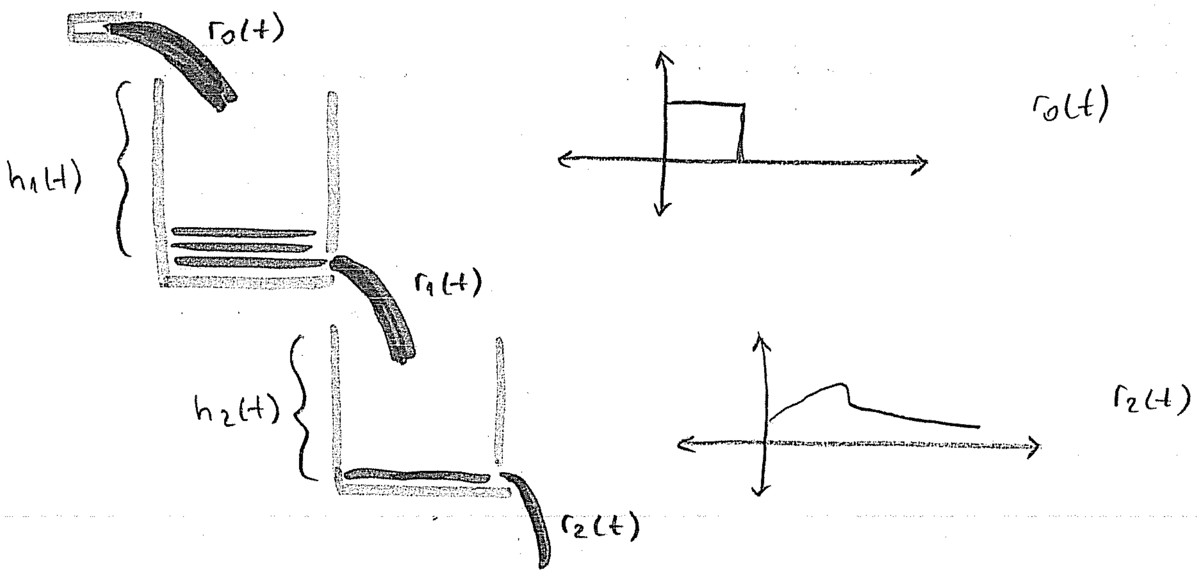
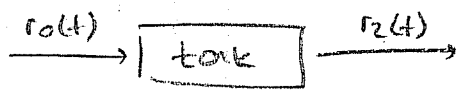
-İşaretler ve Sistemler-

{ Yaz Okulu } 1

∝ → Bilginin karşı tarafa matematiksel ya da fiziksel olarak aktarılması — 23.06.2016 Perşembe

→ Fırın, mikrofon, telefon





∝ → İşaret: Fiziksel sistemin durumuna veya davranışına ilişkin bilgi taşıyan ve bununla birlikte bir ya da daha fazla bağımsız parametreye bağlı olarak ...

?

∝ # Analogta sürekli genlik ve zaman değişiyor.

$x(t) \longrightarrow x(n)$ Analog → ayrık

∝ # Veri yolu ne kadar büyük olursa o kadar ayrıntılı incelenir.

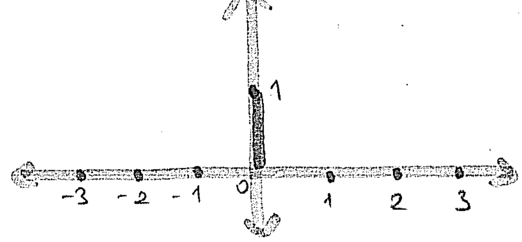
ayrıkla süreklinin benzerliği → genliklerinin herhangi bir sayısal değere sahip olması.

AYRIK ZAMANLI İŞARETLER

$x(n)$ ↳ zaman indisi

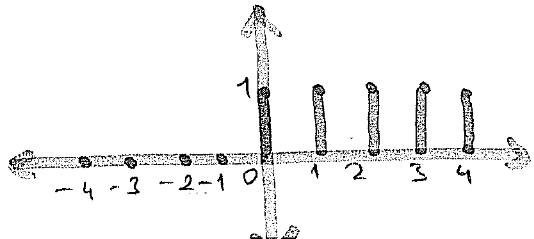
$$\delta(n) = \begin{cases} 1 & , \ n=0 \\ 0 & , \ n\neq 0 \end{cases}$$

(birim örnek veya impuls dizisi)



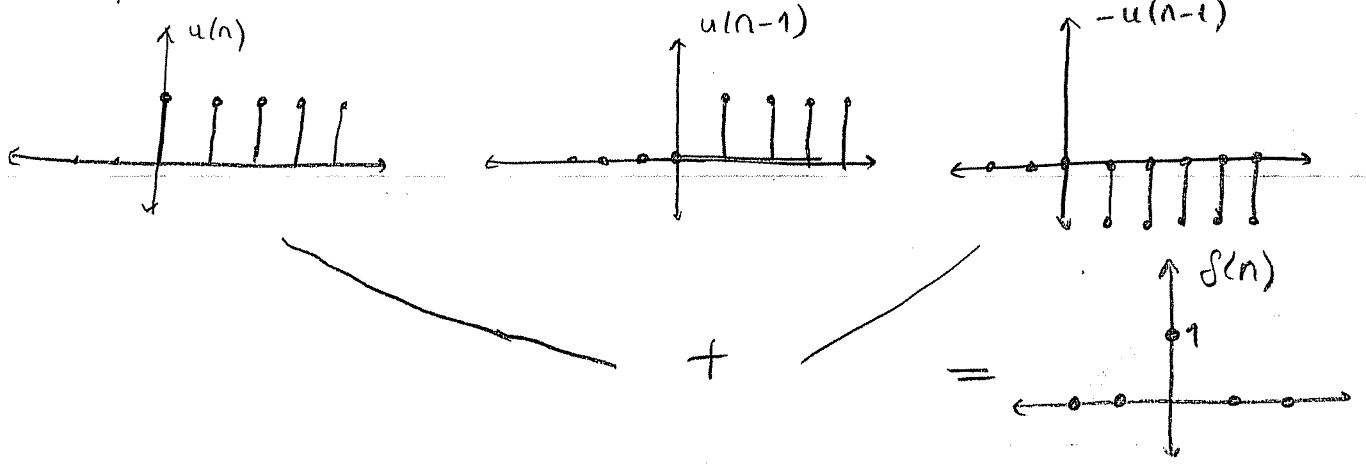
$$u(n) = \begin{cases} 1 & , \ n \geq 0 \\ 0 & , \ n<0 \end{cases}$$

(birim basamak dizisi)

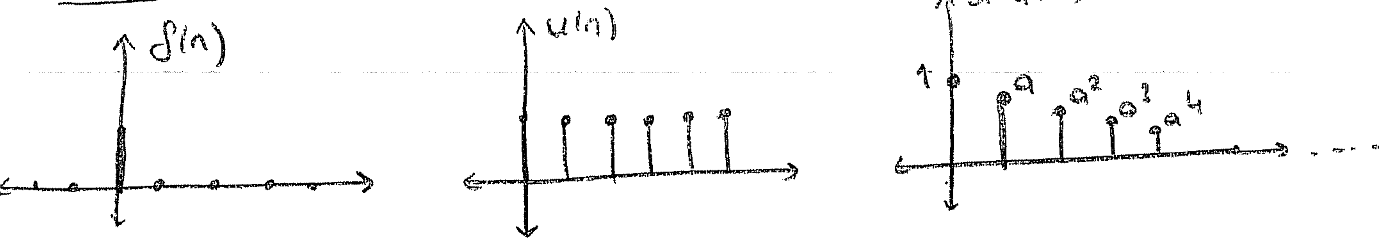


1. $u(n) = \sum_{k=-\infty}^{n} \delta(k)$ 2. $u(n) = \sum_{k=0}^{\infty} \delta(n-k)$

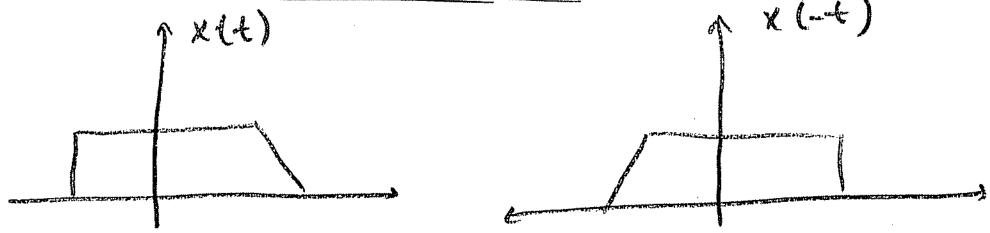
$\longrightarrow$ $u(n)$'den $\delta(n)$ elde etmek için 1 birim sağa kaydırıp ters çevirip toplarız.

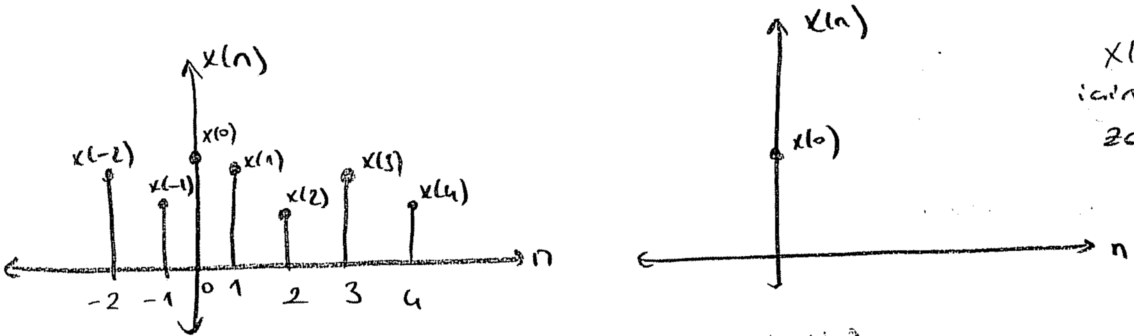


$a^n u(n) \longrightarrow$ n'ye bağlı

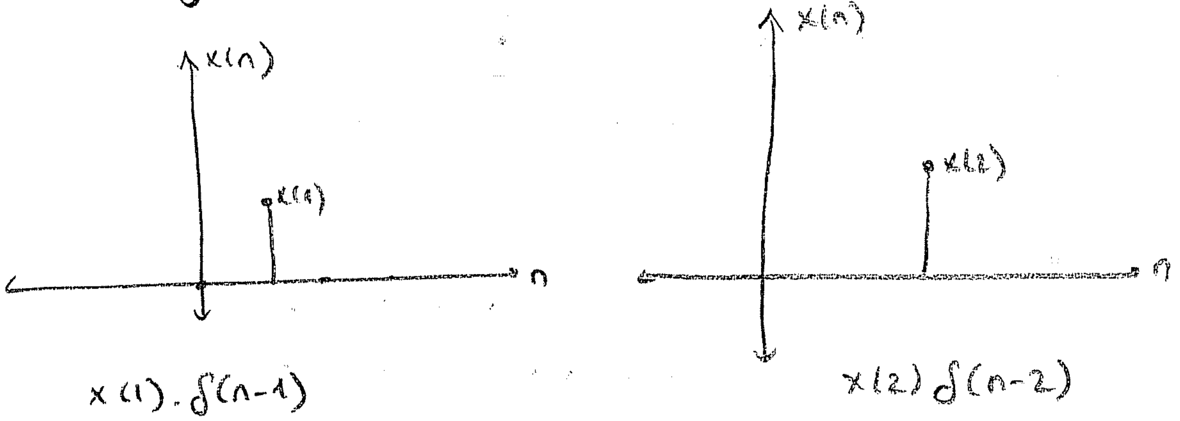


Time reversal (zamanda ters çevirme)





$x(0)$'ın değerini göstermek için $x(0).\delta(n)$ şeklinde yazarız.



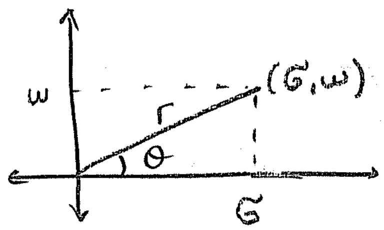
$x(1).\delta(n-1)$ $x(2)\,\delta(n-2)$

$$x(n) = \cdots + x(-1)\delta(n+1) + x(0)\delta(n) + x(1)\delta(n-1) + \cdots$$

$$= \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)$$

Magnitude & Phase Representation:

$\sigma + j\omega$



★ $r = \sqrt{\omega^2 + \sigma^2}$

★ $\theta = \tan^{-1} \dfrac{\omega}{\sigma}$

★ $\sigma + j\omega = r e^{j\theta}$

★ $e^{j\theta} = (\cos\theta + j\sin\theta)$

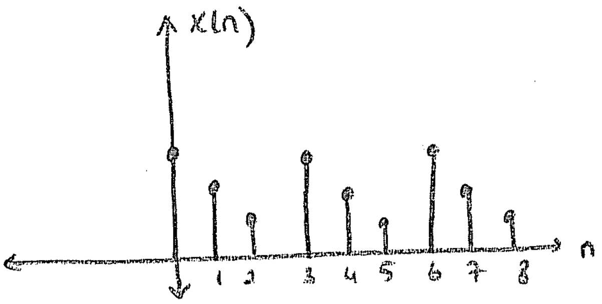
★ $e^{-j\theta} = (\cos\theta - j\sin\theta)$

★ $\cos\theta = \dfrac{e^{j\theta} + e^{-j\theta}}{2}$ ★ $\sin\theta = \dfrac{e^{j\theta} - e^{-j\theta}}{2j}$

PERİYODİK DİZİLER

→ kendini bir periyotta tekrar eden.

$x(n) = x(n+N)$



$x(n) = e^{j\omega_0 n}$

$x(n) = x(n+N)$

$e^{j\omega_0 n} = e^{j\omega_0 (n+N)}$

$\cancel{e^{j\omega_0 n}} = \cancel{e^{j\omega_0 n}} \cdot e^{j\omega_0 N}$

$1 = e^{j\omega_0 N} \Rightarrow 1 = e^{j2\pi k}$

$e^{j0}, e^{j2\pi}, e^{j4\pi}$ → katları 0 olduğundan $e^{j2\pi} = 1$ olur.

$e^{j\omega_0 N} = e^{j2\pi k}$

$\omega_0 N = 2\pi k$ $\Rightarrow$ $\boxed{N = \dfrac{2\pi k}{\omega_0}}$

Ör: $x(n) = e^{j\left(\frac{\pi}{8}\right)n}$ periyodik mi?

$N = \dfrac{2\pi}{\pi/8} k \Rightarrow 16k$ periyodiktir ve periyodu 16'dır.

Ör: $x(n) = e^{j\left(\frac{6\pi}{25}\right)n} = ?$

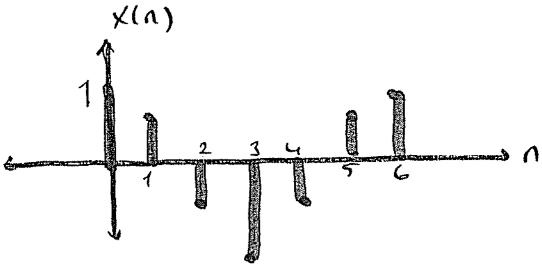
$N = \dfrac{2\pi}{\frac{6\pi}{25}} k \Rightarrow \dfrac{25}{3} k$

$k=3$ olursa $N=25$

periyodik ve periyodu 25.

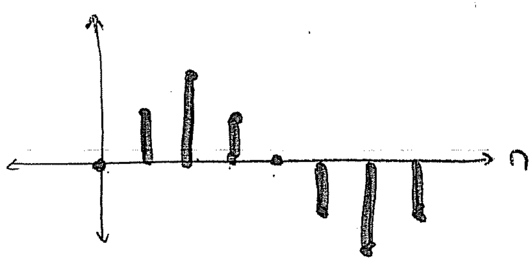
Ör: $x(n) = \cos \frac{\pi}{3} n = ?$

$$N = \frac{2\pi}{\pi/3} = 6k$$



Ör: $x(n) = \sin \frac{\pi}{4} n = ?$

$$N = \frac{2\pi}{\pi/4} = 8k$$



Ör: $x(n) = \underbrace{\cos \frac{\pi}{3} n}_{} + \underbrace{\sin \frac{\pi}{4} n}_{} = ?$

$N_1 = \frac{2\pi}{\pi/3} = 6k$ $\quad N_2 = \frac{2\pi}{\pi/4} = 8k$

$x(n) = x_1(n) + x_2(n)$

$\quad\quad \hookrightarrow N_1 \quad \hookrightarrow N_2$

$x_1(n) = x_1(n+N_1) = x_1(n+kN_1)$

$x_2(n) = x_2(n+N_2) = x_2(n+mN_2)$

$x(n) = x(n+N) = x_1(n+N) + x_2(n+N)$

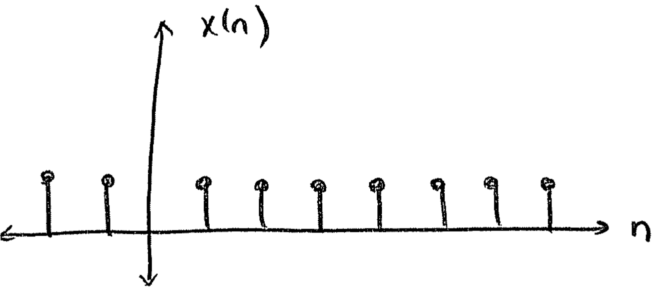
$x_1(n+N) + x_2(n+N) = x_1(n+kN_1) + x_2(n+mN_2)$

$\boxed{N = kN_1 = mN_2}$ eşit olması için k ile m'yi ortak kat yapacak şekilde almamız lazım.

$6k = 8m \Rightarrow k = 4 \quad m = 3$' tür. Periyodu 24 tür.

Ör: $x(n) = \cos^2\left(\frac{\pi}{8} n\right) = ?$ $\quad \cos^2\theta = \frac{1+\cos 2\theta}{2} \Rightarrow \frac{1+\cos(\pi/4\, n)}{2}$

$x(n) = 1$



$$x(n) = \underbrace{\frac{1}{2}}_{x_1(n)} + \underbrace{\frac{1}{2}\cos\left(\pi/4\, n\right)}_{x_2(n)}$$

$N_1 = 1 \quad\quad N_2 = \frac{2\pi}{\pi/4} = 8k$

periyodu 8

periyodu 1, 1 örnekte 1 tekrar ediyor.

#α# $x(n) = 1/2$, $x(n) = 3/4$ de olsa, 1 örnekte 1 tekrar ettiği için periyot yine 1 olurdu!

Ör: $x(n) = \cos\left(\frac{\pi}{8} n^2\right) = ?$ #ödev#

24.06.2016
Cuma

$x(t)$ → zaman
$x(n)$ → zaman indisi

$x(n - n_0)$ → geçmişteki
$x(n + n_0)$ → gelecekteki

özet

Ör: $x(n) = \cos\left(\frac{\pi}{6} n\right) = ?$

$$N = \frac{2\pi}{\pi/6} = 12k \qquad \boxed{N = 12}$$

Ör: $x(n) = \cos\left(\frac{n}{6}\right) = ?$

$w_0 = 1/6 \qquad N = \frac{2\pi}{1/6} = 12\pi k \Rightarrow$ N ifadesini tamsayı yapabilen bir k değeri yok. O yüzden periyodik değil.

— $x(n)$ girişli ve $y(n)$ çıkışlı ayrık zamanlı sistemler —

$$x(n) \longrightarrow \boxed{\text{sistem}} \longrightarrow y(n)$$

$$\{x(n)\} \xrightarrow{\text{sistem}} \{y(n)\}$$

$$y(n) = T[x(n)]$$ → dönüşüm kuralı

doğrusallık: Bir sistemin doğrusallığı, çarpımsallık ve toplamsallık ilkelerinin sağlanmasıyla tanımlanır.

$$T[x_1(n)] = y_1(n)$$
$$T[x_2(n)] = y_2(n)$$
$$x_3(n) = a x_1(n) + b x_2(n)$$
$$T[x_3(n)] = y_3(n) = a y_1(n) + b y_2(n)$$ ise doğrusaldır.

Ör: $n=0$, %1'le yatırılan paranın çıkış değeri $(y(n)) = ?$

$$y(n) = 0.01\, y(n-1) + y(n-1)$$ → önceki ay

$$= 1.01\, y(n-1) \Longrightarrow \text{faizli para}$$

$1.01\, y(n-1) + x(n) \Rightarrow$ yeni yatırılanla birlikte

$$\left.\begin{aligned} y(0) &= 1{,}01\cdot\underbrace{y(-1)}_{0} + x(0) \\ y(0) &= x(0) = 1000 \end{aligned}\right\} n=0$$

1000 yatırıldığını varsayıyorsak

$$\left.\begin{aligned} y(1) &= 1{,}01\ y(0) + x(1) \\ &= 1{,}01 \times 1000 \\ &= 1010 \end{aligned}\right\} n=1$$

$$\left.\begin{aligned} y(2) &= 1{,}01\ y(1) + x(2) \\ &= 1010 \times 1{,}01 \\ &= 1020{,}1 \end{aligned}\right\} n=2$$

→ sadece n anındaki işareti buluyorsak hafızasız sistem

→ önceki, şu anki ve sonraki işaretleri buluyorsak hafızalı sistem

#sistemin diğer bir özelliği ters çevrilebilir olması $T[y(n)] = x(n)$

$$y(n) = \sum_{k=-\infty}^{n} x(k) \qquad u(n) = \sum_{k=-\infty}^{n} \delta(k)$$

$$x(n) = y(n) - y(n-1)$$

$$\delta(n) = u(n) - u(n-1)$$

<u>nedensellik</u>: sistemin gerçeklenebilir olması anlamına gelir.

$y(n) = x(n) - x(n+1)$ → $n=0$ anında $y(0) = x(0) + x(1)$ giriş olarak 0'dayız ama $x(1)$ yüzünden pratikte gerçeklenemeyen sistem.

$y(t) = x(t+1)$

↓ gelecekteki bilgiyi kullanıyorsa nedensel değil.

<u>kararlılık</u>: sisteme uygulanan giriş işaretinin genliği sınırlı bir aralıktaysa çıkış da aynı şekilde sınırlıysa, kararlılık gösterir.

<u>zamanda değişmezlik</u>: $x_1(n) = x(n-n_0)$

$$T[x_1(n)] = y_1(n) = y(n-n_0)$$

doğrusallık: $T[x_1(n)] = y_1(n)$

$T[x_2(n)] = y_2(n)$

$x_3(n) = a x_1(n) + b x_2(n)$

$T[x_3(n)] = y_3(n) = a y_1(n) + b y_2(n)$

Ör: $y(n) = 2x(n) + 3 = T[x(n)]$ $\Rightarrow$ zamanda değişmezlik?

$x_1(n) = x(n-n_0)$

$y_1(n) = 2x_1(n) + 3$

$= 2x(n-n_0) + 3$ $\Rightarrow$ $y_1(n) \overset{?}{=} y(n-n_0)$ olduğuna bakıyoruz.

$y(n-n_0) = 2x(n-n_0) + 3$ $\Rightarrow$ aynı olduğu için zamanda değişmez.

$\Rightarrow$ doğrusal mıdır?

$y_1(n) = 2x_1(n) + 3$

$y_2(n) = 2x_2(n) + 3$

$y_3(n) = 2x_3(n) + 3$

$x_3(n) = a x_1(n) + b x_2(n)$

$y_3(n) = 2(a x_1(n) + b x_2(n)) + 3$ $\Rightarrow$ aynı değil doğrusal değil.

$y_3(n) = a y_1(n) + b y_2(n)$

$= a(2x_1(n) + 3) + b(2x_2(n) + 3) = 2a x_1(n) + 3a + b\,2x_2(n) + 3b$

Ör: $y(n) = 6x^2(n-3) = ?$

* nedenseldir. (geçmişteki bilgi)
* kararlıdır. (girişe en fazla verdiğimiz değer, çıkışı da sınırlıyorsa kararlıdır.)

$\rightarrow$ zamanda değişmezlik

$y(n) = 6x^2(n-3)$

$y_1(n) = 6x_1^2(n-3)$

$= 6x^2(n-n_0-3)$

$y(n-n_0) = 6x^2(n-n_0-3)$

aynı, zamanda değişmez.

$\rightarrow$ doğrusallık

$y_1(n) = 6x_1^2(n-3)$

$y_2(n) = 6x_2^2(n-3)$

$y_3(n) = 6x_3^2(n-3)$

$x_3(n) = a x_1(n) + b x_2(n)$

$y_3(n) = 6(a x_1(n-3) + b x_2(n-3))^2$

$a y_1(n) + b y_2(n) = a(6x_1^2(n-3)) + b(6x_2^2(n-3))$

aynı değil, doğrusal değil

Ör: $y(n) = n^2 x(n+2)$

* nedensel değil (gelecekteki bilgiye ihtiyaç duyar)
* kararsızdır. (x sonsuza giderken y'de sonsuza gider.

→ zamanda değişmezlik

$y_1(n) = n^2 x_1(n+2)$

$x_1(n) = x(n-n_0)$

$\boxed{y_1(n) = n^2 x(n+2-n_0)}$

$x_1(n+2) = x(n+2-n_0)$

$\boxed{y(n-n_0) = (n-n_0)^2 x(n-n_0+2)}$

aynı değil, zamanla değişir//

→ doğrusallık

$y_1(n) = n^2 x_1(n+2)$

$y_2(n) = n^2 x_2(n+2)$

$y_3(n) = n^2 x_3(n+2)$

$x_3(n) = a x_1(n) + b x_2(n)$

$y_3(n) = n^2 (a x_1(n+2) + b x_2(n+2))$ aynı

$a y_1(n) + b y_2(n) = a(n^2 x_1(n+2) + b(n^2 x_2(n+2))$

doğrusal//

Ör: $y(n) = 8n\, x(n)$

* nedenseldir.
* kararlılık !

→ zamanda değişmezlik

$y_1(n) = 8n\, x_1(n) = 8n\, x(n-n_0)$

$x_1(n) = x - n_0)$

$y(n-n_0) = 8(n-n_0)\, x(n-n_0)$

aynı değil zamanda değişir.

→ doğrusallık

$y_1(n) = 8n x_1(n)$

$y_2(n) = 8n x_2(n)$

$y_3(n) = 8n x_3(n)$

$x_3(n) = a x_1(n) + b x_2(n)$

$y_3(n) = 8n(a x_1(n) + b x_2(n))$ aynı

$a y_1(n) + b y_2(n) = a(8n x_1(n)) + b(8n x_2(n))$

doğrusal//

Ör: $y(n) = T[x(n)] = x(n) + 4x(n-3)$

* nedenseldir.
* kararlı

→ zamanda değişmezlik

$y_1(n) = x_1(n) + 4x_1(n-3) = x(n-n_0)$

$x_1(n) = x(n-n_0)$

$x_1(n-3) = x(n-3-n_0)$

$y(n-n_0) = x(n-n_0) + 4x(n-3-n_0)$

aynı, zamanda değişmez

→ doğrusallık

$y_1(n) = x_1(n) + 4x_1(n-3)$

$y_2(n) = x_2(n) + 4x_2(n-3)$

$y_3(n) = x_3(n) + 4x_3(n-3)$

$x_3(n) = a x_1(n) + b x_2(n)$

$x_3(n-3) = a x_1(n-3) + b x_2(n-3)$

$y_3(n) = a x_1(n) + b x_2(n) + 4a\, x_1(n-3) + 4b x_2(n-3)$ aynı

$a y_1(n) + b y_2(n) = a(x_1(n) + 4x_1(n-3)) + b(x_2(n) + 4x_2(n-3))$

doğrusal//

Ayrık zamanlı – doğrusal – zamanla değişmeyen sistemler

yöntemler:
- birim impuls cevabı
- fark denklemleri
- durum denklemleri

→ DZD sistemlerin giriş–çıkış ilişkisinin birim impuls cevabı yöntemiyle belirlenmesi

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)$$

$$x[n] \longrightarrow \boxed{h(n)} \longrightarrow y(n)$$

$$T[x(n)] = y(n)$$

$$T\left[\sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)\right]$$

$$\begin{aligned} y_3 &= T[x_3(n)] \\ &= T[a x_1(n) + b x_2(n)] \\ &= T[a x_1(n)] + T[b x_2(n)] \\ &= a\underbrace{T[x_1(n)]}_{y_1(n)} + b\underbrace{T[x_2(n)]}_{y_2(n)} \\ &= a y_1(n) + b y_2(n) \end{aligned}$$

sistem doğrusalsa bu ikisi eşit

doğrusallık

$$\text{-----} + T[x(-1)\,\delta(n+1)] + T[x(0)\,\delta(n)] + \text{-----}$$

$$\text{----} + x(-1)\,T[\delta(n+1)] + x(0)\,T[\delta(n)] + x(1)\,T[\delta(n-1)] + \text{----}$$

$$\text{---} + x(-1)\,h(n+1) + x(0)\,h(n) + x(1)\,h(n-1) + \text{----}$$

$$T[\delta(n-1)] = h(n-1)$$

$$T[\delta(n-k)] = h(n-k)$$

$$\boxed{y(n) = \sum_{k=-\infty}^{\infty} x(k)\,h(n-k)}$$

→ konvolüsyon toplama denir

$x(n) * h(n)$ şeklinde gösterilir.

$$T[x(n)] = y(n)$$

$$T[x(n-n_0)] = y(n-n_0)$$

} zamanda değişmezlik

$$\boxed{y(n) = x_1(n)\,.\,x_2(n)}$$

$$y(0) = x_1(0)\,x_2(0)$$

$$y(1) = x_1(1)\,x_2(1)$$

$$\boxed{y(n) = x_1(n)\,x_2(n-1)}$$

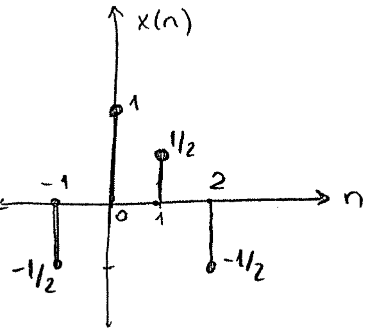
$$y(0) = x_1(0)\,.\,x_2(-1)$$

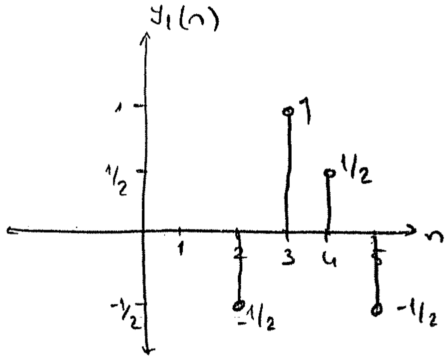
$$y(1) = x_1(1)\,x_2(0)$$

30 Haziran 2016

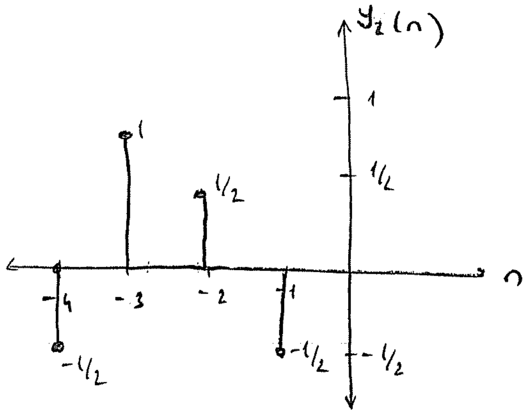
* zamanda öteleme

$x(n) \longrightarrow x(n+k)$





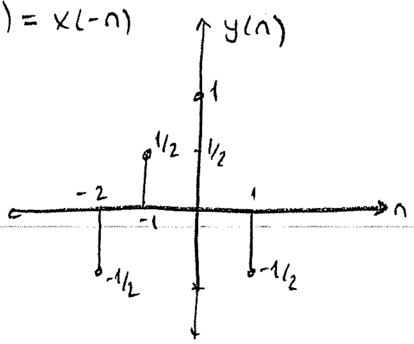
$y_1(n) = x(n-3)$



$y_2(n) = x(n+3)$

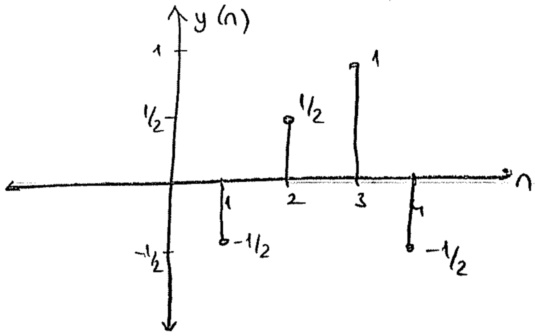
* zamanda ters çevirme

$y(n) = x(-n)$



$\Longrightarrow$

$y(n) = x(-n+3)$



$y(0) = x(3) = 0$

$y(1) = x(2) = -1/2$

$y(2) = x(1) = 1/2$

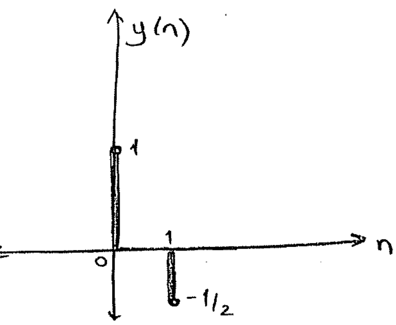
$y(3) = x(0) = 1$

$y(4) = x(-1) = -1/2$

! önce ters çevirip sonra öteliyoruz.

* zamanda ölçekleme

$y(n) = x(2n)$



hafızalı sistem	-	hafızasız sistem
$x(n \mp k)$		$x(n)$

! katsayı olunca $(x(2n))$ hafızalı sistem olur.

* Ters çevirme

$y(n) = T[x(n)]$

$x(n) = T_1[y(n)]$

$x(n) \longrightarrow \boxed{T} \longrightarrow y(n)$

$y(n) \longrightarrow \boxed{T_1} \longrightarrow x(n)$

T_1 ; ters çevrilebilir " sistem

* impuls cevabı

$x \longrightarrow \boxed{} \longrightarrow y$

$\delta(n) \longrightarrow \boxed{} \longrightarrow h(n)$

$$\left.\begin{aligned} x(n) &= \delta(n) \\ y(n) &= h(n) \end{aligned}\right\} \text{konvolüsyon toplamı}$$

$x(n) = x(-1)\delta(n+1) + x(0)\delta(n) + x(1)\delta(n-1) + \cdots$

$$\boxed{x(n) = \sum_{k=-\infty}^{\infty} x(k)\delta(n-k)}$$

impuls dizilerinin toplamı

örneğin yazımı:

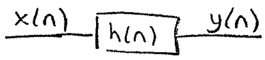
$$
\begin{aligned}
y(n) &= T[x(n)] \\
&= T[x(-1)\,\delta(n+1) + x(0)\delta(n) + x(1)\delta(n-1) + x(2)\delta(n-2)] \\
&= T[x(-1)\delta(n+1)] + T[x(0)\delta(n)] + T[x(1)\delta(n-1)] + T[x(2)\delta(n-2)] \\
&= x(-1)\,T[\delta(n+1)] + x(0)T[\delta(n)] + x(1)T[\delta(n-1)] + x(2)T[\delta(n-2)]
\end{aligned}
\Bigg\}
$$

doğrusallık özelliğinden dolayı yapabildik.

$$y(n-k) = T[x(n-k)]$$
$$h(n) = T[\delta(n)]$$
$$h(n-k) = T[\delta(n-k)]$$

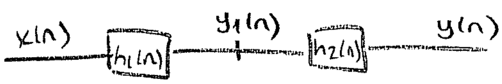
$$y(n) = x(-1)\,h(n+1) + x(0)h(n) + x(1)h(n-1) + x(2)\,h(n-2)$$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k) = x(n) * h(n) = h(n) * x(n)$$



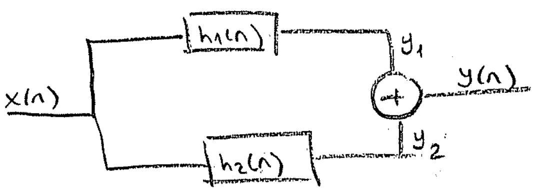
$\Rightarrow x(n) * h(n)$

* seri bağlı sistemlerde sırayla konvolüsyon yapıyoruz.



$$\left.\begin{aligned} y_1(n) &= x(n) * h_1(n) \\ y(n) &= y_1(n) * h_2(n) \end{aligned}\right\} x(n) * h_1(n) * h_2(n) \;//$$

* paralel bağlı sistemlerde ise;



$$\left.\begin{aligned} y(n) &= y_1(n) + y_2(n) \\ y_1(n) &= x(n) * h_1(n) \\ y_2(n) &= x(n) * h_2(n) \end{aligned}\right\} y(n) = x(n) * (h_1 + h_2) \;//$$

ödev sorusunun çözümü:

$$x(n) = \cos(\pi/8\; n^2)$$

$$x(n) = x(n+N)$$

$$\cos(\pi/8\; n^2) = \cos\left(\pi/8\,(n+N)^2\right) = \cos\Big(\pi/8\; n^2 + \underbrace{\pi/8\; 2nN}_{2\pi n k} + \underbrace{\pi/8\; N^2}_{2\pi m}\Big)$$

$$\pi/8\; 2nN = 2\pi n k \qquad \pi/8\; N^2 = 2\pi m$$
$$N = 8k \qquad N^2 = 16m$$

$k=1$ $m=4$ ise

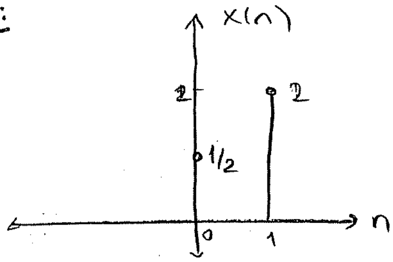
$N = 8$'dir.

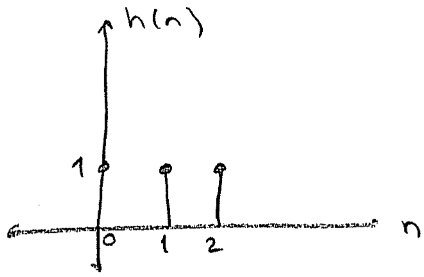
!

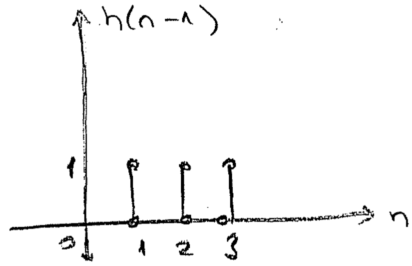
$\cos(2n) \longrightarrow$ periyodik değil

$$\cos(2n + 2\pi k) = \cos(2n + 2N)$$

Ör:





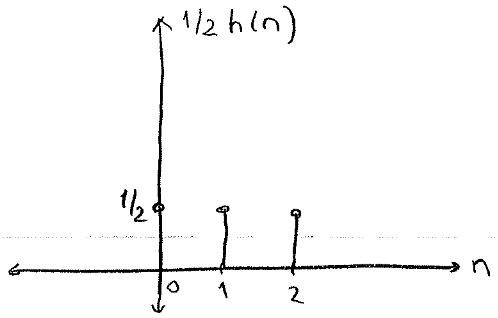


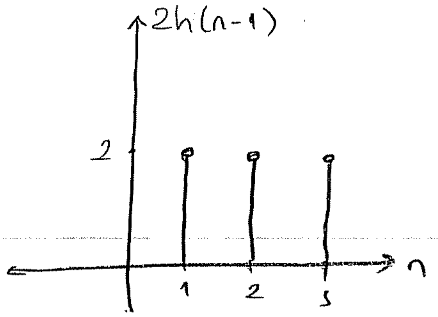
$x(n) \longrightarrow \boxed{h(n)} \longrightarrow y(n)$ $\qquad y(n) = ?$

$$y(n) = x(n) * h(n)$$

$$y(n) = \sum_{0}^{1} x(k)\, h(n-k) \Rightarrow = x(0)\,h(n) + x(1)\,h(n-1)$$

$$= \tfrac{1}{2} h(n) + 2h(n-1)$$





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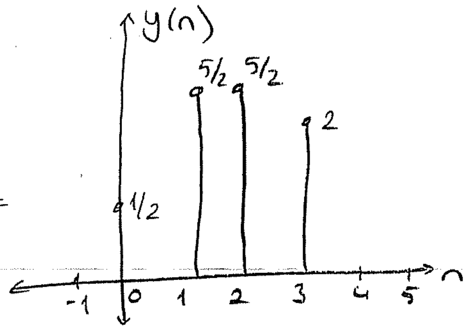
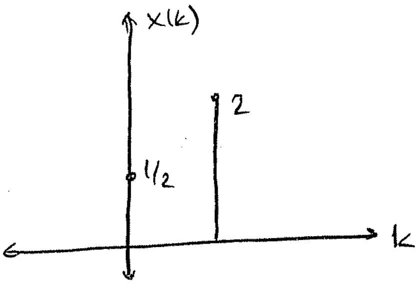
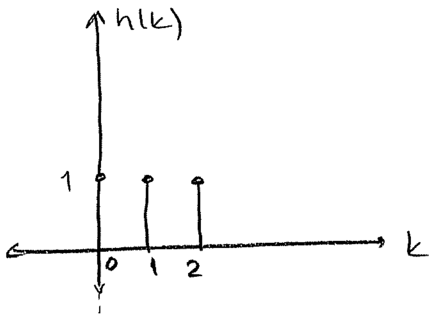
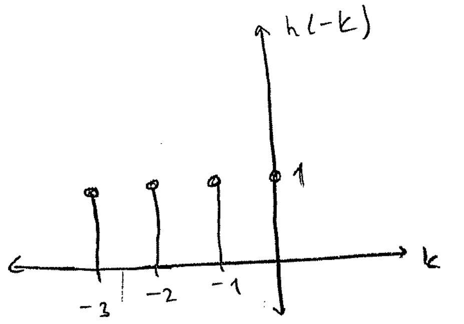
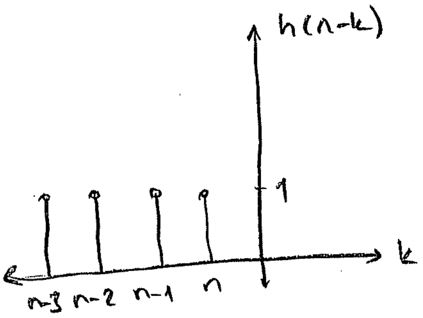


→ konvolüsyon işlemi $x(k)$ ve $h(k)$ olsa;









$y(n) = 0 \qquad n < 0$

$n = 0 \Rightarrow \quad y(0) = 1/2$

$n = 1 \Rightarrow \quad y(1) = 1/2 + 2 = 5/2$

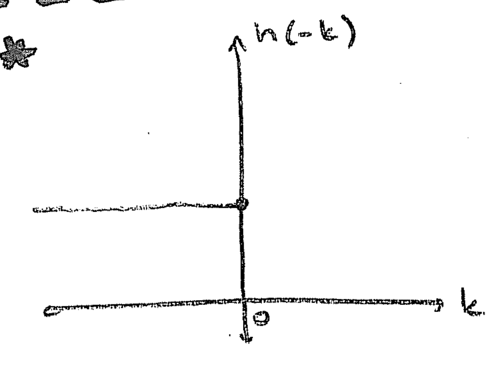
$n = 2 \Rightarrow \quad y(2) = 1/2 \cdot 1 + 2 \cdot 1 + 0 \cdot 1 = 5/2$

$n = 3 \Rightarrow \quad y(3) = 1/2 \cdot 0 + 2 \cdot 1 + 0 \cdot 1 + 0 \cdot 1 = 2$

$n = 4 \Rightarrow \quad y(4) = 1/2 \cdot 0 + 1/2 \cdot 0 + 0 \cdot 0 + 0 \cdot 0 = 0$ //

---

*



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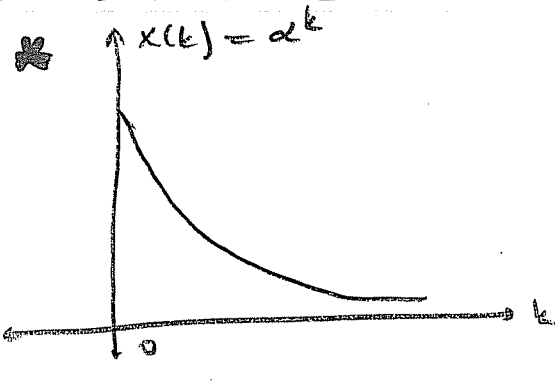
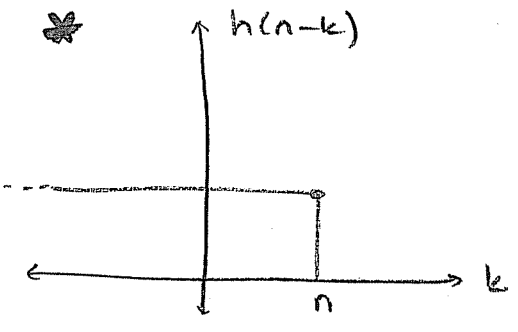
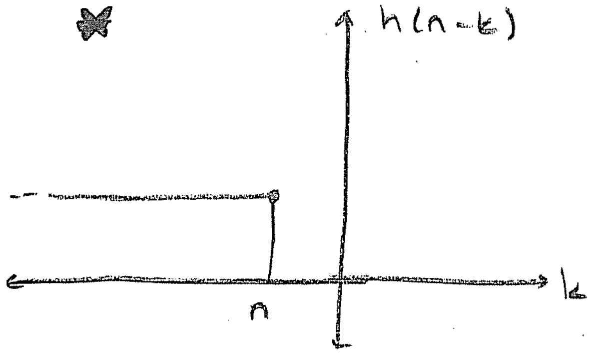




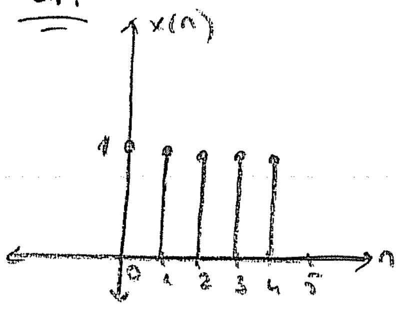


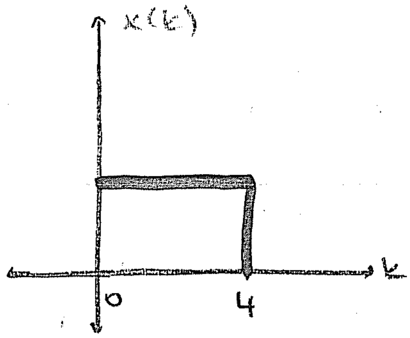
$n<0$
$y(0)=0$?
$y(n)=0$?

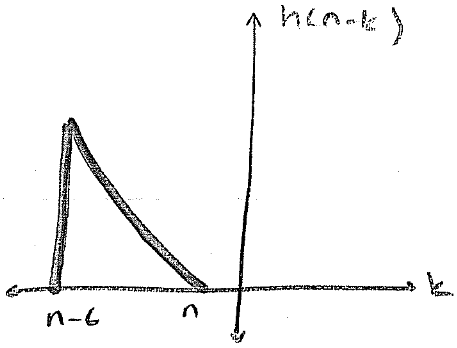
$n>0$

$$y(n) = \sum_{k=0}^{n} \alpha^k \cdot 1 = \frac{1-\alpha^{n+1}}{1-\alpha} \qquad n \geq 0$$

Ör:

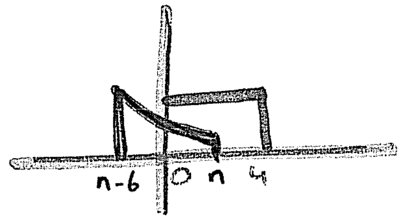




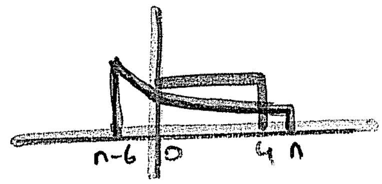


$x[n]=1 \quad 0 \leq n \leq 4$

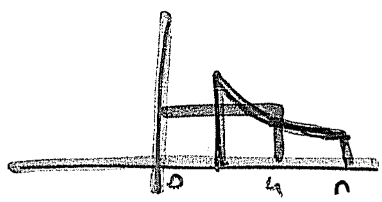
$$0 \leq n < 4 \Rightarrow y(n) = \sum_{k=0}^{n} \alpha^{n-k} = \frac{1-\alpha^{n+1}}{1-\alpha}$$



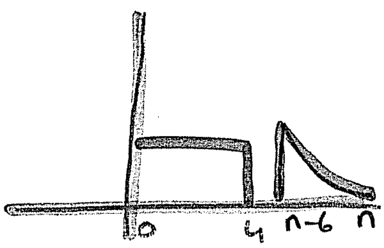
$$4 \leq n < 6 \Rightarrow y(n) = \sum_{k=0}^{4} \alpha^{n-k} = \frac{\alpha^{n-4} - \alpha^{n+1}}{1-\alpha}$$



$$6 \leq n < 10 \Rightarrow y(n) = \sum_{k=n-6}^{4} \alpha^{n-k} = \frac{\alpha^{n-4} - \alpha^{7}}{1-\alpha}$$



$$n \geq 11 \Rightarrow y(n) = 0$$



Ör: $x(n) = 2^n u(-n)$

ÖDEV $h(n) = u(n)$

$n \geq 0 \quad y(n) = \sum_{k=-\infty}^{0} x(k)\,h(n-k) = \sum_{k=-\infty}^{0} 2^k$

Birim impuls cevabı ve karalılık

$x(n)\ ,\ h(n) = \delta(n)\ ,\ y(n) = \sum_{k=-\infty}^{\infty} x(k)\,h(n-k) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)$

$\cdots\ \delta(-1)\,x(n+1) + \delta(0)\,x(n) + \delta(1)\,x(n-1) + \cdots = x(n)$

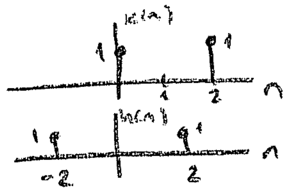
$x(n)\ ,\ h(n) = \delta(n-a)\ ,\ y(n) = x(n-a)$

$\boxed{x(n) * \delta(n) = x(n)}\ ,\ \boxed{x(n) * \delta(n-1) = x(n-1)}$

* $x(n) \longrightarrow \boxed{\delta(n-1)} \longrightarrow x(n-1)$
* $x(n-1) \longrightarrow \boxed{\delta(n+1)} \longrightarrow x(n)$

Ör: $x(n) = \delta(n) + \delta(n-2) \Rightarrow$

$h(n) = \delta(n-2) + \delta(n+2) \Rightarrow$



2 br sağa
2 br sola
kaydırarak
topla

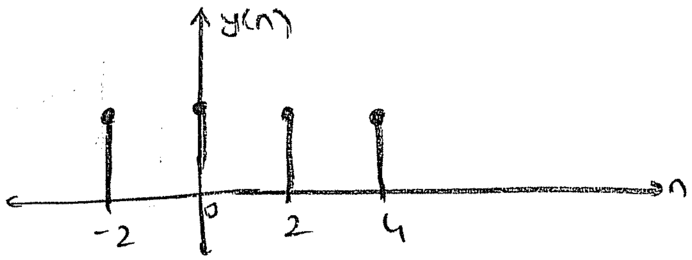
$y(n) = ?\quad x(n) * h(n)$

$= x(n) * \left(\delta(n-2) + \delta(n+2)\right)$

$= x(n) * \delta(n-2) + x(n) * \delta(n+2)$

$= x(n-2) + x(n+2)$

$= \delta(n-2) + \delta(n-4) + \delta(n+2) + \delta(n)$



ÖDEV: $x(n)$ ifadesini açıp $h(n)$ ile konvolüsyon yap.

Ör: $a^n u(n)$ kararlı mı?

$h(n) = u(n)$ kararsız çünkü, $u(n)$ sonsuza kadar gidiyor.

* $\sum_{k=0}^{\infty} |h(k)| = \sum_{k=0}^{\infty} 1 = \infty$

* $\sum_{k=0}^{\infty} a^k$ $0<a<1$ arasındaysa kararlı, $a>1$ kararsız.

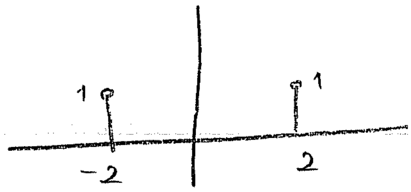
$$y(n) = x(n) * h(n) = h(n) * x(n)$$

$$= \sum_{k=-\infty}^{\infty} h(k)x(n-k) \qquad \cdots + h(-1)x(n+1) + h(0)x(n) + h(1)x(n-1) + \cdots$$

artılı ifadeleri kaldırmak için $h(n)=0$ $n<0$

Ör: $h(n) = \delta(n-2) + \delta(n+2)$

$n=-2$ değerinde genlik 1.
sistem nedensel değil



$h(n) = u(n) \Rightarrow$ nedensel

Sınırlı sayıda impuls cevabı FIR
Sonsuz sayıda impuls cevabı IIR

Fark denklemleriyle belirlenen kısımlar:

doğal çözüm:

$y_d(n)$, $x(n)=0$ için fark denkleminin çözümüdür.

başlangıç koşulları: $y(-1)$, $y(-2)$

zorlanmış çözüm:

$x(n)$ verilen
$y(-1) = y(-2) = \dots = 0$

Toplam Çözüm = Doğal Ç. + Zorlanmış Ç.

$$\sum_{k=0}^{N} b_k y(n-k) = 0 \qquad y(n) = \lambda^n$$

$$b_0 y(n) + b_1 y(n-1) + \dots + b_N y(n-N) = 0$$

$$b_0 \lambda^n + b_1 \lambda^{n-1} + b_2 \lambda^{n-2} + \dots + b_N \lambda^{n-N} = 0$$

$$y_d(n) = C_1 \lambda_1^n + C_2 \lambda_2^n + \dots + C_N \lambda_N^n$$

Ör: $y(n) - 2y(n-1) - 3y(n-2) = x(n)$ $\quad y(-1)=2,\ y(-2)=2$ $\quad n \geq 0$ için doğal çözüm?

$$\left.\begin{array}{l} y(n)=\lambda^n \\ x(n)=0 \end{array}\right\} \quad y(n) - 2y(n-1) - 3y(n-2) = 0$$

$$\lambda^n - 2\lambda^{n-1} - 3\lambda^{n-2} = 0$$

$$\lambda^{n-2}(\lambda^2 - 2\lambda - 3) = 0$$

$$\boxed{\lambda_1 = -1} \quad \boxed{\lambda_2 = 3}$$

bu durumda; $y_d(n) = c_1\lambda_1^n + c_2\lambda_2^n \Rightarrow c_1(-1)^n + c_2 3^n$

$n=0$ için; $y(0) - 2y(-1) - 3y(-2) = 0$

$y(0) - 2.2 - 3.2 = 0 \Rightarrow y(0) = 10 = c_1 + c_2$

$n=1$ için; $y(1) - 2y(0) - 3y(-1) = 0$

$y(1) - 2.10 - 3.2 = 0$

$y(1) = 26 = -c_1 + 3c_2$

$$\left.\begin{array}{l} -c_1 + 3c_2 = 26 \\ c_1 + c_2 = 10 \end{array}\right\} \quad \left.\begin{array}{l} c_1 = 1 \\ c_2 = 9 \end{array}\right\} \quad y_d(n) = (-1)^n + 9.3^n$$

Çift katlı kök çıkarsa

$$c_1\lambda_1^n + c_2 n \lambda_1^n$$

Zorlanmış çözüm:

1. doğal çözümle aynı çözüm
2. özel çözüm

$$y_z(n) + y_d(n) + y_ö(n) = y_T(n)$$
$$= c_3\lambda_1^n + c_4\lambda_2^n + y_ö(n)$$

X_n	$y_ö(n)$
$A.u(n)$	$K.u(n)$
$A.m^n$	$K.m^n$
$A.n^m$	$K_0 n^m + K_1 n^{m-1} + K_2 n^{m-2} + \dots + K_m$
$A^n.n^m$	$A^n(K_0 n^m + K_1 n^{m-1} + \dots + K_m)$
$A\cos w_0 n$ $A\sin w_0 n$	$K_1\cos w_0 n + K_2 \sin w_0 n$

Ör: fark denklemi $y(n) - 2y(n-1) - 3y(n-2) = x(n)$

$x(n) = 10u(n)$, $y(-1) = 2$ $y(-2) = 2$ $\qquad y_t(n) = ?$

$\Rightarrow$ $y_ö(n) = K.u(n)$

$Ku(n) - 2Ku(\cancel{n-1}) - 3Ku(\cancel{n-2}) = 0$

$K - 2K - 3K = 10$

$\boxed{K = -5/2}$

$y_z(n) = c_3(-1)^n + c_4 3^n - 5/2\, u(n)$

$y(0) - 2\cancel{y(-1)} - 3\cancel{y(-2)} = x_0$

Başlangıç koşullarını 0 kabul ediyoruz.

$n=0$ için; $y(0) = x(0) = 10 = c_3 + c_4 - 5/2$

$n=1$ için; $y(1) - 2y(0) - 3\cancel{y(-1)} = x(1)$

$y(1) = 10 + 20 = 30 = -c_3 + 3c_4 - 5/2$

$$\left.\begin{array}{l} c_3 + c_4 - 5/2 = 10 \\ -c_3 + 3c_4 - 5/2 = 30 \end{array}\right\} \left.\begin{array}{l} c_3 = 0.875 \\ c_4 = 11.125 \end{array}\right\} \begin{array}{l} y_z(n) = 0.875(-1)^n + 11.125(3)^n - 5/2\, u(n) \\ y_d(n) = (-1)^n + 9(3)^n \quad + \end{array}$$

$y_t(n) = 1.875(-1)^n + 20.125(3)^n - 5/2\, u(n)$ //

Ör: $y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$ $\qquad$ doğal çözüm = ?

$x(n) = 4^n u(n)$ $\qquad y(-2) = 0$, $y(-1) = 5$

$$\left.\begin{array}{l} \lambda^n - 3\lambda^{n-1} - 4\lambda^{n-2} = 0 \\ \lambda^{n-2}(\lambda^2 - 3\lambda - 4) = 0 \end{array}\right\} \begin{array}{l} \boxed{\lambda_1 = -1} \\ \boxed{\lambda_2 = 4} \end{array}$$

$\boxed{y_d(n) = c_1(-1)^n + c_2(4)^n}$

$n=0$ için; $y(0) - 3y(-1) - 4y(-2) = 0$

$y(0) = 15 = c_1 + c_2$

$n=1$ için: $y(1) - 3y(0) - 4y(-1) = 0$

$y(1) = 65 = -c_1 + 4c_2$

$$\left.\begin{array}{l} -c_1 + 4c_2 = 65 \\ c_1 + c_2 = 15 \end{array}\right\} \left.\begin{array}{l} \boxed{c_1 = -1} \\ \boxed{c_2 = 16} \end{array}\right\} \begin{array}{l} y_d(n) = -(-1)^n + 16(4)^n \\ = (-1)^{n+1} + (4)^{n+2} \end{array}$$ //

zorlanmış çözüm; $y_z(n) = c_3(-1)^n + c_4(4)^n + y_ö(n)$

$y_ö(n) = K4^n u(n)$ katlı kutup çıktığı için $K.n.4^n$ şeklinde yazmamız gerekiyor.

$y_ö(n) = K.n.4^n u(n)$

$\rightarrow Kn4^n \cancel{u(n)} - 3K(n-1)4^{n-1}\cancel{u(n-1)} - 4K(n-2)4^{n-2}\cancel{u(n-2)} = 4^n \cancel{u(n)} + 2.4^{n-1}\cancel{u(n-1)}$

$\rightarrow Kn4^n - 3K(n-1)4^{n-1} - 4K(n-2)4^{n-2} = 4^n + 2.4^{n-1}$

$\rightarrow 4^{n-2}(Kn4^2 - 3K(n-1)4 - 4K(n-2)) = 4^{n-1}(4+2) = 24$

$n=2$ için;

$\rightarrow Kn4^2 - 3K(n-1)4 - \cancel{4K(n-2)} = 4(4+2) = 24$

$32K - 12K = 24$

$20K = 24 \quad \boxed{K = 6/5}$

özel çözüm; $6/5\, n.4^n u(n)$

$y_z(n) = c_3(-1)^n + c_4(4)^n + 6/5\, n 4^n u(n)$

$n=0, \quad y(0) = x(0) + 2x(-1) = 1 + 0 = 1 = c_3 + c_4$

$n=1, \quad y(1) = x(1) + 2x(0) + 3y(0) = 4+2+3 = 9 = -c_3 + 4c_4 + 6/5 \cdot 4$

$c_3 = -1/25 \qquad c_4 = 26/25$

$y_T(n) = (-1)^{n+1} + (4)^{n+2} + -1/25(-1)^n + 26/25\,(4)^n + 6/5\,u(n)$

$= -26/25(-1)^n + 426/25\,(4)^n + 6/5\,u(n)$ //

1 Temmuz 2016
Cuma

doğal çözüm:

$\lambda_1, \lambda_2, \lambda_3 \implies y_d(n) = c_1\lambda_1^n + c_2\lambda_2^n + c_3\lambda_3^n$

$\lambda_1 = \lambda_2, \lambda_3 \implies y_d(n) = c_1\lambda_1^n + c_2 n \lambda_1^n + c_3\lambda_3^n$

$\lambda_1 = \lambda_2 = \lambda_3 \implies y_d(n) = c_1\lambda_1^n + c_2 n \lambda_1^n + c_3 n^2 \lambda_1^n$

$$y(0) = y_d(0) = c_1 + c_2 + c_3$$
$$= c_1 + c_3$$
$$= c_1$$

$$y(1) = y_d(1) = c_1\lambda_1 + c_2\lambda_2 + c_3\lambda_3$$
$$= c_1\lambda_1 + c_2\lambda_1 + c_3\lambda_3$$
$$= c_1\lambda_1 + c_2\lambda_1 + c_3\lambda_1$$

$$y(2) = y_d(2) = c_1\lambda_1^2 + c_2\lambda_2^2 + c_3\lambda_3^2$$
$$= c_1\lambda_1^2 + 2c_2\lambda_1^2 + c_3\lambda_3^2$$
$$= c_1\lambda_1^2 + 2c_2\lambda_1^2 + 4c_3\lambda_1^2$$

zorlanmış çözüm:

$y_z(n) = y_d(n) + y_ö(n)$

$y_ö(n) \longrightarrow$ giriş işaretine $(x(n))$ bağlı olarak

$x(n) = A\,u(n)$	$x(n) = A.m^n u(n)$	$x(n) = \cos w_0 n$, $\sin w_0 n$
$y_ö(n) = K.u(n)$	$y_ö(n) = K.m^n u(n)$	$y_ö(n) = K_1 \cos w_0 n + K_2 \sin w_0 n$

özet

Ör: fark denklemi $y(n) + 0.5\,y(n-1) = x(n)$

$x(n) = u(n)$, $y(-1) = 2$

$\Rightarrow \lambda^n + \frac{1}{2}\lambda^{n-1} = 0$

$\lambda^{n-1}(\lambda + 1/2) = 0$

$\boxed{\lambda_1 = -1/2}$

$y_d(n) = c_1(-1/2)^n$

$n=0$ için; $y(0) + \frac{1}{2}y(-1) = 0$

$y(0) = -1 = c_1$

$\boxed{y_d(n) = -(-1/2)^n}$

$y_z(n) = c_2(-1/2)^n + y_ö(n)$

$y_ö(n) = K.u(n)$

$K.u(n) + 1/2\,K\,u(n-1) = u(n)$

$n \geq 1$

$K + \frac{1}{2}K = 1 \Rightarrow \boxed{K = 2/3}$

$y(0) + \frac{1}{2}\cancel{y(-1)} = x(0)$

$y(0) = x(0) = 1 = c_2 + 2/3 \Rightarrow \boxed{c_2 = 1/3}$

$y_z(n) = 1/3\,(-1/2)^n + 2/3\,u(n) \quad + \quad y_d(n) = -(-1/2)^n$

$$\boxed{y_t(n) = -2/3\,(-1/2)^n + 2/3\,u(n)}$$

Ör: $y(n) + 0.5y(n-1) = x(n)$ birim impuls cevabı = ?

(doğal çözümü bulur gibi buluyoruz.)

$n \geq 0$

$y_d(n) = c(-1/2)^n u(n)$

$x(n) = \delta(n)$

$h(n) + \frac{1}{2}h(n-1) = \delta(n)$

$h(0) + \frac{1}{2}h(-1) = \delta(0) = 1 = c$

$h(n) = (-1/2)^n u(n)$

Ör: $y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$ birim impuls cevabı = ?

$y_d(n) = c_1(-1)^n + c_2(4)^n$

$h(n) - 3h(n-1) - 4h(n-2) = \delta(n) + 2\delta(n-1)$

$n=0;\quad h(0) - 3\cancel{h(-1)}^{0} - 4\cancel{h(-2)}^{0} = \delta(0)^{\to 1} + 2\cancel{\delta(-1)}$

$h(0) = 1 = c_1 + c_2$

$n=1;\quad h(1) - 3h(0) - 4\cancel{h(-1)}^{0} = \cancel{\delta(1)}^{0} + 2\delta(0)^{\to 1}$

$h(1) - 3 = 2 \Rightarrow h(1) = 5 = -c_1 + 4c_2$

$\boxed{c_1 = -1/5} \quad \boxed{c_2 = 6/5}$

$h(n) = \left[-1/5\,(-1)^n + 6/5\,(4)^n\right]u(n)$

[fark denklemlerinde birim zamanda $\frac{\Delta q}{\Delta t}$

durum denklemlerinde $q(n+1) = q(n)$
bir sonraki değer]

Durum değişkenleri yöntemi :

$$\underset{2\times1}{q(n+1)} = \underset{2\times2}{\overset{\to\text{matris}}{A}}\,\underset{2\times1}{q(n)} + \underset{2\times1}{\overset{\to\text{vektör.}}{B}}\,x(n)$$

$$\underset{1\times1}{y(n)} = \underset{1\times2}{C}\,\underset{2\times1}{q(n)} + \underset{1\times1}{D}\,x(n)$$

Ör: $y(n)-3y(n-1)-4y(n-2)=x(n)+2x(n-1)$

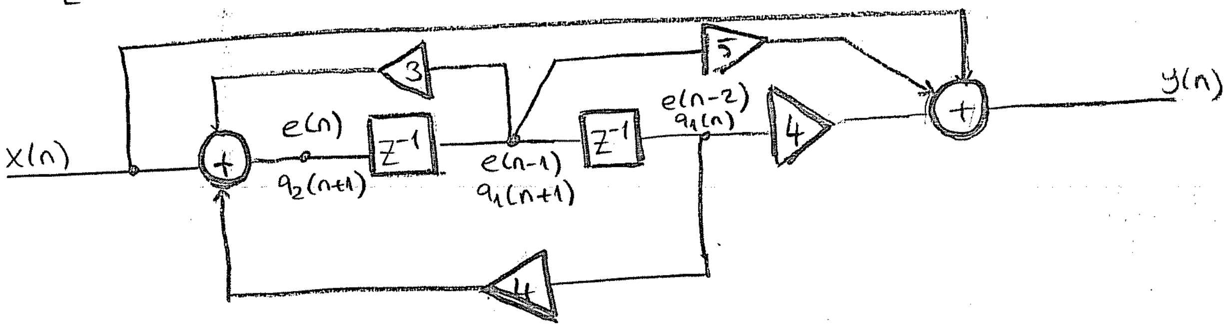
$e(n)-3e(n-1)-4e(n-2)=x(n)+2x(n-1)$

1. $\rightarrow$ $x(n)=e(n)-3e(n-1)-4e(n-2)$

$e(n)=x(n)+3e(n-1)+4e(n-2)$

durum değişkenleri	durum denklemleri
$q_1(n)=e(n-2)$	$q_1(n+1)=e(n-1)=q_2(n)$
$q_2(n)=e(n-1)$	$q_2(n+1)=e(n)=x(n)+3q_2(n)+4q_1(n)$

$$\begin{bmatrix} q_1(n+1) \\ q_2(n+2) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} q_1(n) \\ q_2(n) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} x(n)$$



2. $\longrightarrow$ $y(n)=e(n)+2e(n-1)$

$=x(n)+3e(n-1)+4e(n-2)+2e(n-1)$

$=x(n)+5e(n-1)+4e(n-2)$

$=x(n)+5q_2(n)+4q_1(n)$

$$y(n)=\underbrace{\begin{bmatrix} 4 & 5 \end{bmatrix}}_{C} \begin{bmatrix} q_1(n) \\ q_2(n) \end{bmatrix} + \underbrace{1}_{D} \cdot x(n)$$

Ör: $y(n)=x(n)+2x(n-1)+x(n-2)$

? $y(n)+y(n-1)-2y(n-2)=x(n)+\dots$

?

1. $x(n)=e(n)+e(n-1)-2e(n-2)$

$e(n)=x(n)-e(n-1)+2e(n-2)$

durum değişkenleri	durum denklemleri
$q_1(n)=e(n-2)$	$q_1(n+1)=e(n-1)=q_2(n)$
$q_2(n)=e(n-1)$	$q_2(n+1)=e(n)=x(n)-q_2(n)+2q_1(n)$

$$\begin{bmatrix} q_1(n+1) \\ q_2(n+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} q_1(n) \\ q_2(n) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} x(n)$$

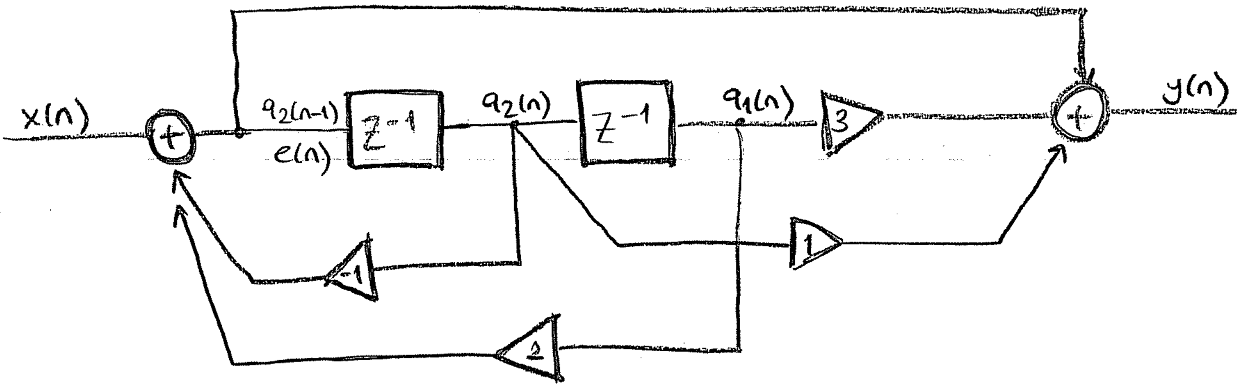
2. $y(n)=e(n)+2e(n-1)+e(n-2)$

$=x(n)-e(n-1)+2e(n-2)+2e(n-1)+e(n-2)$

$= x(n) + e(n-1) + 3e(n-2)$

$= x(n) + q_2(n) + 3q_1(n)$

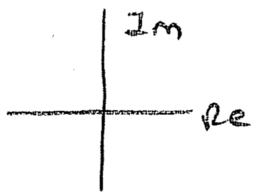
$$y(n) = \begin{bmatrix} 3 & 1 \end{bmatrix} \begin{bmatrix} q_1(n) \\ q_2(n) \end{bmatrix} + 1\, x(n)$$



✻ $y(k+2) + \dots \Rightarrow$ gibi ifade varsa, $k+2=n$ yapıyoruz.

Z Dönüşümü

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} = Z(x(n))$$



$z \rightarrow re^{j\theta}$ gibi bir karmaşık sayı

$x(n) \longleftrightarrow X(z)$

$y(n) = x(n) * h(n)$

$Y(z) = X(z) H(z)$

Yakınsaklık Bölgesi:

Toplama işleminin ∞'a gitmesini engelleyen karmaşık düzlemdeki bölgeye denir.

$$\sum_{n=-\infty}^{\infty} |x(n) z^{-n}| < \infty$$

Ör: $x(n) = \{1, 2, 5, 7, 0, 1\}$ Z dönüşümü ve yakınsama bölgesi?

$X(z) = x(0) z^{0} + x(1) z^{-1} + x(2) z^{-2} + \dots + x(5) z^{-5}$

$1.1 + 2.z^{-1} + 5z^{-2} + 7.z^{-3} + 0.z^{-4} + 1.z^{-5}$

$z \neq 0$ olduğu için tüm karmaşık düzlem

eğer 5, $x(0)$ olsaydı

$x(-2) z^{+2} + x(-1) z^{+1} + x(0).1 + x(1) z^{-1} + x(2) z^{-2} + x(3) z^{-3}$

$= z^2 + 2z + 5 + 7z^{-1} + z^{-3}$

$z \neq 0, \quad z \neq \infty$

Ör: $x(n)=\delta(n)$

$-\infty$ ve $+\infty$'a gidildikçe 0 olacağından $n=0$

$X(z)=1$

Yakınsama 0'dan farklı değerler $z\neq 0$

Ör: $x(n)=\delta(n+k)$

$X(z)=z^{k}$, $z\neq\infty$

* Sola gidildikçe; $z\neq\infty$
  sağa gidildikçe; $z\neq 0$

Ör: $x(n)=\alpha^{n}u(n)$

$$X(z)=\sum_{n=0}^{\infty}\alpha^{n}z^{-n}$$

* * sağ taraflı dizi
  → $n>0$ olduğu durumlarda genlik değerine sahip

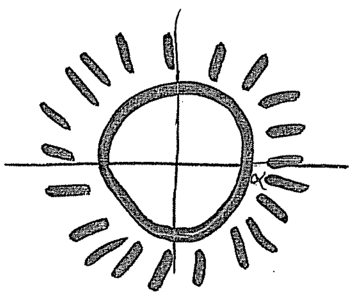
Z dönüşümü ve yakınsama bölgesi?

* * 0'sa genlik yok demektir.

$$X(z)=\sum_{n=0}^{\infty}\alpha^{n}z^{-n}=\sum_{n=0}^{\infty}(\alpha z^{-1})^{n}=\frac{1}{1-\alpha z^{-1}}$$

$|\alpha z^{-1}|<1$

$|z|>\alpha$



α yarıçaplı bölgenin dışında kalan alan (sağ taraflı dizilerde çemberin dışı olur.)

Ör: $x(n)=-\alpha^{n}u(-n-1)$

* sol taraflı dizi
→ $n<0$ olduğu durumlarda genlik değerine sahip

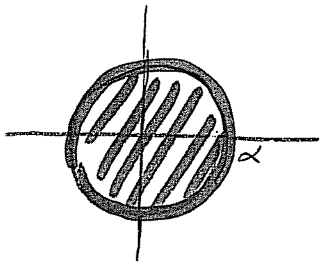
$$X(z)=\sum_{n=-\infty}^{-1}-\alpha^{n}z^{-n}$$

$$\Rightarrow -\sum_{n=-1}^{-\infty}\alpha^{n}z^{-n}$$ $\boxed{n=-\ell}$ dönüşümü yaptık

$$\Rightarrow -\sum_{\ell=1}^{\infty}\alpha^{-\ell}z^{\ell} \Rightarrow -\sum_{\ell=0}^{\infty}\alpha^{-\ell}z^{\ell}+1=1-\sum_{\ell=0}^{\infty}\alpha^{-\ell}z^{\ell}$$

0'dan başlatmak için 1 ekledik.

$$= \frac{1}{1-\alpha z^{-1}} \qquad |z| < \alpha$$



(sol taraflı dizilerde yakınsama bölgesi çemberin içi.)

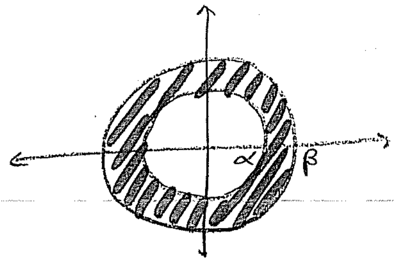
$$\alpha^n u(n) \longleftrightarrow \frac{1}{1-\alpha z^{-1}} \qquad |z| > \alpha \quad \text{dışı}$$

$$-\alpha^n u(-n-1) \longleftrightarrow \frac{1}{1-\alpha z^{-1}} \qquad |z| < \alpha \quad \text{içi}$$

Ör: $x(n) = \alpha^n u(n) + \beta^n u(-n-1)$

$$X(z) = \frac{1}{1-\alpha z^{-1}} - \frac{1}{1-\beta z^{-1}}$$

$$|z| > \alpha \cap |z| < \beta$$



Ör: $x(n) = (\frac{1}{2})^n u(n) - u(n-10)$, $X(z) = ?$

sağ taraflı ve sınırlı old. için $z \neq 0$, yakınsama bölg.

$$x(n) = \{1, \tfrac{1}{2}, \tfrac{1}{4}, \tfrac{1}{8}, \dots (\tfrac{1}{2})^9\}$$

$$X(z) = 1 + \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2} + \dots + (\frac{1}{2})^9 z^{-9}$$

Ör: $x(n) = 2^n u(n)$, $X(z) = ?$

$$X(z) = \frac{1}{1-2z^{-1}} \qquad |z| > 2$$

Ör: $x(n) = (-1/2)^n u(n)$, $X(z) = ?$

$$X(z) = \frac{1}{1+\frac{1}{2}z^{-1}} \qquad |z| > 1/2$$

$x(n) \longleftrightarrow X(z)$ bize verildiyse,

$x(n-k) \longrightarrow z^{-k}X(z)$ yakınsama bölgesi

Ör: $x(n) = \alpha^{n-1} u(n-1)$

$x_1(n) = \alpha^n u(n) \qquad X_1(z) = \dfrac{1}{1-\alpha z^{-1}} \qquad |z| > \alpha$

$x(n) = x_1(n-1) \qquad X(z) = z^{-1} X_1(z) = \dfrac{z^{-1}}{1-\alpha z^{-1}} \qquad |z|>|\alpha|$

Ör: $x(n) = [3(2)^n - 4(3)^n]\, u(n)$

$$X(z) = \frac{3}{1-2z^{-1}} - \frac{4}{1-3z^{-1}}$$

$|z|>2 \qquad |z|>3 \qquad \Longrightarrow \quad |z|>3$

Ör: $x(n) = \cos(w_0 n)\, u(n)$, $X(z) = ?$

$$\cos(w_0 n) = \left(\frac{e^{jw_0 n} + e^{-jw_0 n}}{2}\right) u(n)$$

$$X(z) = \frac{1}{2}\left(\frac{1}{1-e^{jw_0} z^{-1}} + \frac{1}{1-e^{-jw_0} z^{-1}}\right)$$

$|z| > \underbrace{|e^{jw_0}|}_{1} \qquad |z| > |e^{-jw_0}|$

$|z|>1$ dışında kalan bölge.

*
$$
\begin{aligned}
x(n) &\longleftrightarrow X(z) \longrightarrow r_1 < |z| < r_2 \\
a^n x(n) &\longleftrightarrow X\left(\frac{a}{z}\right) \longrightarrow |a| r_1 < |z| < |a| r_2 \\
x(-n) &\longleftrightarrow X(z^{-1}) \longrightarrow \frac{1}{r_2} < |z| < \frac{1}{r_1} \\
n\,x(n) &\longleftrightarrow -z \frac{\partial}{\partial z}(X(z)) \longrightarrow r_1 < |z| < r_2
\end{aligned}
$$

Ör: $x(n) = n \alpha^n u(n)$, z dönüşümü?

$x_1(n) = \alpha^n u(n) \Rightarrow X_1(z) = \dfrac{1}{1-\alpha z^{-1}} \qquad |z|>|\alpha|$

$x(n) = n\, x_1(n)$

$$X(z) = -z\frac{\partial}{\partial z}(X_1(z))$$

$$= -z \frac{-\alpha z^{-2}}{(1-\alpha z^{-1})^2} = \frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2} \qquad |z|>|\alpha|$$

Ör: $x(n) = |n| \left(\frac{1}{2}\right)^{|n|}$ (mutlak değerde hem poz, hem neg. tarafı düşüneceğiz)

$$= \overbrace{n\left(\frac{1}{2}\right)^n u(n)}^{n\geq 0} - \overbrace{n \cdot 2^n u(-n)}^{n<0}$$

$$-z\frac{d}{dz}\left(\frac{1}{1-\frac{1}{2}z^{-1}}\right) \qquad |z|>1/2$$

$x_2(n) = (1/2)^n u(n)$

$x_1(n) = x_2(-n)$

$x_1(z) = x_2(z^{-1})$

$n x_1(n) \longrightarrow -z\frac{d}{dz}(x_1(z))$

$n 2^n u(-n) = n x_1(n)$

$x_1(n) = 2^n u(-n) = x_2(-n) \longrightarrow x_1(z) = x_2(z^{-1}) \quad |z|<2$

$x_2(n) = (1/2)^n u(n) \longrightarrow x_2(z) = \frac{1}{1-\frac{1}{2}z^{-1}} \quad |z|>\frac{1}{2}$

$$x(z) = -z\frac{d}{dz}\left(\frac{1}{1-\frac{1}{2}z^{-1}}\right) + z\frac{d}{dz}(x_1(z))$$

$$|z| > 1/2 \cap |z| < 2$$

Ör: $x(n) = n(1/2)^n u(n-2)$

$x_1(n) = (1/2)^n u(n)$

$x_1(n-2) = (1/2)^{n-2} u(n-2)$

$x(n) = n \cdot \frac{1}{4} \cdot \left(\frac{1}{2}\right)^{n-2} u(n-2)$

① $= n\frac{1}{4} x_1(n-2)$

$x(z) = \frac{1}{1-\frac{1}{2}z^{-1}} \qquad |z|>\frac{1}{2}$

$\frac{z^{-2}}{1-\frac{1}{2}z^{-1}}$ → 2 birim ötelenmiş $\qquad |z| > \frac{1}{2}$

② $= \frac{1}{4} n x_1(n-2)$

$= \frac{1}{4}\left(-z\frac{d}{dz}\left(\frac{z^{-2}}{1-\frac{1}{2}z^{-1}}\right)\right)$

$|z| > 1/2$

Ör: $x(n) = \frac{1}{n} u(n-1)$, $x(z) = ?$

$n x(n) = u(n-1)$

$-z\frac{d}{dz}(x(z)) = \frac{z^{-1}}{1-z^{-1}}$

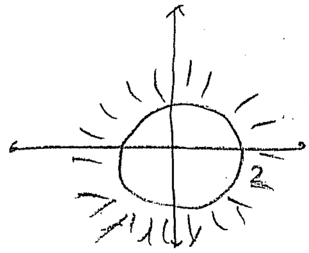
$$\boxed{u(n) \longrightarrow \frac{1}{1-z^{-1}} \quad |z|>1}$$

14.07.2016
Perşembe

Ör: $x(n) = 2^n u(n)$

$$X(z) = \frac{1}{1-2z^{-1}} \quad |z| > 2$$

yakınsama bölgesi çemberin dışı



$\rightarrow$ $|z| = 2$ olursa $\quad X(z) = \sum_{0}^{\infty} 2^n z^{-n} = \sum_{0}^{\infty} 1$

Ör: $(2^n - 1)u(n)$

$x(n) = 2^n u(n) - u(n)$

$$X(z) = \frac{1}{1-2z^{-1}} - \frac{1}{1-z^{-1}}$$

$|z|>2 \cap |z|>1$ $\Big\}$ $|z|>2$ yakınsama bölgesi

Ör: $x(n) = 2^n u(n)$

$y(n) = x(n-2)$

$$Y(z) = z^{-2} \cdot \frac{1}{1-2z^{-1}} \quad |z|>2$$

$$\boxed{y(n) = 2^{n-2} u(n-2)}$$

$2^n u(n)$ ötelenmiş.

$x(n \mp k) \longleftrightarrow z^{\pm k} X(z)$

yakınsama bölg. işaret ötelense de aynı olur.

Ör: $y(n) = n x(n)$

$$n x(n) \longleftrightarrow -z \frac{d}{dz}(X(z))$$

$$Y(z) = -z \frac{-2z^{-2}}{(1-2z^{-1})^2} \quad |z|>2$$

Ör: $y(n) = n x(n-2)$

tüm ifadenin türevini almamız lazım $\rightarrow$ $z^{-2} \dfrac{1}{1-2z^{-1}}$

$$Y_1(z) = -z \frac{d}{dz}\left(\frac{z^{-2}}{1-2z^{-1}}\right)$$

Ör: $x(n) * h(n)$
$X(z)H(z)$ $\Big\}$ $y(n) = \frac{1}{2} y(n-1) + 2x(n)$ fark denk. verilmiş ifadenin transfer fonk ? $(H(z)) = ?$

$\rightarrow$ $Y(z) = \frac{1}{2} z^{-1} Y(z) + 2X(z)$

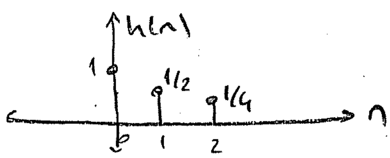
$Y(z) - \frac{1}{2} z^{-1} Y(z) = 2X(z)$

$Y(z)\left(1 - \frac{1}{2} z^{-1}\right) = 2X(z)$

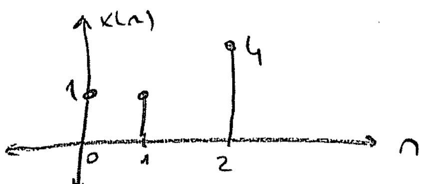
$$H(z) = \frac{Y(z)}{X(z)} = \frac{2}{1-\frac{1}{2}z^{-1}}$$

$$h(n) = 2 \cdot \left(\tfrac{1}{2}\right)^n u(n) \quad |z| > 1/2$$

Ör: $h(n) = \begin{cases} (1/2)^n, & 0 \leq n \leq 2 \\ 0, & \text{diğer} \end{cases}$



$x(n) = \delta(n) + \delta(n-1) + 4\delta(n-2)$



$y(n) = ?$

(z dönüşümü kullanarak)

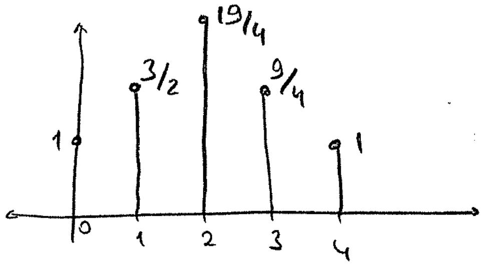
$$H(z) = 1 + \frac{1}{2} z^{-1} + \frac{1}{4} z^{-2}$$
$$X(z) = 1 + z^{-1} + 4 z^{-2}$$

$$Y(z) = X(z) H(z) = (1 + z^{-1} + 4z^{-2})\left(1 + \frac{1}{2} z^{-1} + \frac{1}{4} z^{-2}\right)$$

$$1 + \frac{3}{2} z^{-1} + \frac{19}{4} z^{-2} + \frac{9}{4} z^{-3} + z^{-4}$$

$$h(n) = \delta(n) + \frac{1}{2}\delta(n-1) + \frac{1}{4}\delta(n-2)$$

$$y(n) = \delta(n) + \frac{3}{2}\delta(n-1) + \frac{19}{4}\delta(n-2) + \frac{9}{4}\delta(n-3) + \delta(n-4)$$



TERS Z DÖNÜŞÜMÜ

Rezidü Metodu :

$$x(n) = \sum_{\substack{\text{tüm}\\\text{rezidüler}}} \left(X(z) z^{n-1}\right) \qquad \sum_{i} (z - z_i) X(z) z^{n-1}$$

→ paydayı 0 yapan z değerlerinden herhangi bir tanesi

$z = z_i$ koyuyoruz.

$$X(z) = \frac{1}{1 - a z^{-1}} \qquad |z| > |a|$$

$$X(z) z^{n-1} = \frac{z^{n-1}}{1 - a z^{-1}}$$

paydayı 0 yapan z değerlerine bakıyoruz.

$n \geq 0$, $n \geq 1$: $z = a$'da payda 0 olur.

$$(z-a)\frac{z^n}{z-a}\Big|_{z=a} = a^n$$

$n = 0$:
$$X(z) z^{n-1} = \frac{1}{z(1 - a z^{-1})} = \frac{1}{z-a} \Rightarrow a^n = a^0 = 1$$

$n = -1$:
$$X(z) z^{n-1} = \frac{1}{z(z-a)} \qquad z=0,\ z=a$$

$$(z-0) X(z) z^{n-1}\Big|_{z=0} + (z-a) X(z) z^{n-1}\Big|_{z=a}$$

$$= \frac{1}{z-a}\Big|_{z=0} + \frac{1}{z}\Big|_{z=a} = -\frac{1}{a} + \frac{1}{a} = 0$$

$\underline{n=-2}$ } $X(z)z^{n-1} = \dfrac{1}{z^2(z-a)}$ $z=0,\ z=a$ $\Big\} \left.\dfrac{-1}{(z-a)^2}\right|_{z=0} + \left.\dfrac{1}{z^2}\right|_{z=a} = 0$

* negatif değerler 0 çıktığı için $x(n) = a^n u(n)$ şeklinde yazabiliriz.

Kuvvet serileri:

$$X(z) = \sum_{-\infty}^{\infty} x(n) z^{-n}$$

Ör: $X(z) = \dfrac{1}{1-az^{-1}}$ $|z| > |a|$ → sağ taraflı işaret ; işaret sağ taraflıysa, z'nin üsleri nasıl değişiyor diye bakıyoruz. (−)'li terimler.

$x(0) + x(1)z^{-1} + x(2)z^{-2} + x(3)z^{-3} + \dots$

$$\begin{array}{l|l} 1 & 1-az^{-1} \\ \hline -\ 1+az^{-1} & 1+az^{-1}+a^2z^{-2}+\dots \\ \hline az^{-1} & \\ az^{-1} \mp a^2z^{-2} & \\ \hline a^2z^{-2} & \\ \vdots & \end{array}$$

sonsuza kadar gider

$$= \sum_{n=0}^{\infty} a^n z^{-n}$$

→ 0'dan ∞'a olduğu için u(n) yazabiliriz.

$$\boxed{x(n) = a^n u(n)}$$

* $|z| > |a|$ olsaydı; sol taraflı işaret, z'nin üsleri (+)'lı

$$\begin{array}{l|l} 1 & -az^{-1}+1 \\ \hline -\ 1-a^{-1}z & -a^{-1}z - a^{-2}z^2 - a^{-3}z^3 + \dots \\ \hline a^{-1}z & \\ a^{-1}z + & \\ \hline \vdots & \end{array}$$

$\Big\}$ $X(z) = -a^{-1}z - a^{-2}z^2 - a^{3}z^3 + \dots$

$$X(z) = \sum_{n=-\infty}^{-1} -a^n z^{-n} \quad //$$

$$\boxed{x(n) = -a^n u(-n-1)}$$

Ör: $X(z) = \dfrac{1}{1-1.5z^{-1}+0.5z^{-2}}$ $\left.\begin{array}{l}|z|>1 \\ |z|<0.5\end{array}\right\}$ yakınsama bölgeleri verilmiş kuvvet serileri yardımıyla bul.

Ödev:

kısmi kesirlere ayırma:

$$G(x) = \frac{8x^2+3x-21}{x^3-7x-6} = \frac{}{(x+2)(x-3)(x+1)} = \frac{\alpha_1}{(x+2)} + \frac{\alpha_2}{(x-3)} + \frac{\alpha_3}{x+1}$$

$$\left.\begin{aligned} \alpha_1 &= (x+2)G(x)\Big|_{x=-2} = \frac{}{(x-3)(x+1)}\Big|_{x=-2} \\ \alpha_2 &= (x-3)\,G(x)\Big|_{x=3} = \frac{}{(x+2)(x+1)}\Big|_{x=3} \\ \alpha_3 &= (x+1)G(x)\Big|_{x=-1} = \frac{}{(x+3)(x+2}\Big|_{x=-1} \end{aligned}\right\} \quad G(x) = \frac{1}{x+2} + \frac{3}{x-3} + \frac{4}{x+1} \,//$$

katlı kutup varsa;

$$G(x) = \frac{N(x)}{(x-b)^r(x-a_1)(x-a_2)\ldots}$$

$$G(x) = \frac{\beta_0}{(x-b)^r} + \frac{\beta_1}{(x-b)^{r-1}} + \frac{\beta_2}{(x-b)^{r-2}} + \cdots + \frac{\beta_{r-1}}{x-b} + \frac{\alpha_1}{x-a_1} + \frac{\alpha_2}{x-a_2} + \cdots$$

$$\alpha_1 = (x-a_1)\,G(x)\Big|_{x=a_1}$$

$$\beta_0 = (x-b)^r G(x)\Big|_{x=b}$$

$$\beta_1 = \frac{d}{dx}(\beta_0)\Big|_{x=b}$$

$$\beta_k = \frac{1}{k}\frac{d}{dx}(\beta_{k-1})\Big|_{x=b}$$

Ör: $X(z) = \dfrac{4-\frac{7}{4}z^{-1}+\frac{1}{4}z^{-2}}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})}$ $\quad |z| > 1/2$

$$\begin{array}{l|l} 1/4\, z^{-2} - \frac{7}{4}z^{-1} + 4 & 1/8\, z^{-2} - \frac{3}{4}z^{-1} + 1 \\ \hline 2 - \frac{1}{4}z^{-1} & 2 \end{array}$$

$$X(z) = 2 + \frac{2 - 1/4\, z^{-1}}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})} = 2 + \frac{A_1}{1-\frac{1}{2}z^{-1}} + \frac{A_2}{1-\frac{1}{4}z^{-1}}$$

$$A_1 = \frac{2-\frac{1}{2}}{1-\frac{1}{2}} = 3 \qquad A_2 = \frac{2-\frac{1}{4}z^{-1}}{1-\frac{1}{2}z^{-1}} = \frac{2-1}{1-2} = -1$$

$$x(n) = 2\delta(n) + 3\left(\frac{1}{2}\right)^n u(n) - \left(\frac{1}{4}\right)^n u(n)$$

Ör: $$X(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})} = \frac{A}{1-2z^{-1}} + \frac{B}{1-z^{-1}}$$

$$A = (1-2z^{-1})X(z)\Big|_{z^{-1}=\frac{1}{2}} = \frac{1}{1-z^{-1}}\Big|_{z^{-1}=1/2} = 2$$

$$B = (1-z^{-1})X(z)\Big|_{z^{-1}=1} = \frac{1}{1-2z^{-1}}\Big|_{z^{-1}=1} = -1$$

$1<|z|<2$ olsaydı;

$|z|<2 \qquad |z|>1$

$$x(n) = -2(2)^n u(-n-1) - u(n)$$

Ör: $$X(z) = \frac{1}{(1-z^{-1})^2(1+z^{-1})} = \frac{A_1}{(1-z^{-1})^2} + \frac{A_2}{(1-z^{-1})} + \frac{B}{1+z^{-1}}$$

$$B = (1+z^{-1})X(z)\Big|_{z^{-1}=-1} = \frac{1}{(1-z^{-1})^2}\Big|_{z^{-1}=-1} = \frac{1}{4}$$

$$A_1 = (1-z^{-1})^2 X(z)\Big|_{z^{-1}=1} = \frac{1}{1+z^{-1}}\Big|_{z^{-1}=1} = \frac{1}{2}$$

$$A_2 = \frac{d}{dz}(A_1)\Big|_{z^{-1}=1} = \frac{0-(-z^{-2})}{(1+z^{-1})^2}\Big|_{z^{-1}=1} = \frac{1}{4}$$

$$X(z) = \underbrace{\frac{1/2}{(1-z^{-1})^2}}_{?} + \underbrace{\frac{1/4}{1-z^{-1}}}_{+\frac{1}{4}u(n)} + \frac{1/4}{1+z^{-1}}$$

$$? \quad + \frac{1}{4}u(n) + \frac{1}{4}(-1)^n u(n)$$

$$\frac{d}{dz}\left(\frac{1}{1-z^{-1}}\right)=\frac{-z^{-2}}{(1-z^{-1})^2}$$ bizim $\frac{1/2}{(1-z^{-1})^2}$ elde etmemiz lazım. o yüzden;

$$\underbrace{\frac{-1}{2}\cdot z^2\frac{d}{dz}\left(\frac{1}{1-z^{-1}}\right)}_{\frac{-z^{-2}}{(1-z^{-1})^2}}=\frac{1/2}{(1-z^{-1})^2}$$

$$\boxed{-z\frac{d}{dz}(X(z))\longleftrightarrow n(x(n))}$$

$$z\underbrace{\left[-z\frac{d}{dz}\underbrace{\left(\frac{1}{1-z^{-1}}\right)}_{u(n)}\right]}_{n.u(n)}=\frac{1}{(1-z^{-1})^2}$$

$$\boxed{\begin{array}{l} z^{k}X(z)\longrightarrow x(n+k)\\ z^{-k}X(z)\longrightarrow x(n-k)\end{array}}$$

$$\frac{1}{2}(n+1)u(n+1)\Rightarrow\frac{1}{2}(n+1)u(n)$$

$$\frac{1}{2}(n+1)u(n)+\frac{1}{4}u(n)+\frac{1}{4}(-1)^n u(n)$$

$$=\left[\frac{1}{2}(n+1)+\frac{1}{4}+\frac{1}{4}(-1)^n\right]u(n)$$

Ör: ödev: $X(z)=\dfrac{1+\frac{1}{4}z^{-1}}{(1-1/2\,z^{-1})^2}$ $\quad |z|>1/2$

Ör: ödev: $X(z)=\dfrac{1}{(1-z^{-1})(1-z^{-2})}\xrightarrow{\text{sağ taraflı}}=\dfrac{1}{(1-z^{-1})(1-z^{-1})(1+z^{-1})}$

$$X(z)=\frac{1}{(1-z^{-1})^2(1+z^{-1})}$$

$$\boxed{\text{Cevap}=\frac{1}{4}\left((-1)^n+2(n+1)\right)u(n)}$$

örnek $H(z)=\dfrac{z}{z+\frac{1}{2}}=\dfrac{1}{1+\frac{1}{2}z^{-1}}=\dfrac{Y(z)}{X(z)}$

$$\left.\begin{array}{l} Y(z)\cdot(1+\frac{1}{2}z^{-1})=X(z)\\ Y(z)+\frac{1}{2}z^{-1}Y(z)=X(z)\end{array}\right\}\ y(n)+\frac{1}{2}y(n-1)=x(n)$$

Ör: $x(n) = e^{-\alpha T n}$ $n \geq 0$ } işaretin z dönüşümü ?

$x(n) = e^{-\alpha T n} u(n)$

$$X(z) = \sum x(n) z^{-n} = \sum_{n=0}^{\infty} (e^{-\alpha T})^n z^{-n} = \frac{1}{1 - e^{-\alpha T} z^{-1}} \qquad |z| > |e^{-\alpha T}|$$

Ör: $e(k) = e^{\alpha(2k-1)}$ $k \leq -1$

$e(k) = e^{-\alpha(2k-1)}$ $k \geq 0$

$$e(n) = e^{-\alpha(2n+1)} u(n) + e^{\alpha(2n+1)} u(-n-1)$$

$$e(n) = e^{-\alpha} e^{-2\alpha n} u(n) + 2^{\alpha n} e^{\alpha} u(-n-1)$$

$$E(z) = \frac{e^{-\alpha}}{1 - e^{-2\alpha} z^{-1}} - \frac{e^{\alpha}}{1 - e^{2\alpha} z^{-1}} \qquad |z| < e^{2\alpha}$$

$$e^{-2\alpha} < |z| < e^{2\alpha}$$

Ör: $y(n) + \frac{1}{3} y(n-1) = x(n) + x(n-1)$, $n \geq 0$ $x(n) = \left(\frac{1}{3}\right)^n$

$$X(z) = \frac{1}{1 - \frac{1}{3} z^{-1}}$$

$$Y(z) + \frac{1}{3} z^{-1} Y(z) = X(z) + z^{-1} X(z)$$

$$Y(z)\left(1 + \frac{1}{3} z^{-1}\right) = X(z)\left(1 + z^{-1}\right)$$

$$Y(z) = \frac{1 + z^{-1}}{\left(1 + \frac{1}{3} z^{-1}\right)\left(1 - \frac{1}{3} z^{-1}\right)} = \frac{A}{1 + \frac{1}{3} z^{-1}} + \frac{B}{1 - \frac{1}{3} z^{-1}}$$

$$A = \left.\frac{1 + z^{-1}}{1 - \frac{1}{3} z^{-1}}\right|_{z^{-1} = -3} = -2 \qquad B = \left.\frac{1 + z^{-1}}{1 + \frac{1}{3} z^{-1}}\right|_{z^{-1} = +3} = 2$$

$$y(n) = -2\left(-\frac{1}{3}\right)^n u(n) + 2\left(\frac{1}{3}\right)^n u(n)$$

Çift taraflı z dönüşümü; $X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$

Tek taraflı z dönüşümü; $X^+(z) = \sum_{n=0}^{\infty} x(n) z^{-n}$

Ör: $y(n) = x(n-2)$

$$y^+(z) = x(-2) + z^{-1}x(-1) + z^{-2}x^+(z)$$

$$y(-1) = 3$$

$$y^+(z) + \frac{1}{3}\left(y(-1) + z^{-1}y^+(z)\right) = x^+(z) + \cancel{x(-1)} + z^{-1}x^+(z)$$

$$y^+(z).\left(1 + \frac{1}{3}z^{-1}\right) = x^+(z)\left(1+z^{-1}\right) - 1 = \frac{1+z^{-1}}{1-\frac{1}{3}z^{-1}} - 1$$

$$= \frac{1+z^{-1} - 1 + \frac{1}{3}z^{-1}}{1-\frac{1}{3}z^{-1}} = \frac{4/3\, z^{-1}}{1-1/3\, z^{-1}} \quad //$$

$$y(z) = \frac{4/3\, z^{-1}}{\left(1-\frac{1}{3}z^{-1}\right)\left(1+\frac{1}{3}z^{-1}\right)} = \frac{A}{1-\frac{1}{3}z^{-1}} + \frac{B}{1+\frac{1}{3}z^{-1}} \quad //$$

Ör: $q(n+1) = Aq(n) + Bx(n)$

$y(n) = Cq(n) + Dx(n)$, $H(z) = ?$

$$\left.\begin{aligned} z.Q(z) &= \overset{\text{matris.}}{A}\,Q(z) + BX(z) \\ z.Q(z) - A.Q(z) &= BX(z) \end{aligned}\right\} \quad Q(z) = (zI - A)^{-1}BX(z)$$

$$\begin{aligned} Y(z) &= C.Q(z) + DX(z) \\ &= C.(z.I.-A)^{-1}.BX(z) + DX(z) \\ &= \left[C(zI-A)^{-1}B + D\right]X(z). \end{aligned}$$

$$H(z) = \frac{Y(z)}{X(z)} = C(zI-A)^{-1}B + D$$

$$zI - A = \begin{bmatrix} z & 0 \\ 0 & z \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} z & -1 \\ -2 & z+1 \end{bmatrix}$$

$$-\det = z(z+1) - 2$$

$$(zI-A)^{-1} = \frac{1}{z(z+1)-2}\begin{bmatrix} z+1 & 1 \\ 2 & z \end{bmatrix}$$

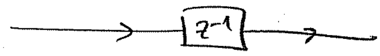
$$\frac{1}{z(z+1)-3}\begin{bmatrix} 3 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ z \end{bmatrix} + 1 = \frac{3+z}{z(z+1)-3} + 1$$

15.07.2016
Cuma

Birim gecikme sistemleinin karşılığı:

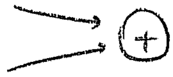
$y(n) = x(n-1)$

$y(n) = x(n) * h(n) = x(n-T)$



$h(n) = \delta(n-1)$ $\quad \boxed{H(z) = z^{-1}}$

* toplayıcı



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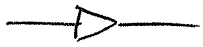
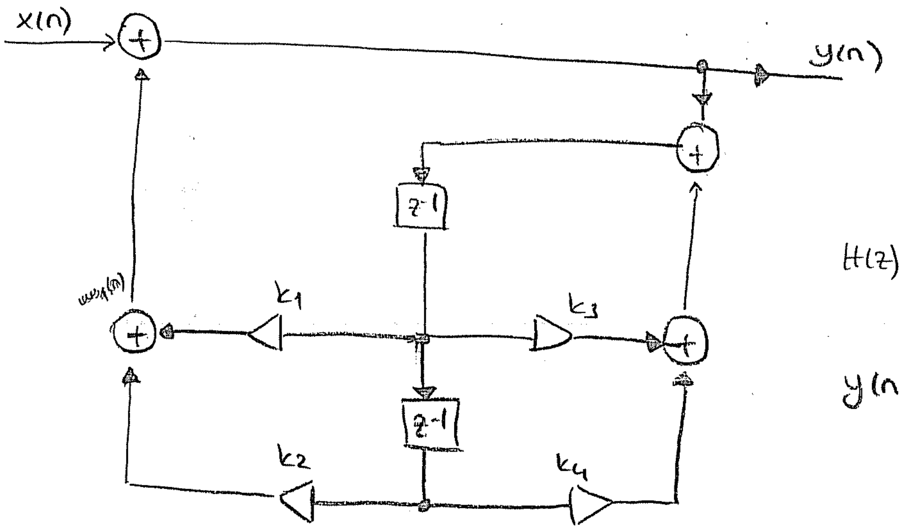


Ör:



$$H(z) = \frac{Y(z)}{X(z)}$$

$y(n) = x(n) + w_1(n)$

$w_1(n) = k_1 V(n-1) + k_2 V(n-2)$

$y(n) = x(n) + k_1 V(n-1) + k_2 V(n-2)$

$Y(z) = X(z) + k_1 z^{-1} V(z) + k_2 z^{-2} V(z)$

$V(n) = y(n) + k_3 V(n-1) + k_4 V(n-2)$

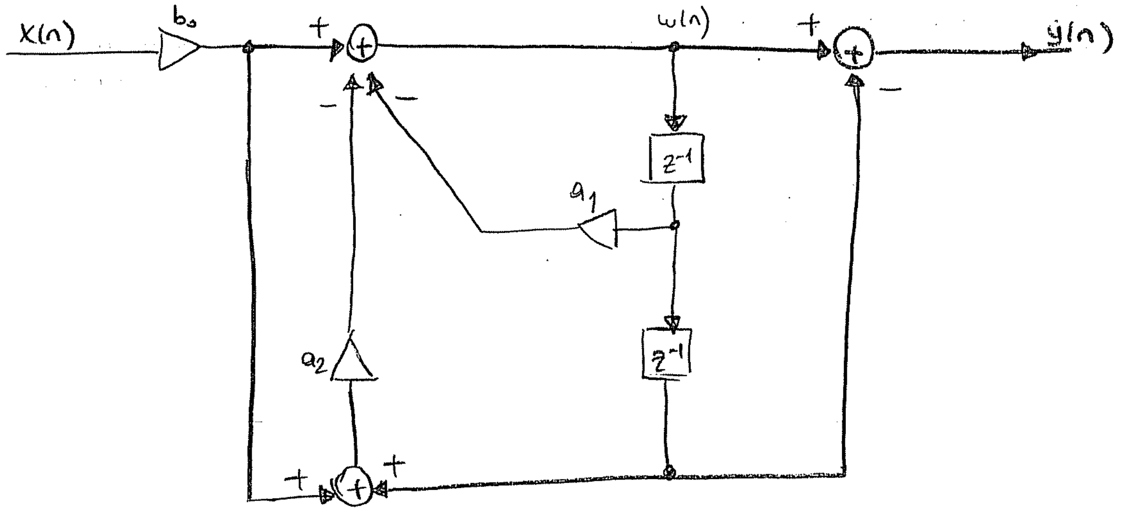
$V(z) = Y(z) + k_3 z^{-1} V(z) + k_4 z^{-2} V(z)$

$V(z)\left(1 - k_3 z^{-1} - k_4 z^{-2}\right) = Y(z)$

$$V(z) = \frac{Y(z)}{1 - k_3 z^{-1} - k_4 z^{-2}}$$

$$X(z) = Y(z) \cdot \left(1 - \frac{k_1 z^{-1} + k_2 z^{-2}}{1 - k_3 z^{-1} - k_4 z^{-2}}\right)$$

Ör:



$$y(n) = w(n) - w(n-2)$$

$$w(n) = b_0 x(n) - a_2 (b_0 x(n) + w(n-2)) - a_1 w(n-1)$$

$$W(z) = b_0 X(z) - a_2 b_0 X(z) - a_2 z^{-2} W(z) - a_1 z^{-1} W(z)$$

$$W(z)(1 + a_2 z^{-2} + a_1 z^{-1}) = X(z)(b_0 - a_2 b_0)$$

$$W(z) = \frac{b_0 - a_2 b_0}{1 + a_1 z^{-1} + a_2 z^{-2}} X(z)$$

$$Y(z) = W(z) - z^{-2} W(z)$$
$$Y(z) = W(z)(1 - z^{-2})$$

$$\frac{Y(z)}{X(z)} = \frac{b_0 - a_2 b_0 (1 - z^{-2})}{1 + a_1 z^{-1} + a_2 z^{-2}} \quad //$$

kararlılık

1. giriş-çıkış sınırlılığı
2. impuls cevabı $\sum |h(n)| < \infty$
3. Yakınsama bölgesi; birim çemberi içeriyorsa kararlı.

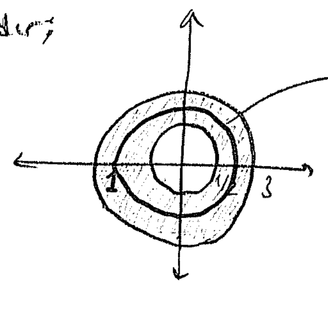
nedensellik:

1. $y(n) = x(n)$ nedensel, $x(n-k)$ nedensel değil.
2. $h(n) = 0 \quad n < 0$
3. $H(z)$ yakınsama bölgesi: herhangi bir çemberin dış bölgesi.

Ör: $H(z) = \frac{1}{1 - \frac{1}{2} z^{-1}} + \frac{2}{1 - 3z^{-1}}$ (sistemin transfer fonk. kısmi basit kesirlere ayrıldıktan sonra bu hale gelmiş.)

$$\frac{1}{2} < |z| < 3$$

- sistem nedensel değil çünkü hem sağ taraflı hem de sol (simit şeklinde)
- kararlıdır;



$$h(n) = \left(\tfrac{1}{2}\right)^n u(n) - 2(3)^n u(-n-1)$$

Ör: $|z|>3$ için ?

nedenseldir ve kararsızdır.

$$h(n) = (1/2)^n u(n) + 2(3)^n u(n)$$

$n<0$ olduğu durumlarda $h(n)=0$

- nedensel

n sonsuza giderken $h(n)$ de sonsuza gider

- kararsız.

Ör: $|z|<1/2$ için ?

nedensel değil ve kararsız.

$$h(n) = -\left(\frac{1}{2}\right)^n u(-n-1) - 2(3)^n u(-n-1)$$

Ör: $$H(z) = \frac{1/4}{1-2z^{-1}} - \frac{1/4}{1+2z^{-1}} \qquad |z|>2$$

nedensel ve kararsız.

$$h(n) = \frac{1}{4}(2)^n u(n) - \frac{1}{4}(-2)^n u(n)$$

Ör: $|z|<2$ için ?

nedensel değil ve kararlı

$$h(n) = -\frac{1}{4}(2)^n u(-n-1) + \frac{1}{4}(-2)^n u(-n-1)$$

Ör: $X(z) = \ln(1+az^{-1}) \quad |z|>a \qquad x(n)=?$

$$\underbrace{-z \cdot \frac{d}{dz} X(z)}_{} = \frac{-a z^{-2} \cdot (-z)}{1+az^{-1}}$$

$$n \cdot x(n) = \frac{a \cdot z^{-1} \cdot 1}{1+az^{-1}}$$

$a^n u(n)$

1 birim ötelenmiş hali

$$n\, x(n) = a \cdot (-a)^{n-1} u(n-1)$$

$$x(n) = \frac{a}{n} (-a)^{n-1} u(n-1)$$

$$y(n) = x(n-1)$$

$$y^+(z) = x(-1) + z^{-1} X^+(z)$$

$$y(n) = x(n+1)$$

$$X^+(z) = \sum_0^\infty x(n) z^{-n} = x(0) + x(1)z^{-1} + x(2)z^{-2} + \dots$$

$$y^+(z) = \sum_0^\infty y(n) z^{-n} = \sum_0^\infty x(n+1) z^{-n}$$

$x^+(z) - x(0) = x_1 z + \dots$

$$\boxed{z\left(x^+(z) - x(0)\right) = y^+(z)}$$

$\boxed{\delta(n) \rightarrow z \text{ domaininde } 1\text{'e eşittir.}}$

Ör: $y(n) - \frac{1}{4}y(n-2) = \delta(n) \quad n \geq 0$

$y(n) = 0$, $y(-1) = ?$ $y(-2) = ?$ (tüm $y(n)$ değerlerinin 0 olması için)

$$y^+(z) - \frac{1}{4}\left(y(-2) + z^{-1}y(-1) + z^{-2}y^+(z)\right) = 1$$

$$y^+(z)\left(1 - \frac{1}{4}z^{-2}\right) = 1 + \frac{1}{4}y(-2) + \frac{1}{4}z^{-1}y(-1)$$

$$y^+(z) = \frac{1 + \frac{1}{4}y(-2) + \frac{1}{4}z^{-1}y(-1)}{1 - \frac{1}{4}z^{-2}} = 0 \quad \text{için}$$

$y(-1) = 0$ $\quad y(-2) = -4$

Ör: $y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$, $y(-1) = 5$, $y(-2) = 0$

$x(n) = 4^n u(n)$

$$y_d(n) = \left((-1)^{n+1} + 4^{n+2}\right)u(n)$$

$$y_z(n) = \frac{-1}{25}(-1)^n + \frac{26}{25}(4)^n + \frac{6}{5}n(4)^n$$

$+$

$$y_t(n) = \frac{-26}{25}(-1)^n + \left(16 + \frac{26}{25}\right)4^n + \frac{6}{5}n(4)^n$$

$$y^+(z) - 3\left[y(-1) + z^{-1}y^+(z)\right] - 4\left[\underset{0}{\cancel{y(-2)}} + z^{-1}y(-1) + z^{-2}y^+(z)\right]$$

$$= \left[x^+(z) + 2(\underset{0}{\cancel{x(-1)}} + z^{-1}x^+(z)\right]$$

$$x(z) = \frac{1}{1 - 4z^{-1}}$$

$$y^+(z)(1 - 3z^{-1} - 4z^{-2}) - 15 - 20z^{-1} = \frac{1}{1 - 4z^{-1}}(1 + 2z^{-1})$$

$$y^+(z)(1 - 3z^{-1} - 4z^{-2}) = \frac{1 + 2z^{-1}}{1 - 4z^{-1}} + 15 + 20z^{-1}$$

$$= \frac{1 + 2z^{-1} + (15 + 20z^{-1})(1 - 4z^{-1})}{1 - 4z^{-1}} = \frac{16 - 38z^{-1} - 80z^{-2}}{1 - 4z^{-2}}$$

$$y^+(z) = \frac{16 - 38z^{-1} - 80z^{-2}}{\underbrace{(1 - 4z^{-1})(1 - 4z^{-1})}_{\text{katlı kutup}}(1 + z^{-1})}$$

$$= \frac{A}{(1-4z^{-1})^2} + \frac{B}{(1-4z^{-1})} + \frac{C}{(1+z^{-1})}$$

$$A = \left.\frac{16 - 38z^{-1} - 80z^{-2}}{1+z^{-1}}\right|_{z^{-1}=1/4} = \frac{16 - 38/4 - 80/16}{1+1/4} = \frac{11 - \frac{38}{4} = \frac{6}{4}}{\frac{5}{4}} = \frac{6}{5}$$

$$C = \left.\frac{16 - 38z^{-1} - 80z^{-2}}{(1-4z^{-1})^2}\right|_{z^{-1}=-1} = \frac{16+38-80}{(1+4)^2}$$

$$B = \left.\frac{d}{dz}(A)\right|_{z^{-1}=1/4}$$

$$\underbrace{\frac{A.z}{4}\Bigg[\underbrace{(-z).\frac{d}{dz}\left(\frac{1}{1-4z^{-1}}\right)}_{4^n u(n)}\Bigg]}_{n4^n u(n)} = \frac{-4z^{-2}.(-z)}{(1-4z^{-1})^2}$$

$$= \frac{4z^{-1}}{(1-4z^{-1})^2} \cdot \frac{A.z}{4} \Big\} A(n+1)4^n u(n+1)$$

Ör: $y(n) = y(n-1) - y(n-2) + \frac{1}{2}x(n) + \frac{1}{2}x(n-1)$ $\quad y_d(n)?$

$y(-1) = 3/4 \quad y(-2) = 1/4 \quad x(n) = (1/2)^n u(n)$

$$y(n) - y(n-1) + y(n-2) = \frac{1}{2}x(n) + \frac{1}{2}x(n-1)$$

$$\lambda^n - \lambda^{n-1} + \lambda^{n-2} = 0$$

$$\lambda^{n-2}(\lambda^2 - \lambda + 1) = 0$$

$$(\lambda - re^{j\theta})(\lambda - re^{-j\theta}) = \lambda^2 - \lambda + 1$$

$$\lambda^2 - \lambda re^{-j\theta} - \lambda re^{j\theta} + r^2$$

$$\frac{re^{-j\theta} + re^{j\theta}}{2} = \frac{1}{2} \quad r\cos\theta = \frac{1}{2} \quad \theta = \mp 60^\circ$$

$$\lambda_1 = e^{-j60^\circ} \qquad \lambda_2 = e^{j60^\circ}$$

$$\cos 60^\circ - j\sin 60^\circ \qquad \cos 60^\circ + j\sin 60^\circ$$

$$\lambda_1 = \frac{1-\sqrt{3}j}{2} \qquad \lambda_2 = \frac{1+\sqrt{3}j}{2}$$

$$y_d(n) = c_1 e^{-j\frac{n\pi}{3}} + c_2 e^{j\frac{n\pi}{3}}$$

$$y_d(n) = c_1(\lambda_1)^n + c_2(\lambda_2)^n$$

$$y(0) = y(-1) - y(-2) = \frac{3}{4} - \frac{1}{4} = \frac{1}{2} = c_1 + c_2$$

$$y(1) = y(0) - y(-1) = \frac{1}{2} - \frac{3}{4} = -\frac{1}{4} = c_1 e^{-j\pi/3} + c_2 e^{j\pi/3}$$

$$= c_1\left(\frac{1 \mp \sqrt{3}j}{2}\right) + c_2\left(\frac{1+\sqrt{3}j}{2}\right)$$

$$= \frac{c_1}{2} + \frac{c_2}{2} = -\frac{1}{4} \qquad -\frac{c_1\sqrt{3}}{2} + \frac{c_2\sqrt{3}}{2} = 0$$

$$c_1 = c_2$$

örnek $x(n) = 2^n u(-n-1)$

$$h(n) = \delta(n) + \delta(n-1)$$

$$y(n) = x(n) * (\delta(n) + \delta(n-1))$$

$$= x(n) + x(n-1)$$

Fourier Seri Açılımı :

21 Temmuz 2016 21
Perşembe

→ frekans bilgisini verir.
→ Periyodik işaretler için kullanıyoruz.

$$x(t)=\sum_{k=-\infty}^{+\infty} a_k \phi_k(t)=\sum_{k=-\infty}^{+\infty} a_k e^{jkw_0 t}=\sum_{k=-\infty}^{+\infty} a_k e^{jk\left(\frac{2\pi}{T}\right)t}$$

$k=+1,-1$, 1. harmonik
$k=+2,-2$, 2. harmonik

* $k=0$ iken DC olur, yani $+5V, -5V$ gibi

$$\boxed{w_0=\frac{2\pi}{T_0} \rightarrow \text{temel frekans}} \qquad \boxed{a_k=\frac{1}{T_0}\int_{T_0} x(t)e^{-jkw_0 t}dt}$$

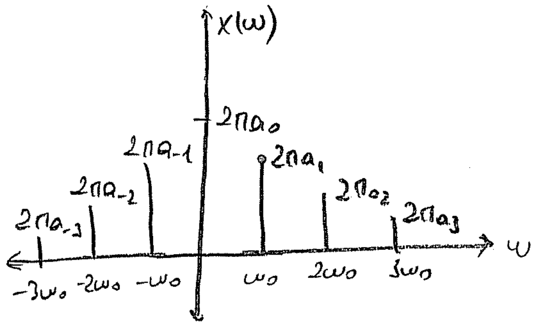
Ör: $x(t)=\sum_{k=-3}^{3} a_k e^{jk2\pi t}$ katsayıları ?

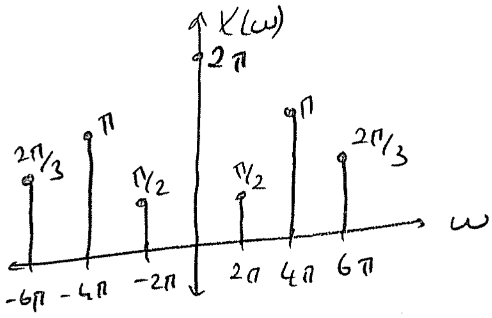
$$x(t)=a_{(-3)}e^{-j6\pi t}+a_{(-2)}e^{-4j\pi t}+a_{(-1)}e^{-j2\pi t}+a_0+a_1 e^{j2\pi t}+a_2 e^{j4\pi t}+a_3 e^{j6\pi t}$$

$$=\frac{1}{3}\left(e^{j6\pi t}+e^{-j6\pi t}\right)+\frac{1}{2}\left(e^{j4\pi t}+e^{-j4\pi t}\right)+\frac{1}{4}\left(e^{j2\pi t}+e^{-j2\pi t}\right)+1$$

$a_0=1$ // $a_1=a_{(-1)}=1/4$ // $a_2=a_{(-2)}=1/2$ // $a_3=a_{(-3)}=1/3$ //

$$=\frac{2}{3}\cos(6\pi t)+\cos(4\pi t)+\frac{1}{2}\cos(2\pi t)+1 \;//$$

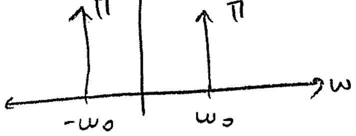




$w_0=2\pi$

$a_1=\frac{1}{4}$

Ör: $\cos(w_0 t)$ → fourier seri açılımı ?

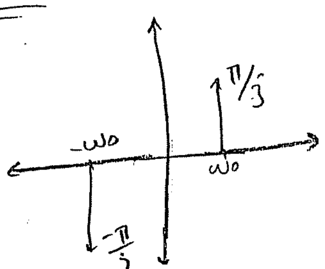


$=\frac{1}{2}\left(e^{jw_0 t}+e^{-jw_0 t}\right)$

w_0 = temel frekans

$a_1=a_{(-1)}=1/2$

Ör: $\sin(w_0 t)$ → fourier seri açılımı ?



$=\frac{1}{2j}\left(e^{jw_0 t}-e^{-jw_0 t}\right)$

w_0 = temel frekans

$a_1=\frac{1}{2j}$ $\quad a_{(-1)}=-\frac{1}{2j}$

Ör: $x(t) = \cos\underbrace{(2t + \pi/4)}$

*t'nin katsayısı bize w_0'ı verir.
$w_0 = 2$

$$= \frac{e^{j(2t+\pi/4)} + e^{-j(2t+\pi/4)}}{2} = \frac{e^{j2t} \cdot e^{j\pi/4}}{2} + \frac{e^{-j2t} \cdot e^{-j\pi/4}}{2}$$

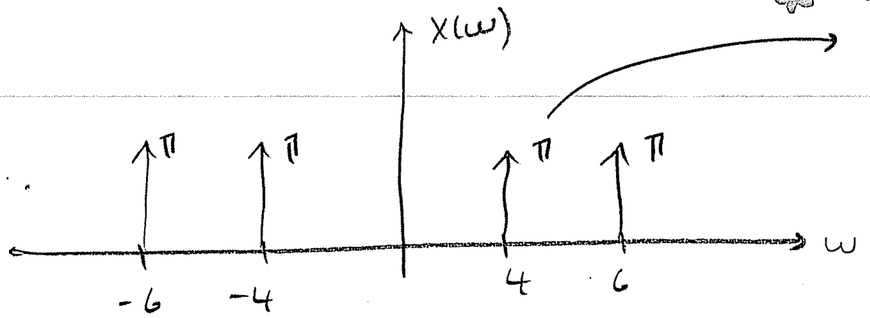
$$a_1 = \frac{e^{j\pi/4}}{2} = \frac{1}{2}\left(\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}\right) \qquad a_{(-1)} = \frac{e^{-j\pi/4}}{2} = \frac{1}{2}\left(\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}\right)$$

Ör: $x(t) = \cos(4t) + \cos(6t)$ $\qquad w_0 = 2$

$k_1 . w_0 = 4 \qquad k_2 . w_0 = 6 \qquad \boxed{k_1 = 2} \; \boxed{k_2 = 3}$

$$= \frac{1}{2}\left(e^{j4t} + e^{-j4t}\right) + \frac{1}{2}\left(e^{j6t} + e^{-j6t}\right)$$

$$a_2 = a_{(-2)} = \frac{1}{2} \qquad a_3 = a_{(-3)} = \frac{1}{2}$$

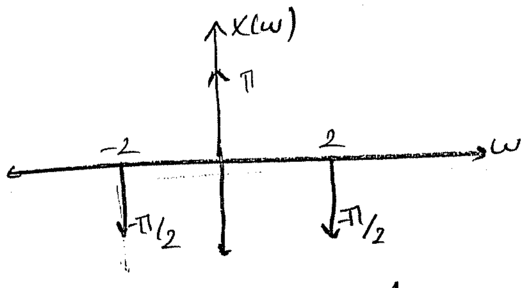


* $\pi/2$ olsaydı $\frac{1}{2}\cos(4t)$ olurdu.

Ör: $x(t) = \sin^2 t$

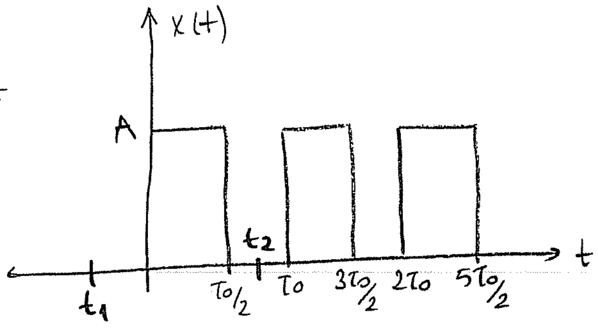
$$= \frac{1 - \cos 2t}{2} = \underbrace{\frac{1}{2}}_{a_0} - \frac{1}{2}\cos(2t) \qquad w_0 = 2$$

$$= \frac{1}{2} - \frac{e^{j2t} + e^{-j2t}}{4}$$



$a_0 = \frac{1}{2} \qquad a_1 = a_{(-1)} = -1/4$

Ör:



$$w_0 = \frac{2\pi}{T_0} \qquad a_k = \frac{1}{T_0}\int_{t_1}^{t_2} x(t)\, e^{-jkw_0t}\, dt$$

$$= \frac{1}{T_0}\left[\cancel{\int_{t_1}^{0} x(t) \ldots}^{\,0} + \int_{0}^{T_0/2} x(t) \ldots + \cancel{\int_{T_0/2}^{t_2} x(t) \ldots}^{\,0}\right]$$

$$= \frac{1}{T_0}\int_0^{T_0/2} A . e^{-jkw_0t}\, dt = \frac{-A}{T_0 . jkw_0} e^{-jkw_0t}\Big|_0^{T_0/2} = \frac{-A}{2\pi jk}\left(e^{-jk\frac{2\pi}{\cancel{T_0}}\cdot\frac{\cancel{T_0}}{\cancel{2}}} - \cancel{e^0}^{\,1}\right)$$

$$\frac{-A}{\cancel{T_0} jk \frac{2\pi}{\cancel{T_0}}} = \frac{-A}{jk2\pi}$$

$$= \frac{-A}{j2\pi k}\left(e^{-j\pi k}-1\right) = \frac{A}{j2\pi k}\left(-e^{j\pi k}+1\right) \qquad * \boxed{e^{j\pi k} = (e^{-j\pi})^k = (-1)^k}$$

$$a_k = \begin{cases} \frac{A}{j\pi k}, & k \text{ tek} \\ 0, & k \text{ çift} \end{cases}$$

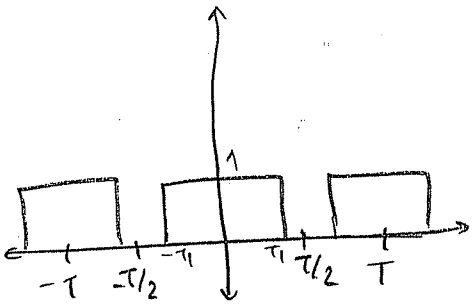
Ör: $x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t} = \sum_{k=-\infty}^{+\infty} a_k e^{jk(2\pi/T)t}$

$$x(t) = 1 + \sin\omega_0 t + 2\cos\omega_0 t + \cos(2\omega_0 t + \pi/4)$$

$$= \underbrace{1}_{a_0} + \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j} + 2\,\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2} + \frac{e^{j(2\omega_0 t + \pi/4)} + e^{-j(2\omega_0 t + \pi/4)}}{2}$$

$$a_0 = 1 \qquad a_1 = \frac{1}{2j} + 1 \qquad a_{(-1)} = 1 - \frac{1}{2j} \qquad a_2 = \frac{e^{j\pi/4}}{2} \qquad a_{(-2)} = \frac{e^{-j\pi/4}}{2}$$

Ör: $a_k = \frac{1}{T}\int_T x(t)\, e^{-jk\omega_0 t}\, dt = \frac{1}{T}\int_T x(t)\, e^{-jk(2\pi/T)t}\, dt$



$$\omega_0 = \frac{2\pi}{T} \qquad a_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{T}\left[\cancel{\int_{-T/2}^{-T_1} \cdots}_{0} + \int_{-T_1}^{T_1} \cdots + \cancel{\int_{T_1}^{T/2} \cdots}_{0}\right]$$

$$= \frac{1}{T}\int_{-T_1}^{T_1} e^{-jk\omega_0 t}\, dt = \frac{1}{\cancel{T}} \cdot \frac{-1}{jk\frac{2\pi}{\cancel{T}}}\, e^{-jk\omega_0 t}\Big|_{-T_1}^{T_1} = \frac{1}{j2\pi k}\left(\frac{-e^{-jk\omega_0 T_1} + e^{jk\omega_0 T_1}}{2j}\right)$$

2j'yi buraya aktarırsak $\longrightarrow \frac{\sin(k\omega_0 T_1)}{\pi k}$

$$a_0 = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, dt \Rightarrow \frac{1}{T}\int_{-T_1}^{T_1} dt = \frac{2T_1}{T}$$

DC bileşen var.

Ör: $Ta_k = T\,\frac{\sin(k2\pi\frac{T_1}{T})}{\pi k} \qquad \omega_0 = \frac{2\pi}{T}$

$$\omega = k\omega_0$$

$$Ta_k = T\,\frac{\sin k\pi/2}{\pi k} \qquad Ta_k = \frac{2\sin(\omega T_1)}{\omega} \qquad \omega T_1 = k\left(\frac{2\pi}{T}\right)T_1 = \frac{2k\pi}{A}$$

doğrusallık özelliği

Eğer ; $\left.\begin{array}{l} x(t) \longleftrightarrow a_k \\ y(t) \longleftrightarrow b_k \end{array}\right\}$ olduğunu biliyorsak $z(t) = ax(t) + by(t)$

$c_k = a a_k + b b_k$

zamanda öteleme özelliği

$x(t-t_0) \longleftrightarrow e^{-jk w_0 t_0} a_k$

zamanda ters çevirme

$x(-t) \longleftrightarrow a_{-k}$ $y(t) = x(-t)$ $b_k = a_{-k}$

* zaman domeninde çarpma işlemi için konvolüsyon yapmalıyız.

Türev:

$\frac{d}{dt} x(t) \longleftrightarrow jk w_0 a_k$

integral

integralden sonra katsayılarına karşılık yaz.

Ör: daha önce bulduğumuz kare dalgada

$a_0 = \frac{2T_1}{T}$, $g(t) = x(t-1) - 1/2$

$T = 4, T_1 = 1$ 'ken

$w_0 = \frac{2\pi}{4} = \pi/2$

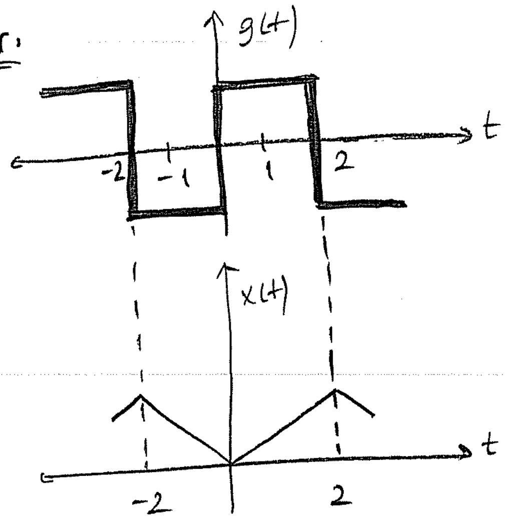
$$a_k = \frac{\sin(k \underbrace{(2\pi/T)}_{\pi/2} T_1)}{k\pi}$$

$x(t-1) \longleftrightarrow e^{-jkw_0} a_k = e^{-jk\pi/2} a_k$

$$\begin{cases} e^{-k\pi/2} a_k & , k \neq 0 \\ b_0 - 1/2 = 0 & , k = 0 \end{cases}$$

$a_0 = \frac{2T_1}{T} = \frac{1}{2} = b_0$

Ör:



$g(t) \longleftrightarrow d_k$

$x(t) \longleftrightarrow e_k$

$g(t) = \frac{d}{dt} x(t)$

$d_k = jk w_0 e_k$

$e_k = \frac{d_k}{jk\pi/2} = \frac{2\sin(k\pi/2)}{j(k\pi)^2} e^{-jk\pi/2}$

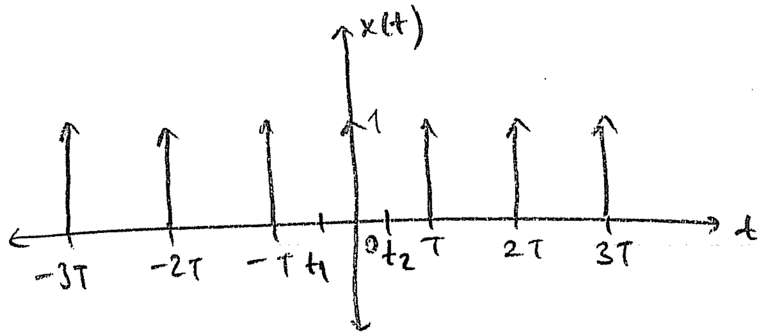
$g(t) = \frac{d}{dt} x(t) \longleftrightarrow d_k = jk(\pi/2) e_k$

$d_k = e^{-jkw_0} a_k$

$= e^{-jk\pi/2} \frac{\sin(k\pi/2)}{jk\pi}$ $\longrightarrow$ L'Hospital yöntemiyle türev alıyoruz

$e_0 = \text{----} = 1/2$

Frequency Shaping Filters :



$$\omega_0 = \frac{2\pi}{T}$$

$$x(t) = \sum a_k e^{jk\omega_0 t} = \frac{1}{T}\sum e^{jk\frac{2\pi}{T}t}$$

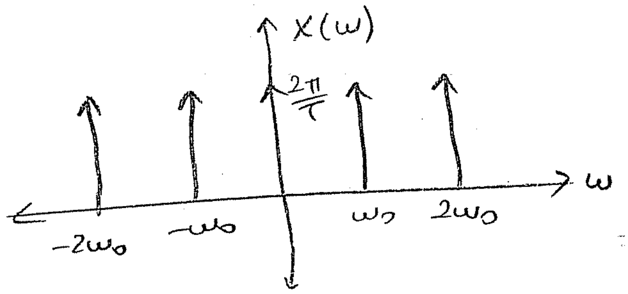
$$x(t) = \sum_{k=-\infty}^{\infty} \delta(t-kT)$$

$$a_k = \frac{1}{T}\int_{t_1}^{t_2} \delta(t)\, e^{-jk\omega_0 t}\,dt = \frac{1}{T} \Big\} \text{DC bileşen}$$

impulsta sınırlar önemli değil

* parantez içini 0 yapan t değerini bulup, integral içinde t gördüğümüz yere yazıyoruz.

* $\delta(t-1)$ olsaydı $\dfrac{e^{-jk\omega_0}}{T}$ olurdu.

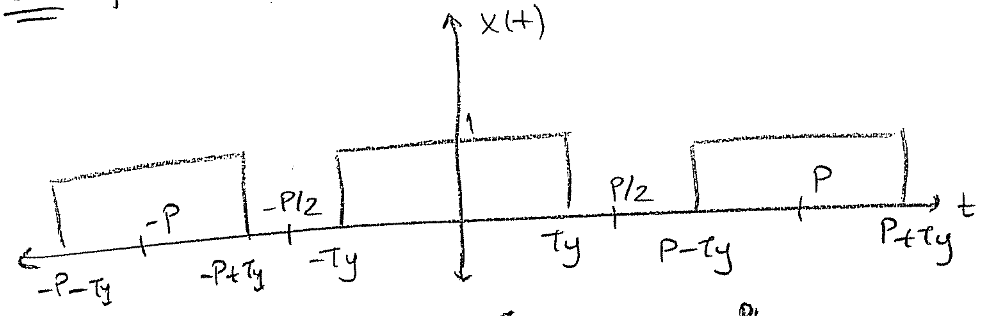


$$X(\omega) = \frac{2\pi}{T}\sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_0)$$

Ör: fourier serisi katsayıları=?

$$\omega_0 = \frac{2\pi}{P}$$

$$a_k = \frac{1}{P}\int_{-P/2}^{P/2} x(t)\, e^{-jk\omega_0 t}\,dt$$



$$= \frac{1}{P}\left[\int_{-P/2}^{-T_y} x(t)\ldots + \int_{-T_y}^{T_y} x(t)\ldots + \int_{T_y}^{P/2} x(t)\ldots\right]$$

$$a_k = \frac{1}{P}\int_{-T_y}^{T_y} 1 . e^{-jk\omega_0 t}\,dt = \frac{-1}{P . jk\omega_0}\, e^{-jk\omega_0 t}\Big|_{-T_y}^{T_y}$$

$$\frac{-1}{\cancel{P}\, jk\frac{2\pi}{\cancel{P}}} . e^{-jk\frac{2\pi}{P}t}\Big|_{-T_y}^{T_y} = \frac{-1}{j2\pi k}\left(e^{-jk\frac{2\pi}{P}T_y} - e^{jk\frac{2\pi}{P}T_y}\right)$$

+ −

$$a_k = \frac{+\sin\left(k\frac{2\pi}{P}T_y\right)}{\pi k} \qquad a_0 = \frac{2T_y}{P} \Rightarrow \frac{\cos\left(\frac{k2\pi}{P}T_y\right)\frac{2\pi T_y}{P}}{\pi}$$

//

Fourier dönüşümü : $\quad$ w_0 yok!

22 Temmuz 2016
Cuma

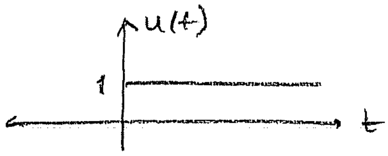
$$x(t) = \sum x(n)\, z^{-n}$$

$$x(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$x(t) = \frac{1}{2\pi} \int x(\omega)\, e^{j\omega t}\, d\omega \rightarrow \text{ters fourier dönüşümü}$$

$$x(\omega) = X(j\omega)$$

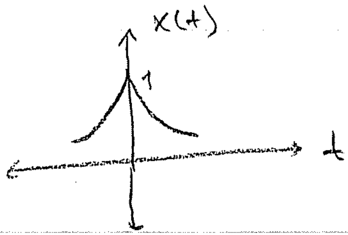
Ör: $x(t) = e^{-at} u(t) \quad a>0$



$$x(\omega) = \int_0^{\infty} e^{-at} \cdot e^{-j\omega t}\, dt$$

$$= \int_0^{\infty} e^{-t(a+j\omega)}\, dt = \frac{-1}{a+j\omega}\, e^{-t(a+j\omega)} \Big|_0^{\infty} = \frac{-1}{a+j\omega}(0-1) = \frac{1}{a+j\omega}$$

Ör: $x(t) = e^{-a|t|} \quad a>0$



$t<0$	$t>0$
e^{at}	e^{-at}

$$x(\omega) = \int_{-\infty}^{0} e^{at} e^{-j\omega t}\, dt + \int_0^{\infty} e^{-at} \cdot e^{-j\omega t}\, dt$$

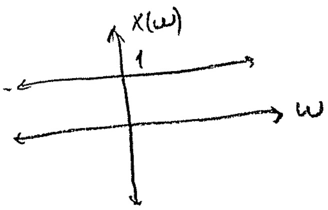
$$= \frac{1}{a-j\omega}\, e^{(a-j\omega)t} \Big|_{-\infty}^{0} + \frac{1}{a+j\omega}\, e^{-(a+j\omega)t} \Big|_0^{\infty}$$

$$= \frac{1}{a-j\omega}(1-0) - \frac{1}{a+j\omega}(0-1) = \frac{1}{a-j\omega} + \frac{1}{a+j\omega} = \frac{2a}{a^2+\omega^2}$$ //

örnek:
$x(t) = \delta(t)$

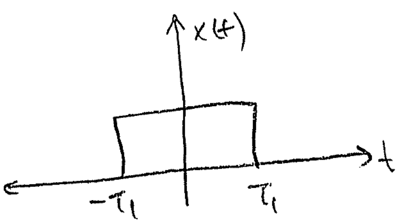
$$x(\omega) = \int_{-\infty}^{\infty} \underbrace{x(t)}_{\delta(t)}\, e^{-j\omega t}\, dt$$

$*\ \delta(t).t = 0$'dır.



t yerine 0 yazınca cevap 1 //

Ör: $x(t) = \begin{cases} 1, & |t|<T_1 \\ 0, & |t|>T_1 \end{cases}$



$$x(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t}\, dt = \int_{-T_1}^{T_1} 1.e^{-j\omega t}\, dt =$$

$$= \frac{-1}{j\omega}\, e^{-j\omega t} \Big|_{-T_1}^{T_1} = \frac{2.1}{j\omega} \left(\frac{-e^{-j\omega T_1} + e^{j\omega T_1}}{2j} \right)$$

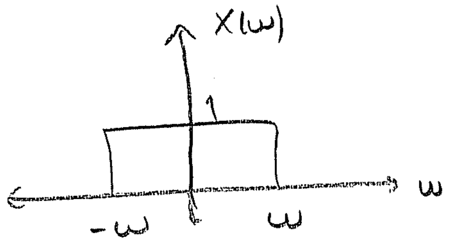
* sin haline getirmek için

$$= \frac{2}{\omega} \sin(\omega T_1) \qquad a_0 = \frac{2T_1 \cos(\omega T_1)}{1} = 2T_1$$ //

→ zaman domeninde sınırlıysa spektrum sınırsız oluyor.

→ spektrum sınırlıysa zaman domeni sınırsız oluyor.

Ör: $X(\omega) = \begin{cases} 1, & |\omega| < W \\ 0, & |\omega| > W \end{cases}$



$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$$

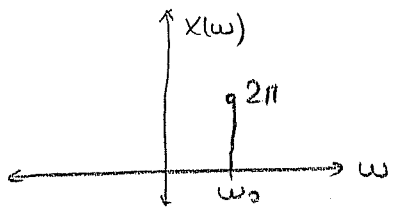
$$= \frac{1}{2\pi} \int_{-W}^{W} 1 . e^{j\omega t} d\omega = \frac{1}{2\pi j t} e^{j\omega t} \Big|_{-W}^{W}$$

$$= \frac{1}{2j\pi t} \left(\frac{e^{jWt} - e^{-jWt}}{2j} \right)$$

*sin hale getirmek için

$$= \frac{\sin(Wt)}{\pi t} \qquad a_0 = \frac{W}{\pi}$$

Ör: $X(\omega) = 2\pi \delta(\omega - \omega_0)$



$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(\omega) e^{j\omega t} d\omega$$

$$= \frac{1}{\cancel{2\pi}} \int \cancel{2\pi} \delta(\omega - \omega_0) e^{j\omega t} d\omega \qquad \omega - \omega_0 = 0 \quad \omega = \omega_0$$

$$= e^{j\omega_0 t}$$

Ör: $X(\omega) = 2\pi \delta(\omega - k\omega_0)$

$$= e^{jk\omega_0 t}$$

$$* \quad X(\omega) = \mathcal{F}\left[\sum a_k e^{jk\omega_0 t} \right]$$

$$= \sum \mathcal{F}(a_k e^{jk\omega_0 t})$$

$$= \sum a_k \mathcal{F}(e^{jk\omega_0 t})$$

$$= \sum a_k 2\pi \delta(\omega - k\omega_0)$$

doğrusallık

$x(t) \longleftrightarrow X(\omega)$
$y(t) \longleftrightarrow Y(\omega)$ olduğunu biliyorsak

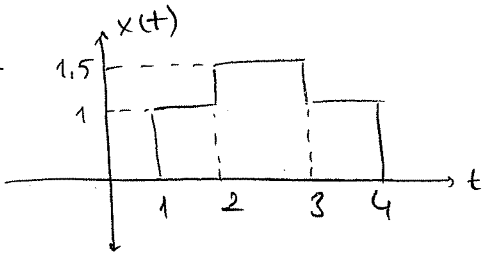
$z(t) = a x(t) + b y(t)$ olduğunu söyleyebiliriz.

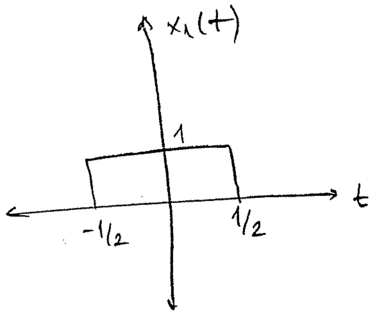
$Z(\omega) = a X(\omega) + b Y(\omega)$

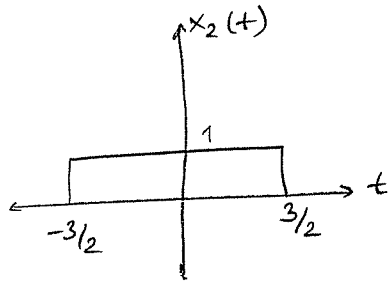
zamanda öteleme

$$x(t-t_0) \longrightarrow e^{-j\omega t_0} X(\omega)$$

Ör:







$$X(\omega) = \frac{2}{\omega} \sin(\omega T_1) \qquad X_1(\omega) = \frac{2}{\omega} \sin\left(\frac{\omega}{2}\right) \qquad X_2(\omega) = \frac{2}{\omega} \sin\left(\frac{3\omega}{2}\right)$$

$$x_1(t-5/2) \longleftrightarrow e^{-j\omega 5/2} X_1(\omega) \qquad x_2(t-5/2) \longleftrightarrow e^{-j\omega 5/2} X_2(\omega)$$

$$X(\omega) = \frac{1}{2} e^{-j\omega 5/2} \cdot \frac{2}{\omega} \sin\left(\frac{\omega}{2}\right) + e^{-j\omega 5/2} \frac{2}{\omega} \cdot \sin\left(\frac{3\omega}{2}\right)$$

türev:

$$\frac{d}{dt} x(t) \xrightarrow{\mathcal{F}} j\omega X(\omega)$$

zamanda ölçekleme:

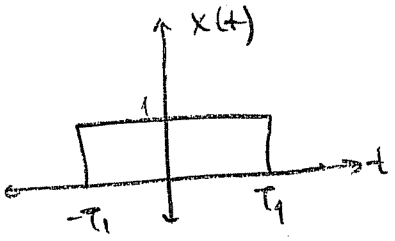
$$x(at) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{|a|} X\left(\frac{j\omega}{a}\right) \qquad \frac{1}{|b|} x\left(\frac{t}{b}\right) \overset{\mathcal{F}}{\longleftrightarrow} X(b\omega)$$

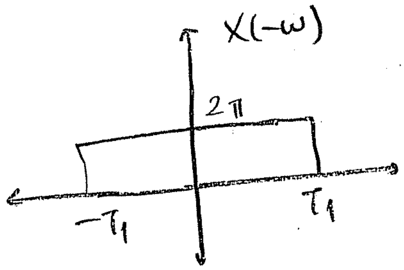
zamanda ters çevirme:

$$x(-t) \xrightarrow{\mathcal{F}} X(-\omega) \qquad X(t) \longleftrightarrow 2\pi x(-\omega)$$

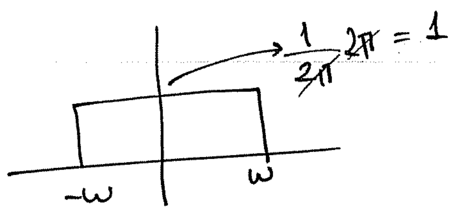
$$X(\omega) = \frac{2}{\omega} \sin(\omega T_1)$$

$$X(t) = \frac{2}{t} \sin(t T_1) \longleftrightarrow 2\pi x(-\omega)$$





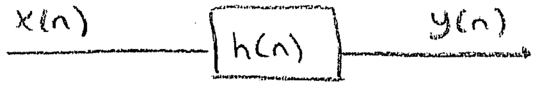
$$\frac{1}{2\pi} \frac{2}{t} \sin(\omega t) \longrightarrow$$

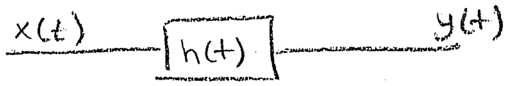


zaman domeninde çarpma işlemi

$$r(t) = s(t) \cdot p(t)$$

$$R(\omega) = \frac{1}{2\pi}\left[S(\omega) * P(\omega)\right]$$



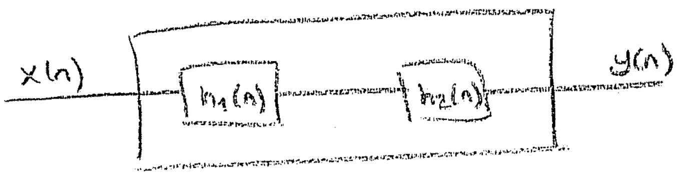


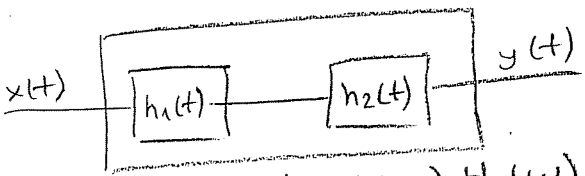
$$y(n) = x(n) * h(n)$$

$$Y(z) = X(z)\, H(z)$$

$$y(t) = x(t) * h(t)$$

$$Y(\omega) = X(\omega)\, H(\omega)$$





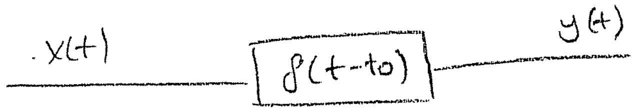
$$h(n) = h_1(n) * h_2(n)$$

$$H(z) = H_1(z) \cdot H_2(z)$$

$$H(\omega) = H_1(\omega) \cdot H_2(\omega)$$

$$Y(\omega) = H(\omega) \cdot X(\omega)$$

Ör:



$$y(t) = x(t) * \delta(t-t_0)$$

$$= x(t-t_0)$$

$$h(t) = \delta(t-t_0)$$

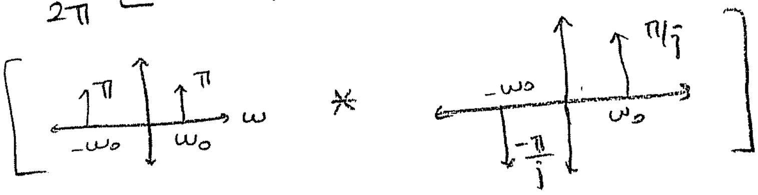
$$H(\omega) = e^{-j\omega t_0} \cdot 1$$

$$Y(\omega) = e^{-j\omega t_0} X(\omega)$$

Ör: $x(t) = \cos(\omega_0 t)\sin(\omega_0 t)$ $\qquad X(\omega) = ?$

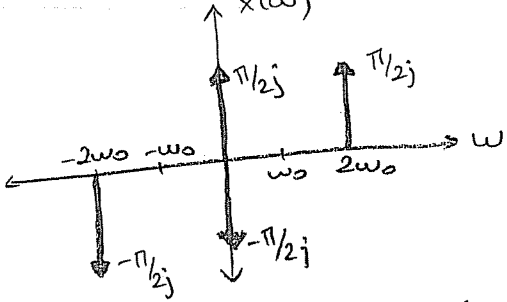
$$X(\omega) = \frac{1}{2\pi}\left[\cos(\omega) * \sin(\omega)\right]$$

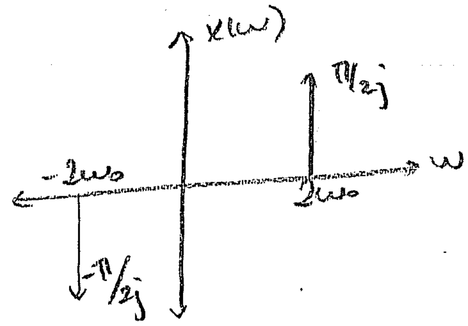
$\frac{1}{2\pi}$



* sin'deki 0 noktasını alıp cos'taki ω_0 noktasının üstüne koyduğumuzu düşünüyoruz-

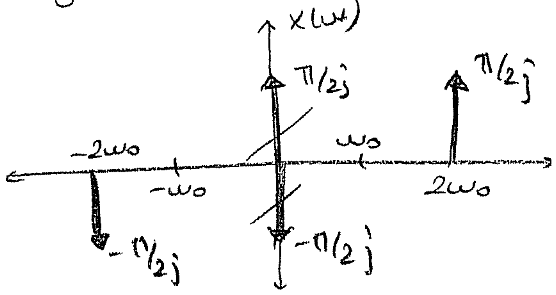
(1) sonra da $-\omega_0$'a koyuyoruz.





$$x(t) = 1/2 \sin(2\omega_0 t)$$

(2) şimdi de referans olarak sin'i alıcaz. cos'un sıfır noktasını sin'deki impulslara yerleştircez.

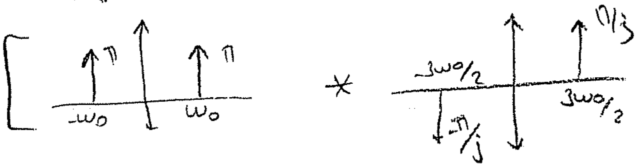


$$= \frac{1}{2}\sin(2\omega_0 t)$$

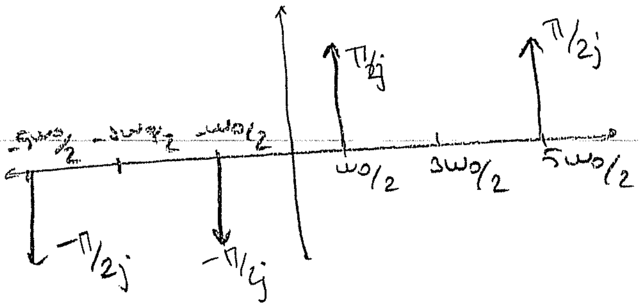
Ör: $x(t) = \cos(\omega_0 t) \cdot \sin\left(\frac{3\omega_0 t}{2}\right)$

$$X(\omega) = \frac{1}{2\pi}\left[\cos(\omega) * \sin(\omega)\right]$$

$$= \frac{1}{2\pi}\left[\quad * \quad\right]$$



$=$



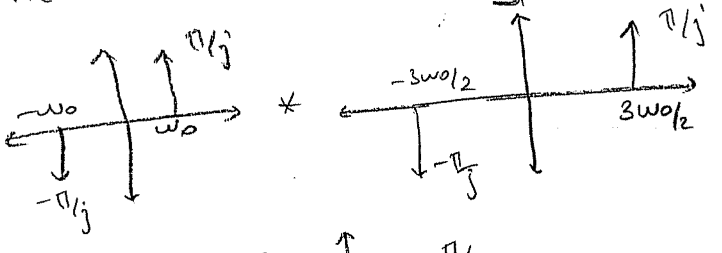
$$x(t) = \frac{1}{2}\sin\left(\frac{5\omega_0 t}{2}\right) + \frac{1}{2}\sin\left(\frac{\omega_0}{2}t\right)$$

Ör: $x(t) = \sin(\omega_0 t) \cdot \sin\left(\frac{3\omega_0}{2}t\right)$

$$X(\omega) = \frac{1}{2\pi}\left[\sin(\omega_0 t) * \sin\left(\frac{3\omega_0}{2}t\right)\right]$$

$$\boxed{j^2 = -1}$$

$$= \frac{1}{2\pi}\left[\quad * \quad\right]$$



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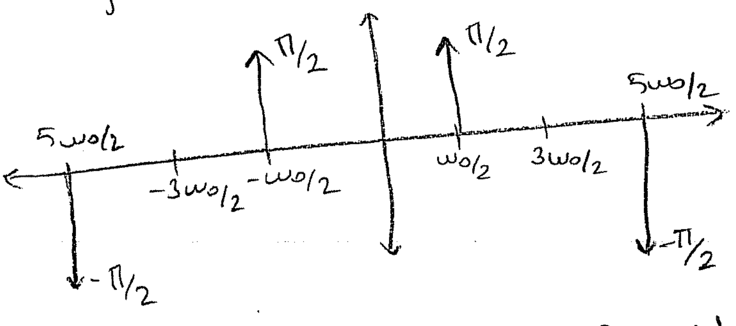
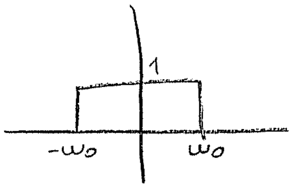


$$x(t) = \frac{1}{2}\left(\cos\left(\frac{\omega_0}{2}t\right) - \cos\left(\frac{5\omega_0}{2}t\right)\right)$$

Ör: $y(t) = \frac{d}{dt}x(t)$ $\qquad H(\omega) = ?$ $\qquad Y(\omega) = j\omega X(\omega)$

$$\frac{Y(\omega)}{X(\omega)} = H(\omega) = j\omega$$

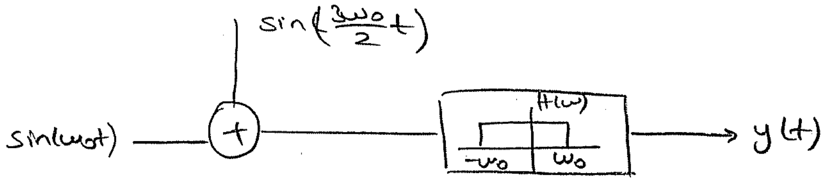
Alçak geçiren filtre :



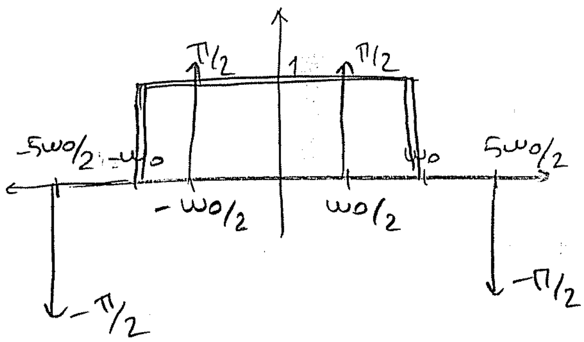
$$h(t) = \frac{1}{2\pi}\int_{-w_c}^{+w_c} e^{jwt}\,dw \qquad H(w) = \begin{cases} 1, & |w| < w_c \\ 0, & |w| > w_c \end{cases}$$

$$= \frac{\sin(w_c t)}{\pi t}$$

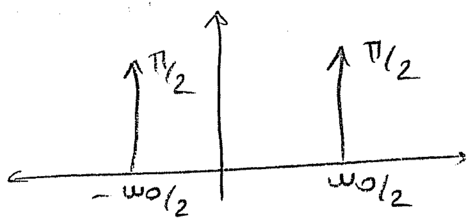
Ör: $x(t) = \frac{1}{2}\left(\cos\left(\frac{w_0}{2}t\right) - \cos\left(\frac{5w_0}{2}t\right)\right)$ alçak geçiren filtre sonucu?



* frekans domeninde çarpma işlemi yaparız.

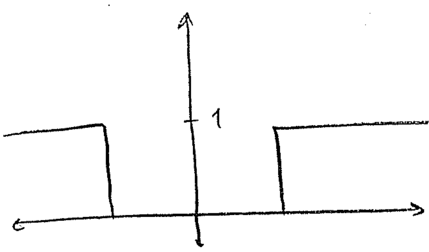


$\Rightarrow$

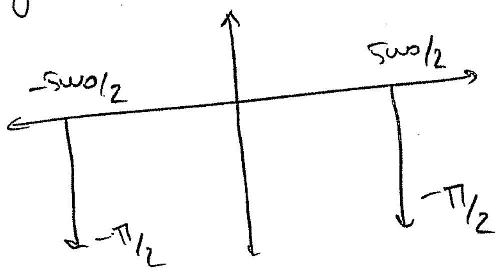


$$y(t) = \frac{1}{2}\cos\left(\frac{w_0}{2}t\right)$$

Yüksek geçiren filtre:

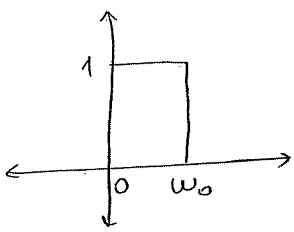


aynı örnek için;

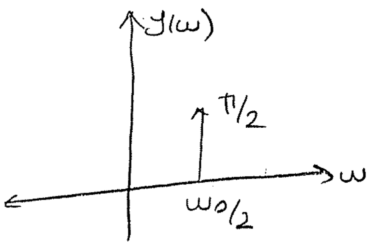


$$y(t) = -\frac{1}{2}\cos\left(\frac{5w_0}{2}t\right)$$

Ör: Eğer filtre



olsaydı;



$$y(t) = \frac{1}{4}e^{jw_0/2\,t}$$

29 Temmuz 2016
Cuma

Ör: $\frac{dy(t)}{dt} + ay(t) = x(t)$ $h(t) = ?$

$$j\omega Y(\omega) + aY(\omega) = X(\omega)$$
$$(a+j\omega)Y(\omega) = X(\omega) \rightarrow H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{1}{a+j\omega}$$

$$h(t) = e^{-at}u(t)$$

Ör: $\frac{d^2y(t)}{dt} + \frac{4dy(t)}{dt} + 3y(t) = \frac{dx(t)}{dt} + 2x(t)$

$$(j\omega)^2 Y(\omega) + 4j\omega Y(\omega) + 3Y(\omega) = j\omega X(\omega) + 2X(\omega)$$

$$Y(\omega)\left((j\omega)^2 + 4j\omega + 3\right) = X(\omega)(2+j\omega)$$

$$\frac{(2+j\omega)}{(j\omega+3)(j\omega+1)} = \frac{Y(\omega)}{X(\omega)} \Rightarrow \frac{A}{j\omega+3} + \frac{B}{j\omega+1}$$

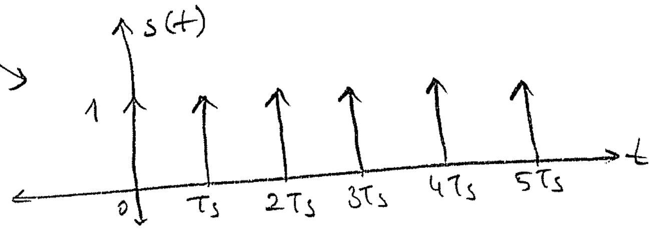
$$A = (j\omega+3)\Big|_{j\omega=-3} = \frac{2+j\omega}{j\omega+1}\Big|_{j\omega=-3} = \frac{-1}{-2} = \frac{1}{2}$$

$$B = (j\omega+1)\Big|_{j\omega=-1} = \frac{2+j\omega}{j\omega+3}\Big|_{j\omega=-1} = \frac{1}{2}$$

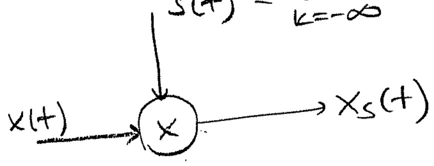
$$= \frac{1}{2}\frac{1}{j\omega+3} + \frac{1}{2}\cdot\frac{1}{j\omega+1} \Rightarrow \boxed{h(t) = \frac{1}{2}e^{-t}u(t) + \frac{1}{2}e^{-3t}u(t)}$$

Örnekleme:

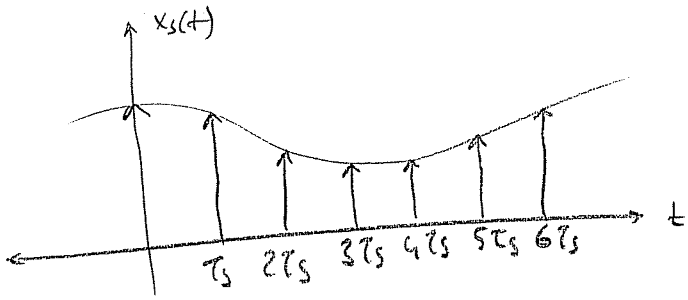
1- Frekans domeni
2- Zaman domeni



$$s(t) = \sum_{k=-\infty}^{\infty} \delta(t-kT_s)$$



$$\omega_s = \frac{2\pi}{T_s} \text{ rad/s} = 2\pi f_s$$
→ örnekleme frekans



$$f_s = \frac{1}{T_s}\text{ Hz } (1/s)$$

$* \; x(n) = x_a(nT_s)$

Ör: $x_a(t) = e^{jw_0 t}$ $\quad T_s$ – örnekleme periyodu

$x(n) = ? \Rightarrow e^{jw_0 n T_s}$ //

$x(n) = x(n+N)$'yi uygulayıp periyodikliğini kontrol ediyoruz.

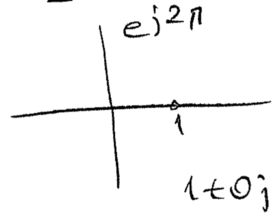
$e^{jw_0 n T_s} = e^{jw_0 (n+N) T_s}$ $\qquad e^{j2\pi n} = (1)^n = 1$

$1 = e^{jw_0 N T_s}$

$w_0 N T_s = 2\pi k$ $\quad \boxed{N = \dfrac{2\pi k}{w_0 T_s}}$ periyodiktir.

$* \; z = re^{j\theta}$

$e^{j2\pi}$



$1 + 0j = 1$ //

Ör: $X_a(t) = \cos(\underbrace{15}_{w_0 = 15} t)$ $\qquad T_s = 0.1\,\pi_s$

$x(n) = \cos(15 n T_s)$

$x(n) = \cos\left(15 \cdot n \cdot \frac{1}{10}\pi_s\right) \Rightarrow \cos\left(\frac{3}{2}\pi n\right)$

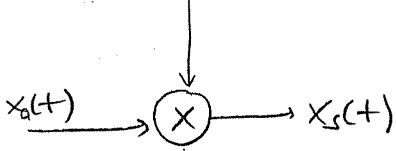
$N = \dfrac{2\pi k}{w_0 T_s} \longrightarrow \dfrac{2\pi k}{15\pi/10} = \frac{4}{3}k \Rightarrow \underset{k=3 \text{ için}}{\boxed{N = 4}}$

Ör: $x_a(t) = e^{-\alpha t}$ $\qquad T_s$

$x(n) = e^{-\alpha n T_s}$

$= \dfrac{1}{1 - e^{-\alpha T_s} z^{-1}}$

$s(t) = \sum \delta(t - kT_s)$

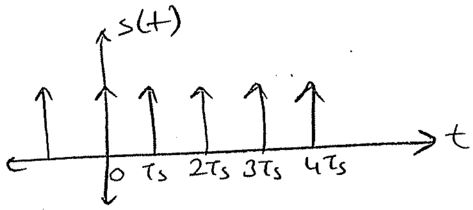


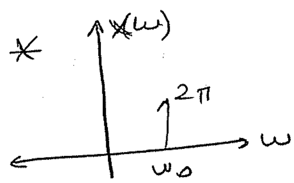
zaman domeni

$X_s(w) = \dfrac{1}{2\pi}\left[X_a(w) * S(w)\right]$ frekans domeni karşılığı

Ör: $s(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_s)$

$w_0 = \dfrac{2\pi}{T_s}$ $\qquad a_k = \dfrac{1}{T_s}\displaystyle\int_{-T_s/2}^{T_s/2} \delta(t)\, e^{-jkw_0 t}\, dt = \dfrac{1}{T_s}$ // $\quad (t=0)$

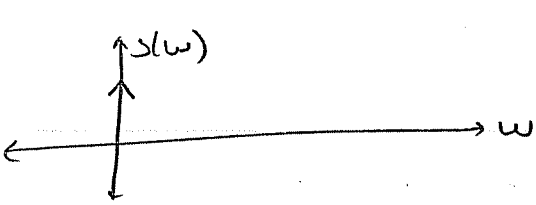




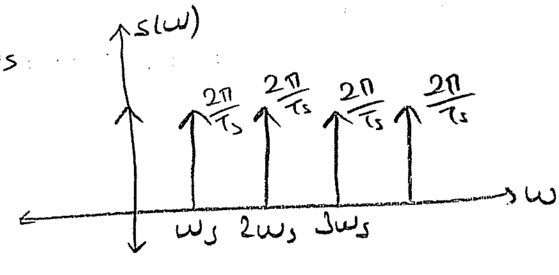
$* \quad X(w) = 2\pi\,\delta(w - w_0)$

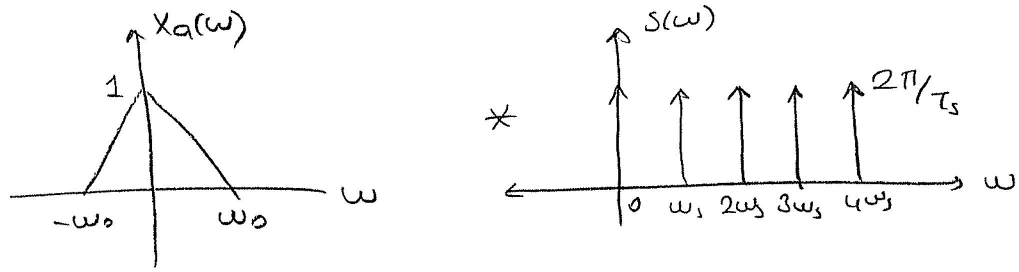
$x(t) = \dfrac{1}{2\pi}\displaystyle\int 2\pi\,\delta(w - w_0)\, e^{jwt}\, dw$

$= e^{jw_0 t}$

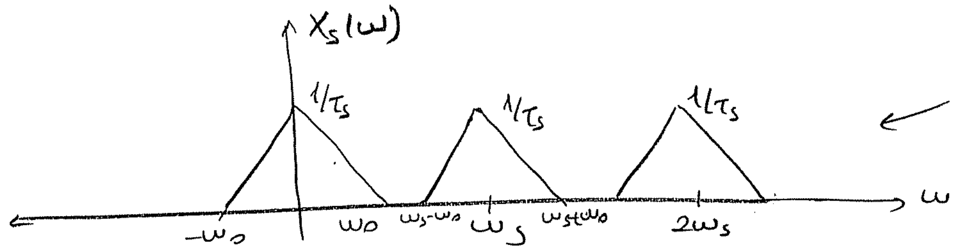


$\dfrac{1}{T_s}\displaystyle\sum_{k=-\infty}^{\infty} e^{jk\overset{w_s}{w_0} t} \longrightarrow \dfrac{2\pi}{T_s}\sum_k \delta(w - k\overset{w_s}{w_0})$





$$X_s(\omega) = \frac{1}{2\pi}\left[X_a(\omega) * S(\omega) \right]$$



$\omega_s - \omega_0 > \omega_0$

$\omega_s - \omega_0 < \omega_0$ olsaydı, iç içe geçen üçgenler olurdu.

* $\omega_s > 2\omega_0$ ⟶ bu şart sağlanmazsa geçişmeler oluşur, bozulma olacağı için orjinal işareti elde edemeyiz.

$$X_s(\omega) = \frac{1}{T_s} \sum x_a(\omega - k\omega_s)$$

Ör: $x(n) = \cos\left(n \frac{\pi}{8}\right)$

$f_s = 10kHz$

$T_s = \frac{1}{10}k$

$X_a(t) = \cos(\omega_0 t)$
$= \cos(2\pi f_0 t)$

$x(n) = \cos(2\pi f_0 n T_s)$

$\omega_0 \cancel{n} T_s = \cancel{n}\pi/8$

$\omega_0 = \frac{10k\pi}{8}$ $\quad f_0 = 625 Hz$ //

Ör: $X_a(t) = \cos(1250\pi t)$ başka frekans değerinde elde edilebilir mi? evet, çünkü 2π'de cos kendini tekrar eder ve 2 ve katlarında doğru sonuç verir.

$x(n) = \cos\left(625\pi n \frac{1}{10k}\right)$ $\quad \frac{0}{1}$ $\quad 1 \quad \cos\left(\cancel{\pi/8}\right) = ?$

$\cos\left(1250\pi n \frac{1}{10k}\right)$ $\quad 1 \quad \cos(\pi/8) = ?$

$\cos\left(2\pi f_0 n \frac{1}{f_s} \mp 2\pi k n.1\right)$ $\quad \cos\left(2\pi \frac{f_0 \mp k f_s}{f_s} n\right)$

$X_a(t) = \cos\left(2\pi\left(f_0 + f_s\right)t\right)$
$= \cos(21250\pi t)$

* her zaman $\omega_s > 2\omega_0$ olması gerekir.

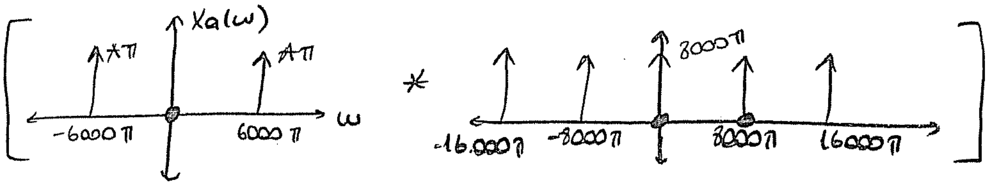
$k = 0 \quad 10.000 > 625$

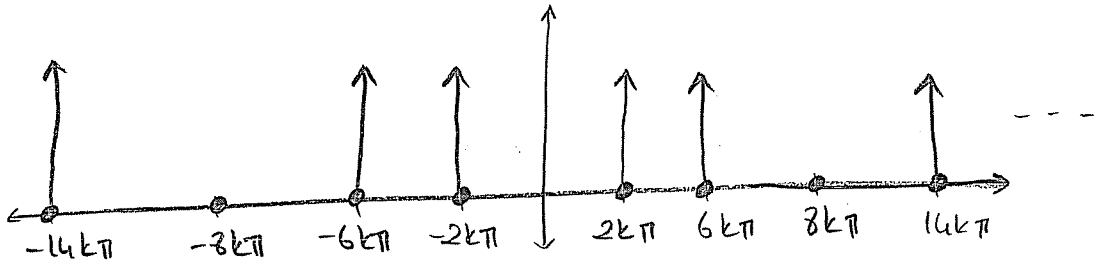
$k = 1 \quad 10.000 > 10.625$ X

frekans	zaman
$X_a(\omega)\,.\,S(\omega) \longrightarrow$	$X_a(t) * s(t)$

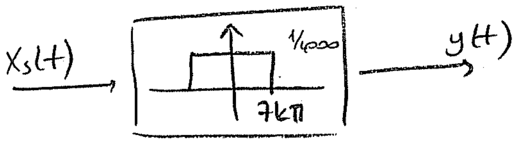
Ör: $x_a(t) = A\cos(6000\pi t)$ $\quad T_s = \dfrac{1}{4000}\,s \quad \omega_s = \dfrac{2\pi}{T_s} = 8000\pi$

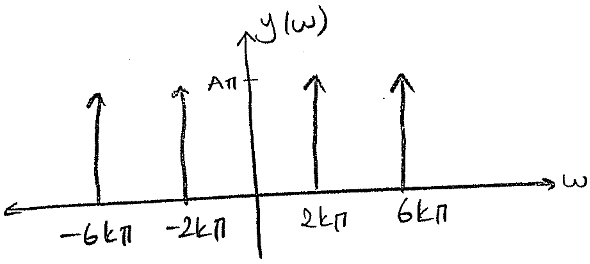
$$= \frac{1}{2\pi}\,\Big[\quad * \quad\Big]$$





filtreden geçirmek istersek;

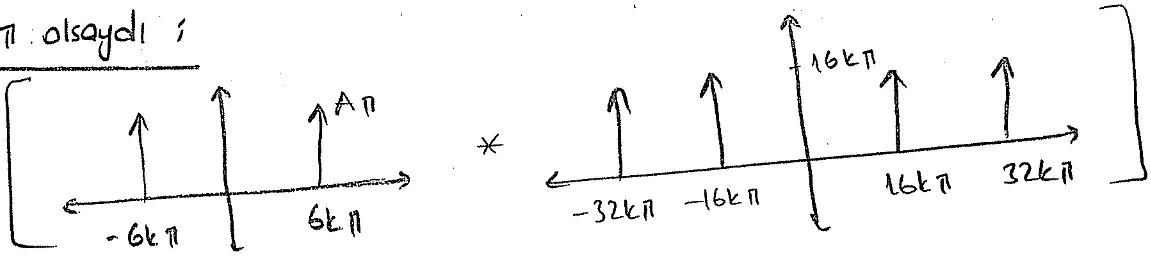


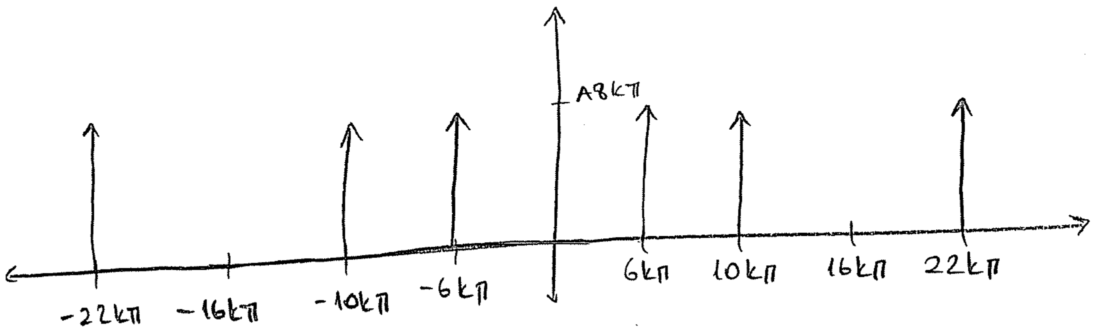


$$y(t) = A\cos(6000\pi t) + A\cos(2000\pi t)$$

* $\omega_s = 16k\pi$ olsaydı ;

$$= \frac{1}{2\pi}\,\Big[\quad * \quad\Big]$$





$\omega_s > 2\omega_0$ olduğu için geçişme olmadı, yani işaretimiz bozulmadı.

* yine $7k\pi$ filtresinden geçirirsek bozulmadığını, aynı kaldığını görürüz.

gamzep