

Chapter 11 Solutions, Susanna Epp Discrete Math

5th Edition

<https://github.com/spamegg1>

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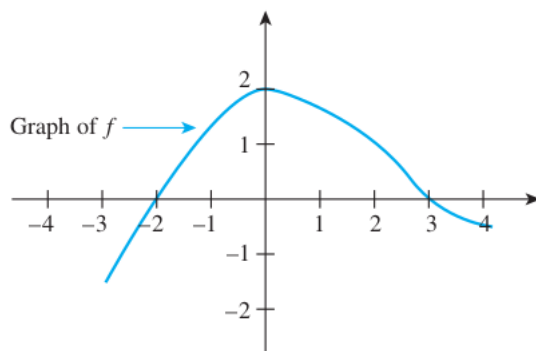
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1 Exercise Set 11.1

1.1 Exercise 1



The graph of a function f is shown above.

1.1.1 (a)

Is $f(0)$ positive or negative?

Proof. positive

□

1.1.2 (b)

For what values of x does $f(x) = 0$?

Proof. $f(x) = 0$ when $x = -2$ and $x = 3$ (approximately)

□

1.1.3 (c)

Find approximate values for x_1 and x_2 so that $f(x_1) = f(x_2) = 1$ but $x_1 \neq x_2$.

Proof. $x_1 = -1$ and $x_2 = 2$ (approximately)

□

1.1.4 (d)

Find an approximate value for x such that $f(x) = 1.5$.

Proof. $x = 1$ or $x = -1/2$ (approximately)

□

1.1.5 (e)

As x increases from -3 to -1 , do the values of f increase or decrease?

Proof. increase

□

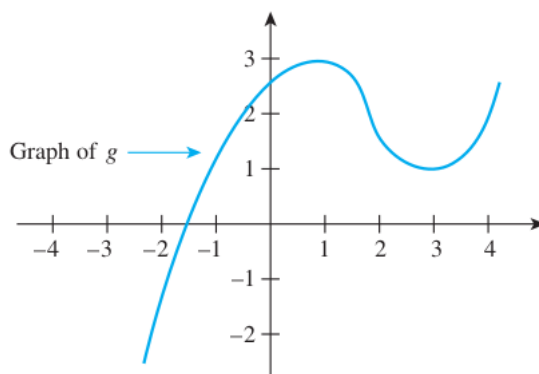
1.1.6 (f)

As x increases from 0 to 4, do the values of f increase or decrease?

Proof. decrease

□

1.2 Exercise 2



The graph of a function g is shown above.

1.2.1 (a)

Is $g(0)$ positive or negative?

Proof. positive

□

1.2.2 (b)

Find an approximate value of x so that $g(x) = 0$.

Proof. -1.5 (approximately)

□

1.2.3 (c)

Find approximate values for x_1 and x_2 so that $g(x_1) = g(x_2) = 1$ but $x_1 \neq x_2$.

Proof. $x_1 = -1, x_2 = 3$ (approximately)

□

1.2.4 (d)

Find an approximate value for x such that $g(x) = -2$.

Proof. $x = -2.2$ (approximately)

□

1.2.5 (e)

As x increases from -2 to 1 , do the values of g increase or decrease?

Proof. increase

□

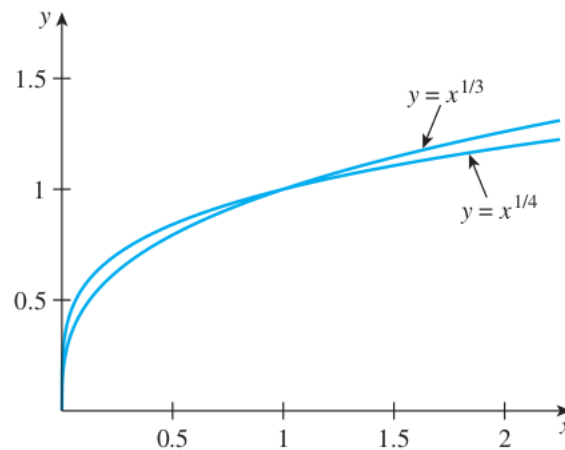
1.2.6 (f)

As x increases from 1 to 3, do the values of g increase or decrease?

Proof. decrease □

1.3 Exercise 3

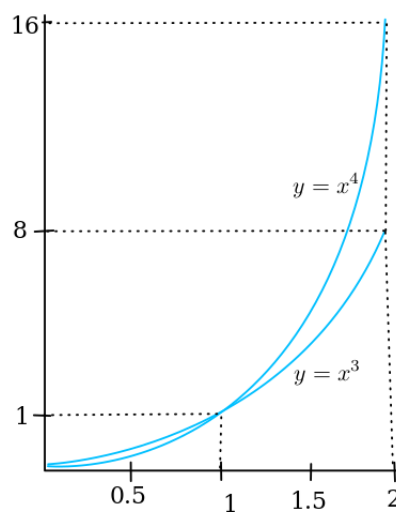
Sketch the graphs of the power functions $p_{1/3}$ and $p_{1/4}$ on the same set of axes. When $0 < x < 1$, which is greater: $x^{1/3}$ or $x^{1/4}$? When $x > 1$, which is greater: $x^{1/3}$ or $x^{1/4}$?



Proof. When $0 < x < 1$, $x^{1/3} < x^{1/4}$. When $1 < x$, $x^{1/4} < x^{1/3}$. □

1.4 Exercise 4

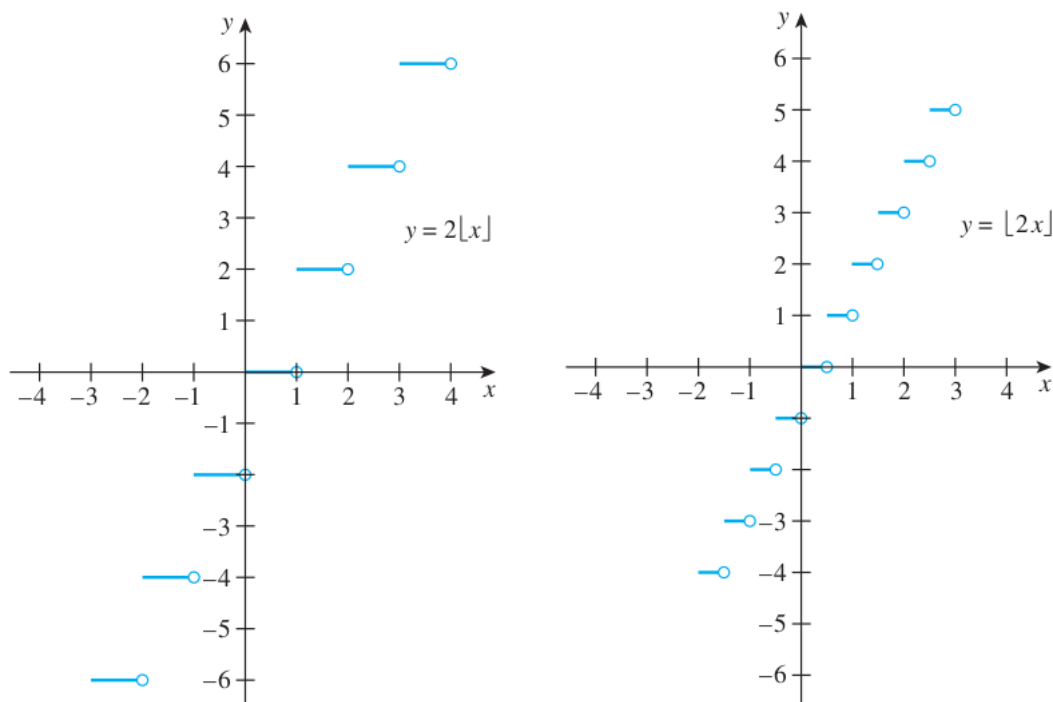
Sketch the graphs of the power functions p_3 and p_4 on the same set of axes. When $0 < x < 1$, which is greater: x^3 or x^4 ? When $x > 1$, which is greater: x^3 or x^4 ?



Proof. When $0 < x < 1$, $x^4 < x^3$. When $1 < x$, $x^3 < x^4$. □

1.5 Exercise 5

Sketch the graphs of $y = 2[x]$; and $y = [2x]$ for each real number x . What can you conclude from these graphs?



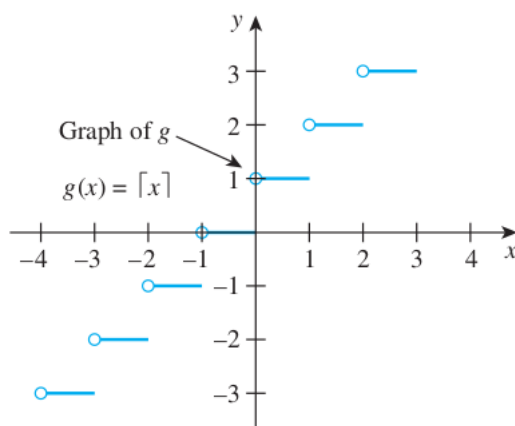
Proof.

The graphs show that $2[x] \neq [2x]$ for many values of x . □

Sketch a graph for each of the functions defined in 6 – 9 below.

1.6 Exercise 6

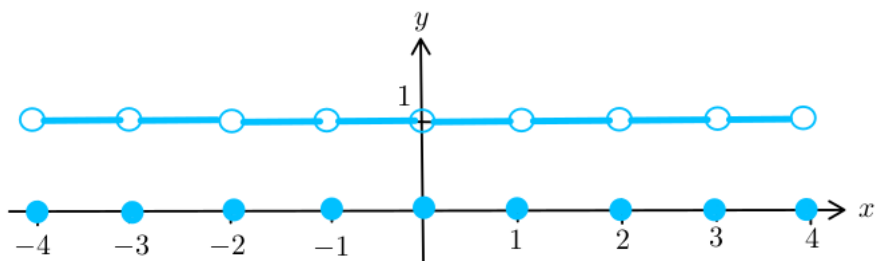
$g(x) = [x]$ for each real number x (Recall that the ceiling of x , $[x]$, is the least integer that is greater than or equal to x . That is, $[x] = n$ is the unique integer n such that $n - 1 < x \leq n$).



Proof. □

1.7 Exercise 7

$h(x) = \lceil x \rceil - \lfloor x \rfloor$ for each real number x

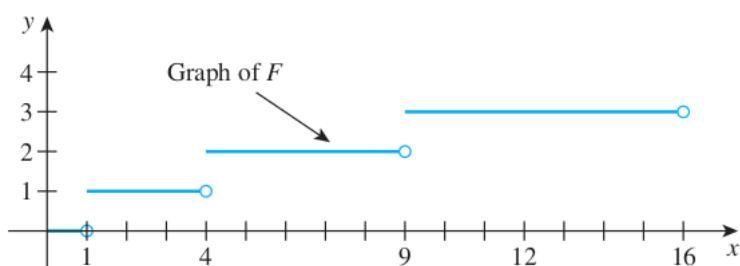


Proof.

□

1.8 Exercise 8

$F(x) = \lfloor x^{1/2} \rfloor$ for each real number x

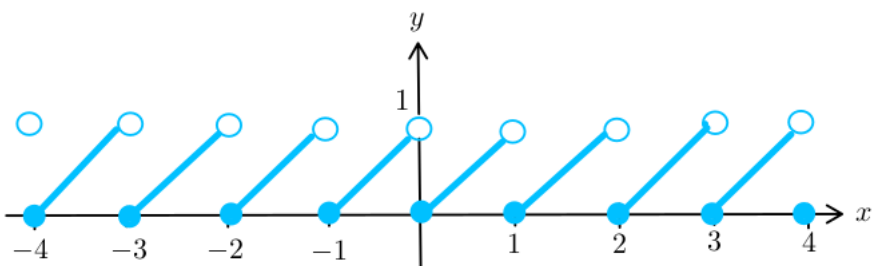


Proof.

□

1.9 Exercise 9

$G(x) = x - \lfloor x \rfloor$ for each real number x



Proof.

□

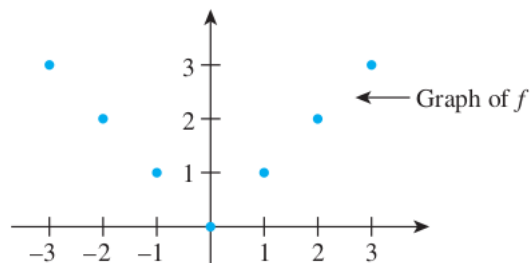
In each of 10 – 13 a function is defined on a set of integers. Sketch a graph for each function.

1.10 Exercise 10

$f(n) = |n|$ for each integer n

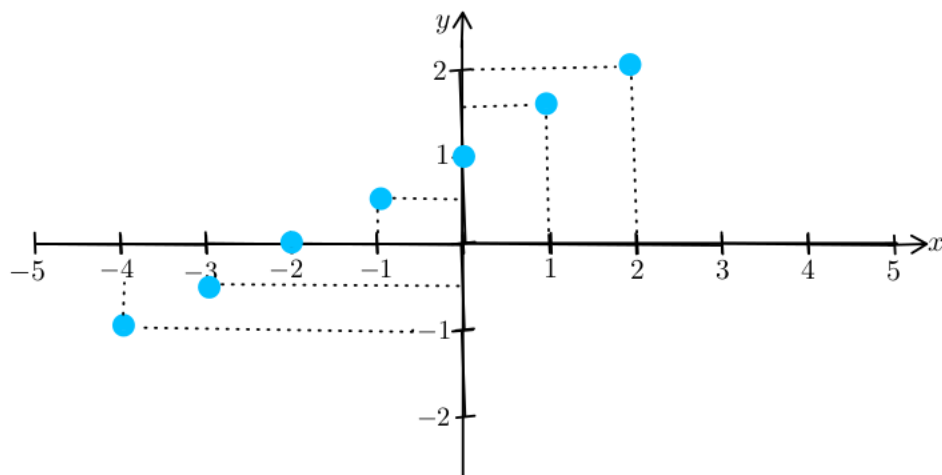
Proof.

□



1.11 Exercise 11

$g(n) = (n/2) + 1$ for each integer n

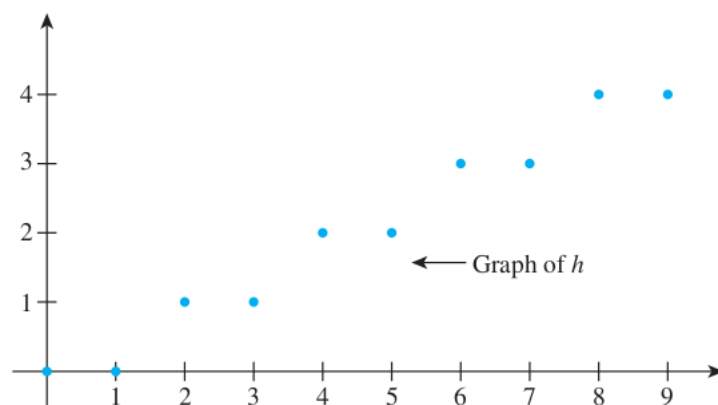


Proof.

□

1.12 Exercise 12

$h(n) = \lfloor n/2 \rfloor$ for each integer $n \geq 0$



Proof.

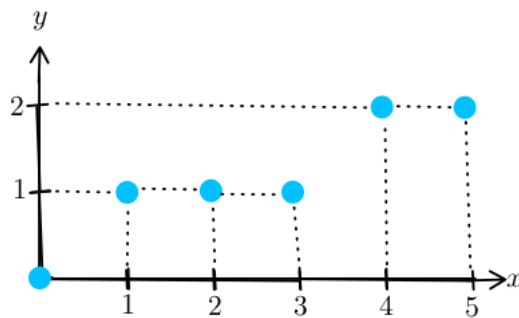
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1.13 Exercise 13

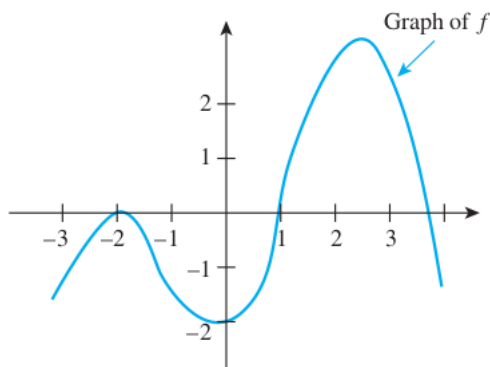
$k(n) = \lfloor n^{1/2} \rfloor$ for each integer $n \geq 0$

Proof.

□



1.14 Exercise 14



The graph of a function f is shown below. Find the intervals on which f is increasing and the intervals on which f is decreasing.

Proof. f is increasing on the intervals $\{x \in \mathbb{R} \mid -3 < x < -2\}$ and $\{x \in \mathbb{R} \mid 0 < x < 2.5\}$, and f is decreasing on $\{x \in \mathbb{R} \mid -2 < x < 0\}$ and $\{x \in \mathbb{R} \mid 2.5 < x < 4\}$ (approximately). \square

1.15 Exercise 15

Show that the function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by the formula $f(x) = 2x - 3$ is increasing on the set of real numbers.

Proof. Suppose that x_1 and x_2 are particular but arbitrarily chosen real numbers such that $x_1 < x_2$. [We must show that $f(x_1) < f(x_2)$.] Since $x_1 < x_2$ then $2x_1 < 2x_2$ and $2x_1 - 3 < 2x_2 - 3$ by basic properties of inequalities. Thus, by definition of f , $f(x_1) < f(x_2)$ [as was to be shown]. Hence f is increasing on the set of all real numbers. \square

1.16 Exercise 16

Show that the function $g : \mathbb{R} \rightarrow \mathbb{R}$ defined by the formula $g(x) = -(x/3) + 1$ is decreasing on the set of real numbers.

Proof.

\square

1.17 Exercise 17

Let h be the function from \mathbb{R} to \mathbb{R} defined by the formula $h(x) = x^2$ for each real number x .

1.17.1 (a)

Show that h is decreasing on the set of real numbers less than zero.

Proof. Suppose that x_1 and x_2 are particular but arbitrarily chosen real numbers such that $x_1 < x_2 < 0$. [We must show that $h(x_1) > h(x_2)$.]

Since $x_1 < x_2 < 0$ then $0 < -x_2 < -x_1$ and multiplying by $-x_1$ (which is a positive number) we get $(-x_1)(-x_2) < (-x_1)(-x_1) = x_1^2$ by basic properties of inequalities.

Similarly, since $x_1 < x_2 < 0$ then $0 < -x_2 < -x_1$ and multiplying by $-x_2$ (which is a positive number) we get $(-x_2)(-x_2) = x_2^2 < (-x_1)(-x_2)$ by basic properties of inequalities.

By combining the two results we get $x_2^2 < (-x_1)(-x_2) < x_1^2$ so $x_2^2 < x_1^2$.

Thus, by definition of h , $h(x_1) > h(x_2)$ [as was to be shown]. Hence h is increasing on the set of all real numbers. \square

1.17.2 (b)

Show that h is increasing on the set of real numbers greater than zero.

Proof. Suppose that x_1 and x_2 are particular but arbitrarily chosen real numbers such that $0 < x_1 < x_2$. [We must show that $h(x_1) < h(x_2)$.]

Since $0 < x_1 < x_2$ then multiplying by x_1 (which is a positive number) we get $x_1x_1 = x_1^2 < x_1x_2$ by basic properties of inequalities.

Similarly, since $0 < x_1 < x_2$ then multiplying by x_2 (which is a positive number) we get $x_1x_2 < x_2x_2 = x_2^2$ by basic properties of inequalities.

By combining the two results we get $x_1^2 < x_1x_2 < x_2^2$ so $x_1^2 < x_2^2$.

Thus, by definition of h , $h(x_1) < h(x_2)$ [as was to be shown]. Hence h is increasing on the set of all real numbers. \square

1.18 Exercise 18

Let $k : \mathbb{R} \rightarrow \mathbb{R}$ be the function defined by the formula $k(x) = (x - 1)/x$ for each real number $x \neq 0$.

1.18.1 (a)

Show that k is increasing for every real number $x > 0$.

Proof. Suppose that x_1 and x_2 are positive real numbers and $x_1 < x_2$. [We must show that $k(x_1) < k(x_2)$.]

$$\begin{array}{ll}
 & x_1 < x_2 & \text{by assumption} \\
 \implies & -x_2 < -x_1 & \text{by multiplying by } -1 \\
 \implies & x_1x_2 - x_2 < x_1x_2 - x_1 & \text{by adding } x_1x_2 \text{ to both sides} \\
 \implies & x_2(x_1 - 1) < x_1(x_2 - 1) & \text{by factoring both sides} \\
 \implies & \frac{x_1 - 1}{x_1} < \frac{x_2 - 1}{x_2} & \text{by dividing both sides by } x_1x_2 > 0 \\
 \implies & k(x_1) < k(x_2) & \text{by definition of } k
 \end{array}$$

□

1.18.2 (b)

Is k increasing or decreasing for $x < 0$? Prove your answer.

Proof. It is increasing. The same proof as in part (a) works. Note that the only place in the proof where the signs of x_1 and x_2 matter is when we divide both sides by x_1x_2 . For the proof to work, x_1x_2 has to be positive. But if both x_1 and x_2 are negative, then x_1x_2 is positive. Therefore the proof still works. □

1.19 Exercise 19

Show that if a function $f : \mathbb{R} \rightarrow \mathbb{R}$ is increasing, then f is one-to-one.

Proof. Suppose $f : \mathbb{R} \rightarrow \mathbb{R}$ is increasing. [We must show that f is one-to-one. In other words, we must show that for all real numbers x_1 and x_2 , if $x_1 \neq x_2$ then $f(x_1) \neq f(x_2)$.] Suppose x_1 and x_2 are real numbers and $x_1 \neq x_2$. By the trichotomy law [Appendix A, T17] $x_1 < x_2$, or $x_1 > x_2$. In case $x_1 < x_2$, then since f is increasing, $f(x_1) < f(x_2)$ and so $f(x_1) \neq f(x_2)$. Similarly, in case $x_1 > x_2$, then $f(x_1) > f(x_2)$ and so $f(x_1) \neq f(x_2)$. Thus in either case, $f(x_1) \neq f(x_2)$ [as was to be shown]. □

1.20 Exercise 20

Given real-valued functions f and g with the same domain D , the sum of f and g , denoted $f + g$, is defined as follows: For each real number x , $(f + g)(x) = f(x) + g(x)$. Show that if f and g are both increasing on a set S , then $f + g$ is also increasing on S .

Proof. Assume $x_1, x_2 \in S$ and $x_1 < x_2$. [We want to show $(f + g)(x_1) < (f + g)(x_2)$.] Since f is increasing, $f(x_1) < f(x_2)$. Since g is increasing, $g(x_1) < g(x_2)$. By definition of $f + g$ we have $(f + g)(x_1) = f(x_1) + g(x_1) < f(x_2) + g(x_2) = (f + g)(x_2)$, [as was to be shown.] □

1.21 Exercise 21

1.21.1 (a)

Let m be any positive integer, and define $f(x) = x^m$ for each nonnegative real number x . Use the binomial theorem to show that f is an increasing function.

Proof. Suppose u and v are nonnegative real numbers with $u < v$. [We must show that $f(u) < f(v)$.] Note that $v = u + h$ for some positive real number h . By substitution and the binomial theorem,

$$v^m = (u + h)^m = \sum_{i=0}^m \binom{m}{i} u^{m-i} h^i = u^m + \sum_{i=1}^m \binom{m}{i} u^{m-i} h^i$$

The last summation is positive because $u \geq 0$ and $h > 0$, and a sum of nonnegative terms that includes at least one positive term is positive. Hence $v^m = u^m +$ a positive number, and so $f(u) = u^m < v^m = f(v)$, [as was to be shown]. \square

1.21.2 (b)

Let m and n be any positive integers, and let $g(x) = x^{m/n}$ for each nonnegative real number x . Prove that g is an increasing function.

Note: The results of exercise 21 are used in the exercises for Sections 11.2 and 11.4.

Proof. Write $f(x) = x^m$. Then $g(x) = (f(x))^{1/n}$ by the law of exponents.

Now assume $0 \leq x_1 < x_2$. In part (a) we showed that f is increasing. Therefore $f(x_1) < f(x_2)$, in other words $x_1^m < x_2^m$. So we need to show that the function $h(x) = x^{1/n}$ is an increasing function. That will imply $g(x_1) = h(x_1^m) < h(x_2^m) = g(x_2)$, in other words $x_1^{m/n} < x_2^{m/n}$, which is what we want.

To show h is increasing, assume $0 \leq z_1 < z_2$. By definition, $h(z_1) = z_1^{1/n} = y_1$ is the real number with the property that $y_1^n = z_1$. Similarly $h(z_2) = z_2^{1/n} = y_2$ is the real number with the property that $y_2^n = z_2$. [We want to show $y_1 < y_2$.]

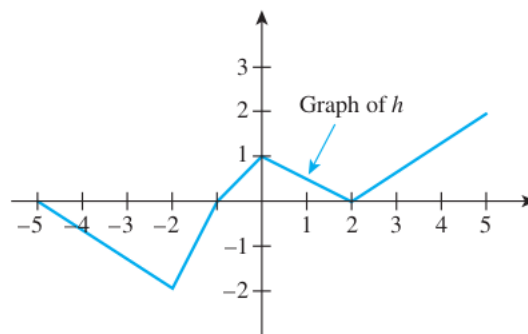
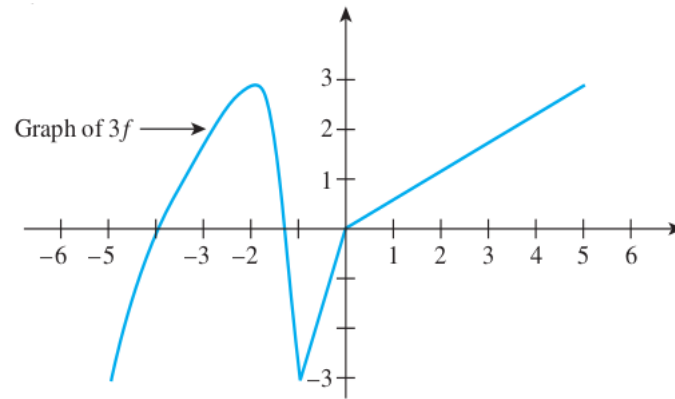
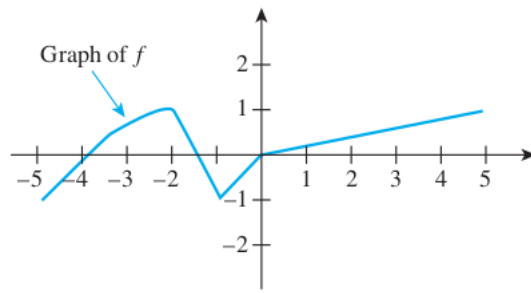
Argue by contradiction and assume $y_2 \leq y_1$. Now consider the function $e(y) = y^n$. This function is also increasing by part (a), since m and n are both any positive integers. Therefore $e(y_2) \leq e(y_1)$, in other words $z_2 \leq z_1$, which is a contradiction!

Therefore $y_1 < y_2$ and h is increasing, and thus g is increasing as a consequence. \square

1.22 Exercise 22

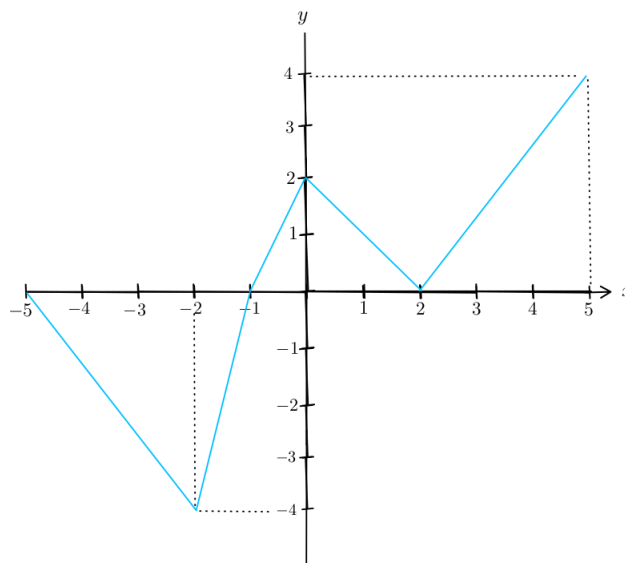
Let f be the function whose graph follows. Sketch the graph of $3f$.

Proof. \square



1.23 Exercise 23

Let h be the function whose graph is shown above. Sketch the graph of $2h$.



Proof.

□

1.24 Exercise 24

Let f be a real-valued function of a real variable. Show that if f is decreasing on a set S and if M is any positive real number, then Mf is decreasing on S .

Proof. Suppose that f is a real-valued function of a real variable, f is decreasing on a set S , and M is any positive real number. [We must show that Mf is decreasing on S . In other words, we must show that for all x_1 and x_2 in S , if $x_1 < x_2$ then $(Mf)(x_1) > (Mf)(x_2)$.] Suppose x_1 and x_2 are in S and $x_1 < x_2$. Since f is decreasing on S , $f(x_1) > f(x_2)$, and since M is positive, $Mf(x_1) > Mf(x_2)$ [because when both sides of an inequality are multiplied by a positive number, the direction of the inequality is unchanged]. It follows by definition of Mf that $(Mf)(x_1) > (Mf)(x_2)$, [as was to be shown]. \square

1.25 Exercise 25

Let f be a real-valued function of a real variable. Show that if f is increasing on a set S and if M is any negative real number, then Mf is decreasing on S .

Proof. The proof is the same as in Exercise 24, except that this time we have $f(x_1) < f(x_2)$ because f is increasing, and multiplying an inequality by a negative number M reverses the direction of the equality, so $Mf(x_1) > Mf(x_2)$. \square

1.26 Exercise 26

Let f be a real-valued function of a real variable. Show that if f is decreasing on a set S and if M is any negative real number, then Mf is increasing on S .

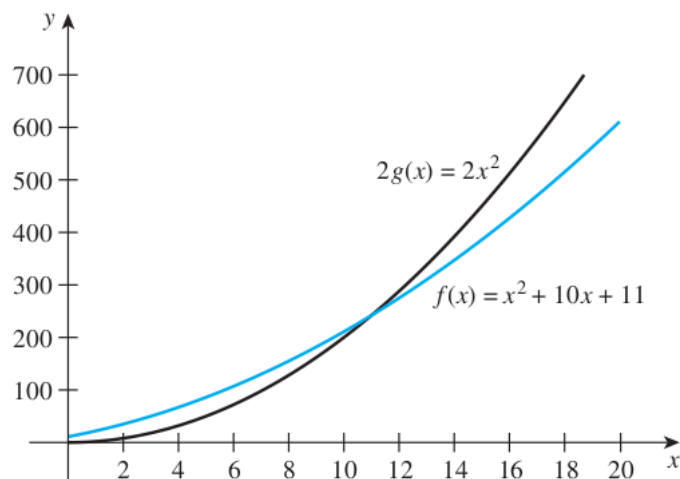
Proof. The proof is the same as in Exercise 24, except that this time multiplying an inequality by a negative number M reverses the direction of the equality, so $Mf(x_1) < Mf(x_2)$. \square

In 27 and 28, functions f and g are defined. In each case sketch the graphs of f and $2g$ on the same set of axes and find a number x_0 so that $f(x) \leq 2g(x)$ for all $x > x_0$. You can find an exact value for x_0 by solving a quadratic equation, or you can find an approximate value for x_0 by using a graphing calculator or computer.

1.27 Exercise 27

$f(x) = x^2 + 10x + 11$ and $g(x) = x^2$ for each real number $x \geq 0$

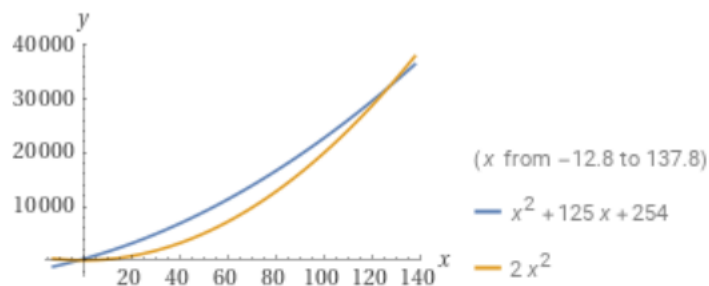
Proof. To find the answer algebraically, solve the equation $2x^2 = x^2 + 10x + 11$ for x . Subtracting x^2 from both sides gives $x^2 - 10x - 11 = 0$, and either using the quadratic formula or factoring $x^2 - 10x - 11 = (x - 11)(x + 1)$ gives $x = 11$ (since $x > 0$). To find an approximate answer with a graphing calculator, plot both $f(x) = x^2 + 10x + 11$ and $2g(x) = 2x^2$ for $x > 0$, as shown in the figure, and find that $2g(x) > f(x)$ when



$x > 11$ (approximately). You can obtain only an approximate answer from a graphing calculator because the calculator computes values only to an accuracy of a finite number of decimal places. \square

1.28 Exercise 28

$f(x) = x^2 + 125x + 254$ and $g(x) = x^2$ for each real number $x \geq 0$



Proof. If we set $f(x) = 2g(x)$ and solve, we get $x^2 + 125x + 254 = 2x^2$ which gives $x^2 - 125x - 254 = 0$ which factors as $(x - 127)(x + 2) = 0$ which has solutions $x = -2, 127$. So let $x_0 = 127$, so that $f(x) < g(x)$ for all $x > x_0 = 127$. \square

2 Exercise Set 11.2

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5 Exercise Set 11.5

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