Solutions for Homework14

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Abstract

In this document we will show the solutions for problems represented in the given homework for this week.

Contents

1 Task 1

1.1 Problem

In the given graph which represents the movement of a car, answer the following:

- Between which point and point the car accelerate, and between which points it decelerate?
- What is the distance traveled at point C? and what is it at point F?
- Indicate line segments with 0 acceleration, does that mean the car is not moving? Explain.
- What are the values of acceleration and deceleration of the car at each segment?

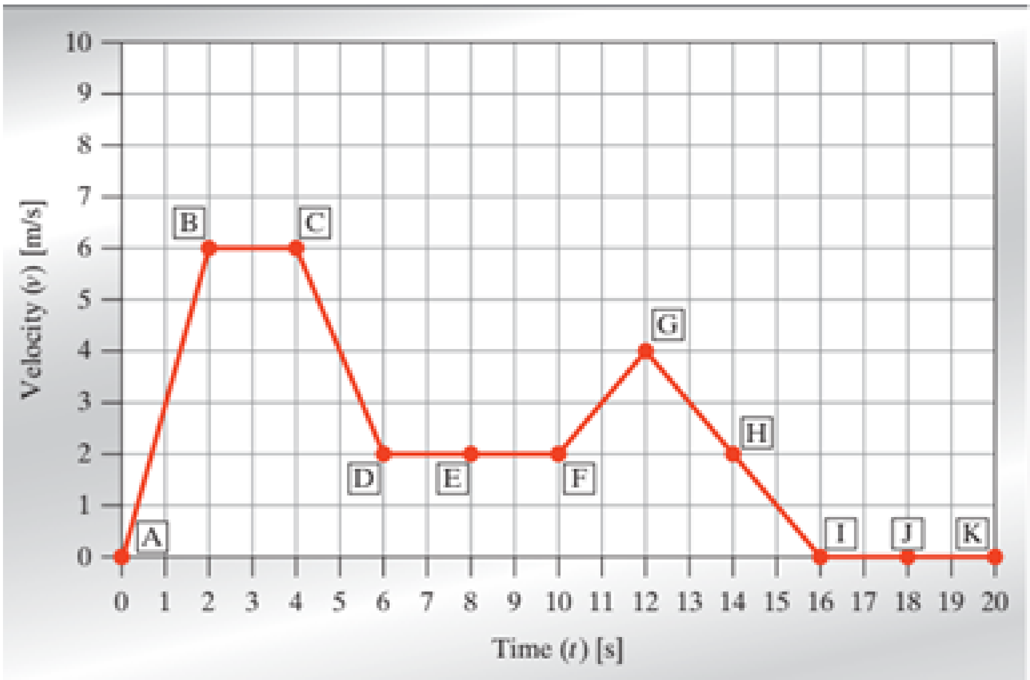

Figure 1: Velocity-Time of the car

1.2 Solution

1. The car accelerates at points:
   - $A \to B$
   - $F \to G$

   And it decelerates at points:
   - $C \to D$
   - $G \to I$ ($G \to H$ & $H \to I$)

2. To get the distance we can just find the integral from point A (0) to whatever points we want. So to see what the distance traveled at point C is we just have to evaluate

$$\int_A^C v * dt = \int_A^B v * dt + \int_B^C v * dt$$

   which is just the area under the graph so from $A \to B$ we have a right triangle with sides 1×6 which means that the area from that triangle is $\frac{6}{2} = 3$, and from $B \to C$ we have a rectangle with sides

2×6 so in total the area of the rectangle is 12 meaning that the distanced traveled from $A \rightarrow C$ is $3 + 12 = 15m$. To find out the distance traveled from $A \rightarrow F$ we have to solve the following integral:

$$\int_A^F v * dt = \int_A^C v * dt + \int_C^B v * dt + \int_D^F v * dt$$

Since we already know that $\int_A^C v * dt = 15m$ we just have to find the other two integrals, which at the end end up being $15 + \frac{2*4}{2} + 2 * 2 + 2 * 4 = 15 + 4 + 4 + 8 = 31m$

3. The line segments with 0 acceleration are

   - $B \rightarrow C$
   - $I \rightarrow K$ ($I \rightarrow J$ & $J \rightarrow K$)

   This does NOT mean that the car isn't moving, this just means that the velocity of the car isn't changing or in other words

$$\frac{dv}{dt} = 0$$

4.

LINE SEGMENT	ACCELERATION VALUE $\left[\frac{m}{s^2}\right]$
$A \rightarrow B$	3
$B \rightarrow C$	0
$C \rightarrow D$	-2
$D \rightarrow E$	0
$E \rightarrow F$	0
$F \rightarrow G$	1
$G \rightarrow H$	-1
$H \rightarrow I$	-1
$I \rightarrow J$	0
$J \rightarrow K$	0

2 Task 2

2.1 Problem

An environmental engineer has obtained a bacteria culture from a municipal water sample and allowed the bacteria to grow. The initial count of Bacteria is A, and their growth formula with time being in hours is given by:

$$B = B_0 e^{Ct}$$

A: is the summation of your birthday digits divided by 0.5
C: is the summation of your IUS ID number divided by 50.

- What is B_0? And what is its value?
- After how many hours, the amount of Bacteria would be 100000?
- Pick up 4 to 5 points in time and draw the graph of Bacteria growth. (This is done by pen and pencil)
- Use Octave to plot the graph of bacteria growth

2.2 Solution

$$A = \frac{1+4+1+2+2+0+0+2}{0.5} = \frac{12}{0.5} = 24$$
$$C = \frac{2+2+0+3+0+2+2+8+9}{50} = \frac{28}{50} = 0.56$$

1. B_0 is the initial amount of bacteria in our system and since our variable A represents the initial count of bacteria we can conclude that $A = B_0$.

2. To figure this out we simply have to figure out the following equation:

$$24 * e^{0.56*t} = 100000$$

$$0.56 * t * \ln 24 * e = \ln 100000$$
$$0.56 * t = \frac{\ln 100000}{\ln 24 * e}$$
$$t = \log_{24*e} 100000 * \frac{1}{0.56}$$
$$t \approx 4.9207[h]$$

3. Using the following code:

```
% Clear previous junk
clear all;
clc;

% Set up needed variables
A = 24;
C = 0.56;
t = linspace(0,20,10000);

% Calculate bacterial growth
B = A.*exp(C.*t);

% Plot the graph
plot(t,B, 'r*');
grid on;legend("Number of ...
    bacteria");xlabel("Time");ylabel("Bacteria");title("Bacterial growth");
```

We get the following graph

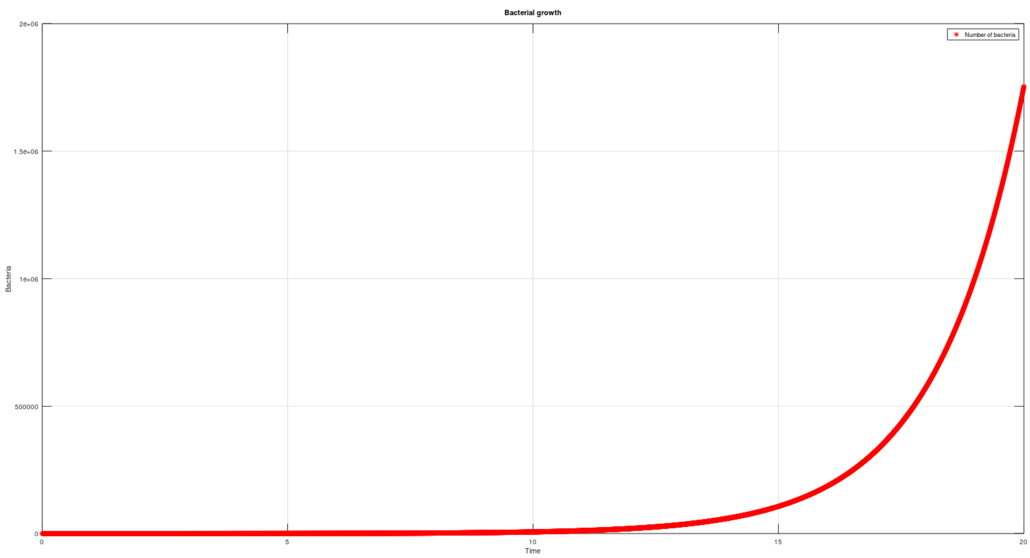

Figure 2: Bacterial growth plot

3 Task 3

3.1 Problem

3.2 Solution

4 Task 4

4.1 Problem

1. Consider the following loop:

```
[r,c]=size(D);
j=c;

i=1;
while j > 0
    T(i,1)=D(i,i); T(i,2)=D(i,j);
    i=i+1; j=j-1;
end
```

   If we were to run the code and generate the following value for T, T = [2 11; 7 16]. What is D

2. What is the output of $M4$ if M = [1 3 2; 6 0 2]

```
[rows,cols]=size(M);
for r=1:1:2*rows
  for c=1:1:2*cols
    M4(r,c)=c;
  end
end
```

4.2 Solution

1. If we take a good look at the code we can spot that the first element is always going to be the element on the main diagonal line of the matrix, and the 2nd element will always be the element of the inverse diagonal of the matrix so if we get T = [2 11; 7 16]. D must be:

$$D = \begin{bmatrix} 2 & 7 \\ 16 & 11 \end{bmatrix}$$

2. All this code does is take the number of rows and columns, afterwards if makes loops that repeat twice the number of rows and columns. So for a 2×3. We would have in total 24 itterations. The final output would be:

$$M4 = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 3 & 4 & 5 & 6 \end{bmatrix}$$

5 Task 5

5.1 Problem

Write a program that will ask the user to input his age in year and it will calculate to him his age in days.

5.2 Solution

```
% Clear useless junk
clear all;
clc;

% Get needed input
age = input("Please enter your age in years : ");

% Print the result
printf("You are %d days old", age * 365);
```

6 Task 6

6.1 Problem

Write a program that takes a vector as it's input and returns the maximum, minimum, and mean of the given vector. And it returns how many positive, negative and 0 numbers in the vector as well.

6.2 Solution

```
%Clear useless junk
clear all;
clc;

% Ask the user to input the vector
inputVector = [];
while true
  inputVector(end+1) = input('Please input a value for your vector: ');
  if !yes_or_no('Do you want to enter another value?');
    break;
  endif
endwhile

% display the vector
display(inputVector)

% Find and display the results
Max = max(inputVector)
min = min(inputVector)
mean = mean(inputVector)
negValues = sum(inputVector < 0)
posValues = sum(inputVector > 0)
zeroValues = sum(inputVector == 0)
```

7 Task 7

7.1 Problem

Show how to inscribe a square inside a circle such that all the square's vertices touch the circles' circumference. Then, calculate the area of the square if the circle's area is 628 cm^2

7.2 Solution

The process for inscribing a square inside of a circle goes as follows:

1. Draw a diameter inside the circle
2. Draw another diameter perpendicular to the previous one.
3. The resulting 4 points that touch the circle are now the four vertices of the square.

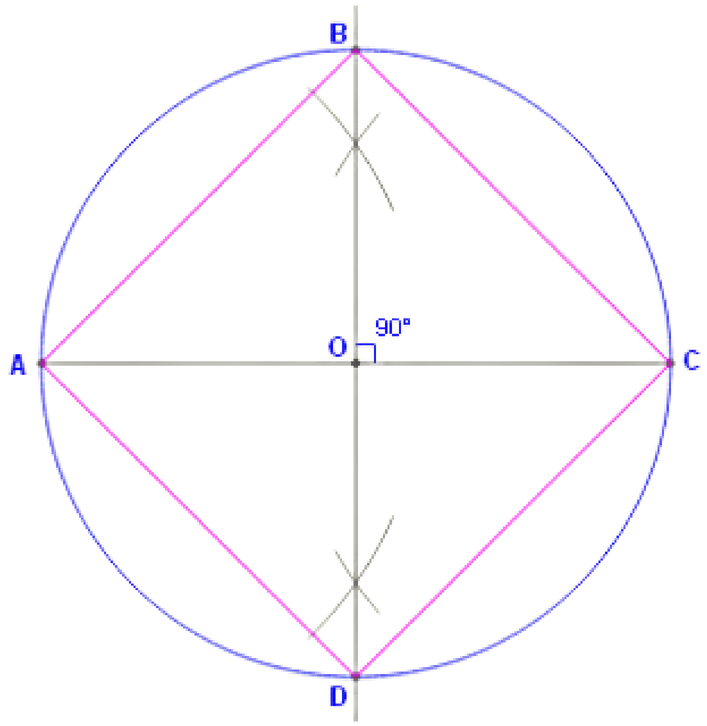

Figure 3: Inscribed square in a circle

As we can see from Figure 3. The diameter of the circle $D = 2r$ is also equal to $D = \sqrt{a^2 * a^2} = \sqrt{2 * a^2}$. Which means the following.

$$D^2 = 2 * a^2 \implies a^2 = A_\square = \frac{D^2}{2} \tag{1}$$

$$A_\circ = r^2 * \pi = 628[cm^2] \implies r^2 = \frac{628[cm^2]}{\pi} \tag{2}$$

$$r = \sqrt{\frac{628}{\pi}}[cm]$$

$$2r = D = 2 * \sqrt{\frac{628}{\pi}}[cm] \approx 28.277[cm]$$

If we now take the result for our D and substitute it into equation 1. We will get that:

$$A_\square \approx \frac{28.277^2}{2} \approx 399.80[cm^2]$$

8 Task 8

8.1 Problem

An ant is crawling on a unit cube, what is the shortest distance for it to get from start to end?

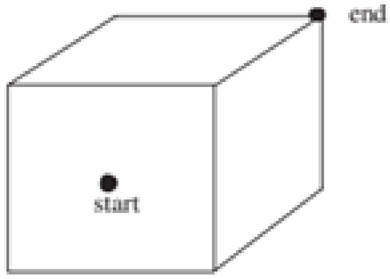

Figure 4: Unit cube that the ant has to traverse

8.2 Solution

First let us take a look at what distance actually is, and lets take a look at non euclidean distance as well. Distance can be confusing because sometimes we use it without even thinking, for example when we play chess we don't say move 5cm to the right we say move 5 checker blocks to the right so how do we know if the word "He moved the chess peace 5 checker blocks away from his starting position" is even correct? Well let us take a look at the mathematical definition of what an metric clamming to be distance actually is.

If we were to define a function (lets say function f) that is defined on an interval I that tells us a distance from two different points, first we need to define some rules that function *must* follow.

1. $f(a,b) = f(b,a) \geq 0, \forall a \wedge b \in Iwherea \neq b$
2. $f(a,b) = 0, \forall a \wedge b \in Iwherea = b$
3. $f(a,b) \leq f(a,c) + f(c,b), \forall c$

Now the 3rd rule is what interests us the most here, it tells us that the distance from point a to point b is always gonna be either shorter or the same as the distance from a to some point c and from c to some point b, aka if we take a detour. Now that we know that our next step is to find a way to somehow make a straight line without going through the cube because we know that a straight line will be the shortest path the ant can take, we just have to transform the cube somehow without making it lose it's information. One thing that we can do that will make this problem much easier will be if we took the cubes faces and layered them down as can be seen in the figure below:

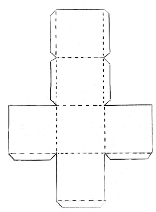

Figure 5: Cube cutout

Now all that is left is to map the blocks on this new model (transform their position) and draw a straight line.

9 Task 9

9.1 Problem

Generally, when a car door is opened, the interior lights come on and turn off again when the door is closed. Some cars turn the interior lights on and off gradually. Suppose that you have a car with 25 watts of interior lights. When a door is opened, the power to the lights increases linearly from 0 to 25 watts over 2 seconds. When the door is closed, the power is reduced to zero in a linear fashion over 5 seconds.

- Create a proper plot of power (P, on the ordinate) and time (t)
- Using the graph, determine the total energy delivered to the interior lights if the door to the car is opened and then closed 10 seconds later

9.2 Solution

1. The code we will use to generate the wanted plots goes as follows:

```
% Clear unwated junk
clear all;
clc;

% Set up variables
t = linspace(0,20,10000);
openDoorSlope = 25/2; closeDoorSlope = -25/5;

% Function
powerOpen = t.*openDoorSlope + 0;
powerClose = t.*closeDoorSlope + 25;

% Clamp all values between 25 and 0
for i = 1:length(t)
  if powerOpen(i) > 25
    powerOpen(i) = 25;
  elseif powerOpen(i) < 0
    powerOpen(i) = 0;
  end

  if powerClose(i) > 25
    powerClose(i) = 25;
  elseif powerClose(i) < 0
    powerClose(i) = 0;
  end
end

% Plotting our results
subplot(2,2, 1:2);
plot(t,powerOpen,'r-', 'linewidth', 5); xlabel("Time[s]"); ylabel("Power[W]"); ...
    title("Power consumption open door"); grid on;

subplot(2,2, 3:4);
plot(t,powerClose,'b-', 'linewidth', 5); xlabel("Time[s]"); ylabel("Power[W]"); ...
    title("Power consumption close door"); grid on;
```

And the plot that we get from this code is:

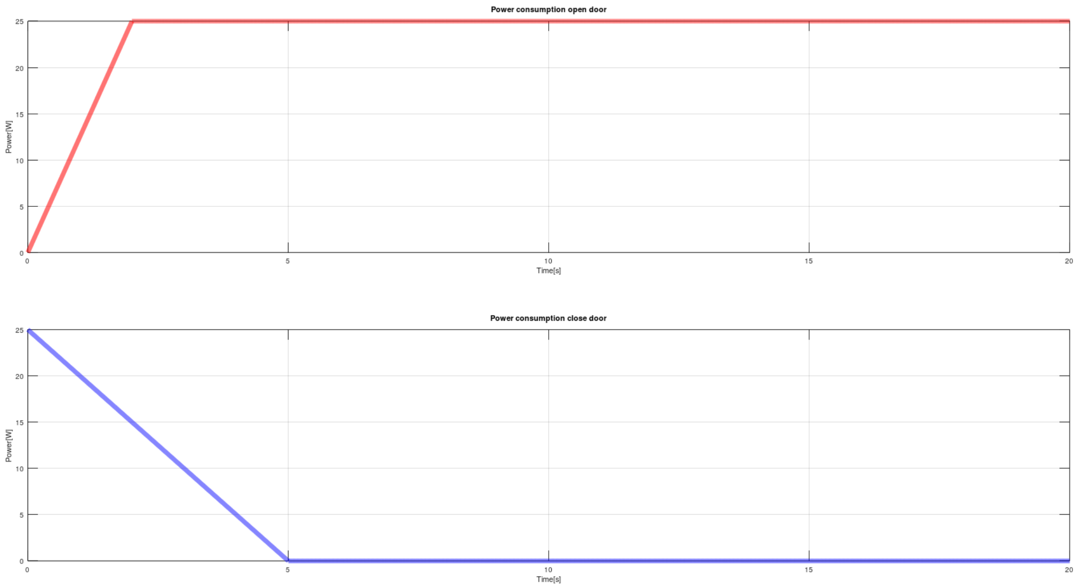

Figure 6: Power plot of both open and close door

2. For the 2nd problem let us first write the code to plot the function.

```
% Clear unwated junk
clear all;
clc;

% Set up variables
t1 = linspace(0,10,10000); t2 = linspace(0,5,5000); t = [t1 10.+t2];
openDoorSlope = 25/2; closeDoorSlope = -25/5;

% Calculate values for when the door is open
valuesOfEnergy = t1.*openDoorSlope;

% Truncate the values
for i = 1:length(t1)
  if valuesOfEnergy(i) > 25
    valuesOfEnergy(i) = 25;
  end
end


% Calculate values for when the door is closed
valuesOfEnergy = [valuesOfEnergy, t2.*closeDoorSlope .+ valuesOfEnergy(length(t1))];

% Truncate the values again
for i = length(t1):length(t)
  if valuesOfEnergy(i) < 0
    valuesOfEnergy(i) = 0;
  end
end

% Plot
plot(t,valuesOfEnergy,'b-', 'linewidth', 5); xlabel("Time[s]"); ...
    ylabel("Power[W]"); title("Power consumption of the door"); grid on;
```

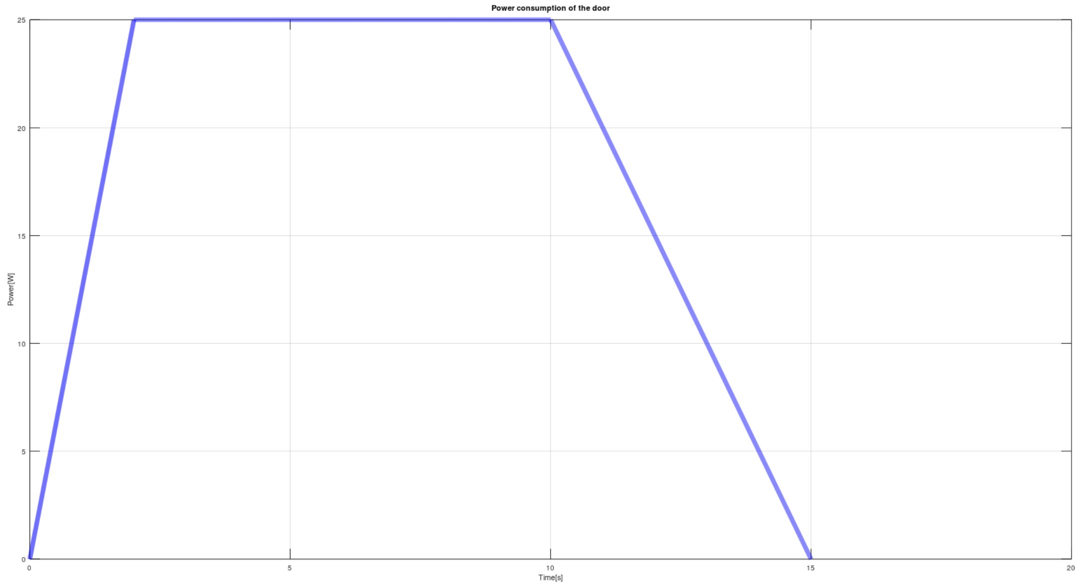

Figure 7: Power plot for 2nd part

Well now all that is left to do is solve the integral

$$E = \int P * dt$$

$$E = \int_0^{15} P * dt = \int_0^{2} P * dt + \int_2^{10} P * dt + \int_{10}^{15} P * dt \tag{3}$$

$$E = 25 + 25 * 8 + \frac{25 * 5}{2} \approx 287.50 \tag{4}$$

10 Task 10

10.1 Problem

10.2 Solution