

Gradient-
Constrained
Knowledge

$$\text{cKG}(x) = \mathbb{E}_{Z_c, Z_y} \left[\max_{x' \in \mathcal{X}} \left\{ (\mu_y^n(x') + \tilde{\sigma}_y(x', x_r^{n+1}, Z_c)) \text{PF}^{n+1}(x'; x_r^n, Z_y) \right\} - \mu_y^n(x_r) \text{PF}^{n+1}(x_r; x_r^{n+1}, Z_y) \right]$$

Reparative metathesis

$$y(x) = \mu^n(x) + \tilde{\sigma}(x, x^{n+1}) z,$$

$$c(x) = \mu_c^n(x) + \tilde{\sigma}_c(x,x^{n+1})z_c + \tilde{\sigma}_c(x,0,I)$$

$$\mathbb{P}^{n+1}(x;x^{n+1},Z_c)=\Pr_{c_j,\mathcal{D}^{n+1}}\left[c_j(x)\leq 0\right]$$

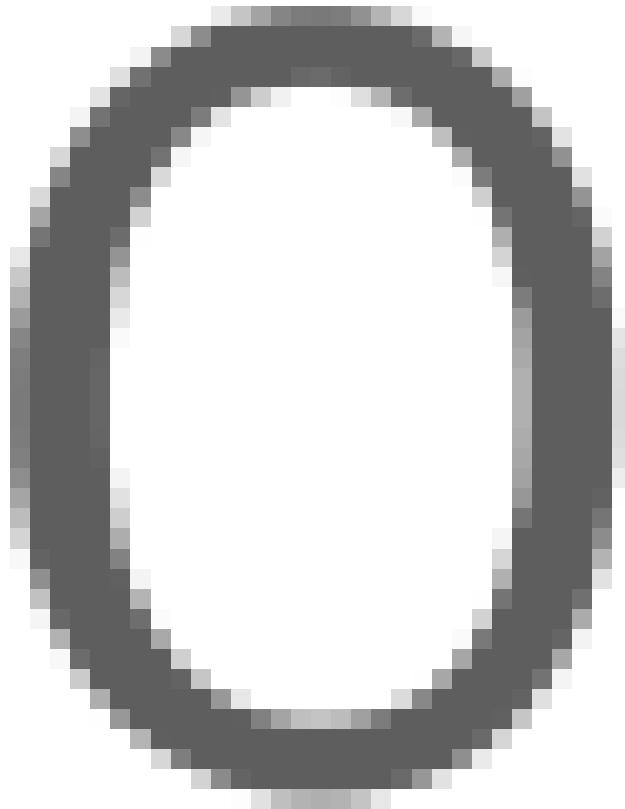
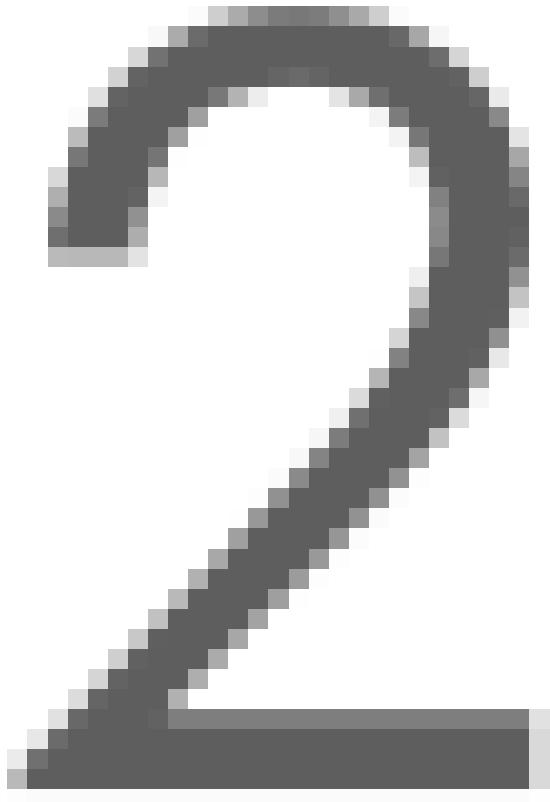
Posterior mean at timestep $n+1$ determined by Z_y



Probability of feasibility at timestep $n+1$ determined by Z_c

The maximum of the penalized posterior mean after getting updated with y^{n+1}

The penalized posterior mean at timestep $n+1$ is evaluated at the point that maximizes the penalized posterior at timestep n . Determined by Z_c



Constrained Knowledge Gradient (cKG) - 2

Recall the reparametrization trick

$$y(x) = \mu_y^n(x) + \tilde{\sigma}_y(x, x^{n+1})Z_y, \quad Z_y \sim \mathcal{N}(0,1) \quad c(x) = \mu_c^n(x) + \tilde{\sigma}_c(x, x^{n+1})Z_c, \quad Z_c \sim \mathcal{N}(0,I)$$

$$\text{PF}^{n+1}(x; x^{n+1}, Z_c) = \Pr [c_j(x) \leq 0 \ \forall j \mid Z_c, \mathcal{D}^{n+1}].$$

The maximum of the penalized posterior mean after getting updated with y^{n+1}

$$\text{cKG}(x) = \mathbb{E}_{Z_c, Z_y} \left[\max_{x' \in \mathcal{X}} \left\{ (\mu_y^n(x') + \tilde{\sigma}_y(x', x^{n+1})Z_y) \text{PF}^{n+1}(x'; x^{n+1}, Z_c) \right\} - \mu_y^n(x_r) \text{PF}^{n+1}(x_r^n; x^{n+1}, Z_c) \mid x^{n+1} = x \right].$$

Posterior mean at timestep $n+1$ determined by Z_y

Probability of feasibility at timestep $n+1$ determined by Z_c

The penalized posterior mean at timestep $n+1$ is evaluated at the point that maximizes the penalized posterior at timestep n . Determined by Z_c

How to Compute ???

$$\text{cKG}(x) = \mathbb{E}_{Z_c, Z_y} \left[\max_{x' \in \mathcal{X}} \left\{ (\mu_y^n(x') + \tilde{\sigma}_y(x', x^{n+1}) Z_y) \text{PF}^{n+1}(x'; x^{n+1}, Z_c) \right\} - \mu_y^n(x_r) \text{PF}^{n+1}(x_r^n; x^{n+1}, Z_c) \mid x^{n+1} = x \right]$$