



Constrained Gradient-Flow

$$\text{cKG}(x) = \mathbb{E}_{Z_c, Z_y} \left[ \max_{x' \in \mathcal{X}} \left\{ \left( \mu_y^n(x') + \tilde{\sigma}_y(x', x'^{n+1}, Z_c) \right) \text{PF}^{n+1}(x'; x'^{n+1}, Z_y) \right\} - \mu_y^n(x) \text{PF}^{n+1}(x; x'^{n+1}, Z_y) \right]$$

Reconnection at the reconnectional interface

$$y(x) = \mu^n(x) z + \tilde{\sigma}_n(x, x^{n+1}) z,$$

$$c(x) = \mu_c^n(x) + \tilde{\sigma}_c(x, x^{n+1}) z_c$$

$$\mathbb{P}^{n+1}(x;x^{n+1},Z_c)$$

$$= \Pr\left[c_j(x) \leq 0 \forall j \middle| Z_c, \mathcal{D}^{n+1}\right]$$



The maximum of the penalized posterior mean after getting updated with  $y^{n+1}$



The penalized posterior mean at timestep  $n+1$  is evaluated at the point that maximizes the penalized posterior at timestep  $n$ . Determined by  $Z_c$

# Constrained Knowledge Gradient (cKG) - 2

## Recall the reparametrization trick

$$y(x) = \mu_y^n(x) + \tilde{\sigma}_y(x, x^{n+1})Z_y, \quad Z_y \sim \mathcal{N}(0,1) \quad c(x) = \mu_c^n(x) + \tilde{\sigma}_c(x, x^{n+1})Z_c, \quad Z_c \sim \mathcal{N}(0,I)$$

$$\text{PF}^{n+1}(x; x^{n+1}, Z_c) = \Pr \left[ c_j(x) \leq 0 \ \forall j \mid Z_c, \mathcal{D}^{n+1} \right].$$

The maximum of the penalized posterior mean after getting updated with  $y^{n+1}$

$$\text{cKG}(x) = \mathbb{E}_{Z_c, Z_y} \left[ \max_{x' \in \mathcal{X}} \left\{ (\mu_y^n(x') + \tilde{\sigma}_y(x', x^{n+1})Z_y) \text{PF}^{n+1}(x'; x^{n+1}, Z_c) \right\} - \mu_y^n(x_r) \text{PF}^{n+1}(x_r^n; x^{n+1}, Z_c) \mid x^{n+1} = x \right].$$

Posterior mean at timestep  $n+1$  determined by  $Z_y$

Probability of feasibility at timestep  $n+1$  determined by  $Z_c$

The penalized posterior mean at timestep  $n+1$  is evaluated at the point that maximizes the penalized posterior at timestep  $n$ . Determined by  $Z_c$

# How to Compute ???

$$\text{cKG}(x) = \mathbb{E}_{Z_c, Z_y} \left[ \max_{x' \in \mathcal{X}} \left\{ (\mu_y^n(x') + \tilde{\sigma}_y(x', x^{n+1}) Z_y) \text{PF}^{n+1}(x'; x^{n+1}, Z_c) \right\} - \mu_y^n(x_r) \text{PF}^{n+1}(x_r^n; x^{n+1}, Z_c) \mid x^{n+1} = x \right]$$