Chapter 11 Solutions, Susanna Epp Discrete Math 5th Edition

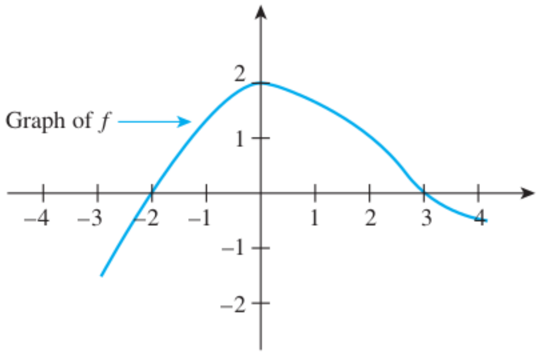
https://github.com/spamegg1

November 25, 2023

Contents

1 Exercise Set 11.1

1.1 Exercise 1



The graph of a function f is shown above.

1.1.1 (a)

Is $f(0)$ positive or negative?

Proof. positive $\square$

1.1.2 (b)

For what values of x does $f(x) = 0$?

Proof. $f(x) = 0$ when $x = -2$ and $x = 3$ (approximately) $\square$

1.1.3 (c)

Find approximate values for x_1 and x_2 so that $f(x_1) = f(x_2) = 1$ but $x_1 \neq x_2$.

Proof. $x_1 = -1$ and $x_2 = 2$ (approximately) $\square$

1.1.4 (d)

Find an approximate value for x such that $f(x) = 1.5$.

Proof. $x = 1$ or $x = -1/2$ (approximately) $\square$

1.1.5 (e)

As x increases from -3 to -1, do the values of f increase or decrease?

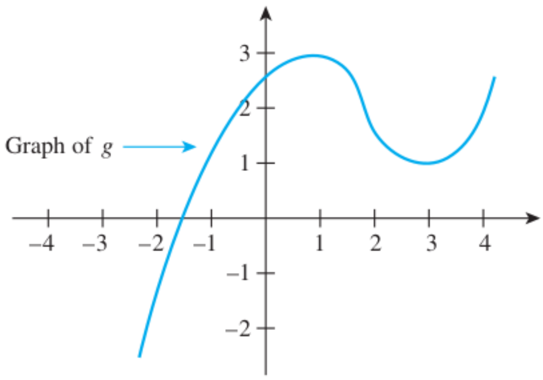
Proof. increase $\square$

1.1.6 (f)

As x increases from 0 to 4, do the values of f increase or decrease?

Proof. decrease ☐

1.2 Exercise 2



The graph of a function g is shown above.

1.2.1 (a)

Is $g(0)$ positive or negative?

Proof. positive ☐

1.2.2 (b)

Find an approximate value of x so that $g(x) = 0$.

Proof. -1.5 (approximately) ☐

1.2.3 (c)

Find approximate values for x_1 and x_2 so that $g(x_1) = g(x_2) = 1$ but $x_1 \neq x_2$.

Proof. $x_1 = -1, x_2 = 3$ (approximately) ☐

1.2.4 (d)

Find an approximate value for x such that $g(x) = -2$.

Proof. $x = -2.2$ (approximately) ☐

1.2.5 (e)

As x increases from -2 to 1, do the values of g increase or decrease?

Proof. increase ☐

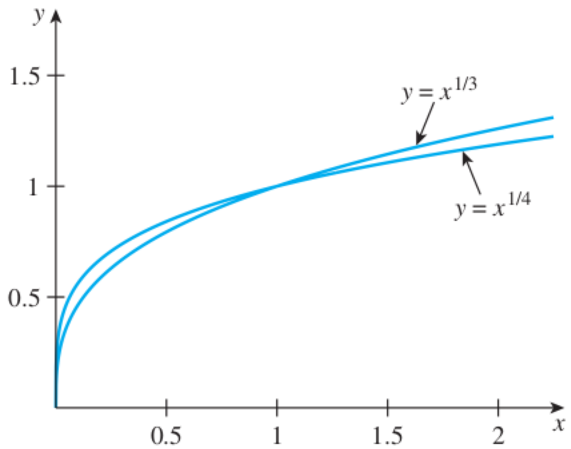
1.2.6 (f)

As x increases from 1 to 3, do the values of g increase or decrease?

Proof. decrease $\square$

1.3 Exercise 3

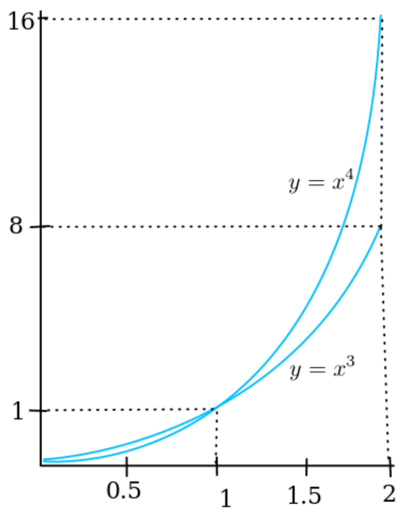
Sketch the graphs of the power functions $p_{1/3}$ and $p_{1/4}$ on the same set of axes. When $0 < x < 1$, which is greater: $x^{1/3}$ or $x^{1/4}$? When $x > 1$, which is greater: $x^{1/3}$ or $x^{1/4}$?



Proof. When $0 < x < 1$, $x^{1/3} < x^{1/4}$. When $1 < x$, $x^{1/4} < x^{1/3}$. $\square$

1.4 Exercise 4

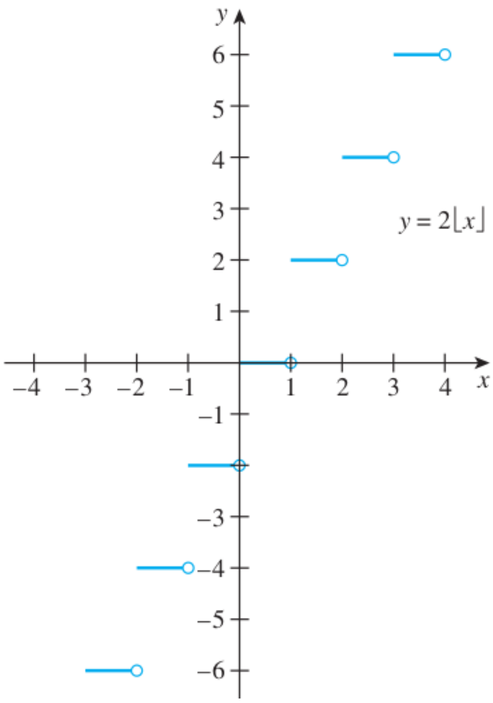
Sketch the graphs of the power functions p_3 and p_4 on the same set of axes. When $0 < x < 1$, which is greater: x^3 or x^4? When $x > 1$, which is greater: x^3 or x^4?

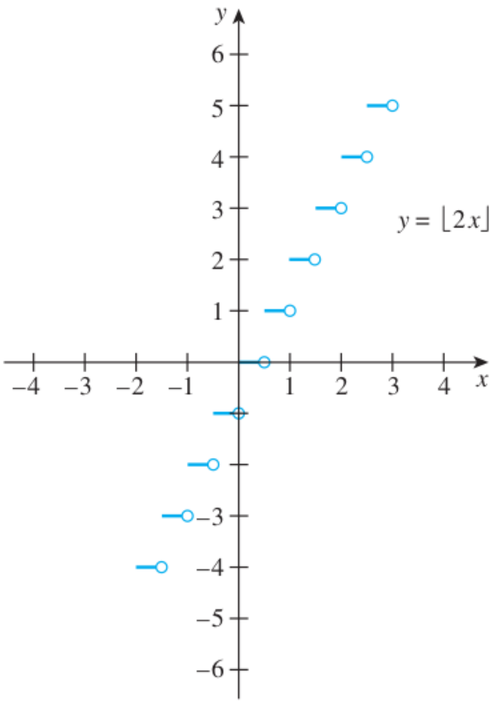


Proof. When $0 < x < 1$, $x^4 < x^3$. When $1 < x$, $x^3 < x^4$. $\square$

1.5 Exercise 5

Sketch the graphs of $y = 2\lfloor x \rfloor$; and $y = \lfloor 2x \rfloor$ for each real number x. What can you conclude from these graphs?





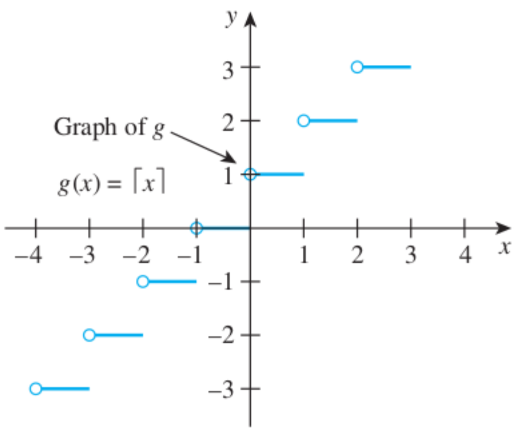
Proof.

The graphs show that $2\lfloor x \rfloor \neq \lfloor 2x \rfloor$ for many values of x. $\square$

Sketch a graph for each of the functions defined in $6-9$ below.

1.6 Exercise 6

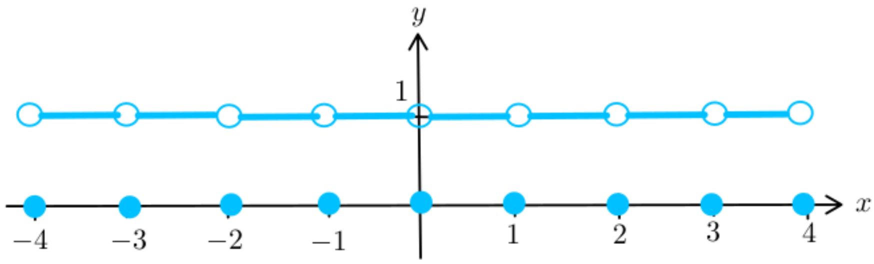
$g(x) = \lceil x \rceil$ for each real number x (Recall that the ceiling of x, $\lceil x \rceil$, is the least integer that is greater than or equal to x. That is, $\lceil x \rceil$ = the unique integer n such that $n - 1 < x \leq n$.



Proof. $\square$

1.7 Exercise 7

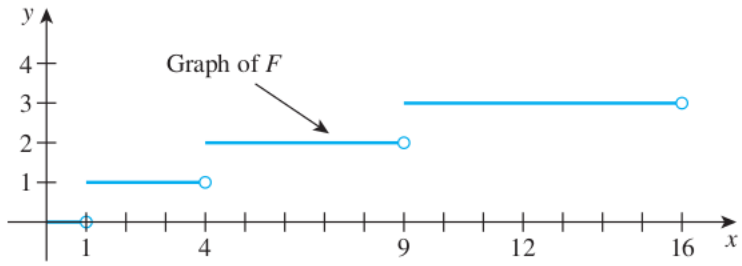
$h(x) = \lceil x \rceil - \lfloor x \rfloor$ for each real number x



Proof. □

1.8 Exercise 8

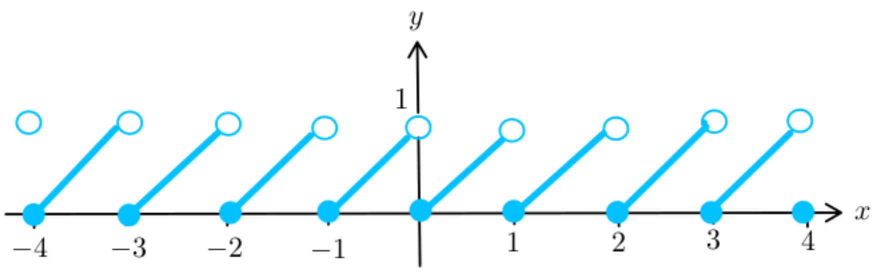
$F(x) = \lfloor x^{1/2} \rfloor$ for each real number x



Proof. □

1.9 Exercise 9

$G(x) = x - \lfloor x \rfloor$ for each real number x



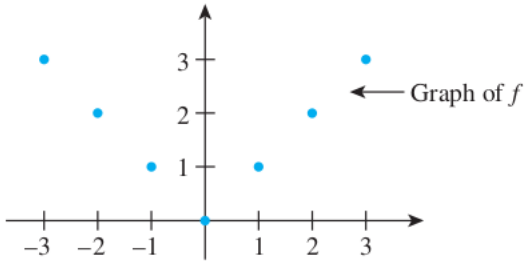
Proof. □

In each of $10 - 13$ a function is defined on a set of integers. Sketch a graph for each function.

1.10 Exercise 10

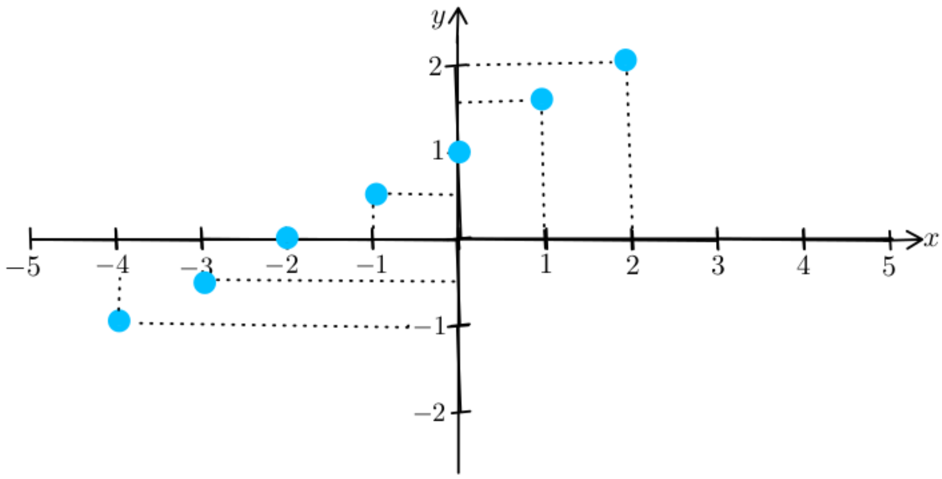
$f(n) = |n|$ for each integer n

Proof. □



1.11 Exercise 11

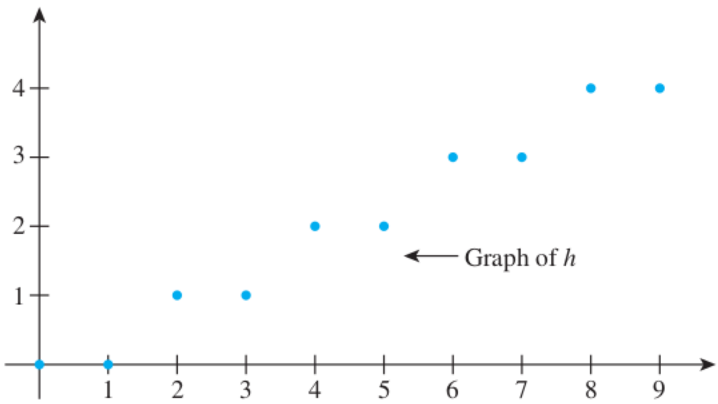
$g(n) = (n/2) + 1$ for each integer n



Proof. □

1.12 Exercise 12

$h(n) = \lfloor n/2 \rfloor$ for each integer $n \geq 0$

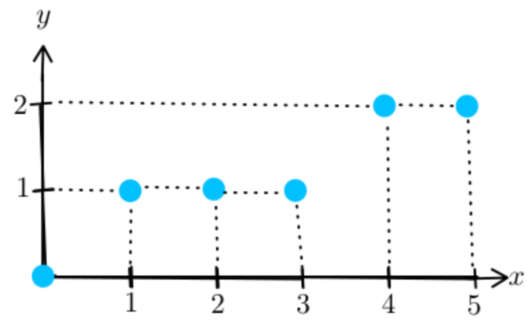


Proof. □

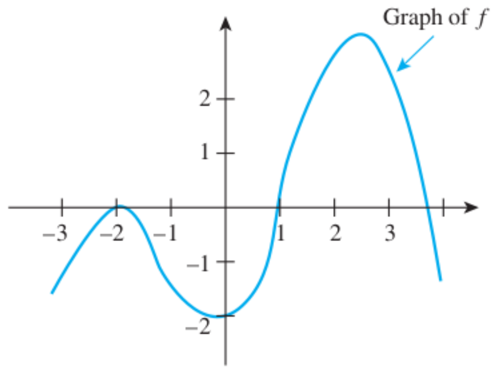
1.13 Exercise 13

$k(n) = \lfloor n^{1/2} \rfloor$ for each integer $n \geq 0$

Proof. □



1.14 Exercise 14



The graph of a function f is shown below. Find the intervals on which f is increasing and the intervals on which f is decreasing.

Proof. f is increasing on the intervals $\{x \in \mathbb{R} \,|\, -3 < x < -2\}$ and $\{x \in \mathbb{R} \,|\, 0 < x < 2.5\}$, and f is decreasing on $\{x \in \mathbb{R} \,|\, -2 < x < 0\}$ and $\{x \in \mathbb{R} \,|\, 2.5 < x < 4\}$ (approximately). $\square$

1.15 Exercise 15

Show that the function $f : \mathbb{R} \to \mathbb{R}$ defined by the formula $f(x) = 2x - 3$ is increasing on the set of real numbers.

Proof. Suppose that x_1 and x_2 are particular but arbitrarily chosen real numbers such that $x_1 < x_2$. *[We must show that $f(x_1) < f(x_2)$.]* Since $x_1 < x_2$ then $2x_1 < 2x_2$ and $2x_1 - 3 < 2x_2 - 3$ by basic properties of inequalities. Thus, by definition of f, $f(x_1) < f(x_2)$ *[as was to be shown]*. Hence f is increasing on the set of all real numbers. $\square$

1.16 Exercise 16

Show that the function $g : \mathbb{R} \to \mathbb{R}$ defined by the formula $g(x) = -(x/3)+1$ is decreasing on the set of real numbers.

Proof. $\square$

1.17 Exercise 17

Let h be the function from $\mathbb{R}$ to $\mathbb{R}$ defined by the formula $h(x) = x^2$ for each real number x.

1.17.1 (a)

Show that h is decreasing on the set of real numbers less than zero.

Proof. Suppose that x_1 and x_2 are particular but arbitrarily chosen real numbers such that $x_1 < x_2 < 0$. *[We must show that $h(x_1) > h(x_2)$.]*

Since $x_1 < x_2 < 0$ then $0 < -x_2 < -x_1$ and multiplying by $-x_1$ (which is a positive number) we get $(-x_1)(-x_2) < (-x_1)(-x_1) = x_1^2$ by basic properties of inequalities.

Similarly, since $x_1 < x_2 < 0$ then $0 < -x_2 < -x_1$ and multiplying by $-x_2$ (which is a positive number) we get $(-x_2)(-x_2) = x_2^2 < (-x_1)(-x_2)$ by basic properties of inequalities.

By combining the two results we get $x_2^2 < (-x_1)(-x_2) < x_1^2$ so $x_2^2 < x_1^2$.

Thus, by definition of h, $h(x_1) > h(x_2)$ *[as was to be shown]*. Hence h is increasing on the set of all real numbers. □

1.17.2 (b)

Show that h is increasing on the set of real numbers greater than zero.

Proof. Suppose that x_1 and x_2 are particular but arbitrarily chosen real numbers such that $0 < x_1 < x_2$. *[We must show that $h(x_1) < h(x_2)$.]*

Since $0 < x_1 < x_2$ then multiplying by x_1 (which is a positive number) we get $x_1x_1 = x_1^2 < x_1x_2$ by basic properties of inequalities.

Similarly, since $0 < x_1 < x_2$ then multiplying by x_2 (which is a positive number) we get $x_1x_2 < x_2x_2 = x_2^2$ by basic properties of inequalities.

By combining the two results we get $x_1^2 < x_1x_2 < x_2^2$ so $x_1^2 < x_2^2$.

Thus, by definition of h, $h(x_1) < h(x_2)$ *[as was to be shown]*. Hence h is increasing on the set of all real numbers. □

1.18 Exercise 18

Let $k : \mathbb{R} \to \mathbb{R}$ be the function defined by the formula $k(x) = (x - 1)/x$ for each real number $x \neq 0$.

1.18.1 (a)

Show that k is increasing for every real number $x > 0$.

Proof. Suppose that x_1 and x_2 are positive real numbers and $x_1 < x_2$. *[We must show that $k(x_1) < k(x_2)$.]*

$$\begin{array}{lll} & x_1 < x_2 & \text{by assumption} \\ \implies & -x_2 < -x_1 & \text{by multiplying by } -1 \\ \implies & x_1x_2 - x_2 < x_1x_2 - x_1 & \text{by adding } x_1x_2 \text{ to both sides} \\ \implies & x_2(x_1-1) < x_1(x_2-1) & \text{by factoring both sides} \\ \implies & \dfrac{x_1-1}{x_1} < \dfrac{x_2-1}{x_2} & \text{by dividing both sides by } x_1x_2 > 0 \\ \implies & k(x_1) < k(x_2) & \text{by definition of } k \end{array}$$

□

1.18.2 (b)

Is k increasing or decreasing for $x < 0$? Prove your answer.

Proof. It is increasing. The same proof as in part (a) works. Note that the only place in the proof where the signs of x_1 and x_2 matter is when we divide both sides by x_1x_2. For the proof to work, x_1x_2 has to be positive. But if both x_1 and x_2 are negative, then x_1x_2 *is* positive. Therefore the proof still works. □

1.19 Exercise 19

Show that if a function $f : \mathbb{R} \to \mathbb{R}$ is increasing, then f is one-to-one.

Proof. Suppose $f : \mathbb{R} \to \mathbb{R}$ is increasing. *[We must show that f is one-to-one. In other words, we must show that for all real numbers x_1 and x_2 , if $x_1 \neq x_2$ then $f(x_1) = f(x_2)$.]* Suppose x_1 and x_2 are real numbers and $x_1 \neq x_2$. By the trichotomy law *[Appendix A, T17]* $x_1 < x2$, or $x_1 > x_2$. In case $x_1 < x_2$, then since f is increasing, $f(x_1) < f(x_2)$ and so $f(x_1) \neq f(x_2)$. Similarly, in case $x_1 > x_2$, then $f(x_1) > f(x_2)$ and so $f(x_1) \neq f(x_2)$. Thus in either case, $f(x_1) \neq f(x_2)$ *[as was to be shown]*. □

1.20 Exercise 20

Given real-valued functions f and g with the same domain D, the sum of f and g, denoted $f + g$, is defined as follows: For each real number x, $(f + g)(x) = f(x) + g(x)$. Show that if f and g are both increasing on a set S, then $f + g$ is also increasing on S.

Proof. Assume $x_1, x_2 \in S$ and $x_1 < x_2$. *[We want to show $(f + g)(x_1) < (f + g)(x_2)$.]* Since f is increasing, $f(x_1) < f(x_2)$. Since g is increasing, $g(x_1) < g(x_2)$. By definition of $f + g$ we have $(f + g)(x_1) = f(x_1) + g(x_1) < f(x_2) + g(x_2) = (f + g)(x_2)$, *[as was to be shown.]* □

1.21 Exercise 21

1.21.1 (a)

Let m be any positive integer, and define $f(x) = x^m$ for each nonnegative real number x. Use the binomial theorem to show that f is an increasing function.

Proof. Suppose u and v are nonnegative real numbers with $u < v$. *[We must show that $f(u) < f(v)$.]* Note that $v = u + h$ for some positive real number h. By substitution and the binomial theorem,

$$v^m = (u+h)^m = \sum_{i=0}^{m} \binom{m}{i} u^{m-i} h^i = u^m + \sum_{i=1}^{m} \binom{m}{i} u^{m-i} h^i$$

The last summation is positive because $u \geq 0$ and $h > 0$, and a sum of nonnegative terms that includes at least one positive term is positive. Hence $v^m = u^m +$ a positive number, and so $f(u) = u^m < v^m = f(v)$, *[as was to be shown]*. □

1.21.2 (b)

Let m and n be any positive integers, and let $g(x) = x^{m/n}$ for each nonnegative real number x. Prove that g is an increasing function.

Note: The results of exercise 21 are used in the exercises for Sections 11.2 and 11.4.

Proof. Write $f(x) = x^m$. Then $g(x) = (f(x))^{1/n}$ by the law of exponents.

Now assume $0 \leq x_1 < x_2$. In part (a) we showed that f is increasing. Therefore $f(x_1) < f(x_2)$, in other words $x_1^m < x_2^m$. So we need to show that the function $h(x) = x^{1/n}$ is an increasing function. That will imply $g(x_1) = h(x_1^m) < h(x_2^m) = g(x_2)$, in other words $x_1^{m/n} < x_2^{m/n}$, which is what we want.

To show h is increasing, assume $0 \leq z_1 < z_2$. By definition, $h(z_1) = z_1^{1/n} = y_1$ is the real number with the property that $y_1^n = z_1$. Similarly $h(z_2) = z_2^{1/n} = y_2$ is the real number with the property that $y_2^n = z_2$. *[We want to show $y_1 < y_2$.]*

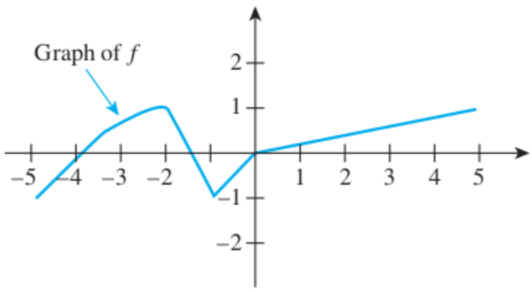
Argue by contradiction and assume $y_2 \leq y_1$. Now consider the function $e(y) = y^n$. This function is also increasing by part (a), since m and n are both any positive integers. Therefore $e(y_2) \leq e(y_1)$, in other words $z_2 \leq z_1$, which is a contradiction!

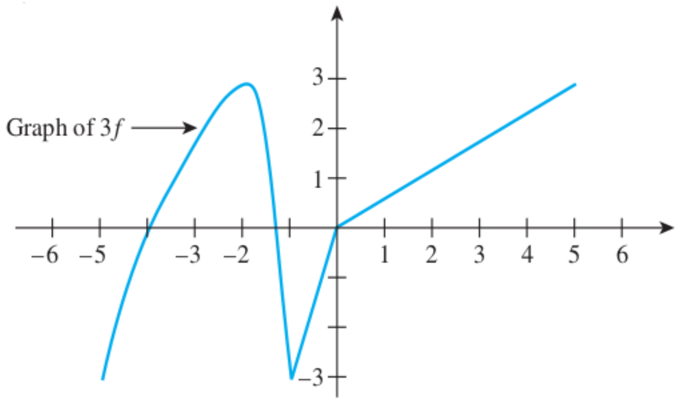
Therefore $y_1 < y_2$ and h is increasing, and thus g is increasing as a consequence. □

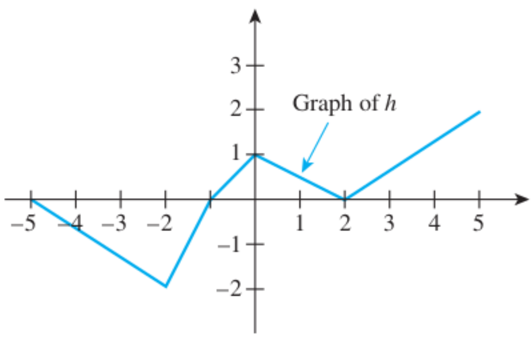
1.22 Exercise 22

Let f be the function whose graph follows. Sketch the graph of $3f$.

Proof. □

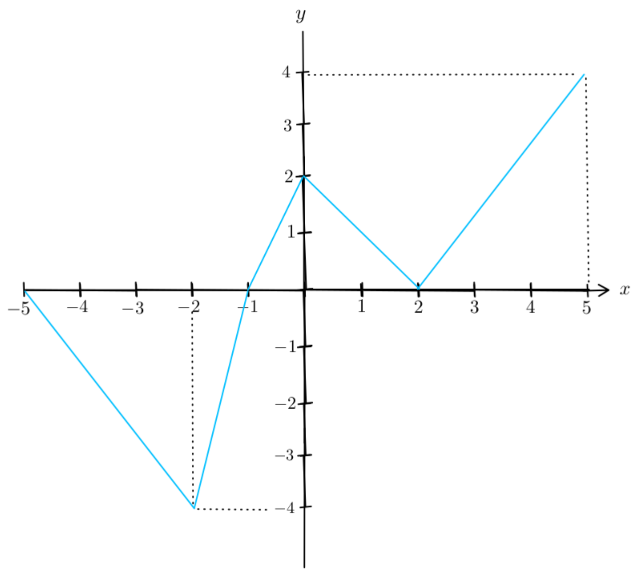






1.23 Exercise 23

Let h be the function whose graph is shown above. Sketch the graph of $2h$.



Proof. □

1.24 Exercise 24

Let f be a real-valued function of a real variable. Show that if f is decreasing on a set S and if M is any positive real number, then Mf is decreasing on S.

Proof. Suppose that f is a real-valued function of a real variable, f is decreasing on a set S, and M is any positive real number. *[We must show that Mf is decreasing on S. In other words, we must show that for all x_1 and x_2 in S, if $x_1 < x_2$ then $(Mf)(x_1) > (Mf)(x_2)$.]* Suppose x_1 and x_2 are in S and $x_1 < x_2$. Since f is decreasing on S, $f(x_1) > f(x_2)$, and since M is positive, $Mf(x_1) > Mf(x_2)$ *[because when both sides of an inequality are multiplied by a positive number, the direction of the inequality is unchanged]*. It follows by definition of Mf that $(Mf)(x_1) > (Mf)(x_2)$, *[as was to be shown]*. □

1.25 Exercise 25

Let f be a real-valued function of a real variable. Show that if f is increasing on a set S and if M is any negative real number, then Mf is decreasing on S.

Proof. The proof is the same as in Exercise 24, except that this time we have $f(x_1) < f(x_2)$ because f is increasing, and multiplying an inequality by a negative number M reverses the direction of the equality, so $Mf(x_1) > Mf(x_2)$. □

1.26 Exercise 26

Let f be a real-valued function of a real variable. Show that if f is decreasing on a set S and if M is any negative real number, then Mf is increasing on S.

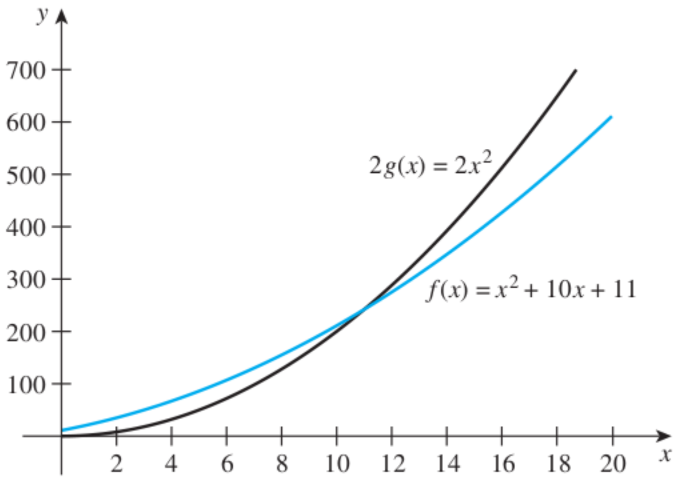
Proof. The proof is the same as in Exercise 24, except that this time multiplying an inequality by a negative number M reverses the direction of the equality, so $Mf(x_1) < Mf(x_2)$. □

In 27 and 28, functions f and g are defined. In each case sketch the graphs of f and $2g$ on the same set of axes and find a number x_0 so that $f(x) \leq 2g(x)$ for all $x > x_0$. You can find an exact value for x_0 by solving a quadratic equation, or you can find an approximate value for x_0 by using a graphing calculator or computer.

1.27 Exercise 27

$f(x) = x^2 + 10x + 11$ and $g(x) = x^2$ for each real number $x \geq 0$

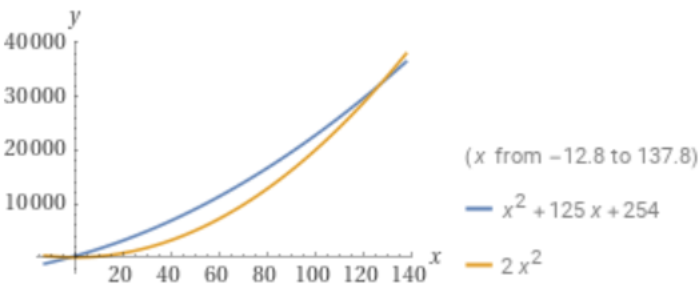
Proof. To find the answer algebraically, solve the equation $2x^2 = x^2 + 10x + 11$ for x. Subtracting x^2 from both sides gives $x^2 - 10x - 11 = 0$, and either using the quadratic formula or factoring $x^2 - 10x - 11 = (x - 11)(x + 1)$ gives $x = 11$ (since $x > 0$). To find an approximate answer with a graphing calculator, plot both $f(x) = x^2 + 10x + 11$ and $2g(x) = 2x^2$ for $x > 0$, as shown in the figure, and find that $2g(x) > f(x)$ when



$x > 11$ (approximately). You can obtain only an approximate answer from a graphing calculator because the calculator computes values only to an accuracy of a finite number of decimal places. □

1.28 Exercise 28

$f(x) = x^2 + 125x + 254$ and $g(x) = x^2$ for each real number $x \geq 0$



Proof. If we set $f(x) = 2g(x)$ and solve, we get $x^2 + 125x + 254 = 2x^2$ which gives $x^2 - 125x - 254 = 0$ which factors as $(x - 127)(x + 2) = 0$ which has solutions $x = -2, 127$. So let $x_0 = 127$, so that $f(x) < g(x)$ for all $x > x_0 = 127$. □

2 Exercise Set 11.2

2.1 Exercise 1

2.1.1 ()

Proof. □

2.2 Exercise 2

2.2.1 ()

Proof. □

2.3 Exercise 3

2.3.1 ()

Proof. □

2.4 Exercise 4

2.4.1 ()

Proof. □

2.5 Exercise 5

2.5.1 ()

Proof. □

2.6 Exercise 6

2.6.1 ()

Proof. □

2.7 Exercise 7

2.7.1 ()

Proof. □

2.8 Exercise 8

2.8.1 ()

Proof. □

2.9 Exercise 9

2.9.1 ()

Proof. □

2.10 Exercise 10

2.10.1 ()

Proof. □

2.11 Exercise 11

2.11.1 ()

Proof. □

2.12 Exercise 12

2.12.1 ()

Proof. □

2.13 Exercise 13

2.13.1 ()

Proof. □

2.14 Exercise 14

2.14.1 ()

Proof. □

2.15 Exercise 15

2.15.1 ()

Proof. □

2.16 Exercise 16

2.16.1 ()

Proof. □

2.17 Exercise 17

2.17.1 ()

Proof. □

2.18 Exercise 18

2.18.1 ()

Proof. □

2.19 Exercise 19

2.19.1 ()

Proof. ◻

2.20 Exercise 20

2.20.1 ()

Proof. ◻

2.21 Exercise 21

2.21.1 ()

Proof. ◻

2.22 Exercise 22

2.22.1 ()

Proof. ◻

2.23 Exercise 23

2.23.1 ()

Proof. ◻

2.24 Exercise 24

2.24.1 ()

Proof. ◻

2.25 Exercise 25

2.25.1 ()

Proof. ◻

2.26 Exercise 26

2.26.1 ()

Proof. ◻

2.27 Exercise 27

2.27.1 ()

Proof. □

2.28 Exercise 28

2.28.1 ()

Proof. □

2.29 Exercise 29

2.29.1 ()

Proof. □

2.30 Exercise 30

2.30.1 ()

Proof. □

2.31 Exercise 31

2.31.1 ()

Proof. □

2.32 Exercise 32

2.32.1 ()

Proof. □

2.33 Exercise 33

2.33.1 ()

Proof. □

2.34 Exercise 34

2.34.1 ()

Proof. □

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3 Exercise Set 11.3

3.1 Exercise 1

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4 Exercise Set 11.4

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5 Exercise Set 11.5

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