Problem set 3 :

Recurrence relation :

$$T(n) = 2T(n/2) + c$$

$$T(n) = 2T(n/2) + c$$

$$= 2[2T(n/4) + c] + c$$

$$= 4T(n/4) + 3c$$

$$= 4[2T(n/8) + 3c] + 3c$$

$$= 8T[(n/8) + 7c]$$

$$= 8[2T(n/16) + c] + 7c$$

$$= 16T(n/16) + 15c$$

$$= 2^k T(n/2^k) + (2^k - 1)c$$

for $\frac{n}{2^k} = 1$

$\frac{n}{2^k} = 1 \quad \therefore n = 2^k \Rightarrow k = \log n$

$$T(n) = 2^{\log n} T(n/2^{\log n}) + (2^{\log n} - 1)c$$

$$= nT(\tfrac{n}{n}) + (n-1)c$$

$$= nT + (n-1)c$$

$$= O(n)$$

$$T(n) = O(n)$$