

Problem set 1 (Binary Search)

$$T(n) = \begin{cases} 1 & n = 1 \\ T(n/2) + 1 & n > 1 \end{cases}$$

Substitution method :

$$T(n) = T(n/2) + 1 \quad \text{--- (i)}$$

$$T(n) = [T(n/2^2) + 1] + 1$$

$$T(n) = T(n/2^2) + 2$$

$$T(n) = T(n/2^3) + 3 \quad \text{--- (ii)}$$

$$T(n) = T\left(\frac{n}{2^k}\right) + k \quad \text{--- (iii)}$$

Assume,

$$\frac{n}{2^k} = 1$$

$$\therefore n = 2^k \Rightarrow k = \log n$$

from (iii) no equation,

$$\begin{aligned} T(n) &= T\left(\frac{n}{2^k}\right) + k \\ &= T(1) + \log n \end{aligned}$$

$$O(\log n)$$

$$T(n) = O(n \log n)$$