Probability of Feasibility (PF)

The **probability of feasibility** for a candidate design $x$is defined as the probability that all constraints are satisfied under the current GP models:

$$\mathrm{PF}^n(x) = \prod_{k=1}^{K} P(c_k(x) \le 0) = \prod_{k=1}^{K} \Phi\left(-\frac{\mu_{c_k}^n(x)}{\sigma_{c_k}^n(x)} \right),$$

The **penalized posterior mean**— the GP mean of the objective scaled by the probability of feasibility:

$$x_r^n = \arg\max_{x \in \mathcal{X}} \ \mu_y^n(x)\, \mathrm{PF}^n(x)\,.$$

Constrained Knowledge Gradient (cKG) -1

$$\mathrm{cKG}(x) = \mathbb{E}\left[\max_{x' \in \mathcal{X}} \left\{\mu_y^{n+1}(x')\,\mathrm{PF}^{n+1}(x')\right\} - \mu_y^{n}(x_r^n)\,\mathrm{PF}^{n+1}(x_r^n) \;\middle|\; x^{n+1} = x\right].$$