

Constrained Knowledge Gradient (ckKG) -1

$$\text{cKKG}(x) = \mathbb{E} \left[\max_{x' \in \mathcal{X}} \left\{ \mu_y^{n+1}(x') \text{PF}^{n+1}(x') \right\} - \mu_y^n(x_r^n) \text{PF}^{n+1}(x_r^n) \mid x^{n+1} = x \right].$$



The maximum of the new penalized posterior
mean after getting updated with y^{n+1}

The penalized posterior mean at timestep $n+1$ is evaluated at the point that maximizes the penalized posterior at timestep n





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$$\text{cKG}(x) = \mathbb{E} \left[\overbrace{\max_{x' \in \mathcal{X}} \{ \mu_y^{n+1}(x') \text{PF}^{n+1}(x') \}}^{\text{max}} - \underbrace{\mu_y^n(x_r^n) \text{PF}^{n+1}(x_r^n)}_{\text{mean}} \mid x^{n+1} = x \right].$$

The maximum of the new penalized posterior mean after getting updated with y^{n+1}

Constrained Knowledge Gradient (cKG) - 2

Recall the reparametrization trick

$$y(x) = \mu_y^n(x) + \tilde{\sigma}_y(x, x^{n+1})Z_y, \quad Z_y \sim \mathcal{N}(0,1) \quad c(x) = \mu_c^n(x) + \tilde{\sigma}_c(x, x^{n+1})Z_c, \quad Z_c \sim \mathcal{N}(0,I)$$

$$\text{PF}^{n+1}(x; x^{n+1}, Z_c) = \Pr [c_j(x) \leq 0 \ \forall j \mid Z_c, \mathcal{D}^{n+1}] .$$

$$\text{cKG}(x) = \mathbb{E}_{Z_c, Z_y} \left[\max_{x' \in \mathcal{X}} \left\{ (\mu_y^n(x') + \tilde{\sigma}_y(x', x^{n+1})Z_y) \text{PF}^{n+1}(x'; x^{n+1}, Z_c) \right\} - \mu_y^n(x_r) \text{PF}^{n+1}(x_r^n; x^{n+1}, Z_c) \mid x^{n+1} = x \right] .$$