DL Homework 1

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1.

$$P(x,y) = \begin{cases} P(c(x)=1)\cdot f(x|c(x)=1)\cdot P(y=0|x,c(x)=1) + \\ P(c(x)=2)\cdot f(x|c(x)=2)\cdot P(y=0|x,c(x)=1), & y=0 \\ \\ P(c(x)=1)\cdot f(x|c(x)=1)\cdot (1-P(y=0|x,c(x)=1)) + \\ P(c(x)=2)\cdot f(x|c(x)=2)\cdot (1-P(y=0|x,c(x)=2)) & y=1 \end{cases}$$

plug in the values

$$\Downarrow$$

$$P(x,y) = \begin{cases} 0.1\cdot f(x|c(x)=1) + 0.35\, f(x|c(x)=2), & y=0 \\ \\ 0.4\cdot f(x|c(x)=1) + 0.15\, f(x|c(x)=2), & y=1 \end{cases}$$

$$f(x|c(x)=i) \sim \frac{1}{2\pi\sigma_{i1}\sigma_{i2}\sqrt{1-\rho_i^2}} \exp\left\{-\frac{z}{2(1-\rho_i^2)}\right\}$$

$$z = \frac{(x_1-\mu_{i1})^2}{\sigma_{i1}^2} + \frac{(x_2-\mu_{i2})^2}{\sigma_{i2}^2} - \frac{2\rho_i(x_1-\mu_{i1})(x_2-\mu_{i2})}{\sigma_{i1}\sigma_{i2}}.$$

2

2.1

Vector u can be seen as flatten matrix A's columns

$$A = \begin{bmatrix} a_{1,1} & \cdots & a_{1,l} \\ \vdots & & \vdots \\ a_{m,1} & \cdots & a_{m,l} \end{bmatrix} \Rightarrow u = \begin{bmatrix} a_{1,1} \\ \vdots \\ a_{m,1} \\ a_{1,2} \\ \vdots \\ a_{m,2} \\ \vdots \\ a_{m,l} \end{bmatrix}$$

Vector v can be constucted using the same way

$$B = \begin{bmatrix} b_{1,1} & \cdots & b_{1,l} \\ \vdots & & \vdots \\ b_{m,1} & \cdots & b_{m,l} \end{bmatrix} \Rightarrow v = \begin{bmatrix} b_{1,1} \\ \vdots \\ b_{1,2} \\ \vdots \\ b_{m,l} \end{bmatrix}$$

$$A \cdot B = tr(A^T B) = \sum_{j=1}^{m} \sum_{i=1}^{l} a_{ij} b_{ij} = \sum_{k=1}^{m \cdot l} a_k b_k = u \cdot v$$

2.2.

$$A \cdot B = tr(A^T B) = \sum_{j=1}^{m} \sum_{i=1}^{l} a_{ij} b_{ij}$$

$$B \cdot A = tr(B^T \cdot A) = \sum_{i=1}^{l} \sum_{j=1}^{m} b_{ij} a_{ij} = A \cdot B$$

2.3 =

$$A = \begin{bmatrix} 1 & -2 \\ -2 & 3 \end{bmatrix} \quad B = \begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix}$$

$$A \cdot B = 1 \cdot -1 + -2 \cdot 1 + -2 \cdot 0 + -2 \cdot 3 = -9$$

$$\|A\| = \sqrt{1^2 + (-2)^2 + (-2)^2 + 3^2} = \sqrt{1+4+4+9} = 3\sqrt{2}$$

$$\|B\| = \sqrt{(-1)^2 + 1^2 + (-2)^2 + 0^2} = \sqrt{1+1+4} = \sqrt{6}$$

$$\cos\angle(v,w) = \cos\theta = \frac{-9}{3\sqrt{2} \cdot \sqrt{6}} = \frac{-9}{6\sqrt{3}} = \frac{-\sqrt{3}}{2}$$

$$\therefore \angle(v,w) = \theta = \frac{5\pi}{6} + 2\pi n \text{ or } -\frac{5}{6}\pi + 2\pi n,\ n \in \mathbb{Z}$$