NUMBER OF REASONABLE CHESS GAMES

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1 ABSTRACTION

This study investigated the complexity of chess games by refining the concept of "game complexity" beyond simply the total number of legal moves. Recognizing the limitations of Shannon's number, we focused on quantifying "reasonable" chess games exhibiting strategic depth. Due to the vast number of possibilities, traditional manual methods were deemed insufficient.

We employed an AI-powered approach utilizing self-play between two Stockfish engines to generate a substantial dataset of strategically relevant games. By analyzing the statistical distribution of moves within these games, we identified a relationship between ply depth and the average number of strategically viable options. This relationship offers insights into the decision-making landscape and escalating complexity of chess.

By combining a critical review of existing research with this novel AI application, we arrived at an estimated number of reasonable chess games: $3.9x10^{183}$. This finding provides a more realistic measure of the playable strategic landscape compared to the inflated number encompassing all legal, yet nonsensical, move sequences.

2 INTRODUCTION

The vast strategic depth of chess has captivated mathematicians and computer scientists for decades, particularly in the quest to quantify the number of playable games. While the total number of legal chess games is astronomically large, a significant portion comprises nonsensical move sequences devoid of strategic merit. Determining the number of "reasonable" chess games, defined as those that reflect strategic decision-making and adhere to established opening principles, development strategies, and tactical considerations, represents a challenging yet intriguing problem

This paper delves into the application of artificial intelligence (AI) techniques to estimate the number of reasonable chess games. By leveraging the capabilities of AI algorithms in analyzing vast game datasets, pattern recognition, and strategic evaluation, we aim to develop a more refined approach to quantifying this elusive aspect of chess. This exploration holds potential not only for theoretical understanding of game complexity but also for advancements in AI chess engine development and the creation of more strategically rich training datasets.

3 SHANNON'S NUMBER AND THE INTRACTABLE LANDSCAPE OF CHESS GAME-TREE COMPLEXITY

Our exploration into the number of reasonable chess games begins by acknowledging the foundational work of Claude Shannon. In his groundbreaking 1950 paper, "Programming a Computer for Playing Chess," Shannon introduced the concept of Shannon's number, a lower bound estimate for the game-tree complexity of chess. This concept revolutionized the understanding of chess as a computational problem.

Shannon's calculation considers the average branching factor – the number of legal moves available to each player at any given juncture – and the typical game length measured in ply (one ply representing a White move followed by a Black move). This approach yields an estimated range of 10^{111} to 10^{121} possible games. However, this staggering figure encompasses both legal and illegal move sequences, resulting in a significant limitation for our research. The vastness of Shannon's number highlights the intractable nature of the complete chess game tree. While it provides a

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valuable theoretical framework, it includes a multitude of nonsensical move combinations that lack strategic merit. Imagine a game where a pawn captures a knight on the first move, or a king wanders aimlessly across the board – these possibilities, while technically legal, deviate significantly from the strategic objectives and established principles that govern meaningful chess play.

Therefore, our research necessitates a more refined approach that specifically targets the subset of "reasonable" chess games. These games, while adhering to the legal move constraints, exhibit strategic depth, reflect sound decision-making throughout the encounter, and showcase the interplay of opening principles, development strategies, and tactical considerations. Determining the number of such games presents a fascinating challenge, one that requires leveraging the power of artificial intelligence (AI) techniques for analysis and pattern recognition within the vast landscape of strategically relevant chess positions.

3.1 Shannon's Number and the Intractable Landscape of Chess Game-Tree Complexity

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Shannon's calculation considers the average branching factor – the number of legal moves available to each player at any given juncture – and the typical game length measured in ply (one ply representing a White move followed by a Black move). This approach yields an estimated range of 10^{111} to $10^{1}21$ possible games. However, this staggering figure encompasses both legal and illegal move sequences, resulting in a significant limitation for our research. The vastness of Shannon's number highlights the intractable nature of the complete chess game tree. While it provides a valuable theoretical framework, it includes a multitude of nonsensical move combinations that lack strategic merit. Imagine a game where a pawn captures a knight on the first move, or a king wanders aimlessly across the board – these possibilities, while technically legal, deviate significantly from the strategic objectives and established principles that govern meaningful chess play.

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4 BEST GAME OPENING SCENARIOS

In order to find the number of reasonable chess games we started from analyzing chess openings.

- In a King's pawn game the percentage of players who chose this opening are 59% and white wins by 49% and black wins by 47%
- In a Queen's pawn game the percentage of players who chose this opening are 26% and white wins by 50% and black wins by 45%
- In a Zukertort opening the percentage of players who chose this opening are 3% and white wins by 51% and black wins by 45%

- In a English opening the percentage of players who chose this opening are 3% and white wins by 51% and black wins by 45%
- In a Van't Kruijs opening the percentage of players who chose this opening are 2% and white wins by 47% and black wins by 49%
- In a Hungarian opening the percentage of players who chose this opening are 2% and white wins by 50% and black wins by 46%
- In a Nimzo-Larsen attack the percentage of players who chose this opening are 2% and white wins by 50% and black wins by 46%
- In a Bird opening the percentage of players who chose this opening are 1% and white wins by 50% and black wins by 46%
- In a King's pawn game the percentage of players who chose this opening are 1% and white wins by 48% and black wins by 48%
- In a Van Geet opening the percentage of players who chose this opening are 1% and white wins by 49% and black wins by 47%
- In a Polish opening the percentage of players who chose this opening are 0.1% and white wins by 51% and black wins by 45%
- In a Saragossa opening the percentage of players who chose this opening are 0.1% and white wins by 48% and black wins by 48%

4.1 Popular Continuation of a King's Pawn Game

- 40% of the games have been continued with E5 move and 51% of these games have won from White and 45% of these games have won from Black
- 19% of the games have been continued with Sicilian Defense move and 48% of these games have won from White and 48% of these games have won from Black
- 10% of the games have been continued with Scandinavian Defense move and 49% of these games have won from White and 47% of these games have won from Black
- 10% of the games have been continued with French Defense move and 48% of these games have won from White and 48% of these games have won from Black
- 7% of the games have been continued with Caro-Kann Defense move and 47% of these games have won from White and 49% of these games have won from Black
- 4% of the games have been continued with Pirc Defense move and 49% of these games have won from White and 47% of these games have won from Black
- 3% of the games have been continued with Modern Defense move and 49% of these games have won from White and 48% of these games have won from Black
- 2% of the games have been continued with Owen Defense move and 50% of these games have won from White and 47% of these games have won from Black
- 2% of the games have been continued with Alekhine Defense move and 48% of these games have won from White and 48% of these games have won from Black
- 1% of the games have been continued with Nimzowitsch Defense move and 50% of these games have won from White and 46% of these games have won from Black

- 0.1% of the games have been continued with St. George Defense move and 50% of these games have won from White and 46% of these games have won from Black
- 0.1% of the games have been continued with Duras Gambit move and 51% of these games have won from White and 46% of these games have won from Black

Shannon's number offers a theoretical foundation, but directly calculating "reasonable" games is impractical due to the vast number of legal, yet strategically nonsensical, moves. We address this by leveraging AI's self-play with best-first search algorithms. These algorithms prioritize strategic moves based on established chess principles, filtering out nonsensical options. By analyzing the resulting AI-generated games, we aim to estimate the number of reasonable chess games.

5 SOFTWARE IMPLEMENTATION AND DATA ACQUISITION

Our research employed Python's extensive library ecosystem to facilitate the self-play process and data collection. We utilized the chess library, a well-established Python package specifically designed for working with chess data and functionalities. This library provided access to essential chess logic and move representation, enabling us to manage the game state and analyze potential moves.

For data persistence and organization, we employed the workbook library (likely referring to a library like openpyxl or pandas). This library facilitated the systematic storage of each game's move sequence within an Excel file, allowing for efficient data retrieval and subsequent analysis.

As the core AI engine responsible for generating moves, we leveraged Stockfish, a highly regarded open-source chess engine renowned for its strategic depth and playing strength. Stockfish's integration into our self-play framework ensured that the generated games exhibited a high degree of strategic decision-making, aligning closely with our research objective of identifying "reasonable" chess games.

5.1 Self-Play Logic and Move Selection

To automate the process of generating chess games, we implemented a self-play mechanism utilizing a while loop. This loop continuously initiated new games between the two Stockfish instances, simulating the game flow until a termination condition was met.

Within each game iteration, the Stockfish engine evaluated the current board position and identified a set of "best" possible moves. This evaluation process likely employed a best-first search algorithm, prioritizing moves that maximize Stockfish's internal evaluation function, which considers factors like piece development, positional advantage, and material threats.

To introduce a degree of exploration and avoid overly deterministic gameplay, we employed a selection mechanism where Stockfish chose randomly its final move from a subset of the top "best" possibilities. This approach, potentially utilizing a probabilistic selection strategy based on centipawn evaluation (a measure of material advantage), balanced the exploration of strategically sound moves while maintaining an element of randomness within the self-play process.

5.2 Game Termination Conditions

To ensure the self-play process generated complete and well-defined games, we incorporated a set of standard chess termination conditions within the loop. These conditions included:

- Checkmate: The game concludes when one player delivers checkmate, rendering the opponent's king unable to escape capture.
- Stalemate: The game ends in a draw when a player has no legal move remaining, but their king is not currently in check.
- Threefold Repetition: The game is drawn if the same position occurs three times with the same player having the move.
- Fifty-Move Rule: If no pawn is captured and no piece is moved for fifty consecutive moves by both players, the game is declared a draw.
- Seventy-Five Move Rule: A similar rule to the fifty-move rule, but with a limit of seventy-five moves.

By incorporating these termination conditions, we ensured that the generated self-play games adhered to established chess rules and represented complete and meaningful game data for our analysis.

6 GRAPH OF THE DATA COMING FROM ARTIFICIAL INTELLIGENCE

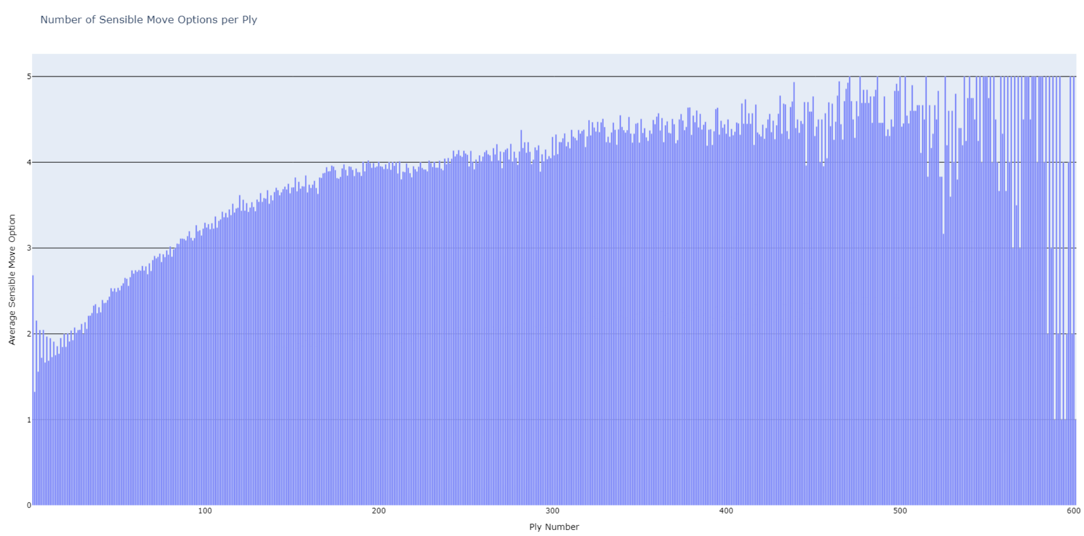

Fig. 1. Number of Sensible Move Options per Ply

We can observe in the graph that if ply number gets larger then number average sensible move option also gets larger but it stops at our artificial intelligence. This gives a relation that number of average sensible move options increases with decreasing momentum.

We see that average number of possible chess games is 3.9^{183}. With percentage of 40% win on White side and 40% win on Black side. 20% of the games are resulted in a draw.

7 CONCLUSION

Our investigation commenced with a comprehensive review of existing literature on the estimation of chess game complexity. While Shannon's number provides a valuable theoretical foundation, its focus on total legal move sequences proved insufficient for our purposes. We recognized the need to refine this approach to specifically target the subset of "reasonable" chess games, those that exhibit strategic depth and adhere to established chess principles.

Given the sheer number of legal move sequences, even within the realm of reasonable games, we acknowledged the limitations of traditional manual calculation methods. Therefore, we opted to leverage the power of artificial intelligence (AI). Our approach involved implementing a self-play technique where two instances of the Stockfish chess engine competed against each other. This facilitated the generation of a vast dataset of strategically relevant chess games.

By analyzing the statistical distribution of moves within these AI-generated games, we aimed to extract a more refined estimate for the number of reasonable chess games. This analysis resulted in the identification of a relationship between the ply number (number of half-moves) and the average number of strategically viable move options at each ply. This relationship, potentially represented as a graph, offers valuable insights into the decision-making landscape of chess and the escalating complexity as the game progresses.

Ultimately, by combining a critical review of existing research with the innovative application of AI-powered self-play, we arrived at an approximate estimate for the number of reasonable chess games: 3.9^{183}. This finding, while an estimate, provides a more realistic representation of the playable strategic landscape of chess compared to the vast number of legal but nonsensical move sequences

REFERENCES

A ONLINE RESOURCES

- https://lichess.org/opening
- https://lichess.org/opening/Kings_Pawn_Game/e4
- https://en.wikipedia.org/wiki/Shannon_number
- https://disservin.github.io/stockfish-docs/stockfish-wiki/Home.html
- https://python-chess.readthedocs.io/en/latest/
- https://xlsxwriter.readthedocs.io/workbook.html
- https://pandas.pydata.org/docs/