Math 100 Cheat Sheet

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Abstract

This document contains a set of identities and equations for Math100

Contents

1 Logarithms

1.1 General definition of a logarithm

Logarithms are inverse function of exponential function meaning that if we want to remove an exponent we can apply an logarithm

$$\log_b(a) = c \implies b^c = a$$

1.1.1 Example

Lets say that we want to figure out 2 to the power of what number gives us 524288, or in other words $2^x = 524288$. Well we can approach this problem in two ways, the first way is to apply the definition of the logarithm and phase the problem as such $x = \log_2(524288)$. Or we can do this by placing the both sides of the equation into logarithms as follows $\log_2(2^x) = \log_2(524288) \implies x = \log_2(524288)$.

1.1.2 Domain

$$\log_b(a) = c$$

The logarithm above is valid for the following values:

$$a \in (0, +\infty)$$

$$b \in (0, 1) \cup (1, +\infty)$$

1.2 Logarithm Terminology

These are some essential logarithm terminologies that can be found in many math books:

1. $\ln(a) = \log_e(a)$
2. $\log(a) = \log_{10}(a)$

1.3 Logarithm Rules

There are a lot of rules that can help us when we are dealing with logarithms, some of these rules are:

1. $\log_b(1) = 0$
2. $\log_b(b) = 1$
3. $\log_b(b^x) = x$
4. $\log_b(a^c) = c * \log_b(a)$
5. $\log_b(a) + \log_b(c) = \log_b(a * c)$ ★
6. $\log_b(a) - \log_b(c) = \log_b(\frac{a}{c})$ ★
7. $\log_b(a) = \frac{\log_c(a)}{\log_c(b)}$ ★

1.4 Logaritmic inequalities

When dealing with inequalities we have to take care about the logarithms base, if the logarithms base is between 0 and 1 once we get rid of the logarithm we have to swap the equality sign, for example:

$$\log_b(a) \geq \log_b(c)$$

If $b \in (0, 1) \vee 0 < b < 1$ then:

$$\log_b(a) \geq \log_b(c) \implies a \leq c$$

else if $b > 1$

$$\log_b(a) \geq \log_b(c) \implies a \geq c$$

2 Trigonometry

2.1 Converting degrees into radians

If we mark the angle in degrees as α_{deg} and angle in radians as α_{rad}, the conversion goes as follows:

$$\alpha_{rad} = \frac{\pi}{180°} * \alpha_{deg}$$

2.2 Converting radians into degrees

Same principal as before just our equation is now:

$$\alpha_{deg} = \frac{180°}{\pi} * \alpha_{rad}$$

2.3 General definition of trigonometric functions

The concept of unit circle helps us to measure the angles of cos, sin and tan directly since the center of the circle is located at the origin and radius is 1. Consider θ to be an angle then

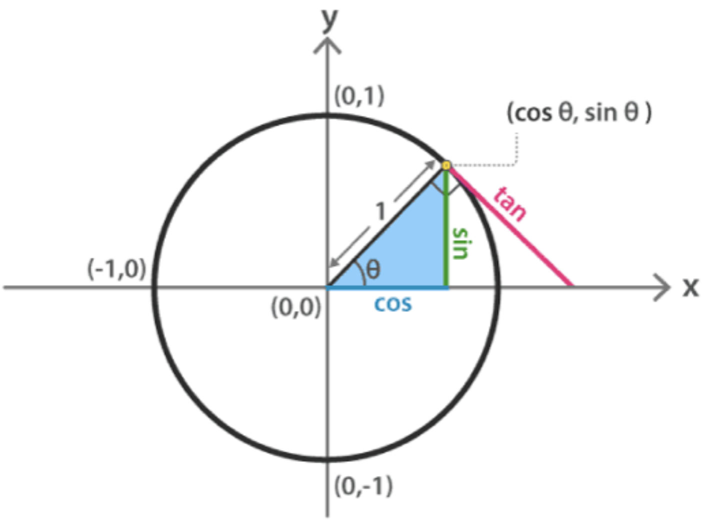


Figure 1: Image of a trigonometric unit circle

2.4 Table of trigonometric values

α_{deg}	0°	30°	45°	60°	90°	180°	270°	360°
α_{rad}	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin(\alpha)$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$\cos(\alpha)$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0	1
$\tan(\alpha)$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	X	0	X	0

2.5 Trigonometric Identities

2.5.1 Trigonometry basis

- $\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$
- $\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$
- $\tan(\theta) = \frac{1}{\cot(\theta)}$
- $\cot(\theta) = \frac{1}{\tan(\theta)}$
- $\sec(\theta) = \frac{1}{\cos(\theta)}$
- $\csc(\theta) = \frac{1}{\sin\theta)}$

2.5.2 Pythagorean Identities

- $\sin^2(\theta) + \cos^2(\theta) = 1$
- $\tan^2(\theta) + 1 = \sec^2(\theta)$
- $\cot^2(\theta) + 1 = \csc^2(\theta)$

2.5.3 Reflections

- $\sin(-\theta) = -\sin(\theta)$
- $\cos(-\theta) = \cos(\theta)$
- $\tan(-\theta) = -\tan(\theta)$
- $\cot(-\theta) = -\cot(\theta)$
- $\sec(-\theta) = \sec(\theta)$
- $\csc(-\theta) = -\csc(\theta)$

2.5.4 Angle Sum and Difference

- $\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \sin(\beta)\cos(\alpha)$
- $\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$
- $\tan(\alpha \pm \beta) = \frac{\tan(\alpha) \pm \tan(\beta)}{1 \mp \tan(\alpha)\tan(\beta)}$
- $\cot(\alpha \pm \beta) = \frac{\cot(\alpha)\cot(\beta) \mp 1}{\cot(\alpha) \pm \cot(\beta)}$

2.5.5 Double Angle

- $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$
- $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$
- $\tan(2\theta) = \frac{2\tan(\theta)}{1 - \tan^2(\theta)}$
- $\cot(2\theta) = \frac{\cot^2(\theta) - 1}{2\cot(\theta)}$
- $\sec(2\theta) = \frac{\sec^2(\theta)}{2 - \sec^2(\theta)}$
- $\csc(2\theta) = \frac{\sec(\theta) csc(\theta)}{2}$

2.5.6 Half Angle

- $\sin(\frac{\theta}{2}) = \pm\sqrt{\frac{1-\cos(\theta)}{2}}$
- $\cos(\frac{\theta}{2}) = \pm\sqrt{\frac{1+\cos(\theta)}{2}}$
- $\tan(\frac{\theta}{2}) = \frac{1-\cos(\theta)}{\sin(\theta)}$

2.6 Finding the 2nd solution

If we get asked to find for what values of x is $\sin(x)$ a value that is between -1 and 1 we know in the range from $0 \to 2\pi$ there are 2 answers. If sin(x) is given to us in the 1^{st} quadrant, then the second solution can be found

$$\sin(180° - x)$$

And if $\cos(x)$ is given in the same quadrant the second solution can be found by using this equation

$$\cos(360° - x)$$

However if you want to find the negative solution but you only have the knows values for the positive values you can do the following. Find the x such that $\sin(x)$ is equal to the value you want to find (if you want to find $-\frac{1}{2}$ you will just take the value for x such that u get $sin(x) = \frac{1}{2}$). Afterwards you will use this equality to find your answer

$$\sin(180° + x) \vee \sin(360° - x)$$

And for $\cos(x)$ the equation is

$$\cos(180° \mp x)$$

3 Quadric Equations

3.1 Introduction

Quadric equations are 2^{nd} degree polynomials meaning they have the form of:

$$a * x^2 + b * x + c$$

Now that we got that out of the way lets talk about some important variables that decide how our parabola will end up looking.

3.1.1 Discriminant

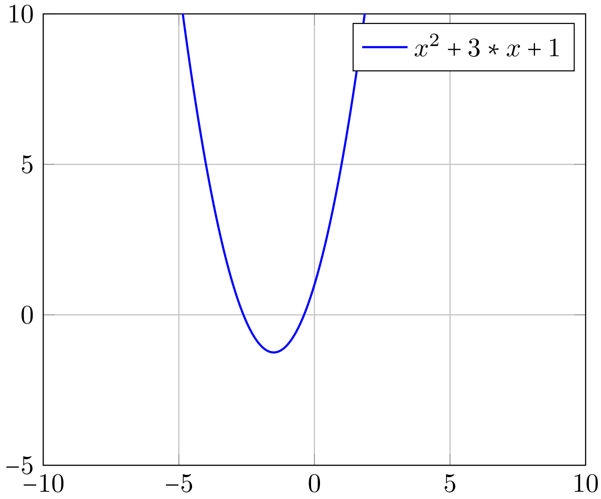
The discriminant is defined in the following way:

$$D = b^2 - 4 * a * c$$

The discriminant together with the constant a can tell us about how the function looks like.

3.2 Function Plots

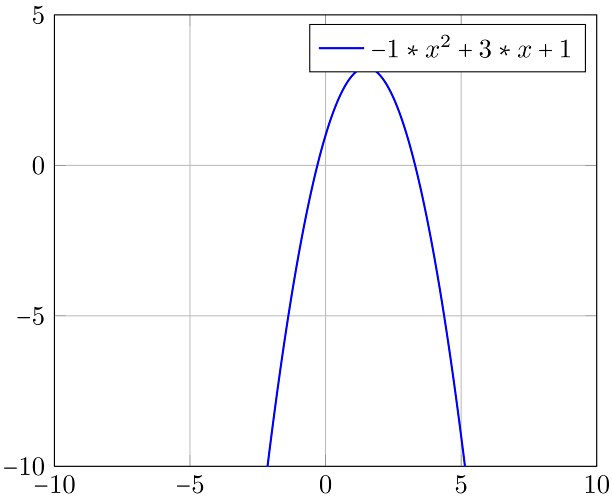
3.2.1 $D > 0 \wedge a > 0$



We can say that the function will have 2 points ($x_{1,2}$) where it's value will be 0 ($y = f(x) \implies y = 0 = f(x_{1,2})$). We can also see that between points x_1 and x_2 the function is negative and everywhere else it is positive. So we can confirm the following:

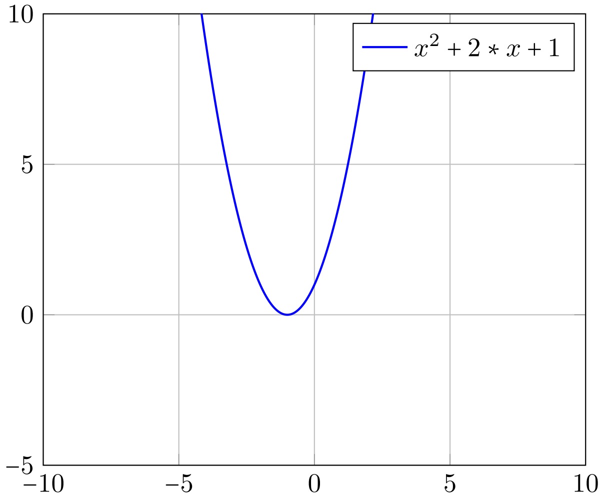
- $f(x) < 0 \rightarrow x \in (x_1, x_2)$
- $f(x) >\rightarrow x \in (-\infty, x_1) \bigcup (x_2, +\infty)$
- $f(x) = 0 \rightarrow x = x_{1,2}$

3.2.2 $D > 0 \wedge a < 0$



- $f(x) > 0 \rightarrow x \in (x_1, x_2)$
- $f(x) < 0 \rightarrow x \in (-\infty, x_1) \bigcup (x_2, +\infty)$
- $f(x) = 0 \rightarrow x = x_{1,2}$

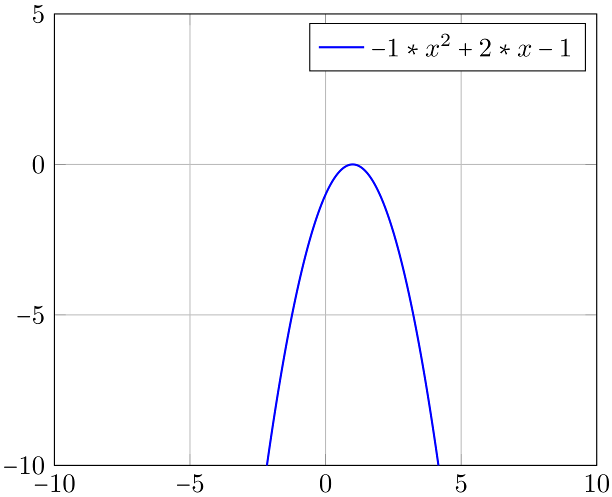
3.2.3 $D = 0 \wedge a > 0$



Here we can see that when $D = 0$ there is only *one* x for which $y = 0$ and we will call this x x_1, and for positive values of a this function is never negative

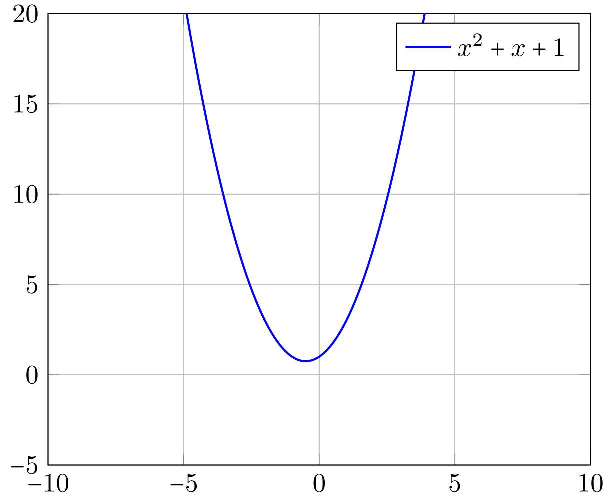
- $f(x) < 0 \rightarrow x \in \varnothing$
- $f(x) > 0 \rightarrow x \in (-\infty, x_1) \bigcup (x_1, +\infty)$
- $f(x) = 0 \rightarrow x = x_1$

3.2.4 $D = 0 \wedge a < 0$



- $f(x) > 0 \rightarrow x \in \varnothing$
- $f(x) < 0 \rightarrow x \in (-\infty, x_1) \bigcup (x_1, +\infty)$
- $f(x) = 0 \rightarrow x = x_1$

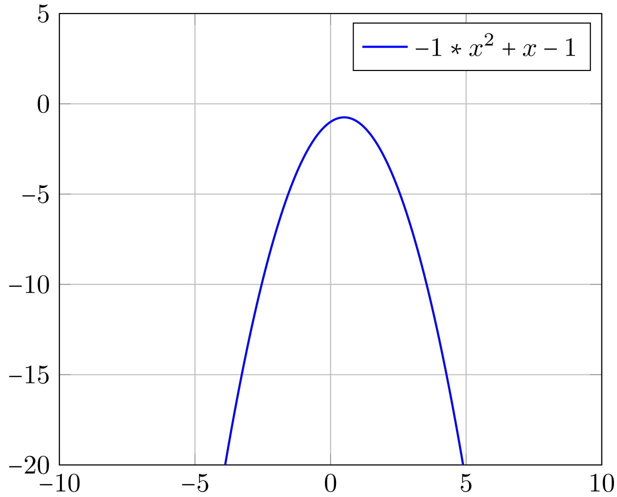
3.2.5 $D < 0 \land a > 0$



In this case we can see that the function is always positive for every values of x.

- $f(x) > 0 \rightarrow x \in (-\infty, +\infty)$
- $f(x) < 0 \rightarrow x \in \varnothing$
- $f(x) = 0 \rightarrow x \in \varnothing$

3.2.6 $D < 0 \land a < 0$



- $f(x) > 0 \rightarrow x \in \varnothing$
- $f(x) < 0 \rightarrow x \in (-\infty, +\infty)$
- $f(x) = 0 \rightarrow x \in \varnothing$

3.3 Finding zero points

To find the values for x for which $y = 0$ we will simply apply the following formula:

$$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

4 Equation of lines

4.1 Introduction

When we talk about straight lines there are 2 ways we describe them.

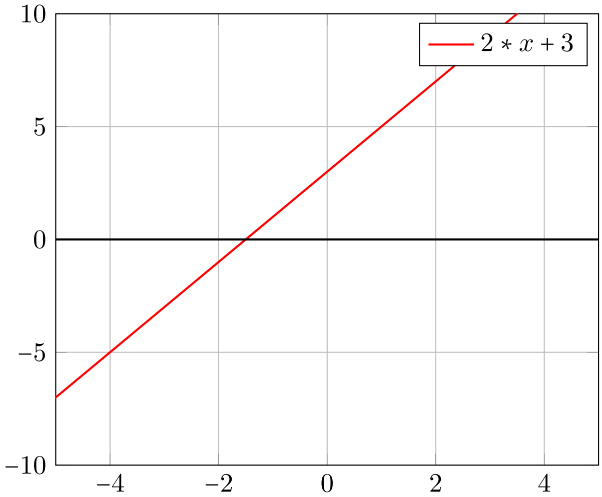
1. $y = kx + n$
2. $y = y_0 + k(x - x_0)$

k is the slope of the line and can be calculated by taking the difference of change over y with respect to x.

$$k = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{\nabla y}{\nabla x} \tag{1}$$

n is the Y intercept and it tells us where does the line touch the y axis when $x = 0$
For the 2nd equation y_0 is just the y value of a starting points and x_0 is the x value of some starting point that we took. Everything else is the same.

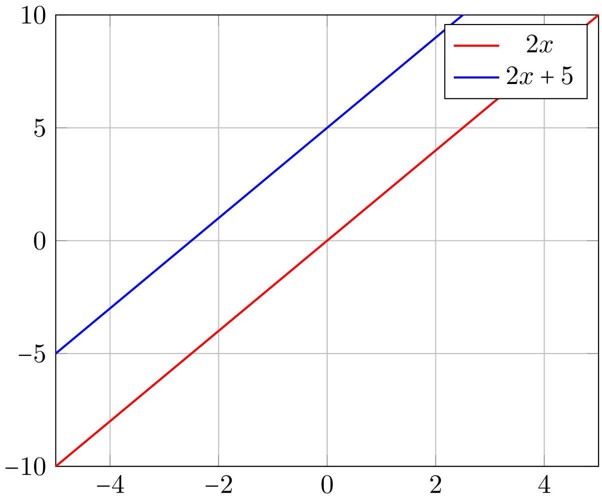
4.1.1 example



As we can see from the equation $y = 2 * x + 3$ we can tell that our $n = 3$ and our $k = 2$ this means at $x = 0$ the line will touch the y axis at point $A(0, 3)$ and for every change in x y doubles, so if x goes from $0 \to 2$ y will go from $0 + n \to 4 + n$

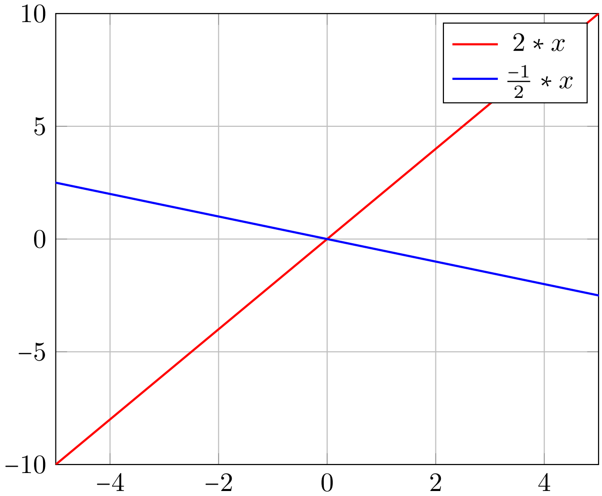
4.2 Conditions for paralel lines

If two lines are paralel that means that their slope k is the same (A.K.A $k_1 = k_2$)



4.3 Conditions for perpendicular lines

For two lines to be perpendicular their slopes must be negative inverses of one another (A.K.A $k_1 = -\frac{1}{k_2}$)



5 Equation of a circle

5.1 Introduction

To understand this part we better take a look at what a circle actually is. By definition a circle is a group of points that have the same distance from once center points; meaning that we will first have to take a look at the equation of distance. The equation of distance between two points goes as follows.

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

If we take x_2 and y_2 to be the cordinants of the center of the circle and write them as x_c and y_c, and if we take r to be the radius we will get the following equation.

$$r = \sqrt{(x_1 - x_c)^2 + (y_1 - y_c)^2}$$

If the center of the circle is at the center of the cordinate system (0,0) then we get the equation.

$$r = \sqrt{x^2 + y^2} \implies r^2 = x^2 + y^2$$

Where x and y are points in the cordinate system.

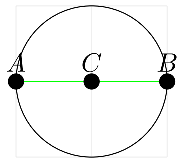
5.2 Problem solving

If we have a question that asks us to find a circle that fits points A and B we have to find a point C such that the distance $\overline{AC}$ must be the same as the distance $\overline{BC}$. To do this we can just find the distance $\overline{AB}$ and divide it by 2 to get our radius, but this will lead us to a complicated way to find the solution; so we will find the point in between A and B. To find a point that is between two other points it is as simple as solving the following set of equations.

$$A(x_1, y_1)$$

$$B(x_2, y_2)$$

$$C(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$$



Another type of problem is when you are given the center of the circle and it's radius and you are suppose to find out if a given point belongs to the circle, but this is simple just plug it into the equation given beforehand and see if the distance between the center and the given point is the same as the radius.