Constrained Knowledge Gradient (cKG) -1

$$\mathrm{cKG}(x) = \mathbb{E}\left[\max_{x' \in \mathcal{X}} \left\{\mu_y^{n+1}(x')\,\mathrm{PF}^{n+1}(x')\right\} - \mu_y^n(x_r^n)\,\mathrm{PF}^{n+1}(x_r^n) \;\middle|\; x^{n+1} = x\right].$$

The maximum of the new penalized posterior mean after getting updated with y^{n+1}

The penalized posterior mean at timestep n+1 is evaluated at the point that maximizes the penalized posterior at timestep n

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The maximum of the new penalized posterior mean after getting updated with y^{n+1}

Constrained Knowledge Gradient (cKG) - 2

Recall the reparametrization trick

$$y(x) = \mu_y^n(x) + \tilde{\sigma}_y(x, x^{n+1})Z_y, \quad Z_y \sim \mathcal{N}(0,1) \qquad\qquad c(x) = \mu_c^n(x) + \tilde{\sigma}_c(x, x^{n+1})Z_c, \quad Z_c \sim \mathcal{N}(0,I)$$

$$\mathrm{PF}^{n+1}(x; x^{n+1}, Z_c) = \Pr\left[c_j(x) \leq 0 \ \forall j \,\middle|\, Z_c, \mathcal{D}^{n+1}\right].$$

$$\mathrm{cKG}(x) = \mathbb{E}_{Z_c, Z_y}\left[\max_{x' \in \mathcal{X}} \left\{ \left(\mu_y^n(x') + \tilde{\sigma}_y(x', x^{n+1})Z_y\right)\mathrm{PF}^{n+1}(x'; x^{n+1}, Z_c) \right\} - \mu_y^n(x_r)\mathrm{PF}^{n+1}(x_r^n; x^{n+1}, Z_c) \ \middle|\ x^{n+1} = x\right].$$