exercise 3

2.3.1 (a)

Proof.



2.3.2 (b)

Proof.



2.3.3 (c)

Proof.



2.3.4 (d)

Proof.



2.4 Exercise 4

2.4.1 (a)

Proof.



2.4.2 (b)

Proof.



2.4.3 (c)

Proof.



2.4.4 (d)

Proof.



2.5 Exercise 5

2.5.1 (a)

Proof.



2.5.2 (b)

Proof.



2.5.3 (c)

Proof.



2.5.4 (d)

Proof.



2.6 Exercise 6

2.6.1 (a)

Proof.



2.6.2 (b)

Proof.



2.6.3 (c)

Proof.



2.6.4 (d)

Proof.



2.7 Exercise 7

2.7.1 (a)

Proof.



2.7.2 (b)

Proof.



2.7.3 (c)

Proof.



2.7.4 (d)

Proof.



2.8 Exercise 8

2.8.1 (a)

Proof.



2.8.2 (b)

Proof.



2.8.3 (c)

Proof.



2.8.4 (d)

Proof.



2.9 Exercise 9

Proof.



2.10 Exercise 10

Proof.



2.11 Exercise 11

Proof.



2.12 Exercise 12

Proof.



2.13 Exercise 13

Proof.



2.14 Exercise 14

Proof.



2.15 Exercise 15

Proof.



2.16 Exercise 16

Proof.



2.17 Exercise 17

Proof.



2.18 Exercise 18

Proof.



2.19 Exercise 19

Proof.



2.20 Exercise 20

Proof.



2.21 Exercise 21

Proof.



2.22 Exercise 22

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2.23 Exercise 23

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2.24 Exercise 24

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2.25 Exercise 25

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2.26 Exercise 26

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2.27 Exercise 27

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2.28 Exercise 28

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2.29 Exercise 29

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2.30 Exercise 30

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2.31 Exercise 31

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2.32 Exercise 32

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2.33 Exercise 33

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2.34 Exercise 34

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2.35 Exercise 35

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2.36 Exercise 36

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2.37 Exercise 37

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2.38 Exercise 38

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2.39 Exercise 39

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2.40 Exercise 40

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2.41 Exercise 41

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2.42 Exercise 42

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2.43 Exercise 43

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2.44 Exercise 44

Proof.



2.45 Exercise 45

Proof.



2.46 Exercise 46

Proof.



2.47 Exercise 47

Proof.



2.48 Exercise 48

Proof.



2.49 Exercise 49

Proof.



2.50 Exercise 50

Proof.



2.51 Exercise 51

Proof.



2.52 Exercise 52

Proof.



2.53 Exercise 53

Proof.



2.54 Exercise 54

Proof.



2.55 Exercise 55

Proof.



2.56 Exercise 56

Proof.



3 Exercise Set 8.3

3.1 Exercise 1

3.1.1 (a)

Proof.



3.1.2 (b)

Proof.



3.1.3 (c)

Proof.



3.1.4 (d)

Proof.



3.2 Exercise 2

3.2.1 (a)

Proof.



3.2.2 (b)

Proof.



3.2.3 (c)

Proof.



3.3 Exercise 3

Proof.



3.4 Exercise 4

Proof.



3.5 Exercise 5

Proof.



3.6 Exercise 6

Proof.



3.7 Exercise 7

Proof.



3.8 Exercise 8

Proof.



3.9 Exercise 9

Proof.



3.10 Exercise 10

Proof.



3.11 Exercise 11

Proof.



3.12 Exercise 12

Proof.



3.13 Exercise 13

Proof.



3.14 Exercise 14

Proof.



3.15 Exercise 15

3.15.1 (a)

Proof.



3.15.2 (b)

Proof.



3.15.3 (c)

Proof.



3.15.4 (d)

Proof.



3.16 Exercise 16

3.16.1 (a)

Proof.



3.16.2 (b)

Proof.



3.17 Exercise 17

3.17.1 (a)

Proof.



3.17.2 (b)

Proof.



3.18 Exercise 18

3.18.1 (a)

Proof.



3.18.2 (b)

Proof.



3.19 Exercise 19

3.19.1 (a)

Proof.



3.19.2 (b)

Proof.



3.20 Exercise 20

Proof.



3.21 Exercise 21

Proof.



3.22 Exercise 22

Proof.



3.23 Exercise 23

Proof.



3.24 Exercise 24

Proof.



3.25 Exercise 25

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3.26 Exercise 26

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3.27 Exercise 27

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3.28 Exercise 28

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3.29 Exercise 29

Proof.



3.30 Exercise 30

Proof.



3.31 Exercise 31

Proof.



3.32 Exercise 32

Proof.



3.33 Exercise 33

Proof.



3.34 Exercise 34

Proof.



3.35 Exercise 35

Proof.



3.36 Exercise 36

Proof.



3.37 Exercise 37

Proof.



3.38 Exercise 38

Proof.



3.39 Exercise 39

Proof.



3.40 Exercise 40

Proof.



3.41 Exercise 41

Proof.



3.42 Exercise 42

3.42.1 (a)

Proof.



3.42.2 (b)

Proof.



3.42.3 (c)

Proof.



3.42.4 (d)

Proof.



3.43 Exercise 43

3.43.1 (a)

Proof.



3.43.2 (b)

Proof.



3.43.3 (c)

Proof.



3.43.4 (d)

Proof.



3.43.5 (e)

Proof.



3.43.6 (f)

Proof.



3.44 Exercise 44

3.44.1 (a)

Proof.



3.44.2 (b)

Proof.



3.44.3 (c)

Proof.



3.44.4 (d)

Proof.



3.44.5 (e)

Proof.



3.44.6 (f)

Proof.



3.44.7 (g)

Proof.



3.45 Exercise 45

Proof.



3.46 Exercise 46

Proof.



3.47 Exercise 47

3.47.1 (a)

Proof.



3.47.2 (b)

Proof.



3.47.3 (c)

Proof.



3.47.4 (d)

Proof.



3.47.5 (e)

Proof.



3.47.6 (f)

Proof.



3.47.7 (g)

Proof.



4 Exercise Set 8.4

4.1 Exercise 1

4.1.1 (a)

Proof.



4.1.2 (b)

Proof.



4.2 Exercise 2

4.2.1 (a)

Proof.



4.2.2 (b)

Proof.



4.3 Exercise 3

4.3.1 (a)

Proof.



4.3.2 (b)

Proof.



4.3.3 (c)

Proof.



4.3.4 (d)

Proof.



4.3.5 (e)

Proof.



4.4 Exercise 4

4.4.1 (a)

Proof.



4.4.2 (b)

Proof.



4.4.3 (c)

Proof.



4.4.4 (d)

Proof.



4.4.5 (e)

Proof.



4.5 Exercise 5

Proof.

☐

4.6 Exercise 6

Proof.

☐

4.7 Exercise 7

4.7.1 (a)

Proof.

☐

4.7.2 (b)

Proof.

☐

4.7.3 (c)

Proof.

☐

4.7.4 (d)

Proof.

☐

4.7.5 (e)

Proof.

☐

4.8 Exercise 8

4.8.1 (a)

Proof.

☐

4.8.2 (b)

Proof.

☐

4.8.3 (c)

Proof.

☐

4.8.4 (d)

Proof.

☐

4.8.5 (e)

Proof.



4.9 Exercise 9

4.9.1 (a)

Proof.



4.9.2 (b)

Proof.



4.10 Exercise 10

Proof.



4.11 Exercise 11

Proof.



4.12 Exercise 12

4.12.1 (a)

Proof.



4.12.2 (b)

Proof.



4.13 Exercise 13

4.13.1 (a)

Proof.



4.13.2 (b)

Proof.



4.14 Exercise 14

Proof.



4.15 Exercise 15

Proof.



4.16 Exercise 16

Proof.



4.17 Exercise 17

Proof.



4.18 Exercise 18

Proof.



4.19 Exercise 19

Proof.



4.20 Exercise 20

Proof.



4.21 Exercise 21

Proof.



4.22 Exercise 22

Proof.



4.23 Exercise 23

Proof.



4.24 Exercise 24

Proof.



4.25 Exercise 25

Proof.



4.26 Exercise 26

Proof.



4.27 Exercise 27

Proof.



4.28 Exercise 28

Proof.



4.29 Exercise 29

Proof.



4.30 Exercise 30

Proof.



4.31 Exercise 31

4.31.1 (a)

Proof.



4.31.2 (b)

Proof.



4.31.3 (c)

Proof.



4.32 Exercise 32

4.32.1 (a)

Proof.



4.32.2 (b)

Proof.



4.33 Exercise 33

Proof.



4.34 Exercise 34

Proof.



4.35 Exercise 35

Proof.



4.36 Exercise 36

Proof.



4.37 Exercise 37

Proof.



4.38 Exercise 38

Proof.



4.39 Exercise 39

Proof.



4.40 Exercise 40

Proof.



4.41 Exercise 41

4.41.1 (a)

Proof.



4.41.2 (b)

Proof.



4.42 Exercise 42

Proof.



4.43 Exercise 43

Proof.



5 Exercise Set 8.5

5.1 Exercise 1

5.1.1 (a)

Proof.



5.1.2 (b)

Proof.



5.1.3 (c)

Proof.



5.1.4 (d)

Proof.



5.2 Exercise 2

Proof.



5.3 Exercise 3

Proof.



5.4 Exercise 4

Proof.



5.5 Exercise 5

Proof.



5.6 Exercise 6

Proof.



5.7 Exercise 7

Proof.



5.8 Exercise 8

Proof.



5.9 Exercise 9

Proof.



5.10 Exercise 10

Proof.



5.11 Exercise 11

5.11.1 (a)

Proof.



5.11.2 (b)

Proof.



5.11.3 (c)

Proof.



5.11.4 (d)

Proof.



5.11.5 (e)

Proof.



5.11.6 (f)

Proof.



5.11.7 (g)

Proof.



5.12 Exercise 12

Proof.



5.13 Exercise 13

Proof.



5.14 Exercise 14

5.14.1 (a)

Proof.



5.14.2 (b)

Proof.



5.15 Exercise 15

Proof.



5.16 Exercise 16

5.16.1 (a)

Proof.



5.16.2 (b)

Proof.



5.17 Exercise 17

Proof.



5.18 Exercise 18

Proof.



5.19 Exercise 19

Proof.



5.20 Exercise 20

Proof.



5.21 Exercise 21

5.21.1 (a)

Proof.



5.21.2 (b)

Proof.



5.22 Exercise 22

Proof.



5.23 Exercise 23

Proof.



5.24 Exercise 24

Proof.



5.25 Exercise 25

Proof.



5.26 Exercise 26

Proof.



5.27 Exercise 27

Proof.



5.28 Exercise 28

Proof.



5.29 Exercise 29

Proof.



5.30 Exercise 30

5.30.1 (a)

Proof.



5.30.2 (b)

Proof.



5.30.3 (c)

Proof.



5.30.4 (d)

Proof.



5.31 Exercise 31

Proof.



5.32 Exercise 32

Proof.



5.33 Exercise 33

Proof.



5.34 Exercise 34

Proof.



5.35 Exercise 35

Proof.



5.36 Exercise 36

Proof.



5.37 Exercise 37

Proof.



5.38 Exercise 38

Proof.



5.39 Exercise 39

Proof.



5.40 Exercise 40

5.40.1 (a)

Proof.



5.40.2 (b)

Proof.



5.41 Exercise 41

5.41.1 (a)

Proof.



5.41.2 (b)

Proof.



5.42 Exercise 42

Proof.



5.43 Exercise 43

Proof.



5.44 Exercise 44

Proof.



5.45 Exercise 45

Proof.



5.46 Exercise 46

Proof.



5.47 Exercise 47

Proof.



5.48 Exercise 48

Proof.



5.49 Exercise 49

5.49.1 (a)

Proof.



5.49.2 (b)

Proof.



5.50 Exercise 50

5.50.1 (a)

Proof.



5.50.2 (b)

Proof.

□

5.51 Exercise 51

5.51.1 (a)

Proof.

□

5.51.2 (b)

Proof.

□