HACETTEPE UNIVERSITY

Department of Nuclear Engineering

NEM 294 ENGINEERING PROJECT IV

Assignment 4 and Analysis of thermal efficiency and net work in Brayton and combined cycles depending on compression ratio

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Delivery Date: 06.06.2025

Due Date: 07.06.2025

1 INTRODUCTION

The Brayton cycle is one of the most fundamental thermodynamic cycles used to produce power in gas turbines. The Brayton cycle can be divided into two main types: open and closed. The open cycle is called internal combustion, and the closed cycle is called external combustion.

Analysis of internal combustion is very difficult because the Working Fluid (WF) changes its composition. Therefore, we use Air Standard assumptions that convert the internal combustion cycle to an external combustion cycle.

The same assumption was used in this project. This assumption is summarized as follows

Air Standard Assumptions:

- The working fluid is modelled as a fixed mass of air circulating continuously in a closed loop.
- Combustion process is replaced by a heat addition from an external source
- Intake and exhaust processes are replaced by heat rejection to the surroundings.
- All processes are assumed to be internally reversible.

In addition, in order to make the results of the project closer to real life, the specific heat of the air was used as a function of temperature, **not** a constant.

Three main applications of Gas Cycles are:

1) Emergency and peak power generation
2) Nuclear Power Plants (Gas Cooled)
3) Aircraft propulsion system
4) Combined Cycle Power Plant
5) Simple Cycle Gas Turbine

The project consists of two main parts.

In the first part of project, the results of a Brayton cycle with certain properties were examined by changing some variables under three different headings.

a) The effect of compression ratio (r) on efficiency is analyzed
b) The r value that gives the maximum network (w_{net}) is calculated
c) For a selected and reasonable value of r, the impact of compressor efficiency and gas turbine efficiency on thermal efficiency was investigated.

Before going into the details of the project, let's create a schematic idea in our minds by looking at some diagrams and schemes of the Brayton cycle.

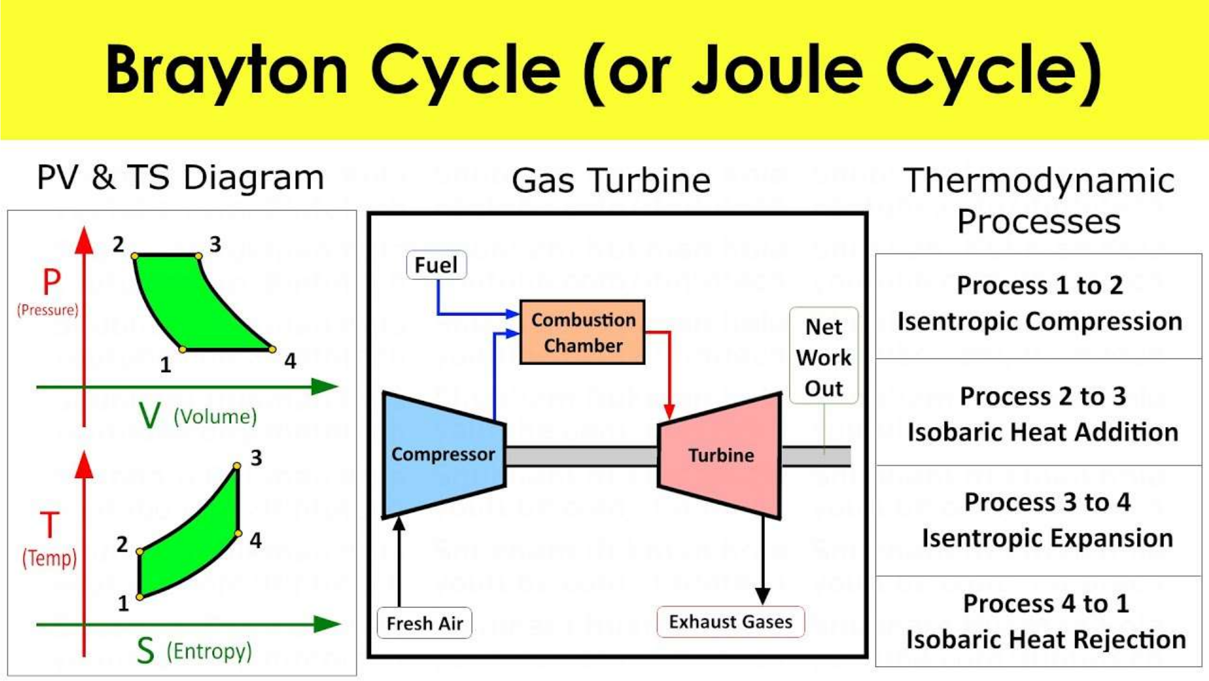


Figure 1: Brayton cycle summary graphs and process steps (Kola, 2022)

In the second part of the project, Combined systems are more complex and more efficient systems where Brayton and Rankine cyclers are used together. In these systems, efficiency can reach up to 60 percent. The specified Brayton cycle and Rankine cycle were connected with a heat exchanger to form a combined cycle. In this combined cycle, the effect of the compression ratio (r) value on the thermal efficiency of the combined cycle was investigated.

2 METHODS AND CALCULATIONS

2.1 The first part of this project examines how a Brayton cycle with air as the working fluid varies under various conditions. The current/steady characteristics of the Brayton cycle are given below. The analyzed parts of the project are, in order, the effect of compression ratio on efficiency, then finding the ideal compression ratio for maximum net work, and examining the effect on the thermal efficiency of the compressor and gas turbine using a selected compression ratio. The efficiency of the compressor and gas turbine were taken at 5 different values: 0.86, 0.88, 0.90, 0.92, and 0.94. Calculations were performed and the AirDataSonntag.txt file was used to find/use the properties of the air while the Python Xsteam library was used to find/use the required enthalpies. In addition, the specific heat of air is used as a function of temperature, not as a constant.

The given current/steady conditions are;

- Turbine inlet temperature (Tmax) = 1200 K
- Compressor inlet temperature (Tmin) = 298.15 K
- Gas Turbine efficiency: 90%
- Compressor efficiency: 90%

The equations used are;

1) **Interpolated Enthalpy Function:**

$$\mathbf{h(T) \approx interp1d(Tdata,\ hdata,\ cubic)}$$

2) **Interpolated Entropy Function:**

$$\boldsymbol{s(T) \approx interp1d(Tdata,\ sdata,\ cubic)}$$

3) **Isentropic Compressor Outlet Entropy:**

$$\boldsymbol{s2s = s1 + R \cdot ln(r)}$$

4) **Numerical Solution for Isentropic Outlet Temperature (Compressor):**

$$\boldsymbol{s(T2s) = s1 + R \cdot ln(r)}$$

5) **Actual Enthalpy at Compressor Exit:**

$$\boldsymbol{h2 = h1 + (h2s - h1/\eta c)}$$

6) **Isentropic Turbine Outlet Entropy:**

$$\boldsymbol{s4s = s3 - R \cdot ln(r)}$$

7) **Solution for Isentropic Outlet Temperature (Turbine):**

$$\boldsymbol{s(T4s) = s3 - R \cdot ln(r)}$$

8) **Actual Enthalpy at Turbine Exit:**

$$\boldsymbol{h4 = h3 - \eta t \cdot (h3 - h4s)}$$

9) **Heat Supplied to the Cycle:**

$$\boldsymbol{q_{in} = h3 - h2}$$

10) **Net Work Output of the Brayton Cycle:**

$$\boldsymbol{wnet = (h3 - h4) - (h2 - h1)}$$

11) **Thermal Efficiency of the Brayton Cycle:**

$$\boldsymbol{\eta_{th} = \frac{w_{net}}{q_{in}}}$$

12) **Efficiency Variation for Different Component Efficiencies:**

$$\boldsymbol{\eta c = \eta t = (0.86,\ 0.88,\ 0.90,\ 0.92,\ 0.94)}$$

2.2 In the second part of the project, a combined system was analyzed. This combined system is connected to each other with a heat exchanger. Here, the air exiting the gas-turbine provides all the heat needed for Rankine, which is an important point for the calculations to be made. The air always exits the heat exchanger at 460 K. The effect of compression ratio on the efficiency of the combined system was analyzed.

The given current/steady conditions are;

- Topping cycle is Brayton in Question 1, with the data:

Tmax = 1200 K, Tmin = 298.15 K, ηgas-turbine = 0.90and ηcompressor = 0.90.

- Bottoming cycle is Rankine:
  - ❖ Steam-turbine inlet = 7 MPa, 400 C
  - ❖ OFWH = 800 kPa
  - ❖ Reheat exit = 800 kPa, 400 C
  - ❖ Condenser = 10 kPa
  - ❖ ηsteam-turbine = ηpumps = 0.90.

The equations are in addition to the Brayton equations used above as follows;

◆ RANKINE CYCLE Equations:

1) **Pump Work (Pump 1 and Pump 2):**

$$\boldsymbol{Wpump = (v \times \Delta P) / \eta\_pump}$$

2) **Pump 1 Output Enthalpy:**

$$\boldsymbol{h2 = h1 + w\_pump1}$$

3) **Pump 2 Output Enthalpy:**

$$\boldsymbol{h4 = h3 + w_{pump2}}$$

4) **Enthalpy After Pump:**

$$\boldsymbol{h\_out = h\_in + wpump}$$

5) **Turbine Expansion with Isentropic Efficiency:**

$$\boldsymbol{h\_out = h\_in - \eta\_turbine \times (h\_in - h\_isentropic)}$$

- Turbine Expansion with Isentropic Efficiency (to OFWH):

$$\boldsymbol{h6 = h5 - \eta\_turbine \times (h5 - h6s)}$$

- Turbine Expansion with Isentropic Efficiency (to condenser):

$$\boldsymbol{h7 = h5 - \eta_{turbine} \times (h5 - h7s)}$$

6) **Extraction Mass Fraction (y):**

$$y = (h3 - h2)/(h6 - h2)$$

7) **Heat Input:**

$$q\_in = h5 - h4$$

8) **Net Work Output:**

$$w\_net = (h5 - h6) + (1 - y)(h6 - h7) - wpump1 - (1 - y) \times wpump2$$

9) **Thermal Efficiency:**

$$\eta\_th = w\_net / q\_in$$

◆ COMBINED CYCLE Equations:

1) **Heat Transfer – Brayton to Rankine:**

$$q_{to\ Rankine} = h_4^{Brayton\ Exit} - h_{stack}$$

2) **Steam Generated (mass ratio per 1 kg air):**

$$\frac{m_{steam}}{m_{air}} = \frac{q_{to\ rankine}}{q_{in\ rankine}}$$

3) **Work from Rankine:**

$$W_{Rankine,\ Total} = \left(\frac{m_{steam}}{m_{air}}\right) \cdot W_{net,\ Rankine}$$

4) **Combined Cycle Efficiency:**

$$\eta_{combined} = \frac{W_{Brayton} + W_{Rankine,\ total}}{q_{in,\ Brayton}} \cdot 100$$

3 RESULTS

Effect of Compression Ratio on Thermal Efficiency:

If we accept the Cp value as constant, there

is a formula for efficiency:

$$\eta_{th=1-\frac{1}{(r_{ps})^{\frac{k-1}{k}}}}$$

When we look at this formula, as the compression ratio (r=P2/P1) value increases, efficiency should also increase. In fact, as can be seen from the table, this is true up to a point, but we need to look carefully at the area in the table where the color changes!

The increase in thermal efficiency does not continue indefinitely, it gradually slows down, as expected in many thermodynamic cycles, but the surprising thing is that after a certain point it is not the rate of increase in efficiency that decreases, but the thermal efficiency itself!

The reason the above formula seems to increase indefinitely is that it is overly theoretical. In real engineering systems, efficiency initially increases as the work provided by the turbine increases. However, after a certain compression ratio, efficiency begins to decrease due to increased compressor work and decreased heat input.

Table 1: Relationship between compression ratio (r) and thermal efficiency. The r value in the increasing trend is shown in blue, in the decreasing trend in red, and the maximum value seen in the table is shown in green.

Compression Ratio (r)	Thermal efficiency (%)
2.0	13.75058
3.473684	22.682131
4.947368	27.475834
6.421053	30.532391
7.894737	32.649463
9.368421	34.188205
10.842105	35.331822
12.315789	36.194139
13.789474	36.843843
15.263158	37.32841
16.736842	37.678127
18.210526	37.918178
19.684211	38.062843
21.157895	38.125376
22.631579	38.118459
24.105263	38.049203
25.578947	37.921954
27.052632	37.740792
28.526316	37.510681
30.0	37.236027

a. Why does efficiency increase as r initially increases?

When the compression ratio increases, the compressor outlet pressure increases. This means that the gas expands at higher pressure, in short, more work is obtained from the turbine. Of course, as the work produced by the turbine increases, the work produced by the compressor, which is negative work for us, also increases, but since the work produced by the turbine is more, the net work increases. Accordingly, efficiency increases.

b. Why does efficiency start to decrease after a certain value of r?

In Brayton, “backwork” is also very important. Here, "backwork" refers to the work done by the compressor.

$w_{net} = (w_{turb}) - w_{comp}$. Remember that $w_{comp} = -\int v\, dP$;

v is large for gases; so, Wcomp is very high about 40-80% of Wturb.

When r increases too much, the compressor outlet temperature increases too much. This increases the enthalpy difference, meaning the compressor consumes more work. As mentioned above, since the compressor consumes such a high rate of work, the speed of this increase now starts to cause a loss of efficiency. In short; even if the turbine produces more work, the work consumed by the compressor increases faster. Although not addressed in this project, these high temperatures mean more difficult conditions for the turbine and compressor. The life of these equipment will be shortened and more complex and durable equipment will be required. Of course, these also mean extra costs, meaning a loss of efficiency. Although this is more a matter of material science, it should be kept in mind when working on efficiency in real life.

In the graph above, we see the compression ratio and thermal efficiency relationship better as a percentage. Both the initial increase rate and the point where it gradually decreases and the diagram formed by these values are seen in the graph.

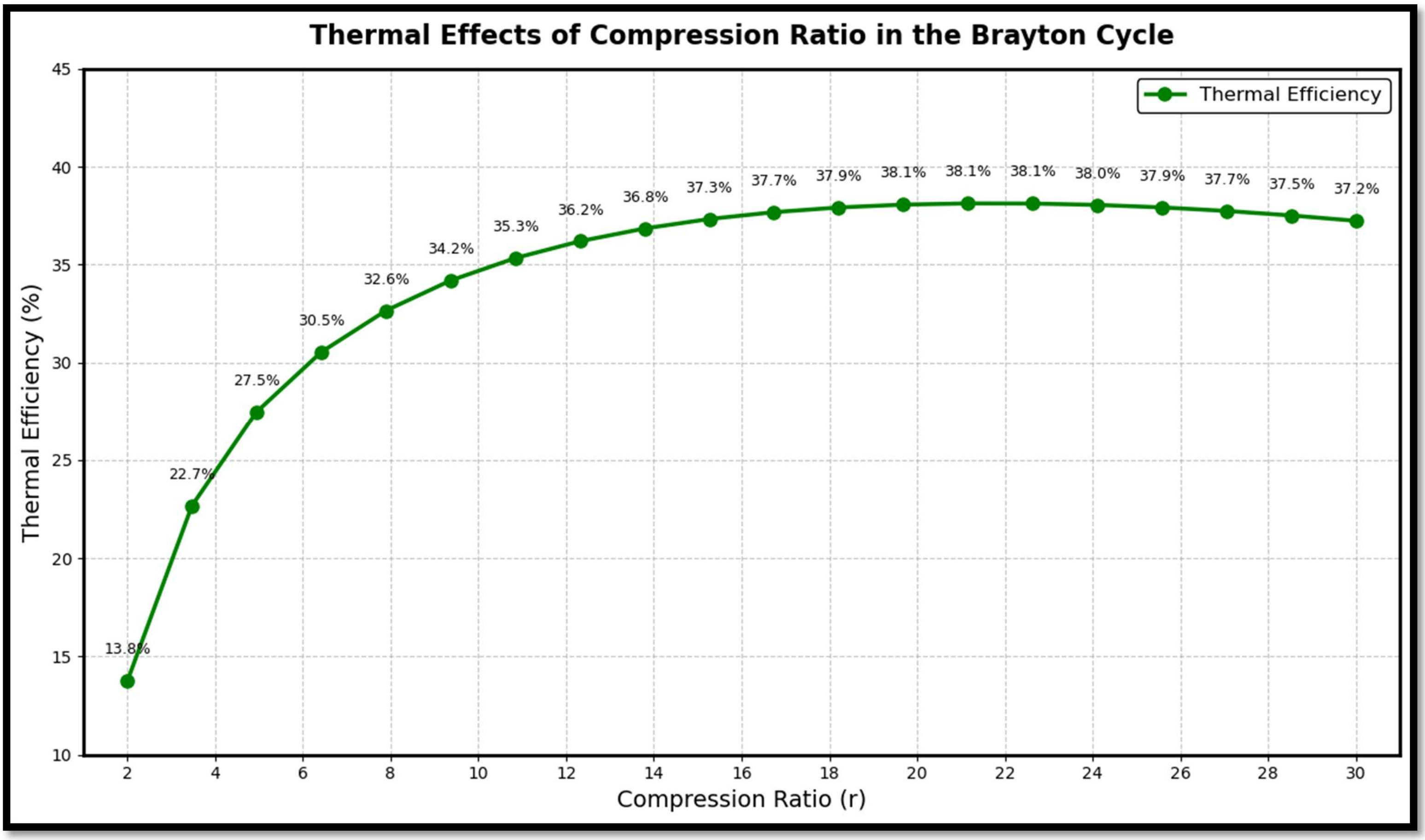


Figure 2: Plots of 20 different r values and their corresponding thermal efficiencies are shown as percentages.

Compression ratio value giving maximum net work:

The r value required for maximum net work in this part of the project was found.

For better understanding, not only the ideal r value and maximum net work were found, but also the net work corresponding to that r value was calculated for many r values, and the graphs and tables of these results were drawn.

Here again, as we saw when examining the effect of the r value on thermal efficiency, the net work does not increase continuously as the r value increases. It reaches a maximum at some point and then starts to decrease. The reason for this, as explained above, is that the consumption of the compressor is gradually increasing and the net work is now starting to decrease.

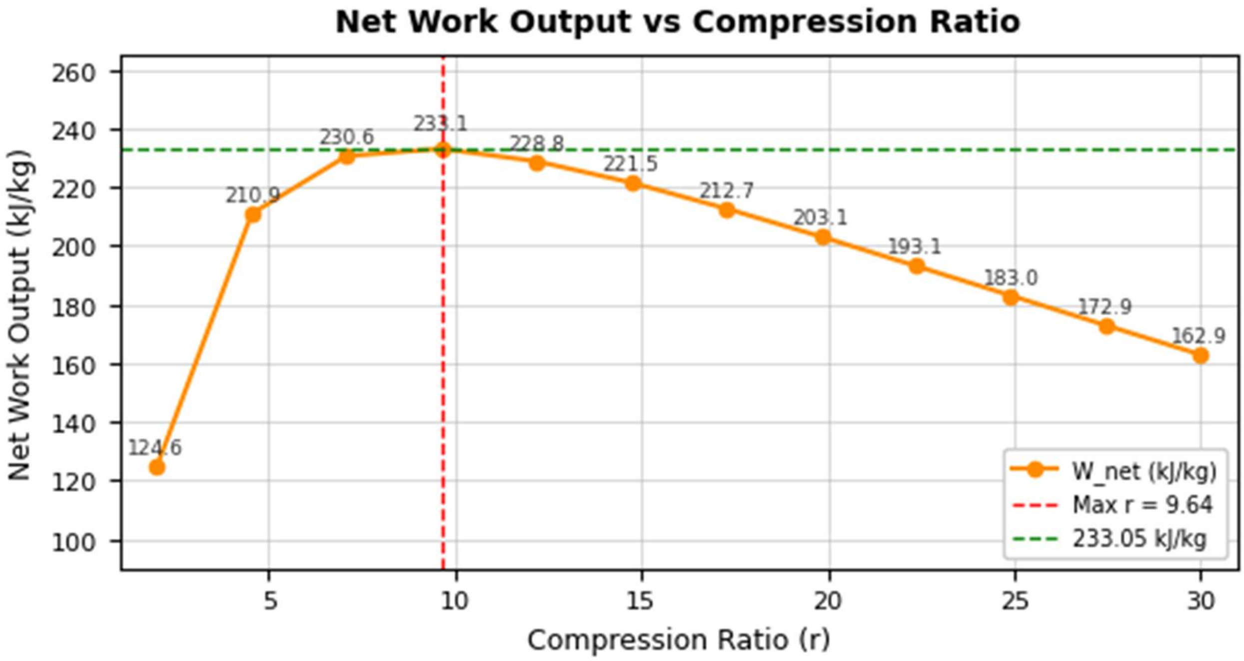


Figure 3: The graph shows the relationship between compression ratio and net work and the desired r value for maximum net work.

On the right, we see the reflection of the values given in the table in the graph above. After reaching the maximum net work, we see that the net work decreases rapidly as the r value continues to increase. From here, we understand that the work consumed by the compressor increases rapidly after that point, while the work produced by the turbine remains more limited. And one of the most striking points is that the r values providing maximum net work and maximum thermal efficiency are very different from each other. This situation is a physical result of the balance between energy quality and energy quantity in the Brayton cycle. Accordingly, different r values are targeted according to the desired engineering process and studies are carried out accordingly. For example, while net work is more important in mobile power systems, thermal efficiency is more important in power plants.

Compression Ratio (r)	Net Work (kJ/kg)
2.0	124.63
4.55	210.94
7.09	230.57
9.64	233.05
12.18	228.82
14.73	221.53
17.27	212.7
19.82	203.11
22.36	193.13
24.91	183.03
27.45	172.91
30.0	162.88

Table 2: It shows how the net work changes as the r value increases, and the ideal r value cell for

If we look at the examples of the use of gas cyclers given in the introduction. Which of these can be more important than net work? Emergency and peak power generation and Aircraft propulsion system. One is an emergency system, as the name suggests, where efficiency is secondary, the priority is instantaneous high power output. Similarly, in Aircraft propulsion, the important thing is thrust. In other words, net work is more important than efficiency. (At least that's true of older military jets. There are high-bypass turbofan aircraft today with much more complex and sophisticated systems, but that's not the point.)

In Gas Cycle Power Plants, since resources must be used efficiently when generating power, both efficiency and power balance are attempted to be achieved. In combined cycle power plants, this r value is as high as possible/as much as the system allows. In short, everything depends on what the desired job is and what your purpose is.

Finally, the graph of the r value that gives the maximum thermal efficiency is given below. It is important both as information and you can compare and examine it with the Net work-compression ratio graph. (Attention! The values in the graph are exactly the same as the values given in Figure 2. The reason why the maximum efficiency is 38.13 here and 38.1 there is due to the rounding to the maximum 3 digits while creating the graph.)

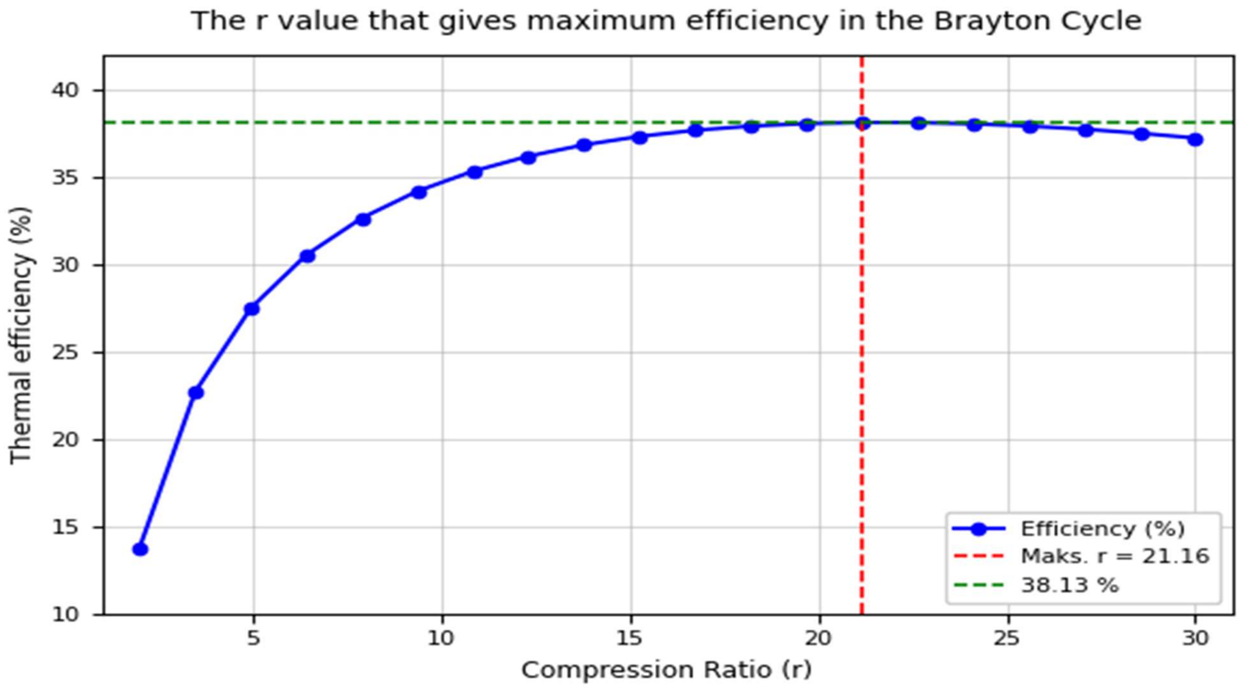


Figure 4: The effect of the compression ratio value on thermal efficiency and the r value that maximizes thermal efficiency.

Effect of turbine and compressor efficiency on thermal efficiency for a fixed r value:

Thermal efficiency was examined by taking the constant r value as 10, and the compressor and turbine efficiencies were changed to 0.86, 0.88, 0.90, 0.92 and 0.94, respectively. As can be seen in the graph, a very significant increase in thermal efficiency was observed.

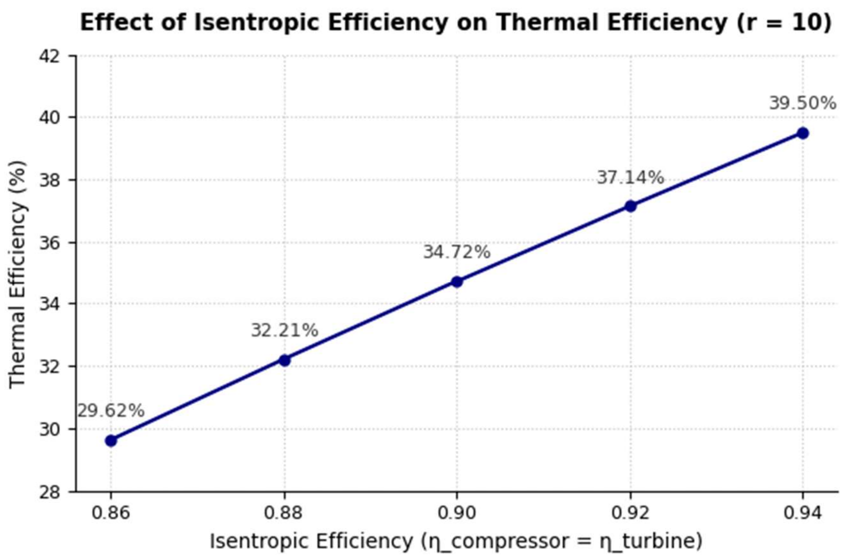


Figure 5: Graph of the effect of compressor and turbine efficiency on thermal efficiency

When the efficiency of the equipment in question was increased from 86% to 94%, the thermal efficiency increased from 29.62% to 39.50%. This increase shows that the reduction in losses experienced by the compressor and turbine directly affects the cycle performance. The increase in efficiency both increases the net work production and reduces fuel consumption. In conclusion, these data show that the increase in thermal efficiency will be achieved not only by increasing the temperature limits, but also by increasing the efficiency of the turbine and compressor themselves. The best result is obtained when the two are done together.

Effect of compression ratio on thermal efficiency in a combined cycle:

In this part of the project, a combined system was investigated. The features of this combined system are as follows.

- Topping cycle is Brayton with the data:

Tmax = 1200 K, Tmin = 298.15 K, ηgas-turbine = 0.90and ηcompressor = 0.90.

- Bottoming cycle is Rankine with the data:

  Steam-turbine inlet = 7 MPa, 673.15K

  OFWH = 800 kPa

  Reheat exit = 800 kPa, 673.15K

  Condenser = 10 kPa

  ηsteam-turbine = ηpumps = 0.90

In this part of the project, all of these values are fixed. Thermal efficiency was examined by changing only the compressor pressure ratio that affects the Brayton cycle.

First, let's look at the following images as a representation. In these images, we see the schematic diagram, connection and T/s diagrams of the combined power plant.

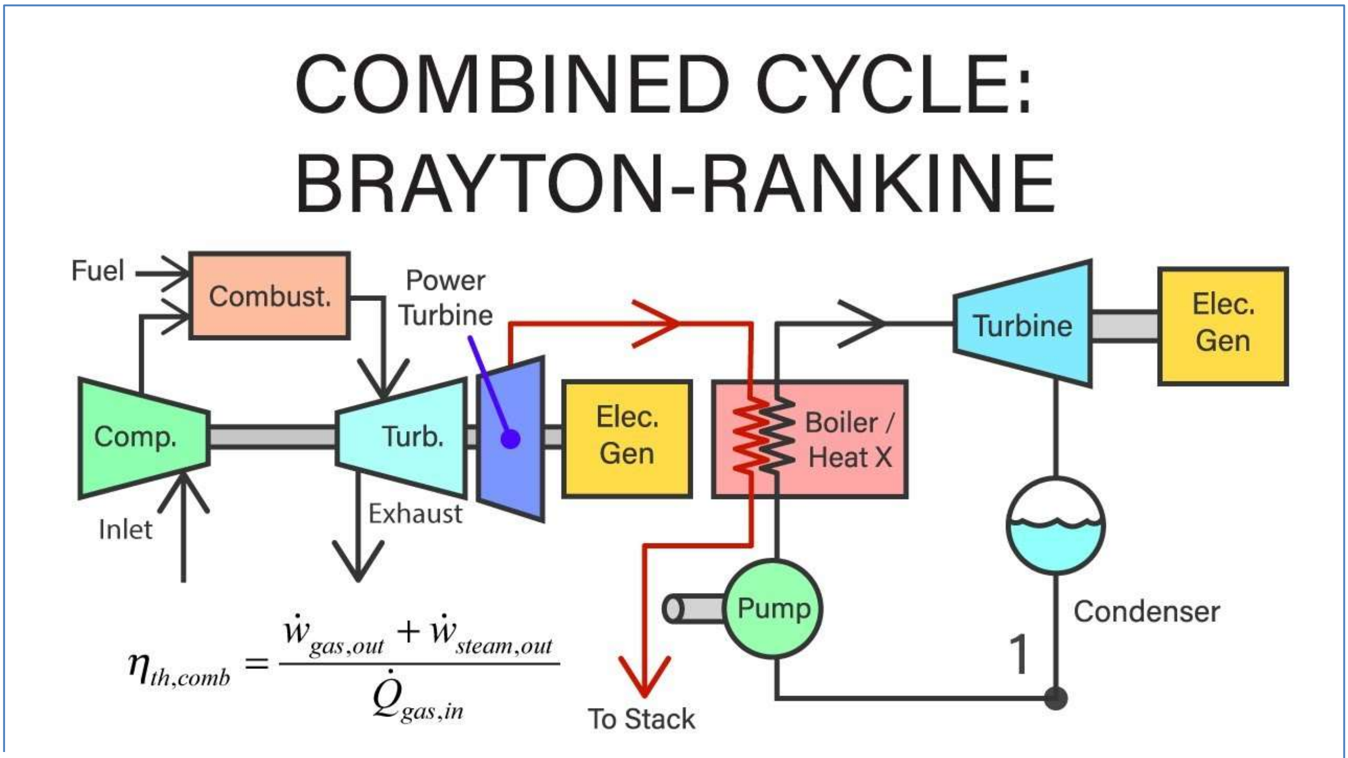
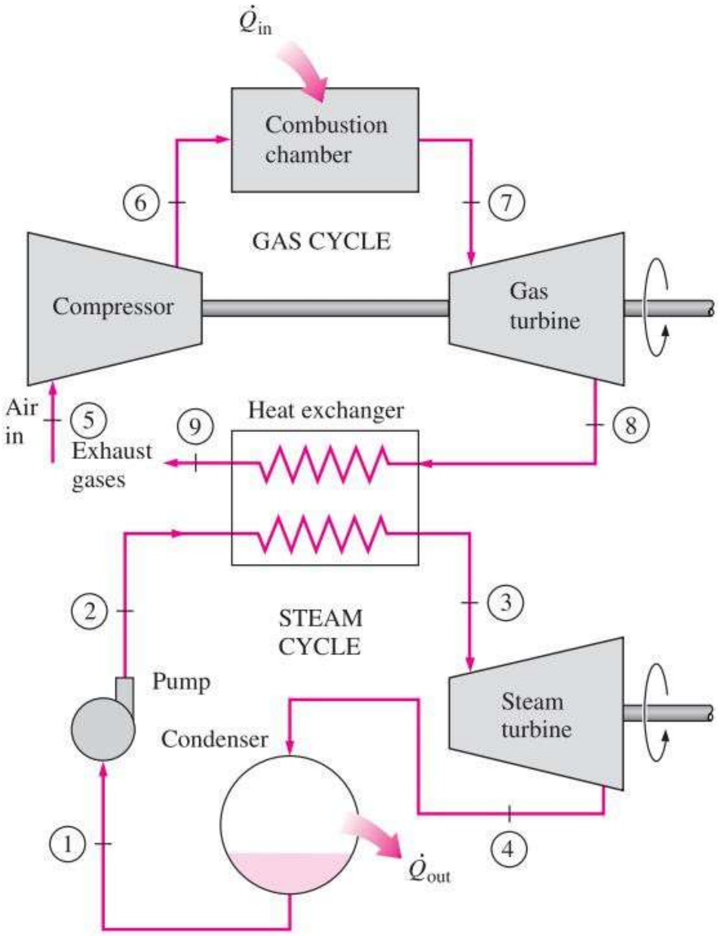


Figure 6: Image of a superficial schematic representation of a simple combined cycle (Hugo, 2014)



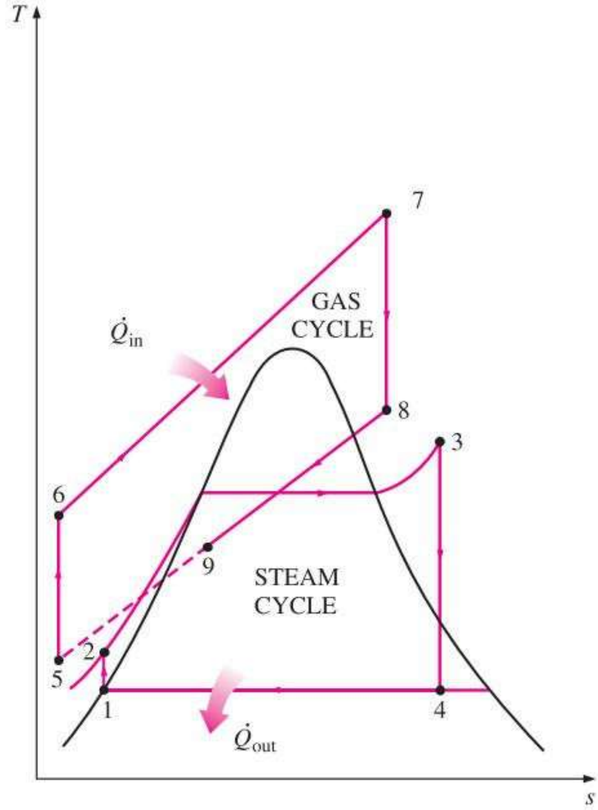


Figure 7: Combined cycle diagram and its T/s diagram (Martech Boiler, 2021)

In this visual, the scheme is shown vertically in addition to the first one and the T/s diagram of the combined cycle is also shown. It should be noted that the visuals here are representative. For example, while a single pump is used in the Rankine cycle section of the visuals, two pumps are used in the Rankine cycle section of our project. These may change, it does not matter. The purpose of the visuals is to better understand the structure/flow.

It should be noted here that all the heat required by the Rankine cycle is provided by the air leaving the Brayton cycle. Therefore, we need to be careful whether the changes we make to the Brayton cycle affect the air leaving the Brayton. So, in this part of the project, let's look at the relationship between the single variable r and the temperature of the air coming out of the Brayton.

Compression Ratio (r)	Exit Temp (K)
2.0	1029.83
4.95	842.57
7.89	759.7
10.84	708.3
13.79	671.8
16.74	643.89
19.68	621.5
22.63	602.95
25.58	587.19
28.53	573.56

Table 3: Change in temperature of air coming out of Brayton according to compression ratio

Although the first thing that comes to mind is to use the values that maximize thermal efficiency in both Rankine and Brayton in a combined cycle (at least in an energy production process where efficiency is important), things are of course different in real life. Since the heat required for the Rankine cycle is provided by Brayton, our r limit here would be either the point where efficiency drops in Brayton or the point where Rankine is disabled. In the table, the values below 673.96 K are colored red because below that value the Rankine cycle does not work and is disabled! Because the temperature of the air coming out of Brayton cannot meet the working conditions of Rankine. the point where Rankine is disabled is much smaller than the point where thermal efficiency drops in Brayton itself. For the best thermal efficiency, r should be increased to the point allowed by Rankine.

Since the Rankine values given in the project are known and the r value does not affect it, its efficiency is fixed. When the necessary calculations are made, it is found that its efficiency is 36.53 percent.

The graph below shows how both this and the combined efficiency change. After the r value exceeds 13.59, the combined cycle is meaningless and only the Brayton cycle works. Therefore, for the conditions given in this project, the maximum thermal efficiency is 51.63% when r is 13.59. While the thermal efficiencies of Rankine and Brayton are around 36%-38%, the efficiency increasing to 50.09% means a very, very serious increase.

In addition to this advantage, the r point where the net work is maximum for Brayton is 9.64. When r is 13.59, the net work value is approximately 5-6 kJ/kg less than the maximum value. That is, the r value that gives the highest efficiency for the combined cycle also remains at the upper limit for net work. It's not the peak, but it's still a very strong point. This is one of the biggest indicators of why combined cycles are preferred.

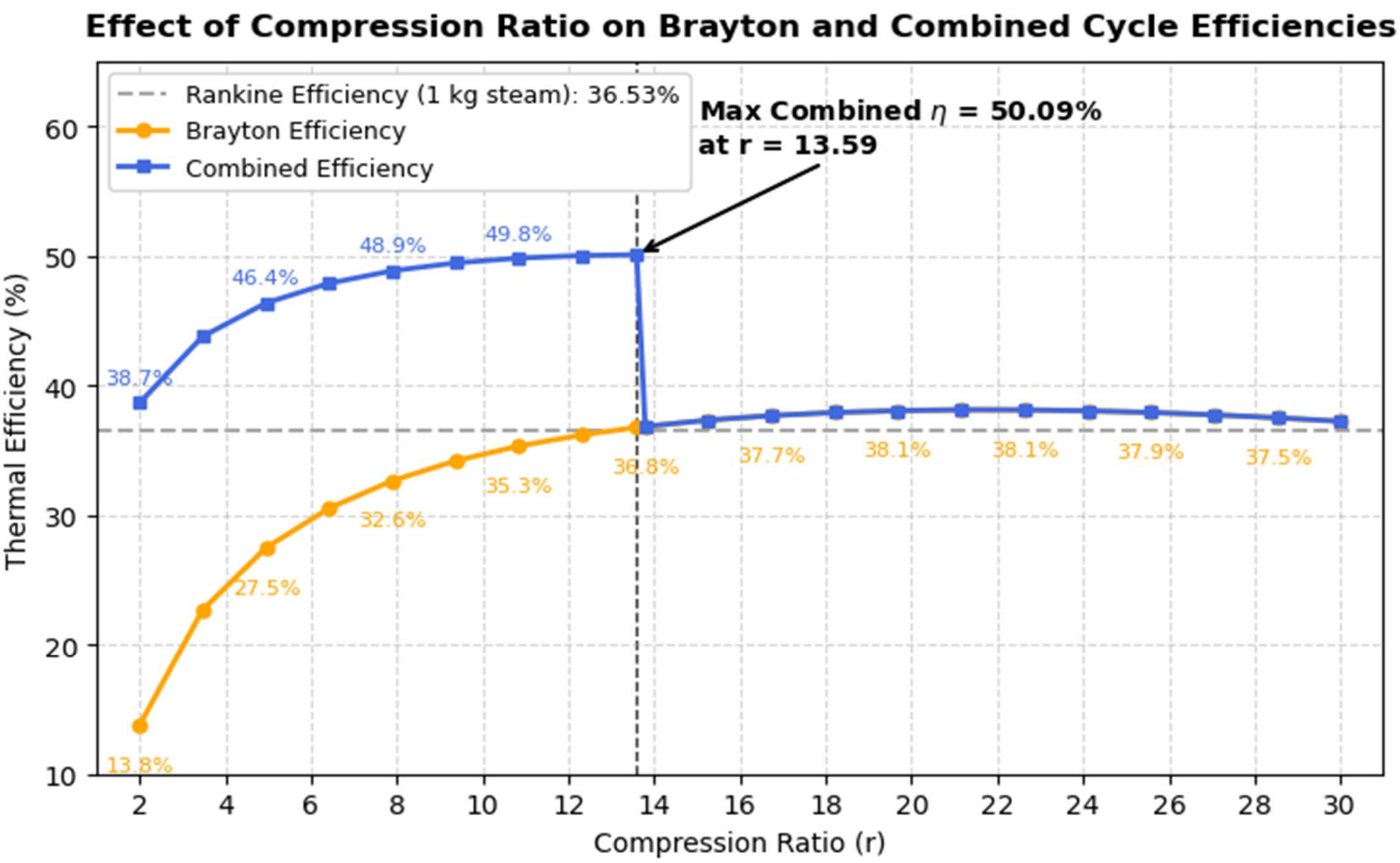


Figure 8: Effect of compression ratio on thermal efficiency of both Brayton and combined cycles and the break-even point of the combined system.

4 CONCLUSION

In summary, the first part of this project was focused on the Brayton Cycle. The compression ratio value of the Brayton cycle was changed to examine the thermal efficiency and net work values. In another case, the thermal efficiency was measured by changing the compressor and turbine efficiencies while keeping the r value constant.

As a result, it is usually important to obtain a larger net work in each power cycle. However, when we consider only efficiency, we can greatly increase the r value in the Brayton cycle. Although in theory, as the r value increases, it seems that the thermal efficiency does not go to infinity in the real engineering approach and decreases after a certain point, in our project, the thermal efficiency reaches 38.13% when the r value is 21.16.

Since the net work and r value are examined in the other part of the project, it would not be reasonable to use this point operationally. When we look at these results, when the r value is 9.64, the maximum net work reaches 233.05 kJ/kg. Also, as seen in the graphs and tables, as the r value increases, net work begins to decline more sharply compared to the efficiency due to the compressor's work increases significantly because it has to create more pressure difference.

The thermal efficiency at the r value that gives the maximum net work is about 34.5%. Considering both the importance of net work in power cycles and the fact that net work decreases faster than efficiency, an optimum r value should be preferred. This optimum r value provides us with an ideal point in terms of both net work and thermal efficiency.

This optimum r value is usually closer to the net work value for the reasons mentioned above. While $r \approx 11–13$ in older generation industrial turbines, the average r value in Brayton cycles used only for power generation is around 12 to 16 based on a more general average.

In another part of the project, the r value was kept constant with the other values given by Brayton and only the efficiency of the turbine and compressor was increased and the thermal efficiency was examined. The results show that the increase in the efficiency of the turbine and compressor also increases the thermal efficiency very significantly. These results show that, the r value is not the only way to increase the thermal efficiency. The thermal efficiency advantage provided by this can be evaluated by decreasing the r value for higher net work.

In addition to all these, although it has not been fully processed in this project, an increase in the efficiency of these components will also increase the net work. In short, a turbine and compressor with good efficiency are essential for a good Brayton cycle. In this way, both thermal efficiency and net work are increased.

The combined cycle was examined in the last part of the project. The most important point here is that Brayton meets the heat required by Rankine. Therefore, this is our first and most important limit point. We saw that the temperature of the air leaving Brayton decreases as the compression ratio increases. We found the r value that gives the minimum temperature required for Rankine to work as 13.59, above this value Rankine is disabled and only Brayton continues to work. Therefore, $r = 13.59$. This should be our considered target value because Rankine continues to work and this r value is a very good point for both thermal efficiency and net work, we saw that in the first part of the project. The thermal efficiency, which is 36-38% for Rankine and Brayton, has increased to 50.09% with the combined system. This value can be achieved not only theoretically but also in real life.

Since two separate complex systems are established, the initial investment cost will be high but all these advantages demonstrate the critical importance and value of combined cycles in modern energy systems. These systems will continue to exist and develop today and in the future.

5 REFERENCES

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6 APPENDIX

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from scipy.interpolate import interp1d
from scipy.optimize import fsolve
import re

# ==========================================
# 1. DATA LOADING AND PREPARATION
# ==========================================
def load_data(filename):
    try:
        with open(filename, 'r') as f:
            content = f.read()
        # Fix potential typo in source file
        content = content.replace("1182.9", "1182.9")
        numbers = np.array([float(x) for x in re.findall(r"[-+]?\d*\.\d+|[-+]?\d+", content)])
```

```
            num_cols = 6
            num_rows = len(numbers) // num_cols
            return          numbers[:num_rows*num_cols].reshape((num_rows,
num_cols))
        except:
            return np.zeros((10, 6))

    data = load_data("AirDataSonntag.txt")

    # Assign Columns (Using Enthalpy h instead of Internal Energy u)
    T_data = data[:, 0]  # Temperature [K]
    h_data = data[:, 1]  # Enthalpy [kJ/kg]
    s_data = data[:, 5]  # Entropy [kJ/kg-K]

    # Interpolation Functions
    h = interp1d(T_data, h_data, kind='cubic', fill_value="extrapolate")
    s = interp1d(T_data, s_data, kind='cubic', fill_value="extrapolate")

    # Constants
    T1 = 298.15
    T3 = 1200.0
    R = 0.287

    print(">>> SIMULATION RESULTS <<<\n")

    # ==========================================
    # PART 1: THERMAL EFFICIENCY (Table 1)
    # ==========================================
    eta_c = 0.90
    eta_t = 0.90

    def thermal_efficiency(r):
        # Compressor
        s1 = s(T1)
        def s_diff(T): return s(T) - s1 - R * np.log(r)
        T2s = fsolve(s_diff, T1 * r ** 0.3)[0]
        h1 = h(T1)
        h2s = h(T2s)
        h2 = h1 + (h2s - h1) / eta_c
```

```
    # Turbine
    s3 = s(T3)
    def s4_diff(T): return s(T) - s3 + R * np.log(r)
    T4s = fsolve(s4_diff, T3 / r ** 0.3)[0]
    h3 = h(T3)
    h4s = h(T4s)
    h4 = h3 - eta_t * (h3 - h4s)

    w_net = (h3 - h4) - (h2 - h1)
    q_in = h3 - h2
    return w_net / q_in

r_values = np.linspace(2, 30, 20)
eta_values_percent = [thermal_efficiency(r) * 100 for r in r_values]

df1 = pd.DataFrame({
   "Compression ratio (r)": r_values,
   "Thermal efficieny (%)": eta_values_percent
})
print("--- PART 1 OUTPUT (Efficiency) ---")
print(df1.to_string(index=False))
print("\n" + "="*30 + "\n")

# ==========================================
# PART 2: NET WORK (Table 2)
# ==========================================
def net_work(r):
    s1 = s(T1)
    T2s = fsolve(lambda T: s(T) - s1 - R * np.log(r), T1 * r**0.3)[0]
    h1 = h(T1)
    h2s = h(T2s)
    h2 = h1 + (h2s - h1) / eta_c

    s3 = s(T3)
    T4s = fsolve(lambda T: s(T) - s3 + R * np.log(r), T3 / r**0.3)[0]
    h3 = h(T3)
    h4s = h(T4s)
    h4 = h3 - eta_t * (h3 - h4s)

    return (h3 - h4) - (h2 - h1)
```

```
r_values_2 = np.linspace(2, 30, 12)
w_net_values = [net_work(r) for r in r_values_2]
w_net_max = max(w_net_values)
r_opt = r_values_2[np.argmax(w_net_values)]

df2 = pd.DataFrame({
   "Compression Ratio (r)": np.round(r_values_2, 2),
   "Net Work (kJ/kg)": np.round(w_net_values, 2)
})
print("--- PART 2 OUTPUT (Net Work) ---")
print(df2.to_string(index=False))
print(f"\n Maximum Net Work: {w_net_max:.2f} kJ/kg")
print(f" Optimal Compression Ratio: r = {r_opt:.2f}")
print("\n" + "="*30 + "\n")

# ==========================================
# PART 3: SENSITIVITY ANALYSIS (r=10)
# ==========================================
def thermal_efficiency_sens(r, eta_c_val, eta_t_val):
    s1 = s(T1)
    T2s = fsolve(lambda T: s(T) - s1 - R * np.log(r), T1 * r**0.3)[0]
    h1 = h(T1)
    h2s = h(T2s)
    h2 = h1 + (h2s - h1) / eta_c_val

    s3 = s(T3)
    T4s = fsolve(lambda T: s(T) - s3 + R * np.log(r), T3 / r**0.3)[0]
    h3 = h(T3)
    h4s = h(T4s)
    h4 = h3 - eta_t_val * (h3 - h4s)

    w_net = (h3 - h4) - (h2 - h1)
    q_in = h3 - h2
    return (w_net / q_in) * 100

r_fixed = 10
isentropic_efficiencies = [0.86, 0.88, 0.90, 0.92, 0.94]
eta_th_list = []
```

```
    for eta in isentropic_efficiencies:
        eta_th     =     thermal_efficiency_sens(r_fixed,     eta_c_val=eta,
eta_t_val=eta)
        eta_th_list.append(eta_th)

    df3 = pd.DataFrame({
        "η_c = η_t": isentropic_efficiencies,
        "Thermal Efficiency (%)": np.round(eta_th_list, 2)
    })
    print("--- PART 3 OUTPUT (Sensitivity r=10) ---")
    print(df3.to_string(index=False))
    print("\n" + "="*30 + "\n")

    # ==========================================
    # PART 4: COMBINED CYCLE
    # ==========================================
    T_stack = 460.0
    T4_limit = 673.15 # 400C (Rankine Limit)
    eta_rankine_1kg = 36.53
    q_in_rankine = 2917.0 # Corrected Rankine Heat Input

    def calculate_efficiencies_comb(r):
        # Brayton Cycle
        s1 = s(T1)
        T2s = fsolve(lambda T: s(T) - s1 - R*np.log(r), 350)[0]
        h1 = h(T1)
        h2s = h(T2s)
        h2 = h1 + (h2s - h1) / eta_c

        s3 = s(T3)
        T4s = fsolve(lambda T: s(T) - s3 + R*np.log(r), 1000)[0]
        h3 = h(T3)
        h4s = h(T4s)
        h4 = h3 - eta_t * (h3 - h4s)

        w_net_brayton = (h3 - h4) - (h2 - h1)
        q_in_brayton = h3 - h2
        eta_brayton = w_net_brayton / q_in_brayton * 100

        T4_real = fsolve(lambda T: h(T) - h4, 800)[0]
```

```
        # Combined Cycle Logic
        if T4_real >= T4_limit:
            h_stack = h(T_stack)
            q_to_rankine = max(0, h4 - h_stack)
            m_steam = q_to_rankine / q_in_rankine
            w_rankine = m_steam * (q_in_rankine * eta_rankine_1kg / 100)
            eta_combined  =  (w_net_brayton  +  w_rankine)  /  q_in_brayton  *
100
        else:
            eta_combined = eta_brayton

        return eta_brayton, eta_combined, T4_real

    r_values_comb = np.linspace(2, 30, 20)
    results_comb = []

    for r in r_values_comb:
        eta_b, eta_cmb, T4_real = calculate_efficiencies_comb(r)
        results_comb.append([r, eta_b, eta_cmb, T4_real])

    df4 = pd.DataFrame(results_comb, columns=[
       "Compression Ratio (r)",
       "Brayton Efficiency (%)",
       "Combined Efficiency (%)",
       "T4 Exit Temp (K)"
    ]).round(2)

    print("--- PART 4 OUTPUT (Combined Cycle) ---")
    print(df4.to_string(index=False))

    # Specific Point r = 13.59
    r_max = 13.59
    eta_b_max,            eta_combined_max,            T4_max             =
calculate_efficiencies_comb(r_max)

    print("\nmax (r = 13.59)")
    print(f"Compression Ratio (r): {r_max}")
    print(f"Brayton Efficiency: {eta_b_max:.2f}%")
    print(f"Combined Efficiency: {eta_combined_max:.2f}%")
    print(f"T4 Exit Temperature: {T4_max:.2f} K")
```

```
    print("\nI found the maximum r value using the minimum allowable temperature for Rankine")
    print(f"r = 13.59:")
    print(f"T4 (Rankine inlet) = {T4_max:.2f} K")
```