Math 100 Cheat Sheet

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Abstract

This document contains a set of identities and equations for Math100

Contents

1 Logarithms

1.1 General definition of a logarithm

Logarithms are inverse function of exponential function meaning that if we want to remove an exponent we can apply an logarithm

$$\log_b(a) = c \implies b^c = a$$

1.1.1 Example

Lets say that we want to figure out 2 to the power of what number gives us 524288, or in other words $2^x = 524288$. Well we can approach this problem in two ways, the first way is to apply the definition of the logarithm and phase the problem as such $x = \log_2(524288)$. Or we can do this by placing the both sides of the equation into logarithms as follows $\log_2(2^x) = \log_2(524288) \implies x = \log_2(524288)$.

1.1.2 Domain

$$\log_b(a) = c$$

The logarithm above is valid for the following values:

$$a \in (0, +\infty)$$

$$b \in (0, 1) \cup (1, +\infty)$$

1.2 Logarithm Terminology

These are some essential logarithm terminologies that can be found in many math books:

1. $\ln(a) = \log_e(a)$
2. $\log(a) = \log_{10}(a)$

1.3 Logarithm Rules

There are a lot of rules that can help us when we are dealing with logarithms, some of these rules are:

1. $\log_b(1) = 0$
2. $\log_b(b) = 1$
3. $\log_b(b^x) = x$
4. $\log_b(a^c) = c * \log_b(a)$
5. $\log_b(a) + \log_b(c) = \log_b(a * c)$ ★
6. $\log_b(a) - \log_b(c) = \log_b(\frac{a}{c})$ ★
7. $\log_b(a) = \frac{\log_c(a)}{\log_c(b)}$ ★

2 Trigonometry

2.1 Converting degrees into radians

If we mark the angle in degrees as α_{deg} and angle in radians as α_{rad}, the conversion goes as follows:

$$\alpha_{rad} = \frac{\pi}{180°} * \alpha_{deg}$$

2.2 Converting radians into degrees

Same principal as before just our equation is now:

$$\alpha_{deg} = \frac{180°}{\pi} * \alpha_{rad}$$

2.3 General definition of trigonometric functions

The concept of unit circle helps us to measure the angles of cos, sin and tan directly since the center of the circle is located at the origin and radius is 1. Consider θ to be an angle then

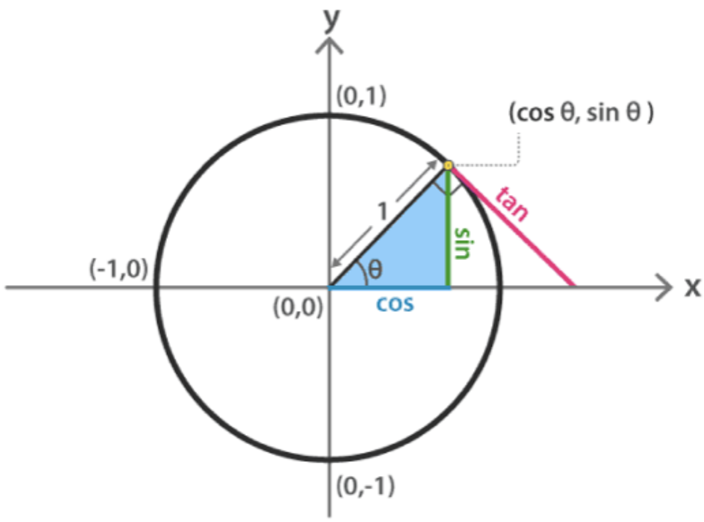


Figure 1: Image of a trigonometric unit circle

2.4 Table of trigonometric values

α_{deg}	0°	30°	45°	60°	90°	180°	270°	360°
α_{rad}	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin(\alpha)$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$\cos(\alpha)$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0	1
$\tan(\alpha)$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	X	0	X	0

2.5 Trigonometric Identities

2.5.1 Trigonometry basis

- $\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$
- $\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$
- $\tan(\theta) = \frac{1}{\cot(\theta)}$
- $\cot(\theta) = \frac{1}{\tan(\theta)}$
- $\sec(\theta) = \frac{1}{\cos(\theta)}$
- $\csc(\theta) = \frac{1}{\sin\theta)}$

2.5.2 Pythagorean Identities

- $\sin^2(\theta) + \cos^2(\theta) = 1$
- $\tan^2(\theta) + 1 = \sec^2(\theta)$
- $\cot^2(\theta) + 1 = \csc^2(\theta)$

2.5.3 Reflections

- $\sin(-\theta) = -\sin(\theta)$
- $\cos(-\theta) = \cos(\theta)$
- $\tan(-\theta) = -\tan(\theta)$
- $\cot(-\theta) = -\cot(\theta)$
- $\sec(-\theta) = \sec(\theta)$
- $\csc(-\theta) = -\csc(\theta)$

2.5.4 Angle Sum and Difference

- $\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \sin(\beta)\cos(\alpha)$
- $\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$
- $\tan(\alpha \pm \beta) = \frac{\tan(\alpha) \pm \tan(\beta)}{1 \mp \tan(\alpha)\tan(\beta)}$
- $\cot(\alpha \pm \beta) = \frac{\cot(\alpha)\cot(\beta) \mp 1}{\cot(\alpha) \pm \cot(\beta)}$

2.5.5 Double Angle

- $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$
- $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$
- $\tan(2\theta) = \frac{2\tan(\theta)}{1-\tan^2(\theta)}$
- $\cot(2\theta) = \frac{\cot^2(\theta)-1}{2\cot(\theta)}$
- $\sec(2\theta) = \frac{\sec^2(\theta)}{2-\sec^2(\theta)}$
- $\csc(2\theta) = \frac{\sec(\theta)csc(\theta)}{2}$

2.5.6 Half Angle

- $\sin(\frac{\theta}{2}) = \pm\sqrt{\frac{1-\cos(\theta)}{2}}$
- $\cos(\frac{\theta}{2}) = \pm\sqrt{\frac{1+\cos(\theta)}{2}}$
- $\tan(\frac{\theta}{2}) = \frac{1-\cos(\theta)}{\sin(\theta)}$

3 Quadric Equations

3.1 Introduction

Quadric equations are 2^{nd} degree polynomials meaning they have the form of:

$$a * x^2 + b * x + c$$

Now that we got that out of the way lets talk about some important variables that decide how our parabola will end up looking.

3.1.1 Discriminant

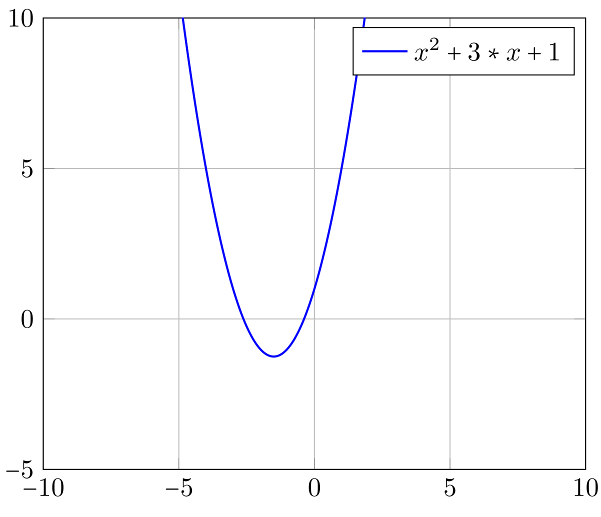
The discriminant is defined in the following way:

$$D = b^2 - 4 * a * c$$

The discriminant together with the constant a can tell us about how the function looks like.

3.2 Function Plots

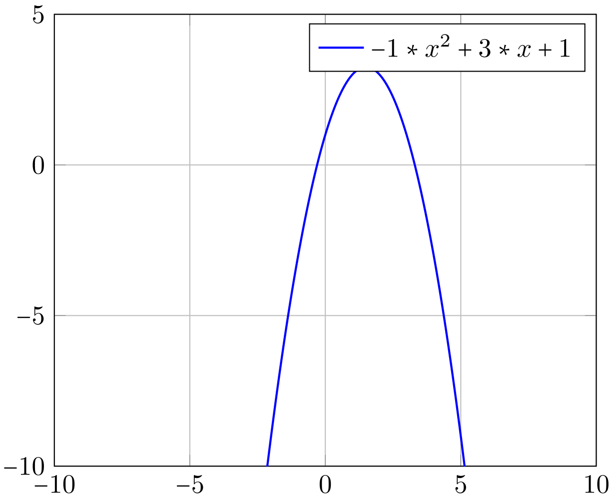
3.2.1 $D > 0 \wedge a > 0$



We can say that the function will have 2 points $(x_{1,2})$ where it's value will be 0 $(y = f(x) \implies y = 0 = f(x_{1,2}))$. We can also see that between points x_1 and x_2 the function is negative and everywhere else it is positive. So we can confirm the following:

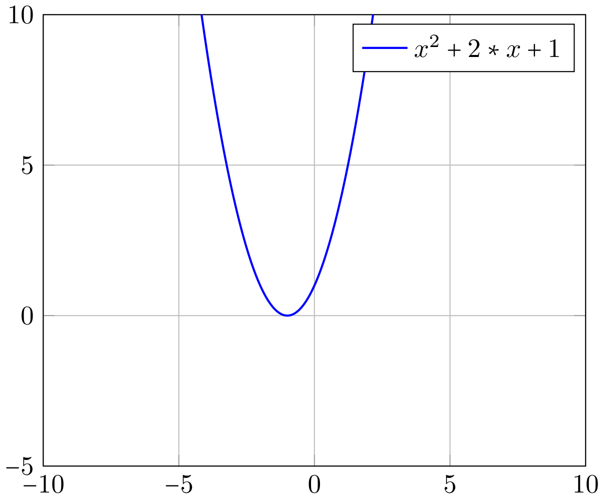
- $f(x) < 0 \rightarrow x \in (x_1, x_2)$
- $f(x) >\rightarrow x \in (-\infty, x_1) \bigcup (x_2, +\infty)$
- $f(x) = 0 \rightarrow x = x_{1,2}$

3.2.2 $D > 0 \wedge a < 0$



- $f(x) > 0 \rightarrow x \in (x_1, x_2)$
- $f(x) < 0 \rightarrow x \in (-\infty, x_1) \bigcup (x_2, +\infty)$
- $f(x) = 0 \rightarrow x = x_{1,2}$

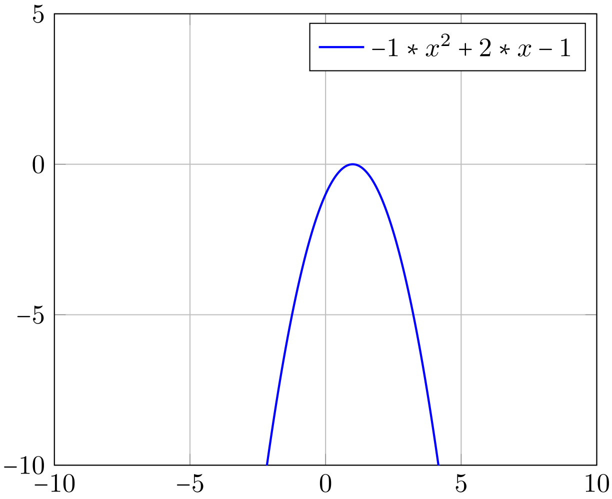
3.2.3 $D = 0 \wedge a > 0$



Here we can see that when $D = 0$ there is only *one* x for which $y = 0$ and we will call this x x_1, and for positive values of a this function is never negative

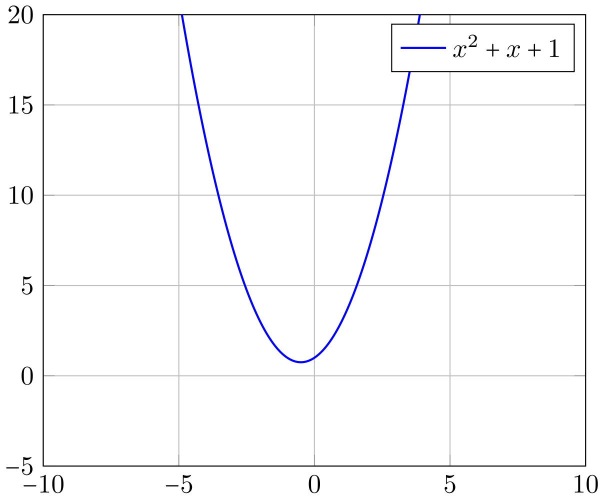
- $f(x) < 0 \rightarrow x \in \varnothing$
- $f(x) > 0 \rightarrow x \in (-\infty, x_1) \bigcup (x_1, +\infty)$
- $f(x) = 0 \rightarrow x = x_1$

3.2.4 $D = 0 \wedge a < 0$



- $f(x) > 0 \rightarrow x \in \varnothing$
- $f(x) < 0 \rightarrow x \in (-\infty, x_1) \bigcup (x_1, +\infty)$
- $f(x) = 0 \rightarrow x = x_1$

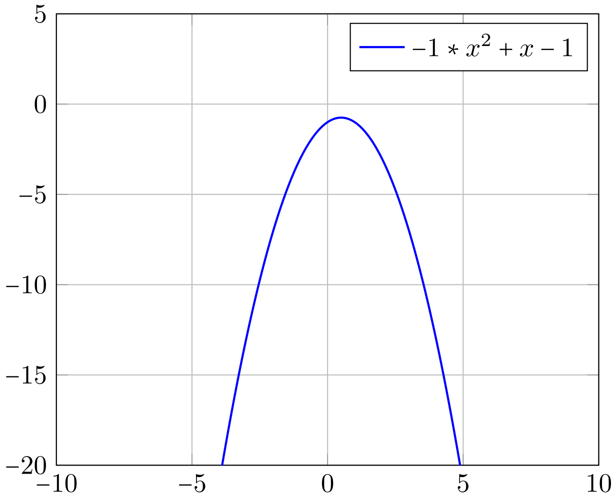
3.2.5 $D < 0 \wedge a > 0$



In this case we can see that the function is always positive for every values of x.

- $f(x) > 0 \rightarrow x \in (-\infty, +\infty)$
- $f(x) < 0 \rightarrow x \in \varnothing$
- $f(x) = 0 \rightarrow x \in \varnothing$

3.2.6 $D < 0 \wedge a < 0$



- $f(x) > 0 \rightarrow x \in \varnothing$
- $f(x) < 0 \rightarrow x \in (-\infty, +\infty)$
- $f(x) = 0 \rightarrow x \in \varnothing$

3.3 Finding zero points

To find the values for x for which $y = 0$ we will simply apply the following formula:

$$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$