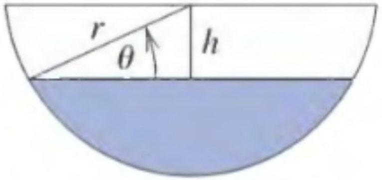
A trough of length L has a cross section in the shape of a semicircle with radius r. (See the accompanying figure.) When filled with water to within a distance h of the top, the volume V of water is

$$V = L\left[0.5\pi r^2 - r^2 \arcsin(h/r) - h(r^2 - h^2)^{1/2}\right].$$



Suppose $L = 10$ ft, $r = 1$ ft, and $V = 12.4$ ft^3. Find the depth of water in the trough to within 0.01 ft.

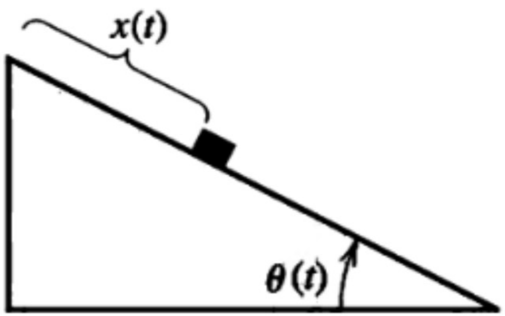
A particle starts at rest on a smooth inclined plane whose angle θ is changing at a constant rate

$$\frac{d\theta}{dt} = \omega < 0.$$

At the end of t seconds, the position of the object is given by

$$x(t) = -\frac{g}{2\omega^2}\left(\frac{e^{wt} - e^{-wt}}{2} - \sin \omega t\right).$$

Suppose the particle has moved 1.7 ft in 1 second. Find, to within 10^{-5}, the rate ω at which θ changes. Assume that $g = 32.17$ ft/s^2.



1. **Derive a function f for which the Bisection method converges to a value that is not a zero of f.**

2. **Derive a function f for which the Bisection method converges to a zero of f but f is not continuous at that point.**

1. Using the bisection method, determine the point of intersection of the curves given by $y = x^3 - 2x + 1$ and $y = x^2$.

4. Find a root of the equation $6(e^x - x) = 6 + 3x^2 + 2x^3$ between -1 and $+1$ using the bisection method.

(**Circuit problem**) A simple circuit with resistance R, capacitance C in series with a battery of voltage V is given by $Q = CV[1 - e^{-T/(RC)}]$, where Q is the charge of the capacitor and T is the time needed to obtain the charge. We wish to solve for the unknown C. For example, solve this problem

$$f(x) = \left[10x\left(1 - e^{-0.004/(2000x)}\right) - 0.00001\right]$$

Plot the curve. *Hint:* You may wish to magnify the vertical scale by using $y = 10^5 f(x)$.