Problem set 2

$$T(n) = T(n/2) + 8T(n/4)$$

$$\Rightarrow T(n) = T(n/4) + 8T(n/8) + 8T(n/8) + 8T(n/16)$$

$$T(n/2) = T(n/4) + 8T(n/8)$$
$$T(n/4) = T(n/8) + 8T(n/16)$$
$$T(n/8) = T(n/16) + 8T(n/32)$$
$$T(n/16) = T(n/32) + 8T(n/64)$$

$$\Rightarrow T(n) = T(n/2) + 8T(n/16) + 8\{T(n/16) + 8T(n/32) + 8T(n/64)\}$$

$$\Rightarrow T(n) = T(n/8) + 2^3 T\left(\frac{n}{2^4}\right) + 2^3\{T(n/2^4) + 2^3 T(n/2^5) + 2^3 T(n/2^6)\}$$

Maximum

$$8\ 2^3\left[2^3 T\left(\frac{n}{2^6}\right)\right]$$

$$\Rightarrow 64T(n/2^6)$$

$$= 2^6 T\left(\frac{n}{2^6}\right)$$

$$\Rightarrow 2^k T\left(\frac{n}{2^k}\right)$$

For,

$$\frac{n}{2^k} = 1$$

$$\Rightarrow n = 2^k \Rightarrow k = \log n$$

$$T(n) = 8+8+8 \cdots nT\left(\frac{n}{n}\right) \text{ -----}$$

$$\Rightarrow T(1)$$

$$\Rightarrow T(n) = O(n)$$

$$\therefore T(n) = O(n)$$