Solutions to Chapter 8, Susanna Epp Discrete Math 5th Edition

https://github.com/spamegg1

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Contents

1 Exercise Set 8.1

1.1 Exercise 1

As in Example 8.1.2, the **congruence modulo 2** relation E is defined from $\mathbb{Z}$ to $\mathbb{Z}$ as follows: For every ordered pair $(m, n) \in \mathbb{Z} \times \mathbb{Z}$, $m\ E\ n \iff m - n$ is even.

1.1.1 (a)

Is $0\ E\ 0$? Is $5\ E\ 2$? Is $(6, 6) \in E$? Is $(21, 7) \in E$?

Proof. $0\ E\ 0$ because $0 - 0 = 0 = 2 \cdot 0$, so $2 \mid (0 - 0)$. $5 \not E\ 2$ because $5 - 2 = 3$ and $3 \neq 2k$ for any integer k, so $2 \nmid (5 - 2)$. $(6, 6) \in E$ because $6 - 6 = 0 = 2 \cdot 0$, so $2 \mid (6 - 6)$.$(-1, 7) \in E$ because $-1 - 7 = -8 = 2 \cdot (-4)$, so $2 \mid (-1 - 7)$. □

1.1.2 (b)

Prove that for any even integer n, $n\ E\ 0$.

Proof. Assume n is even. By definition of even, $n = 2k$ for some integer k. Then $n - 0 = 2k - 0 = 2k$ is also even. Therefore by definition of E, $n\ E\ 0$. □

1.2 Exercise 2

Prove that for all integers m and n, $m - n$ is even if, and only if, both m and n are even or both m and n are odd.

Proof. $\implies$: Assume $m - n$ is even. *[We want to prove that both m and n are even or both m and n are odd.]* By definition of even, $m - n = 2k$ for some integer k. There are 4 cases:

Case 1: both m and n are even: Nothing to prove.

Case 2: both m and n are odd: Nothing to prove.

Case 3: m is even, n is odd: By definitions of even and odd, $m = 2k, n = 2l + 1$ for some integers k, l. So $m - n = 2k - 2l - 1 = 2(k - l - 1) + 1$ where $k - l - 1$ is an integer. So by definition of odd, $m - n$ is odd, a contradiction. So this case is impossible.

Case 4: m is odd, n is even: By definitions of even and odd, $m = 2k+1, n = 2l$ for some integers k, l. So $m - n = 2k + 1 - 2l = 2(k-l) + 1$ where $k - l$ is an integer. So by definition of odd, $m - n$ is odd, a contradiction. So this case is impossible.

$\Longleftarrow$: Assume both m and n are even or both m and n are odd. *[We want to prove that $m - n$ is even.]* There are 2 cases:

Case 1: both m and n are even: By definition of even, $m = 2k, n = 2l$ for some integers k, l. Then $m - n = 2k - 2l = 2(k - l)$ where $k - l$ is an integer. So by definition, $m - n$ is even.

Case 2: both m and n are odd: By definition of even, $m = 2k+1, n = 2l+1$ for some integers k, l. Then $m - n = 2k + 1 - 2l - 1 = 2(k - l)$ where $k - l$ is an integer. So by definition, $m - n$ is even. □

1.3 Exercise 3

The congruence modulo 3 relation, T, is defined from $\mathbb{Z}$ to $\mathbb{Z}$ as follows: For all integers m and n, $m \; T \; n \iff 3 \mid (m - n)$.

1.3.1 (a)

Is $10 \; T \; 1$? Is $1 \; T \; 10$? Is $(2, 2) \in T$? Is $(8, 1) \in T$?

Proof. $10 \; T \; 1$ because $10 - 1 = 9 = 3 \cdot 3$, and so $3 \mid (10 - 1)$.

$1 \; T \; 10$ because $1 - 10 = -9 = 3 \cdot (-3)$, and so $3 \mid (1 - 10)$.

$2 \; T \; 2$ because $2 - 2 = 0 = 3 \cdot 0$, and so $3 \mid (2 - 2)$.

$8 \not{T} \; 1$ because $8 - 1 = 7 \neq 3k$, for any integer k. So $3 \nmid (8 - 1)$. □

1.3.2 (b)

List five integers n such that $n \; T \; 0$.

Proof. One possible answer: 3, 6, 9, -3, -6 □

1.3.3 (c)

List five integers n such that $n \; T \; 1$.

Proof. One possible answer: 4, 7, 10, -2, -5 □

1.3.4 (d)

List five integers n such that $n \; T \; 2$.

Proof. One possible answer: 5, 8, 11, -1, -4 □

1.3.5 (e)

Make and prove a conjecture about which integers are related by T to 0, which integers are related by T to 1, and which integers are related by T to 2.

All integers of the form $3k+1$, for some integer k, are related by T to 1.

Proof. All integers of the form $3k$, for some integer k, are related by T to 0.

All integers of the form $3k+1$, for some integer k, are related by T to 1.

All integers of the form $3k+2$, for some integer k, are related by T to 2. □

1.4 Exercise 4

Define a relation P on $\mathbb{Z}$ as follows: For every ordered pair $(m,n) \in \mathbb{Z} \times \mathbb{Z}$, $m\ P\ n \iff m$ and n have a common prime factor.

1.4.1 (a)

Is $15\ P\ 25$?

Proof. Yes, because 15 and 25 are both divisible by 5, which is prime. □

1.4.2 (b)

Is $22\ P\ 27$?

Proof. No, because 22 and 27 have no common prime factor. □

1.4.3 (c)

Is $0\ P\ 5$?

Proof. Yes, because 0 and 5 are both divisible by 5, which is prime. □

1.4.4 (d)

Is $8\ P\ 8$?

Proof. Yes, because 8 and 8 are both divisible by 2, which is prime. □

1.5 Exercise 5

Let $X = \{a,b,c\}$. Recall that $\mathscr{P}(X)$ is the power set of X. Define a relation $\mathbf{S}$ on $\mathscr{P}(X)$ as follows: For all sets A and B in $\mathscr{P}(X)$, $A\ \mathbf{S}\ B \iff A$ has the same number of elements as B.

1.5.1 (a)

Is $\{a, b\}$ **S** $\{b, c\}$?

Proof. Yes, because both $\{a, b\}$ and $\{b, c\}$ have two elements. □

1.5.2 (b)

Is $\{a\}$ **S** $\{a, b\}$?

Proof. No, one has 1 element, the other has 2 elements. □

1.5.3 (c)

Is $\{c\}$ **S** $\{b\}$?

Proof. Yes, because both $\{c\}$ and $\{b\}$ have one element. □

1.6 Exercise 6

Let $X = \{a, b, c\}$. Define a relation **J** on $\mathscr{P}(X)$ as follows: For all sets A and B in $\mathscr{P}(X)$, $A \mathbf{J} B \iff A \cap B \neq \varnothing$.

1.6.1 (a)

Is $\{a\}$ **J** $\{c\}$?

Proof. No, because $\{a\} \cap \{c\} = \varnothing$. □

1.6.2 (b)

Is $\{a, b\}$ **J** $\{b, c\}$?

Proof. Yes, because $\{a, b\} \cap \{b, c\} = \{b\} \neq \varnothing$. □

1.6.3 (c)

Is $\{a, b\}$ **J** $\{a, b, c\}$?

Proof. Yes, because $\{a, b\} \cap \{a, b, c\} = \{a, b\} \neq \varnothing$. □

1.7 Exercise 7

Define a relation R on $\mathbb{Z}$ as follows: For all integers m and n, $m\, R\, n \iff 5 \mid (m^2 - n^2)$.

1.7.1 (a)

Is $1 \; R \; (-9)$?

Proof. Yes. $1 \; R \; (-9) \iff 5 \mid (1^2 - (-9)^2)$. But $1^2 - (-9)^2 = 1 - 81 = -80$, and $5 \mid (-80)$ because $-80 = 5 \cdot (-16)$. $\square$

1.7.2 (b)

Is $2 \; R \; 13$?

Proof. Yes, $2^2 - (13)^2 = 4 - 169 = -165 = 5 \cdot (-33)$. So $5 \mid 2^2 - (13)^2$. $\square$

1.7.3 (c)

Is $2 \; R \; (-8)$?

Proof. Yes, $2^2 - (-8)^2 = 4 - 64 = -60 = 5 \cdot (-12)$. So $5 \mid 2^2 - (-8)^2$. $\square$

1.7.4 (d)

Is $(-8) \; R \; 2$?

Proof. Yes, $(-8)^2 - 2^2 = 64 - 4 = 60 = 5 \cdot 12$. So $5 \mid (-8)^2 - 2^2$. $\square$

1.8 Exercise 8

Let A be the set of all strings of a's and b's of length 4. Define a relation R on A as follows: For every $s, t \in A$, $s \; R \; t \iff s$ has the same first two characters as t.

1.8.1 (a)

Is *abaa* R *abba*?

Proof. Yes, because both *abaa* and *abba* have the same first two characters *ab*. $\square$

1.8.2 (b)

Is *aabb* R *bbaa*?

Proof. No, because the first two characters of *aabb* are different from the first two characters of *bbaa*. $\square$

1.8.3 (c)

Is *aaaa* R *aaab*?

Proof. Yes, because both *aaaa* and *aaab* have the same first two characters *aa*. $\square$

1.8.4 (d)

Is $baaa\ R\ abaa$?

Proof. No, because the first two characters of $baaa$ are different from the first two characters of $abaa$. □

1.9 Exercise 9

Let A be the set of all strings of 0's, 1's, and 2's of length 4. Define a relation R on A as follows: For every $s, t \in A$, $s\ R\ t \iff$ the sum of the characters in s equals the sum of the characters in t.

1.9.1 (a)

Is $0121\ R\ 2200$?

Proof. Yes, because the sum of the characters in 0121 is 4 and the sum of the characters in 2200 is also 4. □

1.9.2 (b)

Is $1011\ R\ 2101$?

Proof. No, because the sum of the characters in 1011 is 3, whereas the sum of the characters in 2101 is 4. □

1.9.3 (c)

Is $2212\ R\ 2121$?

Proof. No, because the sum of the characters in 2212 is 7, whereas the sum of the characters in 2121 is 6. □

1.9.4 (d)

Is $1220\ R\ 2111$?

Proof. Yes, because the sum of the characters in 1220 is 5 and the sum of the characters in 2111 is also 5. □

1.10 Exercise 10

Let $A = \{3, 4, 5\}$ and $B = \{4, 5, 6\}$ and let R be the "less than" relation. That is, for every ordered pair $(x, y) \in A \times B, x\,R\,y \iff x < y$. State explicitly which ordered pairs are in R and R^{-1}.

Proof. $R = \{(3,4),(3,5),(3,6),(4,5),(4,6),(5,6)\}$

$R^{-1} = \{(4,3),(5,3),(6,3),(5,4),(6,4),(6,5)\}$ □

1.11 Exercise 11

Let $A = \{3,4,5\}$ and $B = \{4,5,6\}$ and let S be the "divides" relation. That is, for every ordered pair $(x,y) \in A \times B, x\, S\, y \iff x \mid y$. State explicitly which ordered pairs are in S and S^{-1}.

Proof. $S = \{(3,6),(4,4),(5,5)\}, S^{-1} = \{(6,3),(4,4),(5,5)\}$ □

1.12 Exercise 12

1.12.1 (a)

Suppose a function $F : X \to Y$ is one-to-one but not onto. Is F^{-1} (the inverse relation for F) a function? Explain your answer.

Proof. No. If $F : X \to Y$ is not onto, then F fails to be defined on all of Y. In other words, there is an element y in Y such that $(y,x) \notin F^{-1}$ for any $x \in X$. Consequently, F^{-1} does not satisfy property (1) of the definition of function. □

1.12.2 (b)

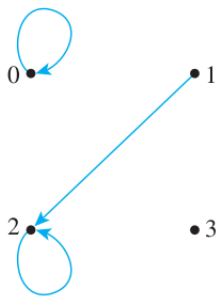
Suppose a function $F : X \to Y$ is onto but not one-to-one. Is F^{-1} (the inverse relation for F) a function? Explain your answer.

Proof. No. If $F : X \to Y$ is not one-to-one, then F for some y in Y, there will be multiple potential values for $F^{-1}(y)$. In other words, there is an element y in Y and elements $x_1, x_2 \in X$ such that $(y,x_1) \in F^{-1}$ and $(y,x_2) \in F^{-1}$. Consequently, F^{-1} does not satisfy property (2) of the definition of function. □

Draw the directed graphs of the relations defined in 13 – 18.

1.13 Exercise 13

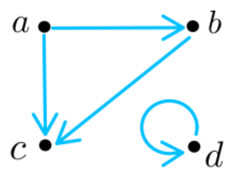
Define a relation R on $A = \{0,1,2,3\}$ by $R = \{(0,0),(1,2),(2,2)\}$.



Proof. □

1.14 Exercise 14

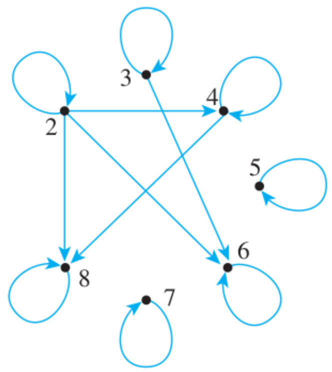
Define a relation S on $B = \{a, b, c, d\}$ by $S = \{(a, b), (a, c), (b, c), (d, d)\}$.



Proof. □

1.15 Exercise 15

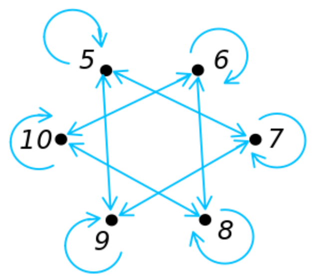
Let $A = \{2, 3, 4, 5, 6, 7, 8\}$ and define a relation R on A as follows: For every $x, y \in A, x\,R\,y \iff x \mid y$.



Proof. □

1.16 Exercise 16

Let $A = \{5, 6, 7, 8, 9, 10\}$ and define a relation S on A as follows: For every $x, y \in A, x\,S\,y \iff 2 \mid (x - y)$.

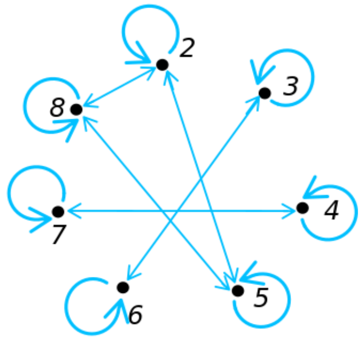


Proof. □

1.17 Exercise 17

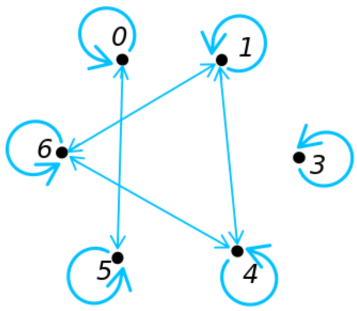
Let $A = \{2, 3, 4, 5, 6, 7, 8\}$ and define a relation T on A as follows: For every $x, y \in A, x\,T\,y \iff 3 \mid (x - y)$.

Proof. □



1.18 Exercise 18

Let $A = \{0, 1, 3, 4, 5, 6\}$ and define a relation V on A as follows: For every $x, y \in A, x\, V\, y \iff 5 \mid (x^2 - y^2)$.



Proof. □

1.19 Exercise 19

Let $A = \{2, 4\}$ and $B = \{6, 8, 10\}$ and define relations R and S from A to B as follows: For every $(x, y) \in A \times B, x\, R\, y \iff x \mid y$ and $x\, S\, y \iff y - 4 = x$. State explicitly which ordered pairs are in $A \times B, R, S, R \cup S$, and $R \cap S$.

Proof. $A \times B = \{(2, 6), (2, 8), (2, 10), (4, 6), (4, 8), (4, 10)\}$

$R = \{(2, 6), (2, 8), (2, 10), (4, 8)\}$, $S = \{(2, 6), (4, 8)\}$, $R \cup S = R, R \cap S = S$ □

1.20 Exercise 20

Let $A = \{-1, 1, 2, 4\}$ and $B = \{1, 2\}$ and define relations R and S from A to B as follows: For every $(x, y) \in A \times B, x\, R\, y \iff |x| \mid |y|$ and $x\, S\, y \iff x - y$ is even. State explicitly which ordered pairs are in $A \times B, R, S, R \cup S$, and $R \cap S$.

Proof. $A \times B = \{(-1, 1), (-1, 2), (1, 1), (1, 2), (2, 1), (2, 2), (4, 1), (4, 2)\}$

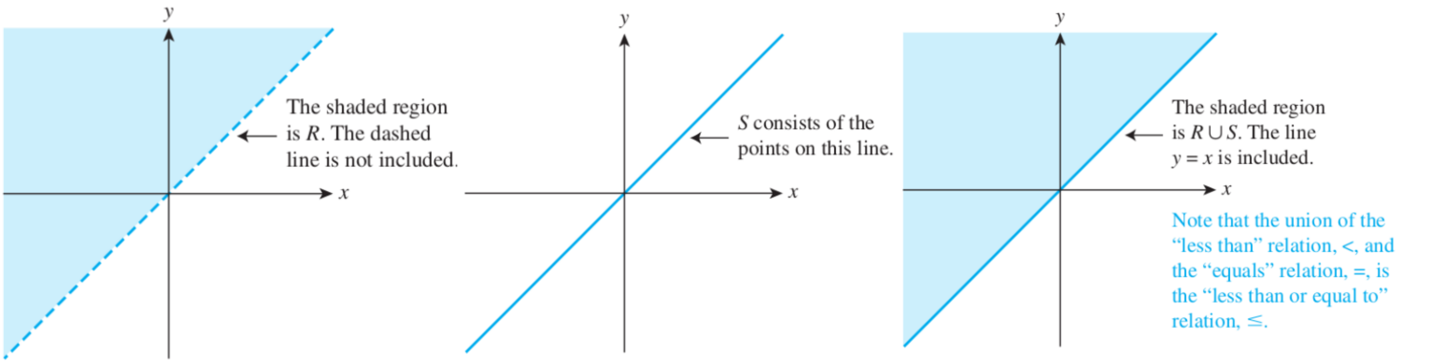
$R = \{(-1, 1), (1, 1), (2, 2)\}$, $S = \{(-1, 1), (1, 1), (2, 2), (4, 2)\}$, $R \cup S = S, R \cap S = R$ □

1.21 Exercise 21

Define relations R and S on R as follows: $R = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid x < y\}$ and

$S = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid x = y\}$. That is, R is the "less than" relation and S is the "equals" relation on $\mathbb{R}$. Graph $R, S, R \cup S$, and $R \cap S$ in the Cartesian plane.

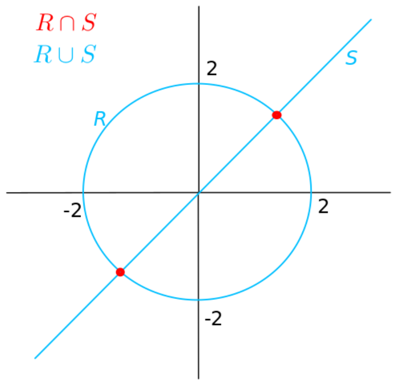
Proof. The graph of the intersection of R and S is obtained by finding the set of all points common to both graphs. But there are no points for which both $x < y$ and $x = y$. Hence $R \cap S = \varnothing$ and the graph consists of no points at all.



$\square$

1.22 Exercise 22

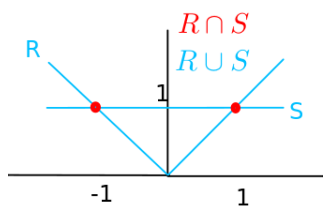
Define relations R and S on R as follows: $R = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid x^2 + y^2 = 4\}$ and $S = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid x = y\}$. Graph $R, S, R \cup S$, and $R \cap S$ in the Cartesian plane.



Proof. $\square$

1.23 Exercise 23

Define relations R and S on R as follows: $R = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid y = |x|\}$ and $S = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid y = 1\}$. Graph $R, S, R \cup S$, and $R \cap S$ in the Cartesian plane.



Proof. $\square$

1.24 Exercise 24

In Example 8.1.7 consider the query SELECT Patient_ID#, Name FROM S WHERE Primary_Diagnosis = X. The response to the query is the projection onto the first two coordinates of the intersection of the database with the set $A_1 \times A_2 \times A_3 \times \{X\}$.

1.24.1 (a)

Find the result of the query SELECT Patient_ID#, Name FROM S WHERE Primary_Diagnosis = pneumonia.

Proof. 574329 Tak Kurosawa, 011985 John Schmidt □

1.24.2 (b)

Find the result of the query SELECT Patient_ID#, Name FROM S WHERE Primary_Diagnosis = appendicitis.

Proof. 466581 Mary Lazars, 778400 Jamal Baskers □

2 Exercise Set 8.2

2.1 Exercise 1

2.1.1 (a)

Proof. □

2.1.2 (b)

Proof. □

2.1.3 (c)

Proof. □

2.1.4 (d)

Proof. □

2.2 Exercise 2

2.2.1 (a)

Proof. □

2.2.2 (b)

Proof. □

2.2.3 (c)

Proof. □

2.2.4 (d)

Proof. □

2.3 Exercise 3

2.3.1 (a)

Proof. □

2.3.2 (b)

Proof. □

2.3.3 (c)

Proof. □

2.3.4 (d)

Proof. □

2.4 Exercise 4

2.4.1 (a)

Proof. □

2.4.2 (b)

Proof. □

2.4.3 (c)

Proof. □

2.4.4 (d)

Proof. □

2.5 Exercise 5

2.5.1 (a)

Proof. □

2.5.2 (b)

Proof. □

2.5.3 (c)

Proof. □

2.5.4 (d)

Proof. □

2.6 Exercise 6

2.6.1 (a)

Proof. □

2.6.2 (b)

Proof. □

2.6.3 (c)

Proof. □

2.6.4 (d)

Proof. □

2.7 Exercise 7

2.7.1 (a)

Proof. □

2.7.2 (b)

Proof. □

2.7.3 (c)

Proof. □

2.7.4 (d)

Proof. □

2.8 Exercise 8

2.8.1 (a)

Proof. □

2.8.2 (b)

Proof. □

2.8.3 (c)

Proof. □

2.8.4 (d)

Proof. □

2.9 Exercise 9

Proof. □

2.10 Exercise 10

Proof. □

2.11 Exercise 11

Proof. □

2.12 Exercise 12

Proof. □

2.13 Exercise 13

Proof. □

2.14 Exercise 14

Proof. □

2.15 Exercise 15

Proof. □

2.16 Exercise 16

Proof. □

2.17 Exercise 17

Proof. □

2.18 Exercise 18

Proof. □

2.19 Exercise 19

Proof. □

2.20 Exercise 20

Proof. □

2.21 Exercise 21

Proof. □

2.22 Exercise 22

Proof. □

2.23 Exercise 23

Proof. □

2.24 Exercise 24

Proof. □

2.25 Exercise 25

Proof. □

2.26 Exercise 26

Proof. □

2.27 Exercise 27

Proof. □

2.28 Exercise 28

Proof. □

2.29 Exercise 29

Proof. □

2.30 Exercise 30

Proof. □

2.31 Exercise 31

Proof. □

2.32 Exercise 32

Proof. □

2.33 Exercise 33

Proof. □

2.34 Exercise 34

Proof. □

2.35 Exercise 35

Proof. □

2.36 Exercise 36

Proof. □

2.37 Exercise 37

Proof. □

2.38 Exercise 38

Proof. □

2.39 Exercise 39

Proof. □

2.40 Exercise 40

Proof. □

2.41 Exercise 41

Proof. □

2.42 Exercise 42

Proof. □

2.43 Exercise 43

Proof. □

2.44 Exercise 44

Proof. □

2.45 Exercise 45

Proof. □

2.46 Exercise 46

Proof. □

2.47 Exercise 47

Proof. □

2.48 Exercise 48

Proof. □

2.49 Exercise 49

Proof. □

2.50 Exercise 50

Proof. □

2.51 Exercise 51

Proof. □

2.52 Exercise 52

Proof. □

2.53 Exercise 53

Proof. □

2.54 Exercise 54

Proof. □

2.55 Exercise 55

Proof. □

2.56 Exercise 56

Proof. □

3 Exercise Set 8.3

3.1 Exercise 1

3.1.1 (a)

Proof. □

3.1.2 (b)

Proof. □

3.1.3 (c)

Proof. □

3.1.4 (d)

Proof. □

3.2 Exercise 2

3.2.1 (a)

Proof. □

3.2.2 (b)

Proof. ☐

3.2.3 (c)

Proof. ☐

3.3 Exercise 3

Proof. ☐

3.4 Exercise 4

Proof. ☐

3.5 Exercise 5

Proof. ☐

3.6 Exercise 6

Proof. ☐

3.7 Exercise 7

Proof. ☐

3.8 Exercise 8

Proof. ☐

3.9 Exercise 9

Proof. ☐

3.10 Exercise 10

Proof. ☐

3.11 Exercise 11

Proof. ☐

3.12 Exercise 12

Proof. ☐

3.13 Exercise 13

Proof. □

3.14 Exercise 14

Proof. □

3.15 Exercise 15

3.15.1 (a)

Proof. □

3.15.2 (b)

Proof. □

3.15.3 (c)

Proof. □

3.15.4 (d)

Proof. □

3.16 Exercise 16

3.16.1 (a)

Proof. □

3.16.2 (b)

Proof. □

3.17 Exercise 17

3.17.1 (a)

Proof. □

3.17.2 (b)

Proof. □

3.18 Exercise 18

3.18.1 (a)

Proof. □

3.18.2 (b)

Proof. □

3.19 Exercise 19

3.19.1 (a)

Proof. □

3.19.2 (b)

Proof. □

3.20 Exercise 20

Proof. □

3.21 Exercise 21

Proof. □

3.22 Exercise 22

Proof. □

3.23 Exercise 23

Proof. □

3.24 Exercise 24

Proof. □

3.25 Exercise 25

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3.26 Exercise 26

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3.27 Exercise 27

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3.28 Exercise 28

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3.30 Exercise 30

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3.37 Exercise 37

Proof. □

3.38 Exercise 38

Proof. □

3.39 Exercise 39

Proof. □

3.40 Exercise 40

Proof. □

3.41 Exercise 41

Proof. □

3.42 Exercise 42

3.42.1 (a)

Proof. □

3.42.2 (b)

Proof. □

3.42.3 (c)

Proof. □

3.42.4 (d)

Proof. □

3.43 Exercise 43

3.43.1 (a)

Proof. □

3.43.2 (b)

Proof. □

3.43.3 (c)

Proof. □

3.43.4 (d)

Proof. □

3.43.5 (e)

Proof. □

3.43.6 (f)

Proof. □

3.44 Exercise 44

3.44.1 (a)

Proof. □

3.44.2 (b)

Proof. □

3.44.3 (c)

Proof. □

3.44.4 (d)

Proof. □

3.44.5 (e)

Proof. □

3.44.6 (f)

Proof. □

3.44.7 (g)

Proof. □

3.45 Exercise 45

Proof. □

3.46 Exercise 46

Proof. □

3.47 Exercise 47

3.47.1 (a)

Proof. □

3.47.2 (b)

Proof. □

3.47.3 (c)

Proof. □

3.47.4 (d)

Proof. □

3.47.5 (e)

Proof. □

3.47.6 (f)

Proof. □

3.47.7 (g)

Proof. □

4 Exercise Set 8.4

4.1 Exercise 1

4.1.1 (a)

Proof. □

4.1.2 (b)

Proof. □

4.2 Exercise 2

4.2.1 (a)

Proof. □

4.2.2 (b)

Proof. ☐

4.3 Exercise 3

4.3.1 (a)

Proof. ☐

4.3.2 (b)

Proof. ☐

4.3.3 (c)

Proof. ☐

4.3.4 (d)

Proof. ☐

4.3.5 (e)

Proof. ☐

4.4 Exercise 4

4.4.1 (a)

Proof. ☐

4.4.2 (b)

Proof. ☐

4.4.3 (c)

Proof. ☐

4.4.4 (d)

Proof. ☐

4.4.5 (e)

Proof. ☐

4.5 Exercise 5

Proof. □

4.6 Exercise 6

Proof. □

4.7 Exercise 7

4.7.1 (a)

Proof. □

4.7.2 (b)

Proof. □

4.7.3 (c)

Proof. □

4.7.4 (d)

Proof. □

4.7.5 (e)

Proof. □

4.8 Exercise 8

4.8.1 (a)

Proof. □

4.8.2 (b)

Proof. □

4.8.3 (c)

Proof. □

4.8.4 (d)

Proof. □

4.8.5 (e)

Proof. □

4.9 Exercise 9

4.9.1 (a)

Proof. □

4.9.2 (b)

Proof. □

4.10 Exercise 10

Proof. □

4.11 Exercise 11

Proof. □

4.12 Exercise 12

4.12.1 (a)

Proof. □

4.12.2 (b)

Proof. □

4.13 Exercise 13

4.13.1 (a)

Proof. □

4.13.2 (b)

Proof. □

4.14 Exercise 14

Proof. □

4.15 Exercise 15

Proof. □

4.16 Exercise 16

Proof. □

4.17 Exercise 17

Proof. □

4.18 Exercise 18

Proof. □

4.19 Exercise 19

Proof. □

4.20 Exercise 20

Proof. □

4.21 Exercise 21

Proof. □

4.22 Exercise 22

Proof. □

4.23 Exercise 23

Proof. □

4.24 Exercise 24

Proof. □

4.25 Exercise 25

Proof. □

4.26 Exercise 26

Proof. □

4.27 Exercise 27

Proof. □

4.28 Exercise 28

Proof. □

4.29 Exercise 29

Proof. □

4.30 Exercise 30

Proof. □

4.31 Exercise 31

4.31.1 (a)

Proof. □

4.31.2 (b)

Proof. □

4.31.3 (c)

Proof. □

4.32 Exercise 32

4.32.1 (a)

Proof. □

4.32.2 (b)

Proof. □

4.33 Exercise 33

Proof. □

4.34 Exercise 34

Proof. □

4.35 Exercise 35

Proof. □

4.36 Exercise 36

Proof. □

4.37 Exercise 37

Proof. □

4.38 Exercise 38

Proof. □

4.39 Exercise 39

Proof. □

4.40 Exercise 40

Proof. □

4.41 Exercise 41

4.41.1 (a)

Proof. □

4.41.2 (b)

Proof. □

4.42 Exercise 42

Proof. □

4.43 Exercise 43

Proof. □

5 Exercise Set 8.5

5.1 Exercise 1

5.1.1 (a)

Proof. □

5.1.2 (b)

Proof. □

5.1.3 (c)

Proof. □

5.1.4 (d)

Proof. □

5.2 Exercise 2

Proof. □

5.3 Exercise 3

Proof. □

5.4 Exercise 4

Proof. □

5.5 Exercise 5

Proof. □

5.6 Exercise 6

Proof. □

5.7 Exercise 7

Proof. □

5.8 Exercise 8

Proof. □

5.9 Exercise 9

Proof. □

5.10 Exercise 10

Proof. □

5.11 Exercise 11

5.11.1 (a)

Proof. □

5.11.2 (b)

Proof. □

5.11.3 (c)

Proof. □

5.11.4 (d)

Proof. □

5.11.5 (e)

Proof. □

5.11.6 (f)

Proof. □

5.11.7 (g)

Proof. □

5.12 Exercise 12

Proof. □

5.13 Exercise 13

Proof. □

5.14 Exercise 14

5.14.1 (a)

Proof. □

5.14.2 (b)

Proof. □

5.15 Exercise 15

Proof. □

5.16 Exercise 16

5.16.1 (a)

Proof. □

5.16.2 (b)

Proof. □

5.17 Exercise 17

Proof. □

5.18 Exercise 18

Proof. □

5.19 Exercise 19

Proof. □

5.20 Exercise 20

Proof. □

5.21 Exercise 21

5.21.1 (a)

Proof. □

5.21.2 (b)

Proof. □

5.22 Exercise 22

Proof. □

5.23 Exercise 23

Proof. □

5.24 Exercise 24

Proof. □

5.25 Exercise 25

Proof. □

5.26 Exercise 26

Proof. □

5.27 Exercise 27

Proof. □

5.28 Exercise 28

Proof. □

5.29 Exercise 29

Proof. □

5.30 Exercise 30

5.30.1 (a)

Proof. □

5.30.2 (b)

Proof. □

5.30.3 (c)

Proof. □

5.30.4 (d)

Proof. □

5.31 Exercise 31

Proof. □

5.32 Exercise 32

Proof. □

5.33 Exercise 33

Proof. □

5.34 Exercise 34

Proof. □

5.35 Exercise 35

Proof. □

5.36 Exercise 36

Proof. □

5.37 Exercise 37

Proof. □

5.38 Exercise 38

Proof. □

5.39 Exercise 39

Proof. □

5.40 Exercise 40

5.40.1 (a)

Proof. □

5.40.2 (b)

Proof. □

5.41 Exercise 41

5.41.1 (a)

Proof. □

5.41.2 (b)

Proof. ☐

5.42 Exercise 42

Proof. ☐

5.43 Exercise 43

Proof. ☐

5.44 Exercise 44

Proof. ☐

5.45 Exercise 45

Proof. ☐

5.46 Exercise 46

Proof. ☐

5.47 Exercise 47

Proof. ☐

5.48 Exercise 48

Proof. ☐

5.49 Exercise 49

5.49.1 (a)

Proof. ☐

5.49.2 (b)

Proof. ☐

5.50 Exercise 50

5.50.1 (a)

Proof. ☐

5.50.2 (b)

Proof. □

5.51 Exercise 51

5.51.1 (a)

Proof. □

5.51.2 (b)

Proof. □