

Probability of Feasibility (PF)

The **probability of feasibility** for a candidate design x is defined as the probability that all constraints are satisfied under the current GP models:

$$\text{PF}^n(x) = \prod_{k=1}^K P(c_k(x) \leq 0) = \prod_{k=1}^K \Phi\left(-\frac{\mu_{c_k}^n(x)}{\sigma_{c_k}^n(x)}\right),$$

The **penalized posterior mean**— the GP mean of the objective scaled by the probability of feasibility:

$$x_r^n = \arg \max_{x \in \mathcal{X}} \mu_y^n(x) \text{PF}^n(x).$$

Constrained Knowledge Gradient (cKG) -1

$$\text{cKG}(x) = \mathbb{E} \left[\max_{x' \in \mathcal{X}} \{ \mu_y^{n+1}(x') \text{PF}^{n+1}(x') \} - \mu_y^n(x_r^n) \text{PF}^{n+1}(x_r^n) \mid x^{n+1} = x \right] .$$