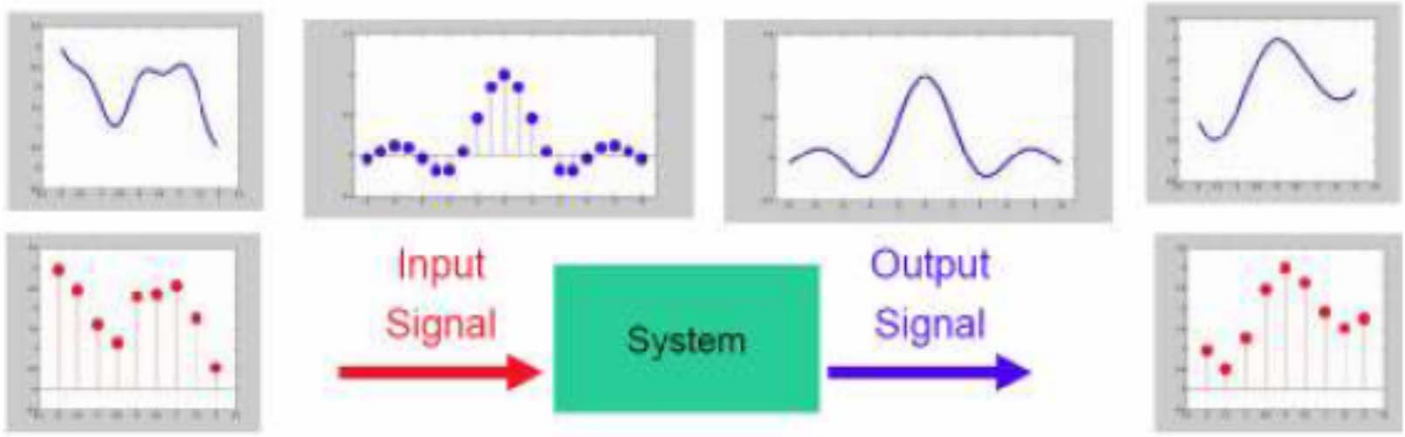
Hafta 1
İŞARETLER VE SİSTEMLER

2019-2020

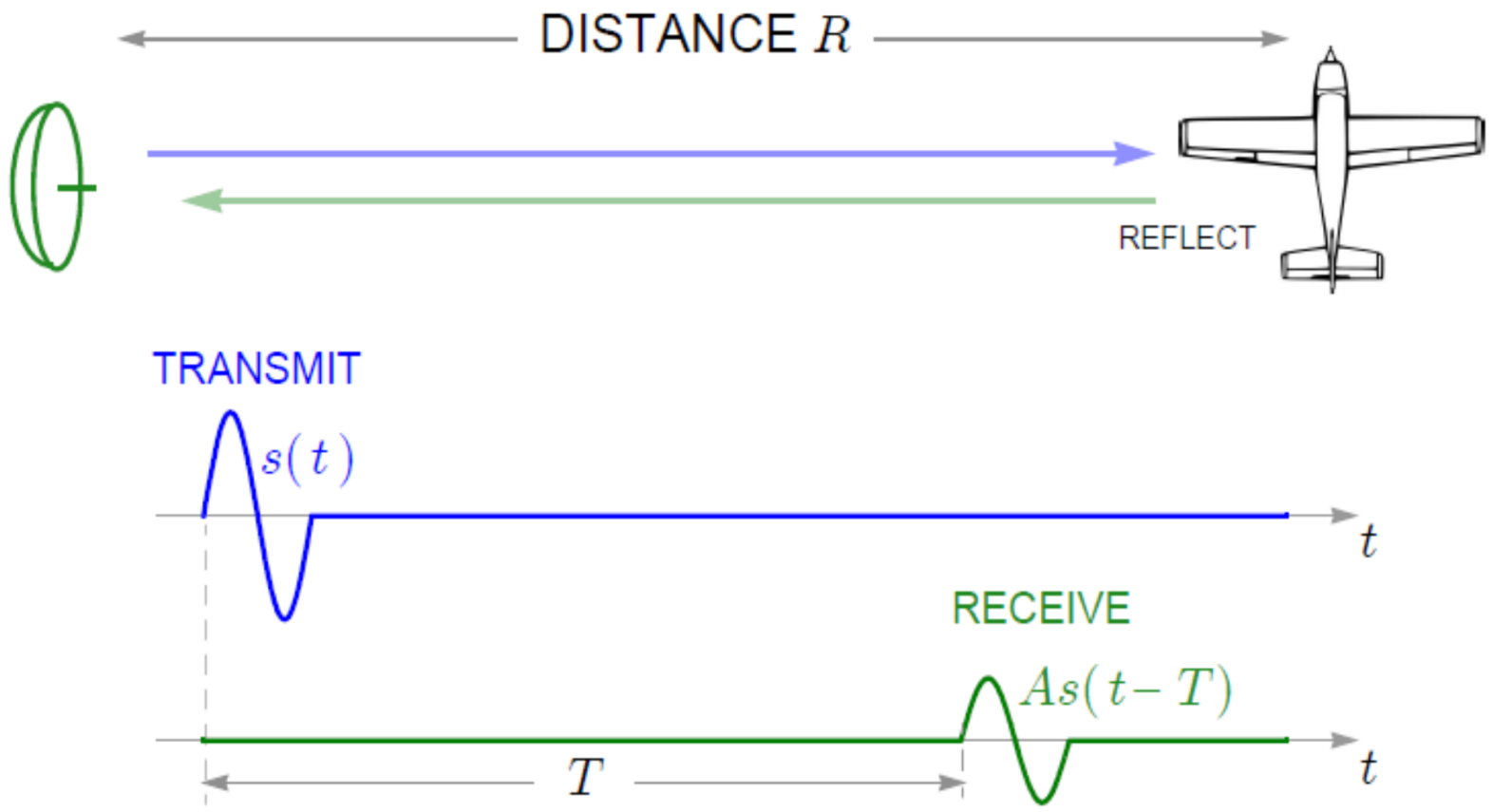
GÜZ

İşaretler ve Sistemler

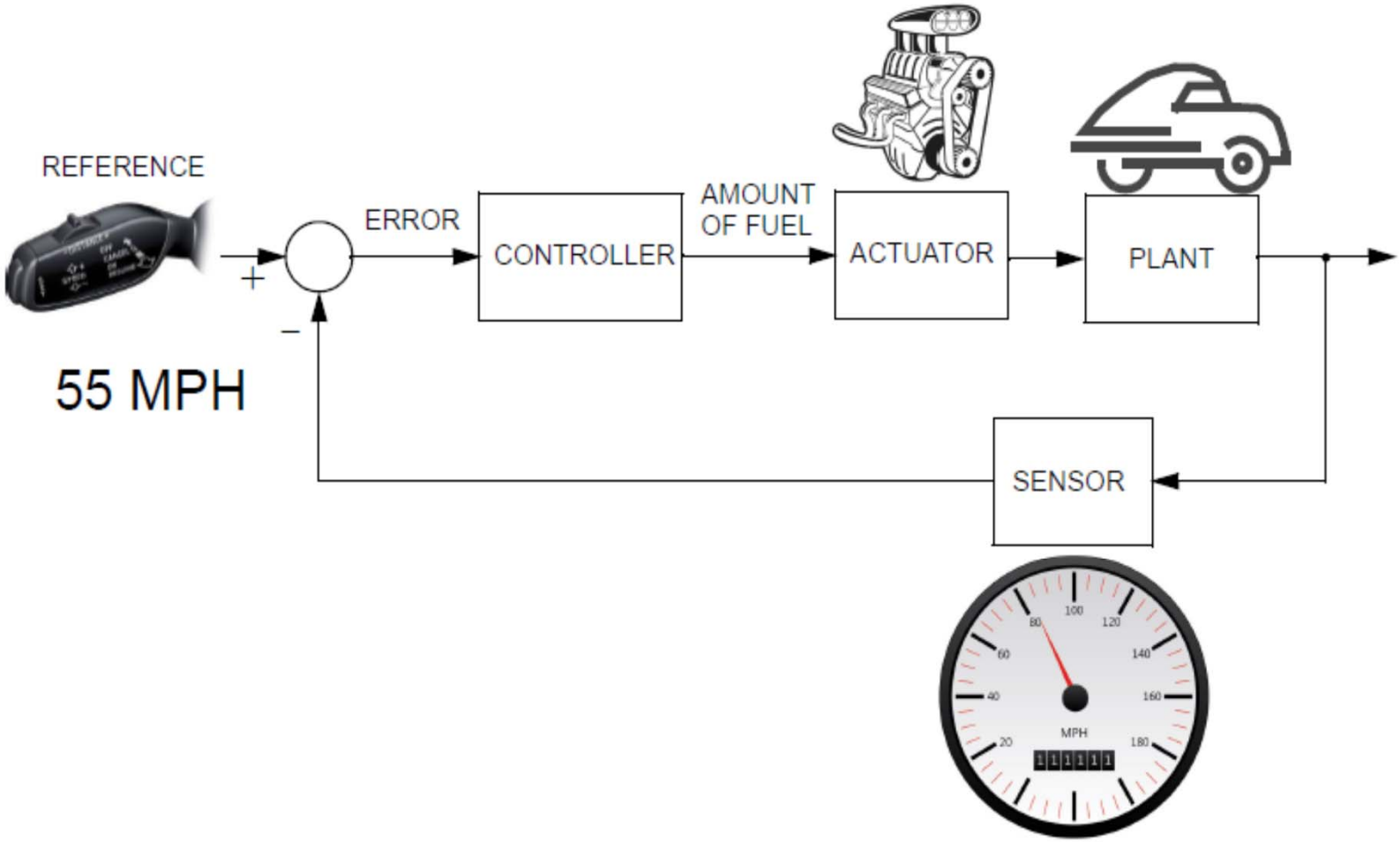
- İşaretler ve Sistemler bilim dalı sinyalleri işleyen sistemleri tanımlamak ve analiz etmek için matematiksel teknikleri kullanır
- İşaret: bilgi taşıyan bir yada daha fazla değişkene bağlı değişen büyüklük



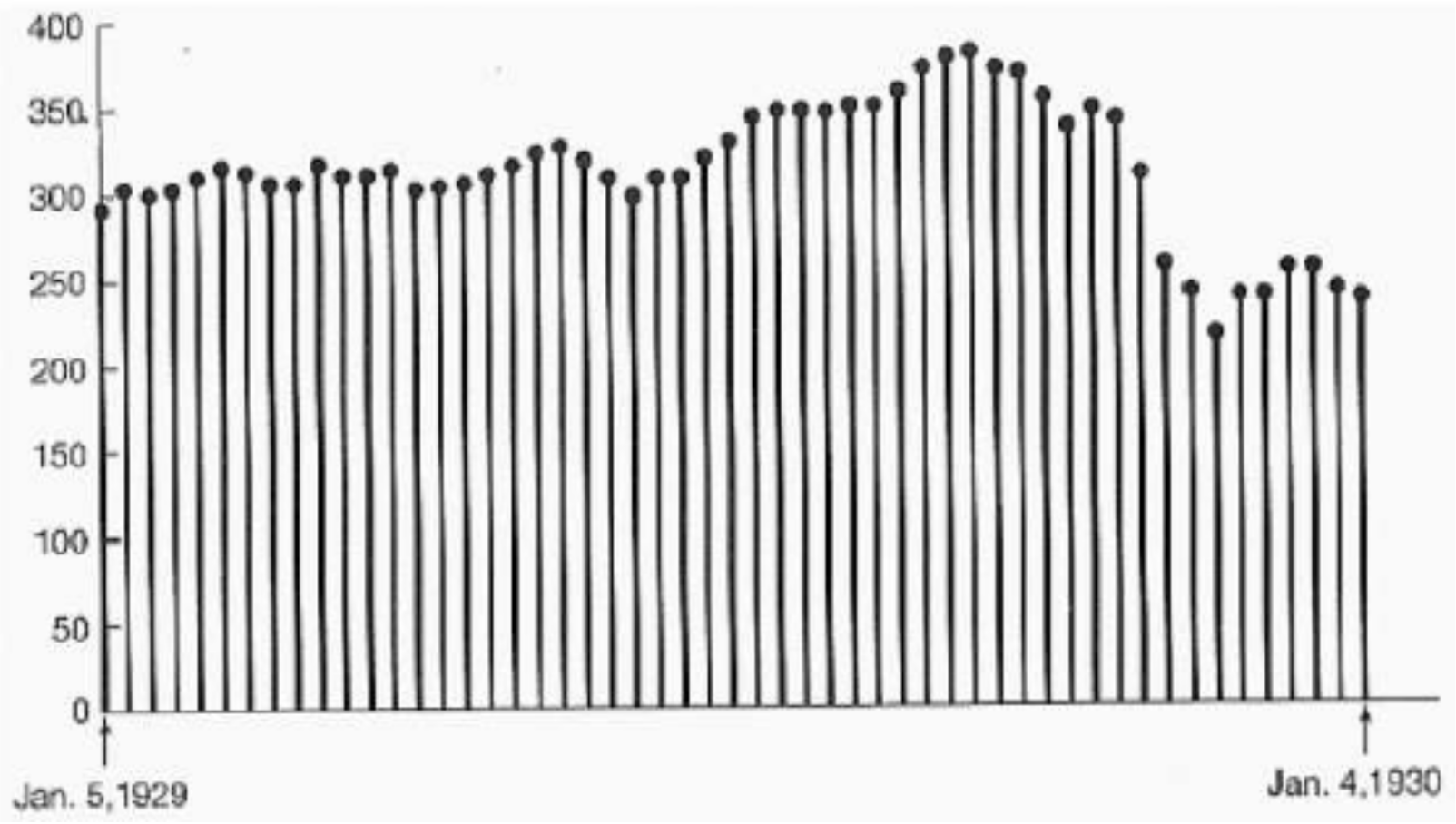
Radar Örneği



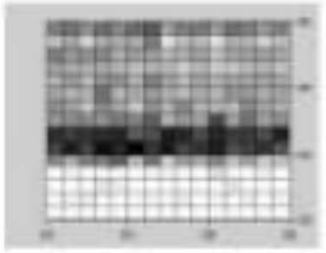
Hız Sabitleyici

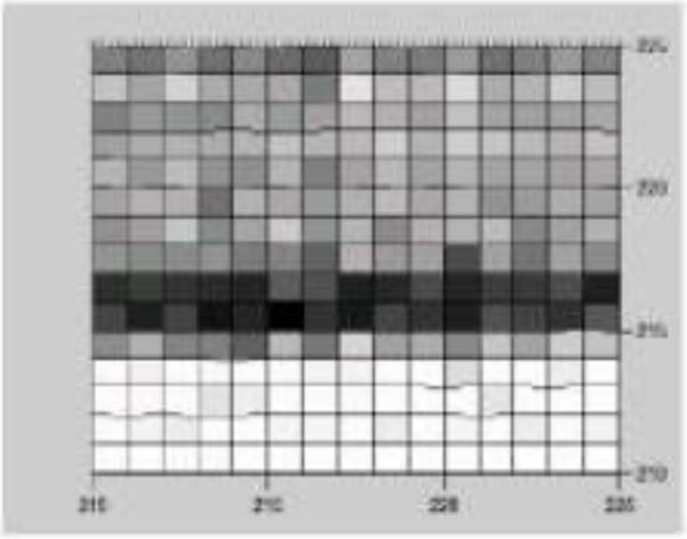


Ayrık-zamanlı işaretler



Ayrık-zamanlı işaret:
Resim pikselleri

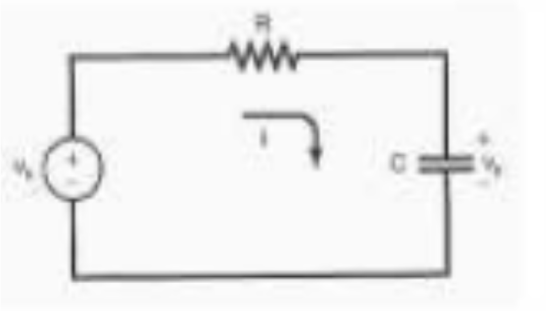




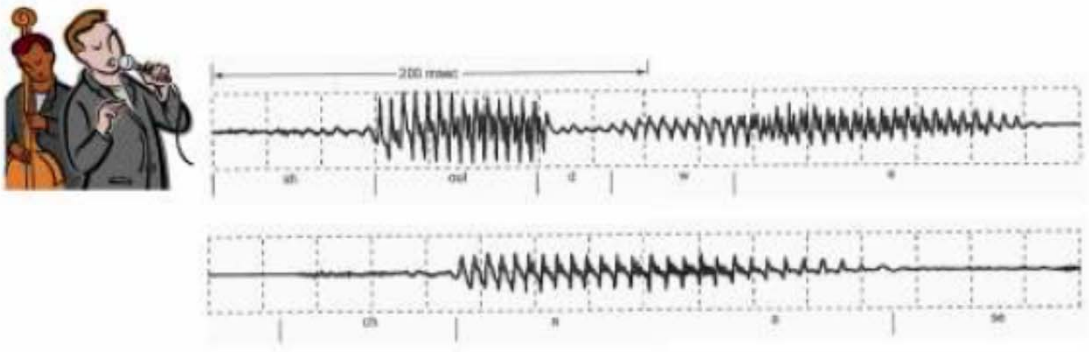
117	151	112	136	110	95	166	131	125	159	114	130	159	133
164	209	173	171	165	120	225	169	184	219	172	160	205	168
158	151	163	178	156	160	186	179	163	193	167	161	167	168
173	179	172	162	195	161	187	205	183	205	196	166	180	193
155	168	169	165	161	131	166	169	166	161	164	164	166	171
176	176	113	178	163	159	177	186	171	162	149	159	174	164
162	198	161	164	102	149	181	142	176	192	189	136	165	176
163	163	160	127	132	97	177	152	160	74	163	119	162	163
62	51	40	32	95	92	28	36	71	25	40	38	77	28
26	67	16	36	3	49	22	69	49	26	91	67	69	91
176	194	129	97	185	120	212	160	173	139	197	166	207	192
243	253	253	253	254	242	242	255	254	222	255	253	239	252
248	253	224	238	255	255	249	255	255	253	230	252	255	253
243	238	255	253	245	247	255	238	255	248	255	237	249	241

Sürekli-zaman işaretleri

- Basit bir RC devresinde kaynak veya kondansatör üzerindeki gerilim

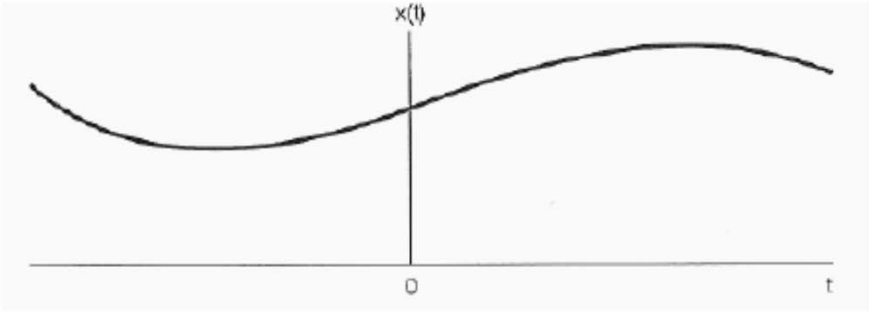


- Ses kaydetme

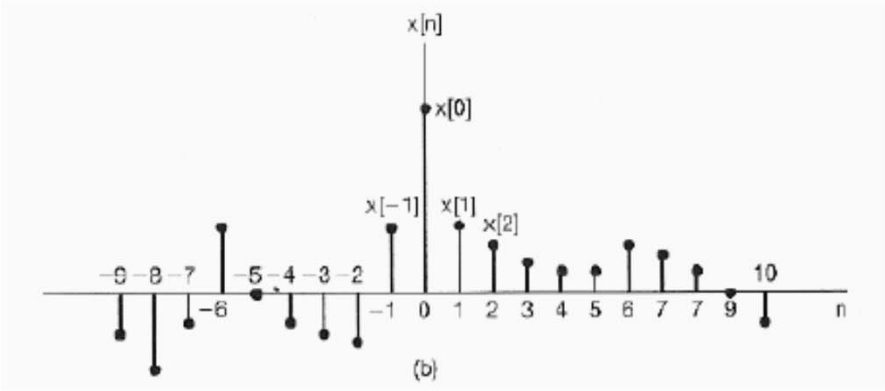


Sinyallerin Grafiksel Gösterimi

- Continuous-time signals $x(t)$ or $x_c(t)$

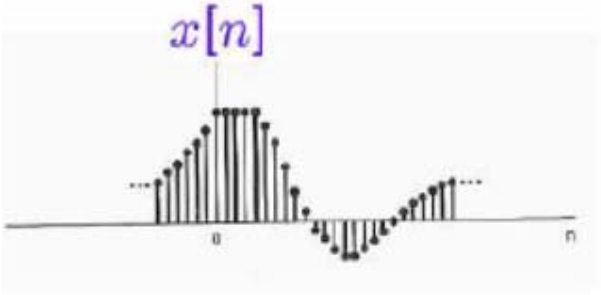


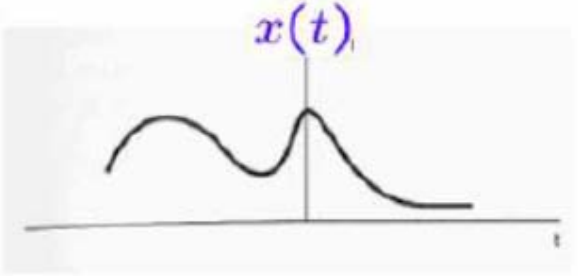
- Discrete-time signals $x[n]$ or $x_d[n]$

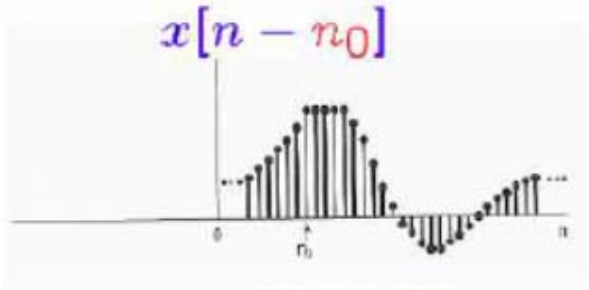


Zaman üzerinde öteleme

$$\begin{cases} n_0,\ t_0 > 0: & \text{delay} \\ n_0,\ t_0 < 0: & \text{advance} \end{cases}$$

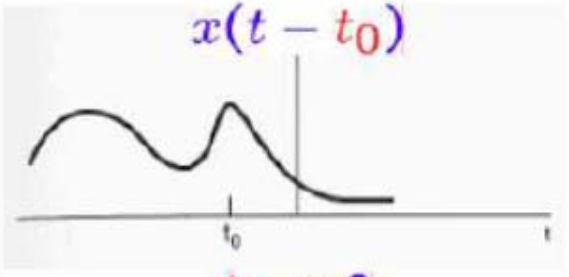






$n_0 > 0$

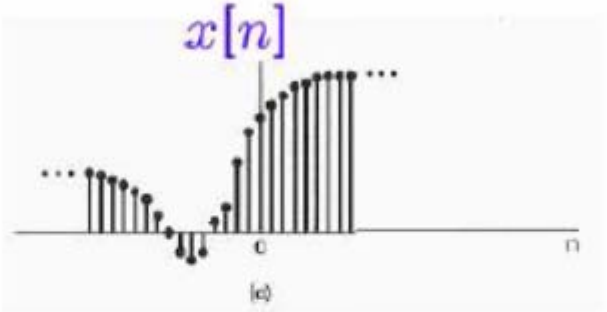
$x[n-8]$

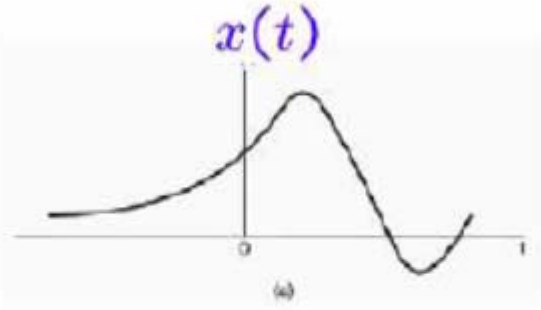


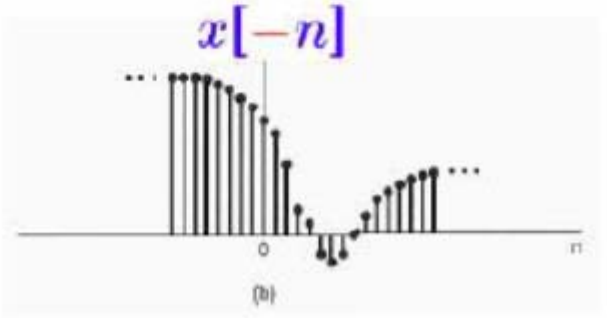
$t_0 < 0$

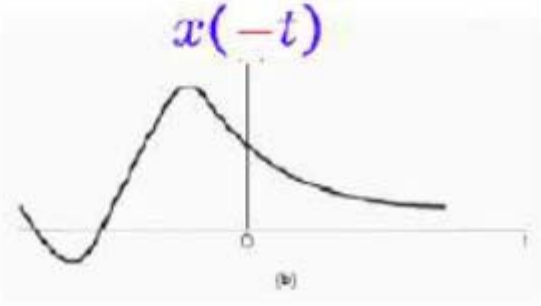
$x(t+5)$

Zaman üzerinde Tersini Alma

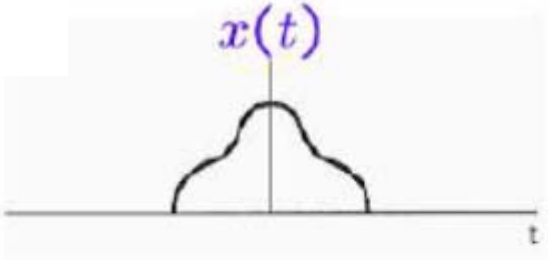




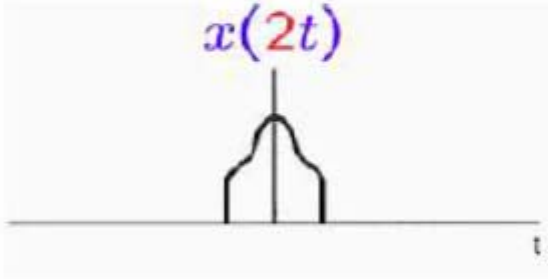




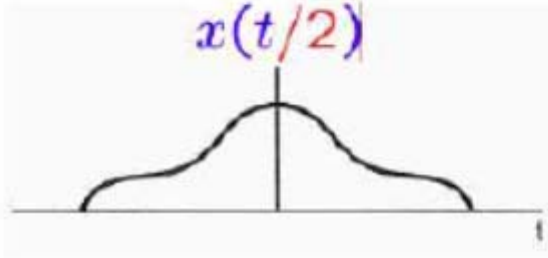
Zaman üzerinde Ölçekleme



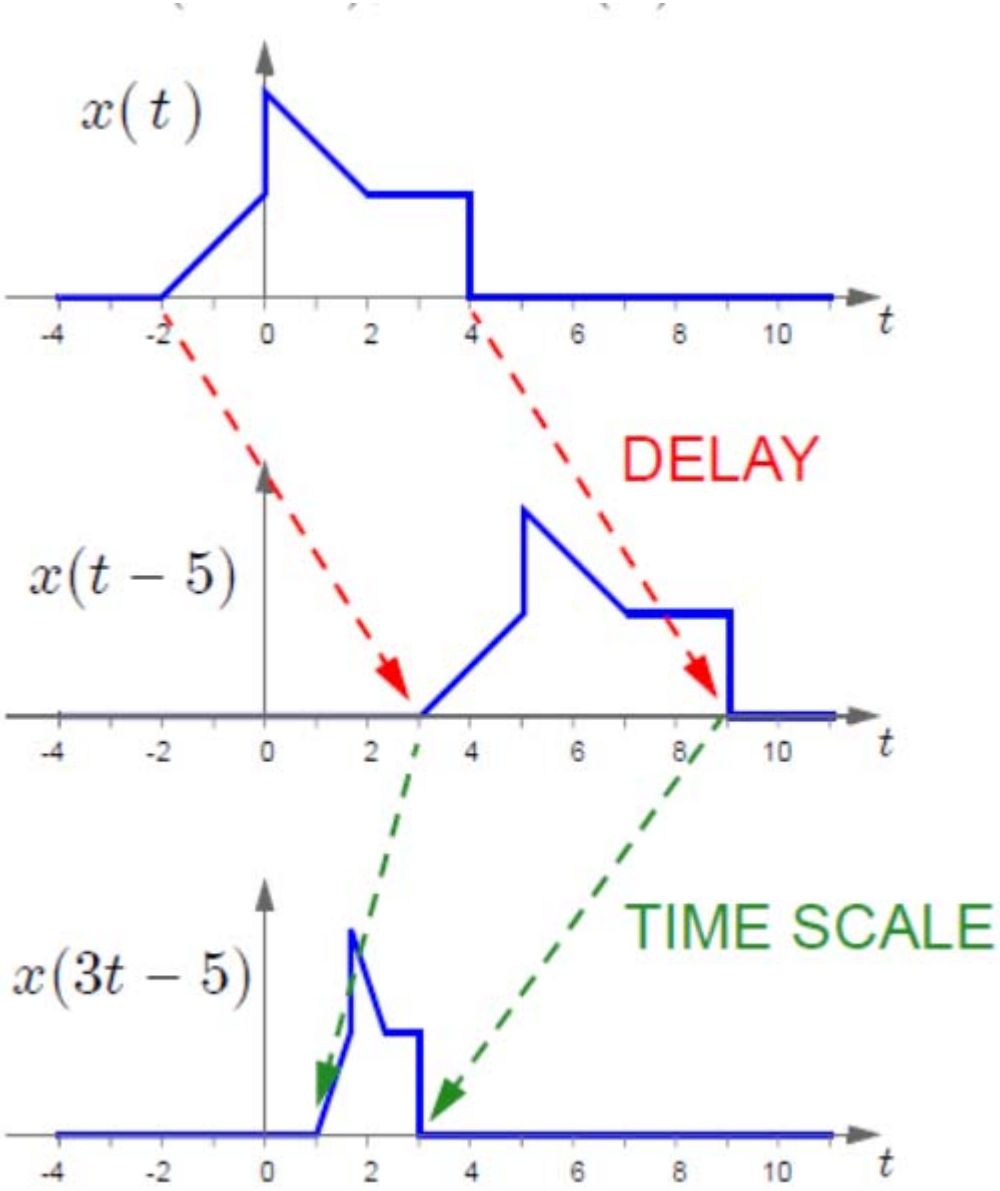
$t \to 2t$



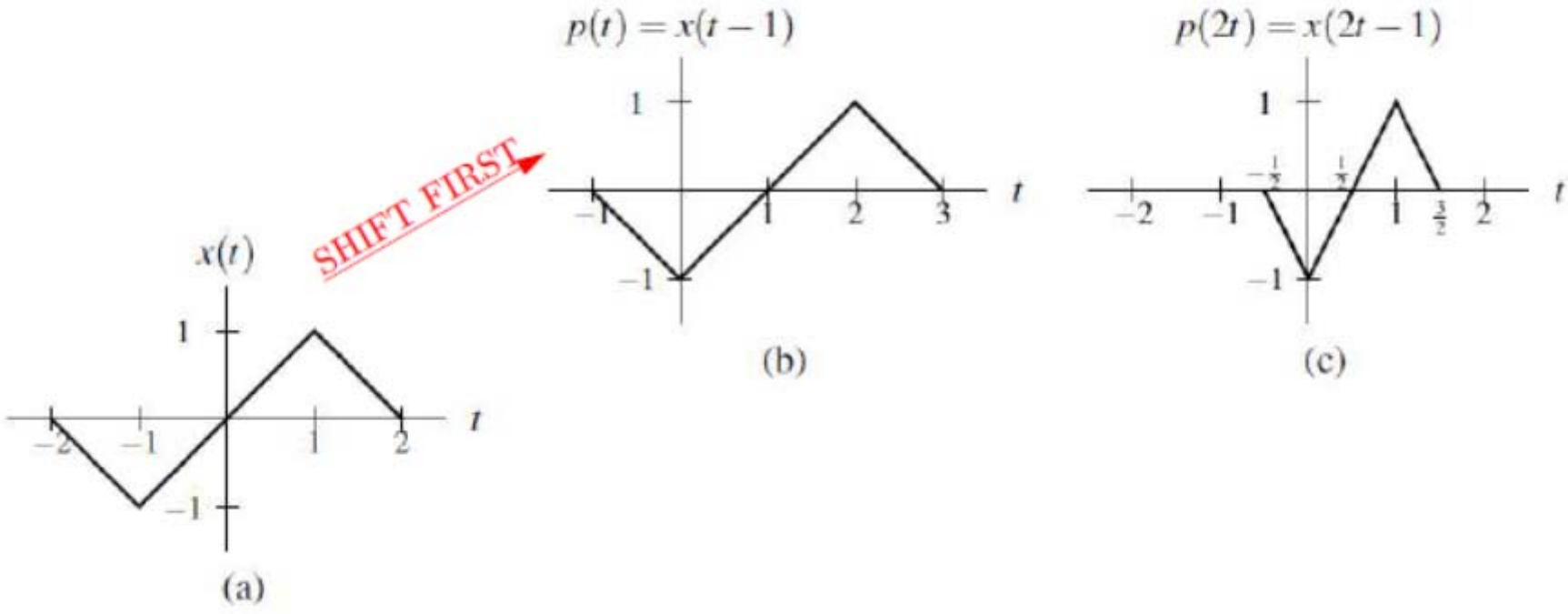
$t \to t/2$



ÖRNEK

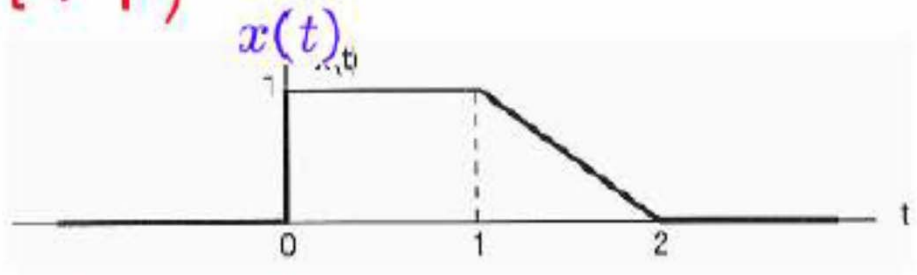

$x(t)$
-4
-2
0
2
4
6
8
10
t
DELAY
$x(t-5)$
-4
-2
0
2
4
6
8
10
t
TIME SCALE
$x(3t-5)$
-4
-2
0
2
4
6
8
10
t

Örnek



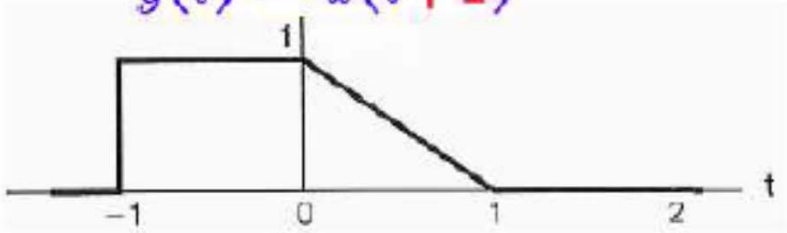
ÖRNEK

- x (t) → x (- t + 1)



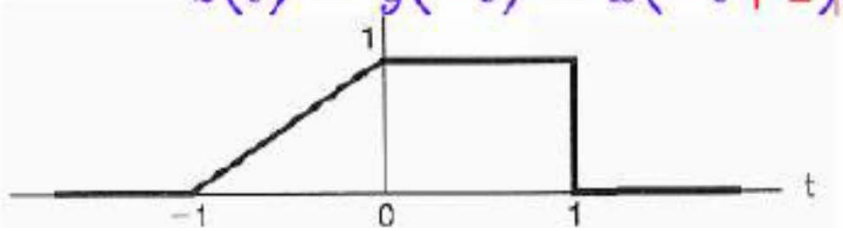
$t \rightarrow t+1$

$$y(t) = x(t+1)$$



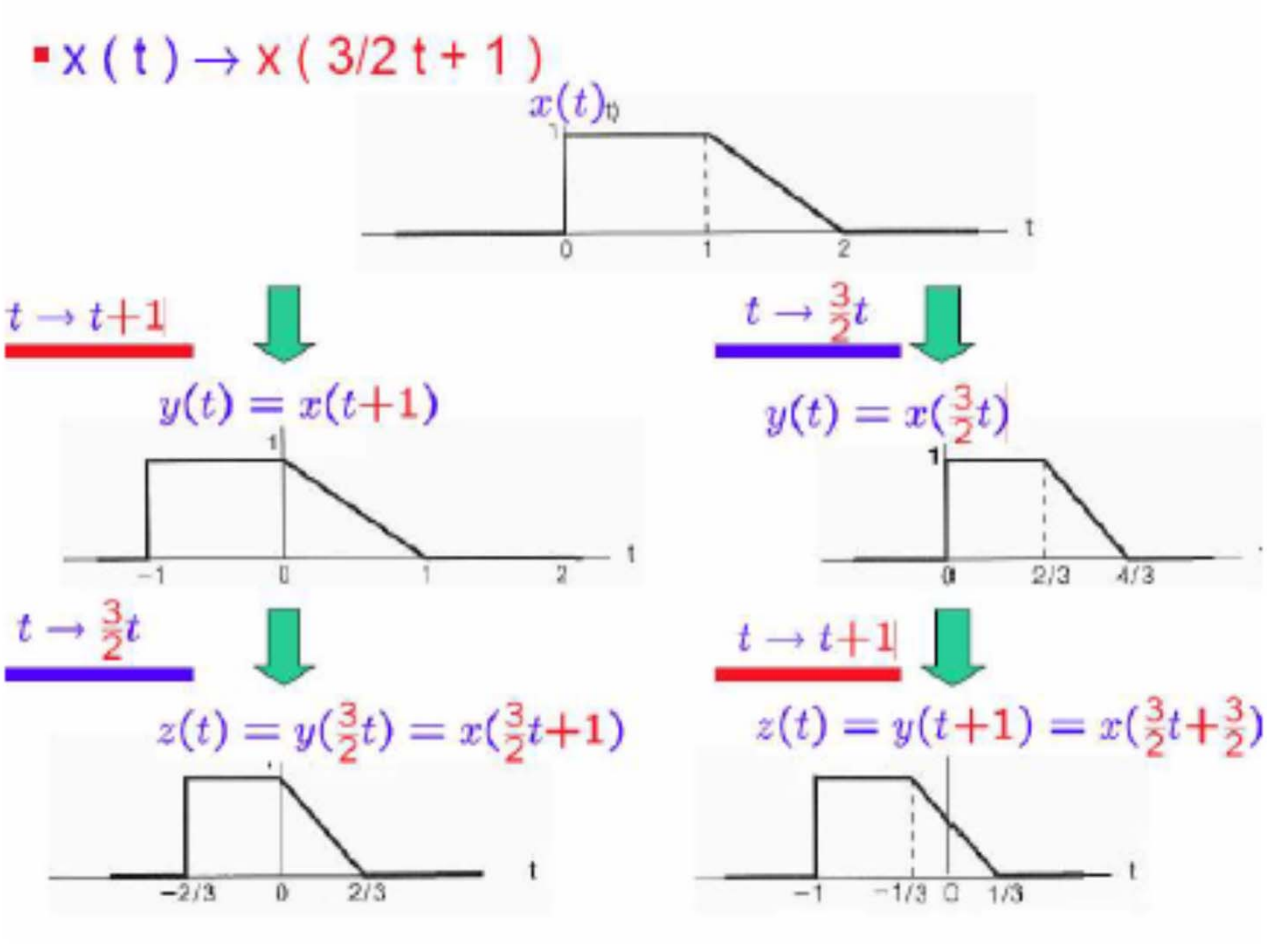
$t \rightarrow -t$

$$z(t) = y(-t) = x(-t+1)$$



ÖRNEK

- x (t) → x (3/2 t + 1)



ÖZET

- $x(t) \rightarrow x(at-b)$

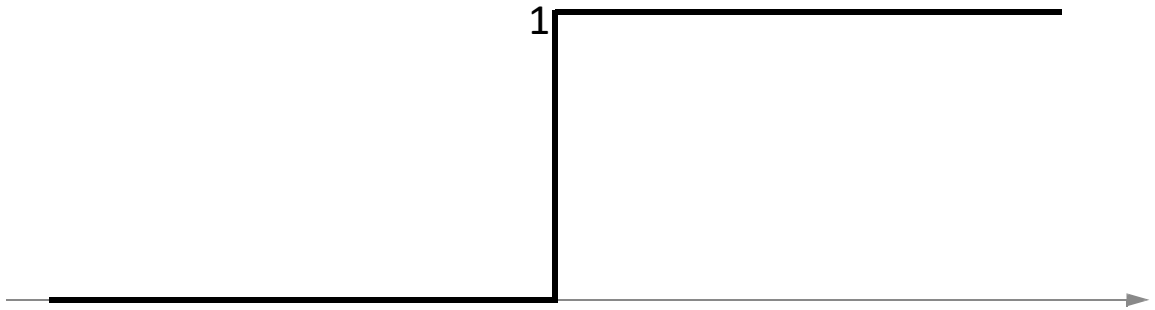
$\|a\|$	< 1	Doğrusal genişleme
$\|a\|$	> 1	Doğrusal sıkıştırma
a	< 1	Zaman üzerinde tersini alma
b	> 0	Geriye doğru zamanı öteleme
b	< 0	İleriye doğru zamanı öteleme

Önemli İşaretler

- Birim Basamak
- Delta Dirak
- Birim Rampa

Önemli İşaretler: Birim Basamak

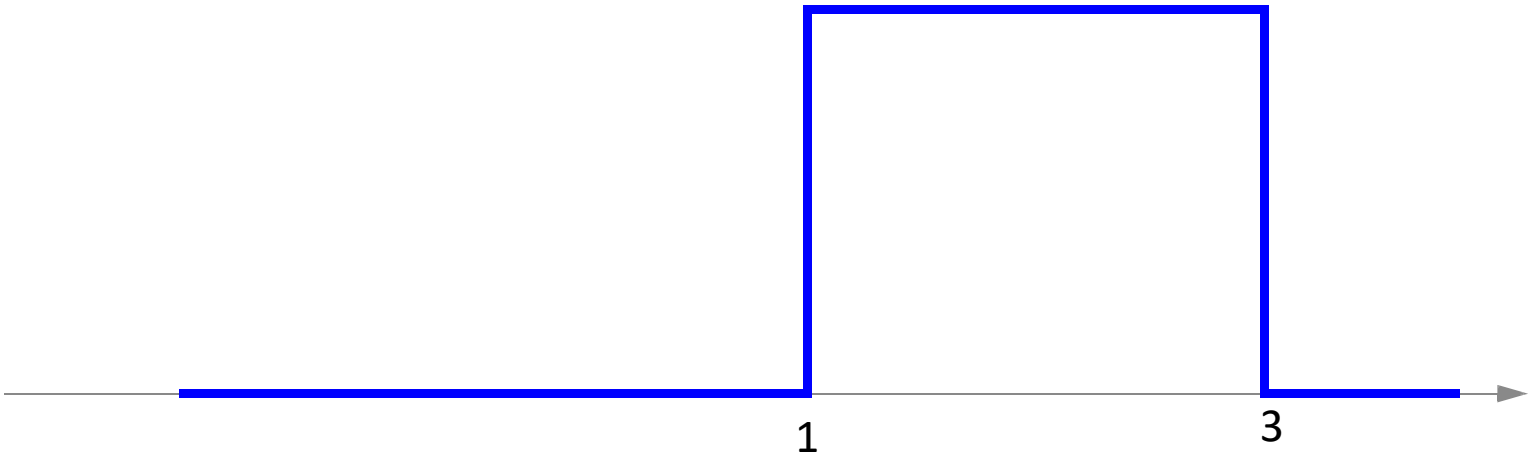
- Birim Basamak: $u(t) = \begin{cases} 1 & \text{for } t > 0, \\ 0 & \text{for } t < 0. \end{cases}$

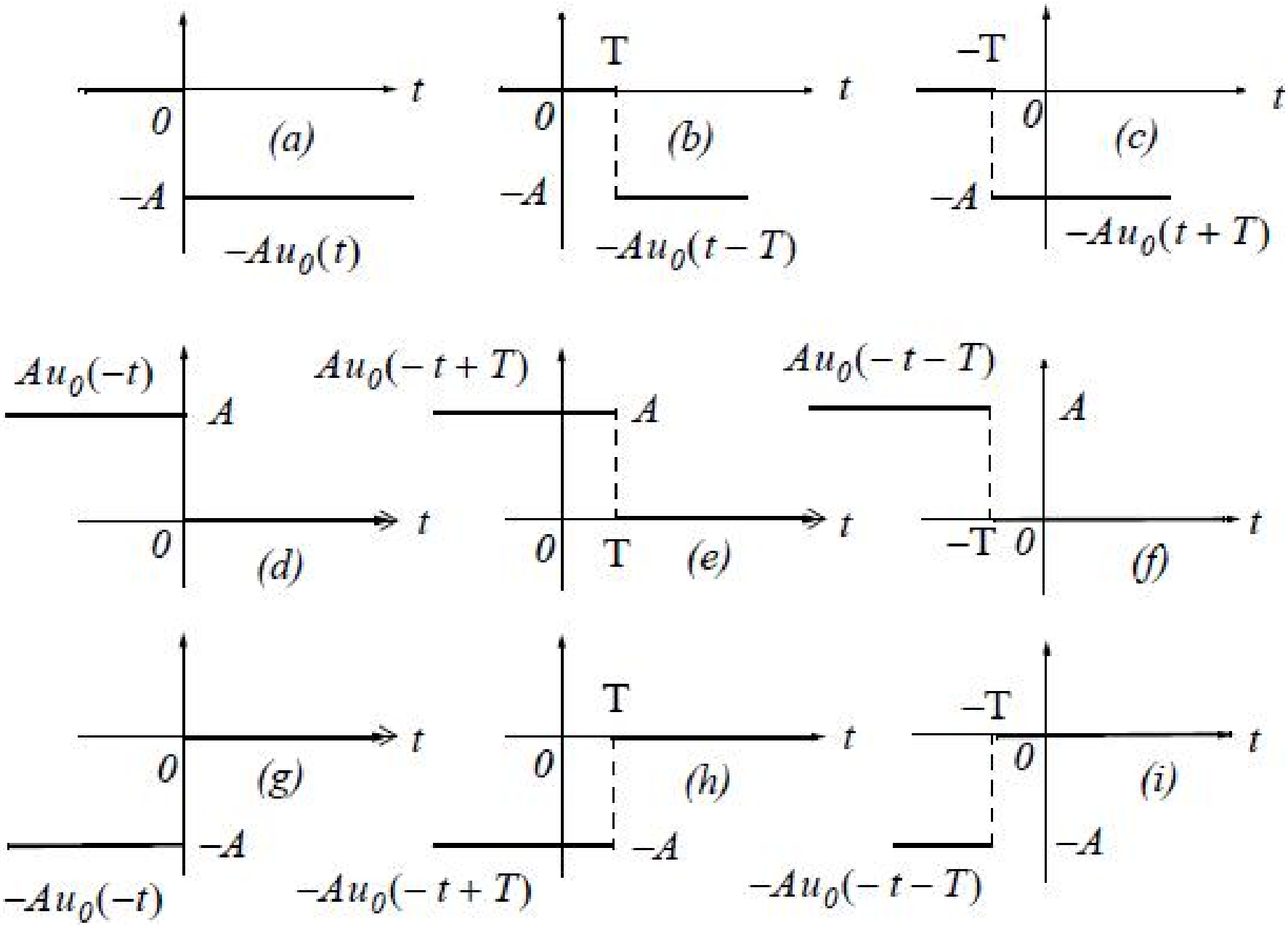


Zamanda öteleme: $u(t-3)$

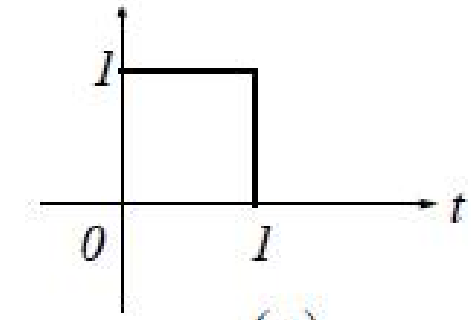
İki adet birim basamak işaretinden kare dalga elde etme

$$u(t-1) - u(t-3)$$



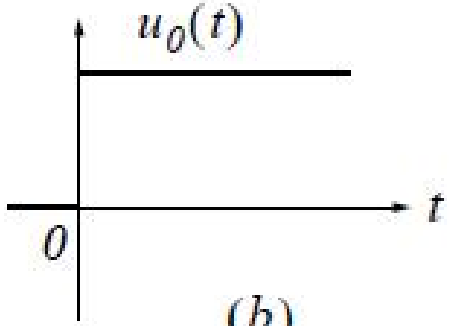

t
0
(a)
−A
−Au0(t)
T
t
0
(b)
−A
−Au0(t − T)
−T
t
0
(c)
−A
−Au0(t + T)
Au0(−t)
A
t
0
(d)
Au0(− t + T)
A
t
0
T
(e)
Au0(− t − T)
A
t
−T
0
(f)
t
0
(g)
−A
−Au0(−t)
T
t
0
(h)
−A
−Au0(− t + T)
−T
t
0
(i)
−A
−Au0(− t − T)

Lojik ‘1’

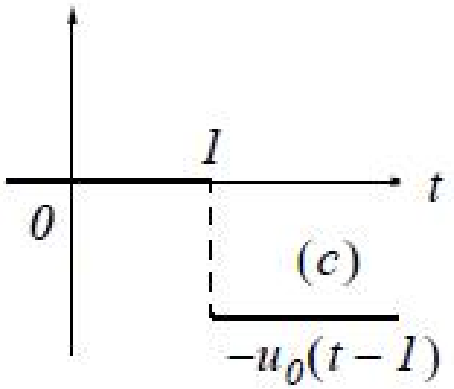


(a)

$u_0(t) - u_0(t-1)$

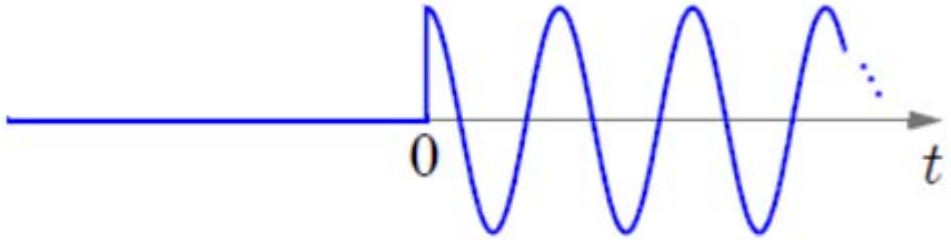


(b)

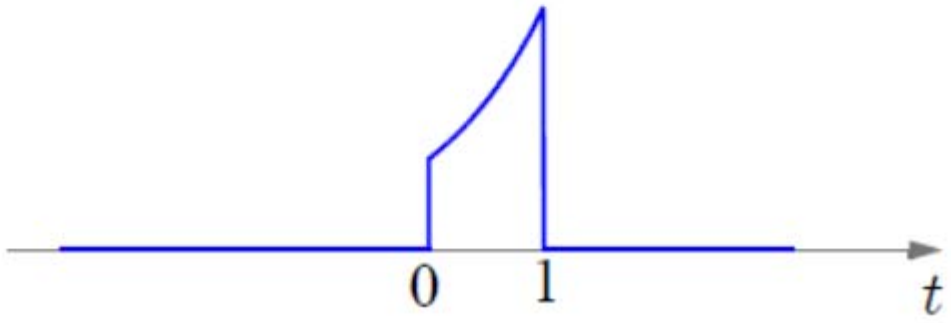


İşaretleri açma / kapama için kullanma

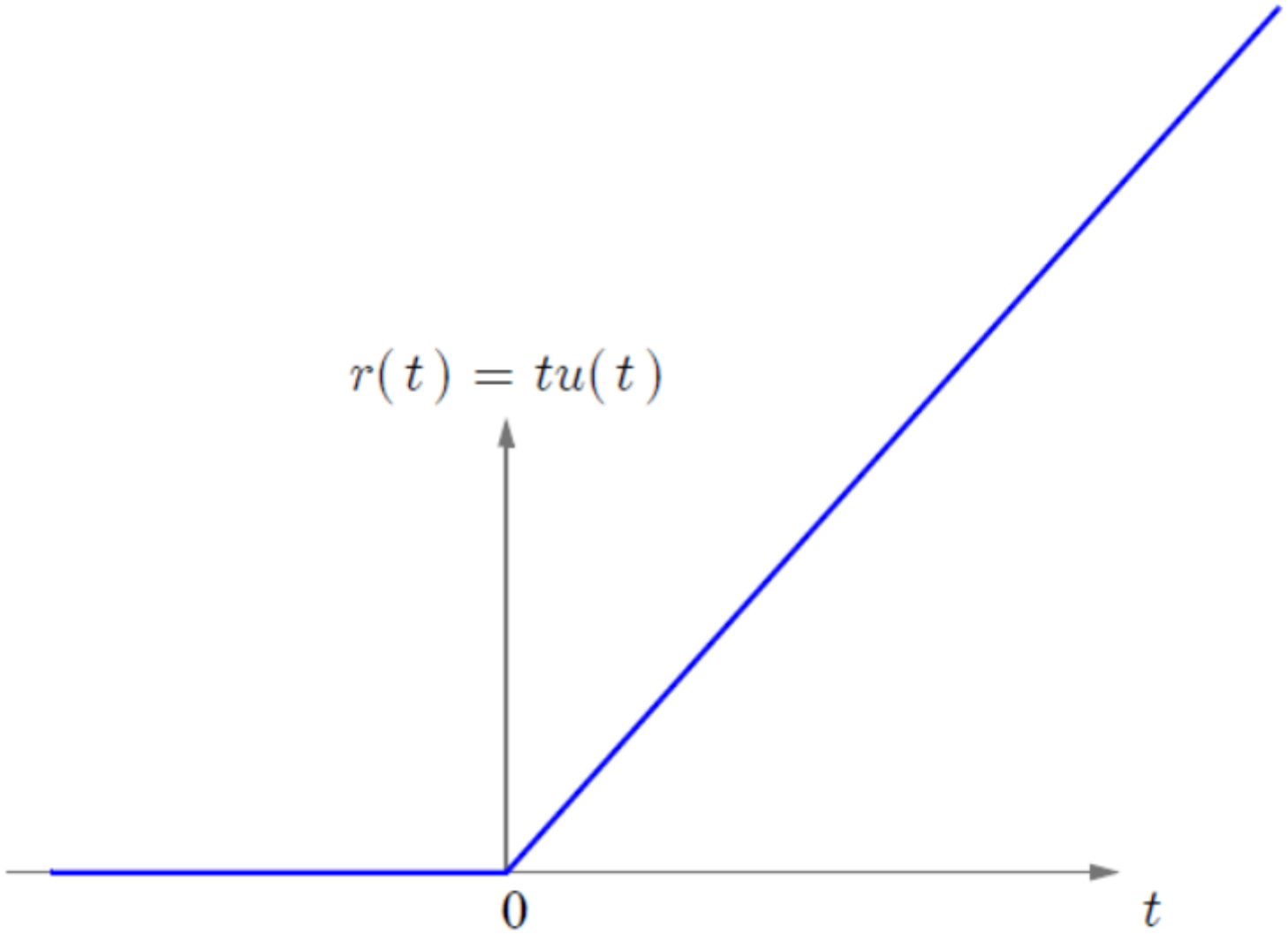
- $x(t) = \cos(2\pi t)u(t)$ sıfır anında açar



- $x(t) = e^{t}(u(t) - u(t-1))$ sadece 0 ve 1 arasında sıfırdan farklıdır :

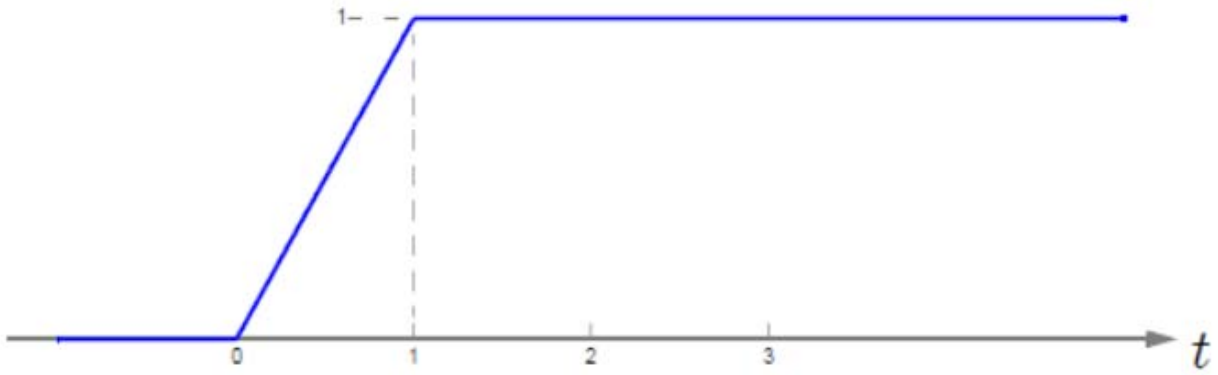


BİRİM RAMPA

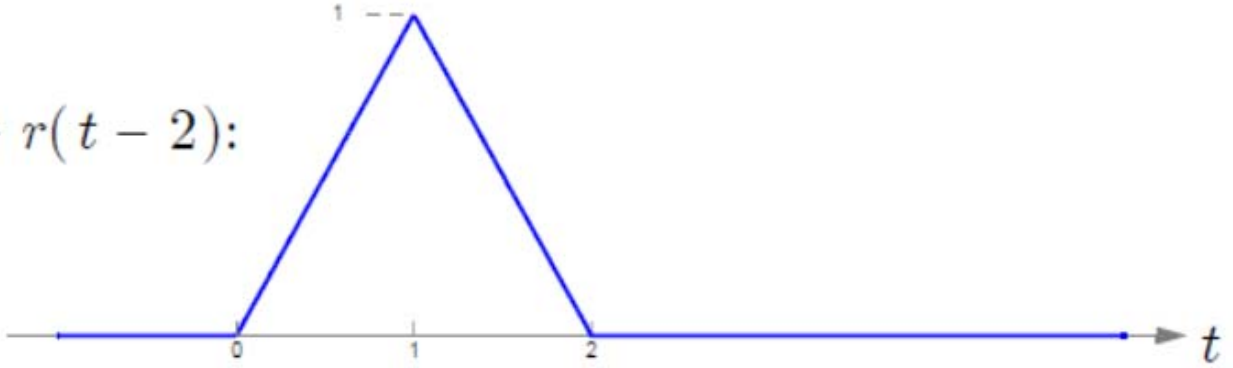


Birim Rampa işaretini kullanarak yeni işaretler üretme

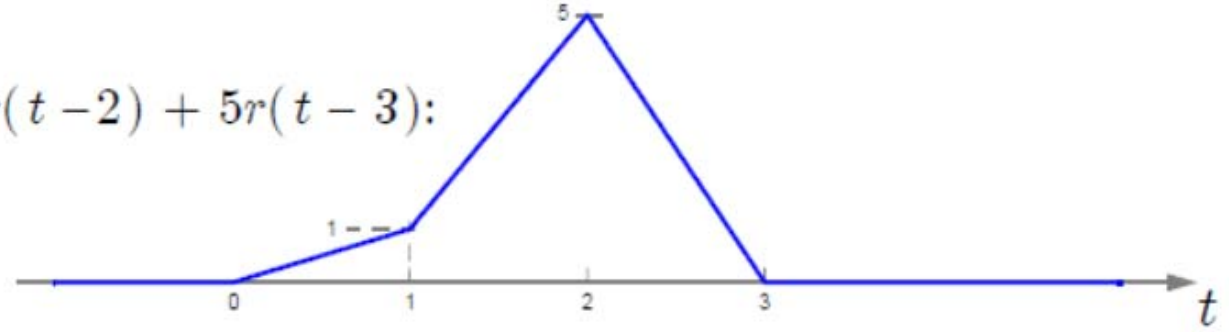
$r(t) - r(t-1)$:

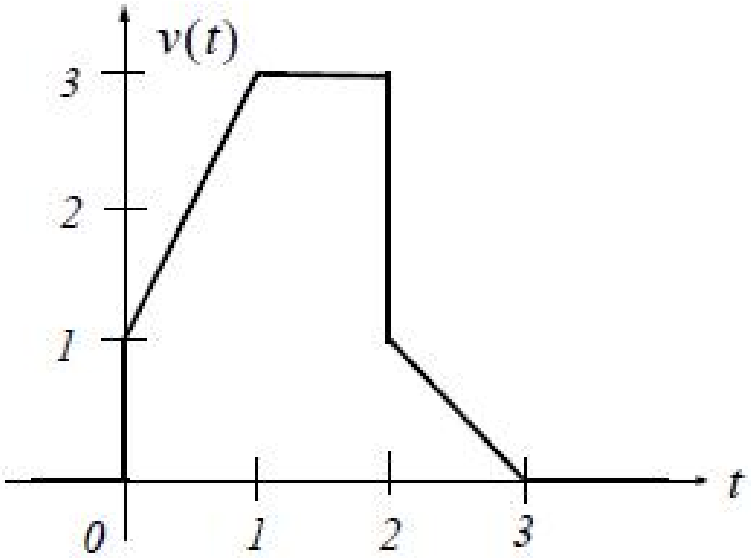


$r(t) - 2r(t-1) + r(t-2)$:



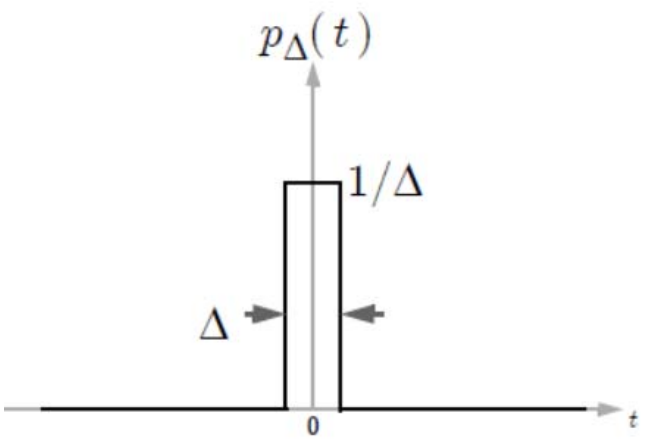
$r(t) + 3r(t-1) - 9r(t-2) + 5r(t-3)$:

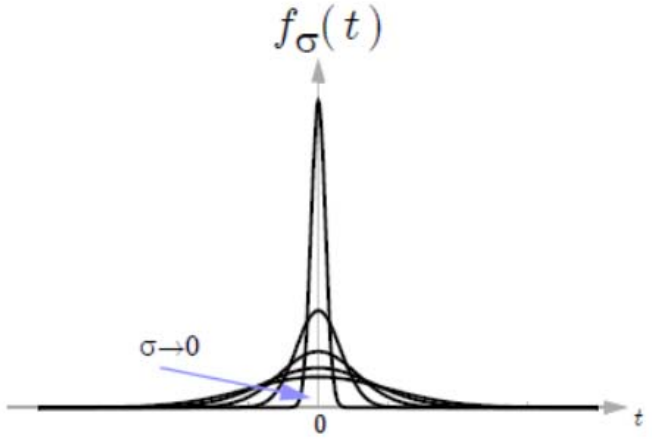


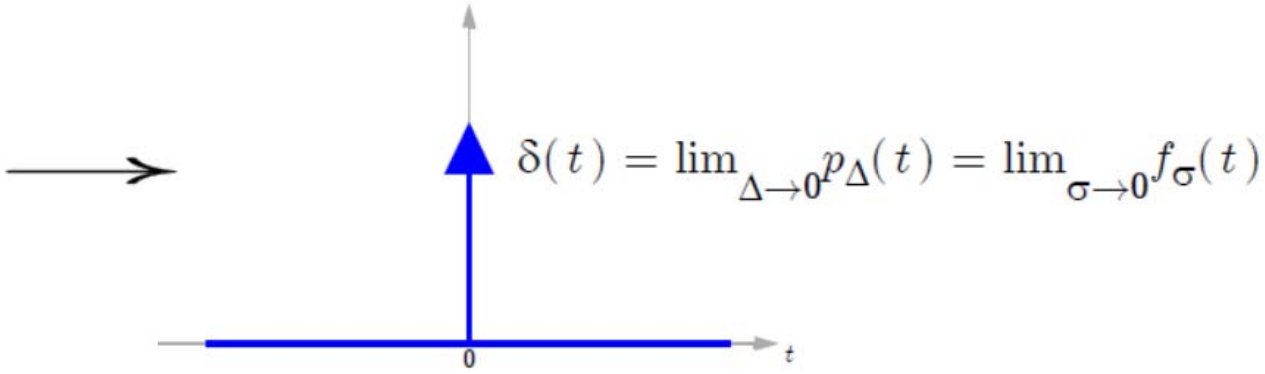


$$v(t) = (2t+1)[u_0(t) - u_0(t-1)] + 3[u_0(t-1) - u_0(t-2)] + (-t+3)[u_0(t-2) - u_0(t-3)]$$

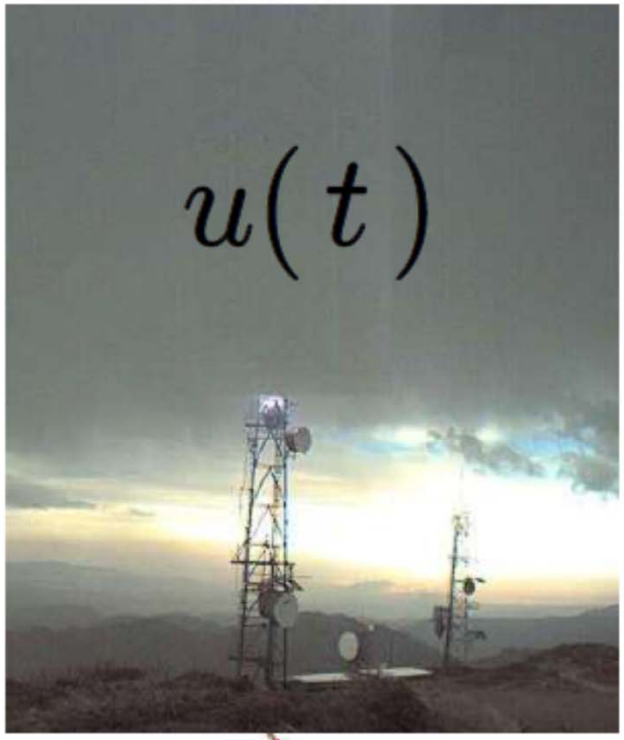
Dirak İmpuls «Fonksiyonu»

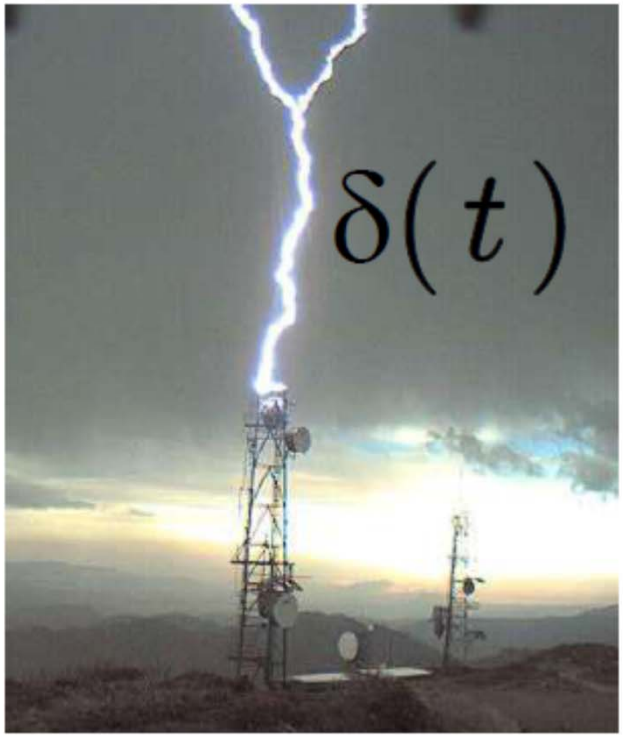






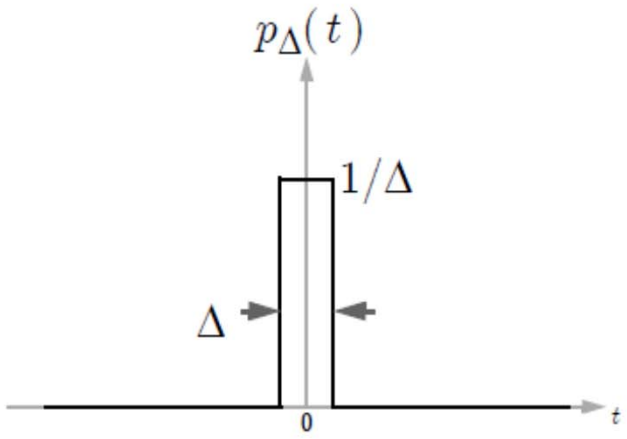
Birim Basamak ve Birim İmpuls arasındaki fark

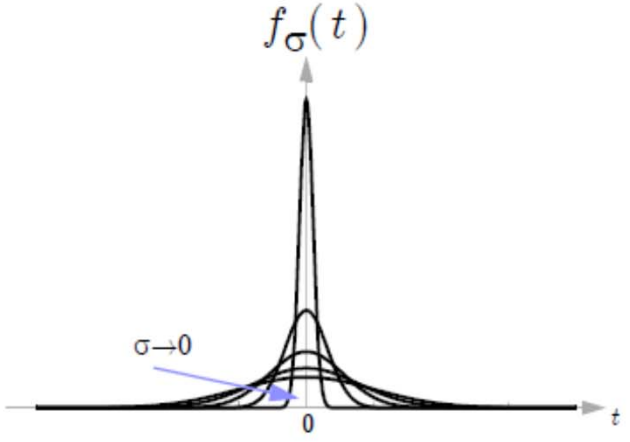


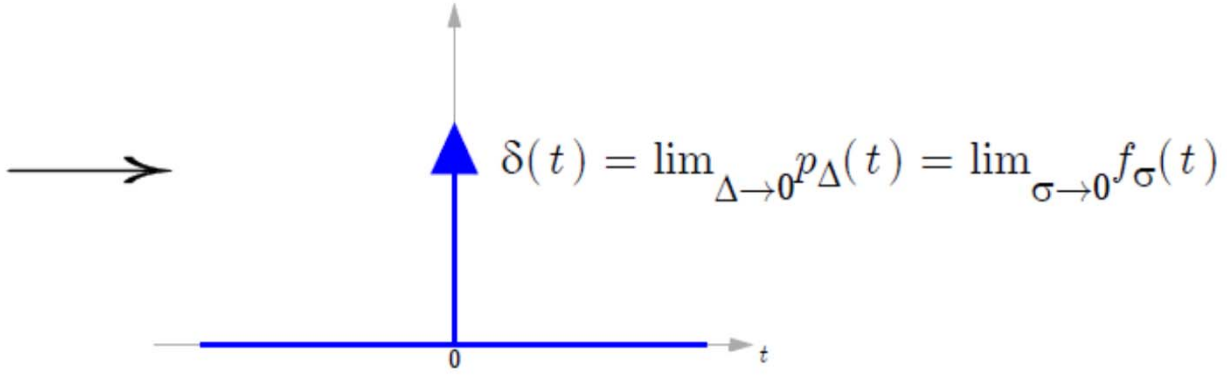


Birim Darbe Fonksiyonu

Toplamı 1 olan son derece kısa bir sinyal







Delta Dirak Fonksiyonu Özellikleri

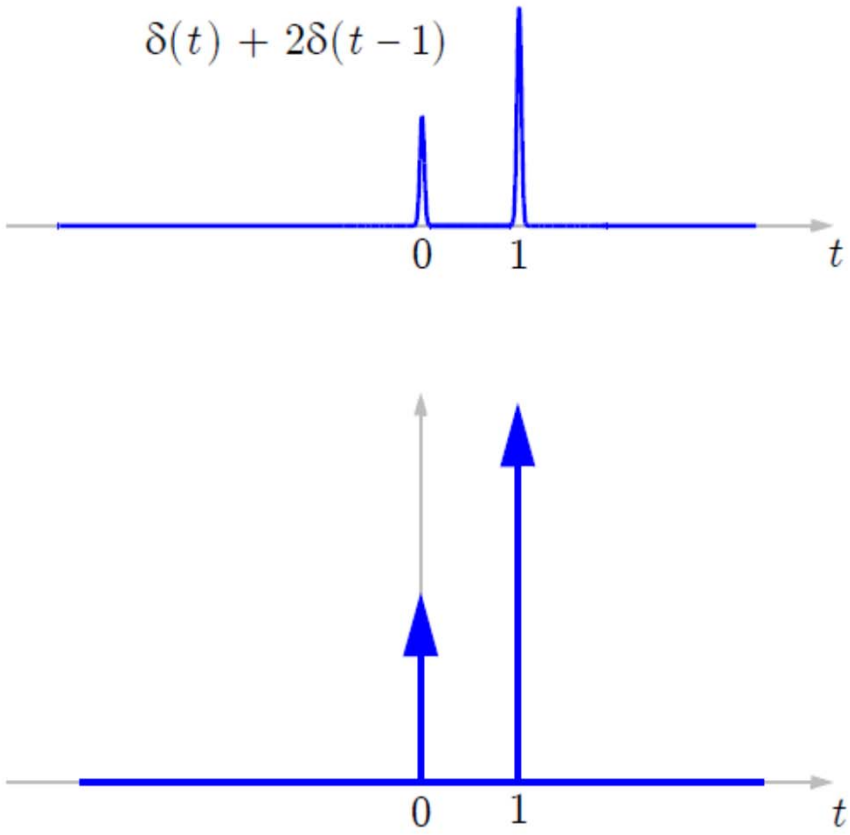
- zero for all $t \neq 0$, yet integrates to one: $\int_{-\infty}^{\infty} \delta(t)\mathrm{d}t = 1$
- Integrates to unit step: $u(t) = \int_{-\infty}^{t} \delta(\tau)\mathrm{d}\tau$
- derivative of unit step: $\delta(t) = \frac{d}{dt}u(t)$
- **Sampling** property:
  Multiplying *anything* by a delta function yields a *scaled* delta function:

$$x(t)\delta(t - t_0) = x(t_0)\delta(t - t_0)$$

- **Sifting** property:

$$\int_{-\infty}^{\infty} x(t)\delta(t - t_0)\mathrm{d}t = x(t_0)$$

Nasıl Çizilir?



İşaretlerin Kategorize Edilmesi

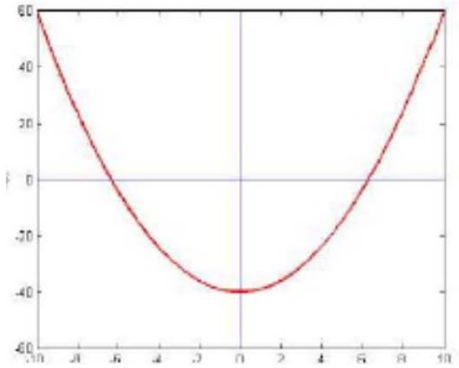
- Tek, Çift, ne tek ne çift
- Periyodik, periyodik olmayan
- nedensel, nedensel olmayan
- “enerji” (sonlu enerji, sıfır enerji)
- “güç” (sonsuz enerji, sonlu güç)

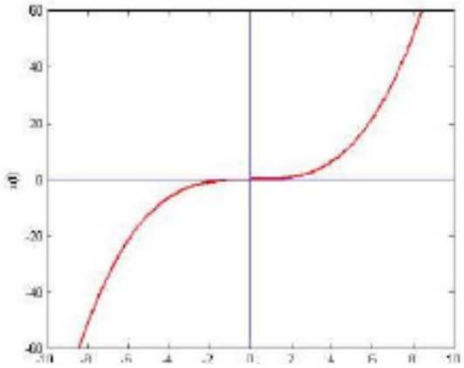
Tek, Çift

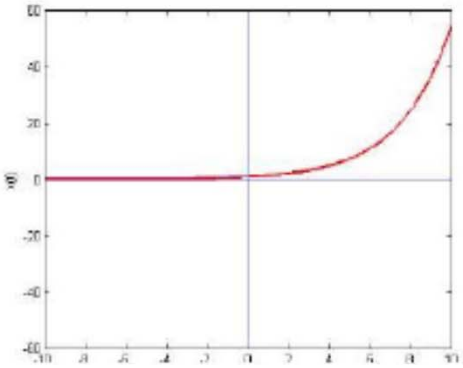
Çift: $x(-t) = x(t)$ örnek: t^4, $e^{-|t|}$, cos(t), ...

Tek: $x(-t) = -x(t)$ örnek: t^3, sin(t), ...

ÇİFT TEK hiçbiri







Teorem: herhangi bir sinyal tek ve çift parçalara ayrılabilir.

$x(t) = x_e(t) + x_o(t)$

$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

$$x_o(t) = \frac{x(t) - x(-t)}{2}$$

Periyodiklik

Bir $x(n)$ işareti tüm n değerleri ve sabit bir N sayısı için,

$$x(n) = x(n + N)$$

koşulunu sağlıyorsa periyodiktir.

$x(n) = e^{j\omega_0 n}$ için $2\pi / \omega_0$ tam sayı olursa periyodiktir.

Periyot: $N = 2\pi / \omega_0$

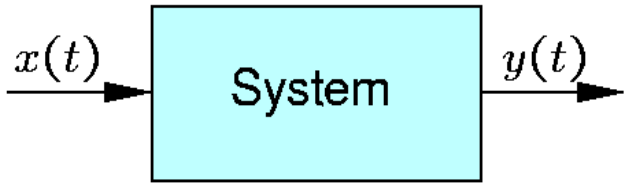
1. $x(n) = e^{j(\frac{\pi}{8})n}$ periyodiktir. $\omega_0 = \frac{\pi}{8}$, $N = \frac{2\pi}{\omega_0} = \frac{2\pi}{\pi/8} = 16$ tam sayı

2. $x(n) = e^{j(\frac{6\pi}{25})n}$ periyodiktir. $\omega_0 = \frac{6\pi}{25}$, $N = \frac{2\pi}{\omega_0} = \frac{2\pi}{6\pi/25} = 25/3$, $N = \frac{25}{3}3 = 25$

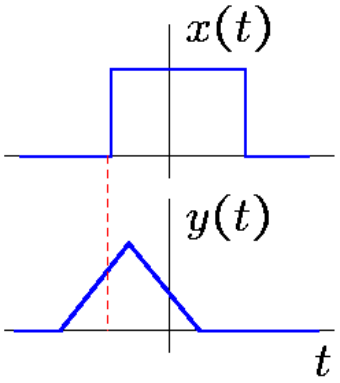
3. $x(n) = e^{j(\frac{n}{8})}$ periyodik değildir. $\omega_0 = \frac{1}{8}$, $N = \frac{2\pi}{\omega_0} = \frac{2\pi}{1/8} = 16\pi$

Nedensel vs. nedensel olmayan

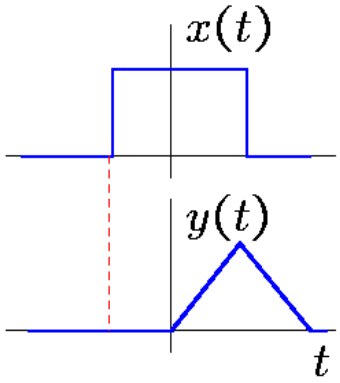
For a causal system the output at time t_o depends only on the input for $t \leq t_o$, i.e., the system cannot anticipate the input.



Non causal



Causal



Enerji ve Güç

- Enerji:

$$E = \int_{-\infty}^{\infty} x^2(t)dt$$

- Güç:

$$P = \lim_{\tau \to \infty} \frac{1}{2\tau} \int_{-\tau}^{\tau} x^2(t)dt$$