

Constrained Knowledge Gradient (ckKG) - 2

$$\text{cKKG}(x) = \mathbb{E}_{Z_c, Z_y} \left[\max_{x' \in \mathcal{X}} \left\{ \left(\mu_y^n(x') + \tilde{\sigma}_y(x', x^{n+1}) Z_y \right) \text{PF}^{n+1}(x'; x^{n+1}, Z_c) \right\} - \mu_y^n(x_r) \text{PF}^{n+1}(x_r^n; x^{n+1}, Z_c) \mid x^{n+1} = x \right].$$

Recall the reparameterization trick

$$y(x) = \mu_y^n(x) + \tilde{\sigma}_y(x, x^{n+1})Z_y, \quad Z_y \sim \mathcal{N}(0, 1)$$

$$c(x) = \mu_c^n(x) + \tilde{\sigma}_c(x, x^{n+1})Z_c, Z_c \sim \mathcal{N}(0, I)$$

$$\text{PF}^{n+1}(x; x^{n+1}, Z_c) = \Pr \left[c_j(x) \leq 0 \, \forall j \, \middle| \, Z_c, \mathcal{D}^{n+1} \right].$$



Posterior mean at timestep $n+1$ determined by Z_y



Probabilty of feasibility at timestep $n+1$ determined by Z_c

The maximum of the penalized posterior mean after getting updated with y^{n+1}



The penalized posterior mean at timestep $n+1$ is evaluated at the point that maximizes the penalized posterior at timestep n . Determined by Z_c

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$$y(x) = \mu_y^n(x) + \tilde{\sigma}_y(x, x^{n+1})Z_y, \quad Z_y \sim \mathcal{N}(0,1) \quad c(x) = \mu_c^n(x) + \tilde{\sigma}_c(x, x^{n+1})Z_c, \quad Z_c \sim \mathcal{N}(0,I)$$

$$\text{PF}^{n+1}(x; x^{n+1}, Z_c) = \Pr [c_j(x) \leq 0 \ \forall j \mid Z_c, \mathcal{D}^{n+1}] .$$

The maximum of the penalized posterior mean after getting updated with y^{n+1}

$$\text{cKG}(x) = \mathbb{E}_{Z_c, Z_y} \left[\overbrace{\max_{x' \in \mathcal{X}} \left\{ \left(\mu_y^n(x') + \tilde{\sigma}_y(x', x^{n+1})Z_y \right) \text{PF}^{n+1}(x'; x^{n+1}, Z_c) \right\}}^{\text{Posterior mean at timestep n+1 determined by } Z_y} - \underbrace{\mu_y^n(x_r) \text{PF}^{n+1}(x_r^n; x^{n+1}, Z_c)}_{\text{Probabilty of feasability at timestep n+1 determined by } Z_c} \mid x^{n+1} = x \right] .$$

Posterior mean at timestep n+1 determined by Z_y

Probabilty of feasability at timestep n+1 determined by Z_c

The penalized posterior mean at timestep n+1 is evaluated at the point that maximizes the penalized posterior at timestep n. Determined by Z_c

How to Compute???

$$\text{cKG}(x) = \mathbb{E}_{Z_c, Z_y} \left[\max_{x' \in \mathcal{X}} \left\{ (\mu_y^n(x') + \tilde{\sigma}_y(x', x^{n+1}) Z_y) \text{PF}^{n+1}(x'; x^{n+1}, Z_c) \right\} - \mu_y^n(x_r) \text{PF}^{n+1}(x_r^n; x^{n+1}, Z_c) \mid x^{n+1} = x \right]$$