Theory Homework 3

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1. 'ijk , i → j,k' , [A, b]
2. 'ijk, ik → j' , [A, b]
3. 'ijkl → ik' , [A]
4. 'ijkl-ki' [A]
5. 'ijk, ijk → i' [A, A]
6. 'i, ij, j → ' [x, A, x]
7. 'de, fe, fl → dl' [A, G, B]
8. 'abcd, bcde, cdef → ab ef' [A, B, E]

2.

$$E_{(x,y)\sim p}[I[f_0(x) \neq y]] = P(Y=+1)\cdot P(x^{(1)}<0 \mid Y=+1) + P(Y=-1)\cdot P(x^{(1)}>0 \mid Y=-1)$$
$$= P(Y=+1)\cdot P(x^{(1)}<0 \mid Y=+1) + (1-P(Y=+1))\cdot(1-P(x^{(1)}<0 \mid Y=-1))$$
$$= P(Y=+1)\cdot 0.5 + (1-P(Y=+1))\cdot 0.5 = 0.5$$

- for first $N/2$ points, we know since $Y=-1$ $\forall x^{(i)}<0$ $i\in\{1,\cdots\frac{n}{2}\}$
  The probability of that is $\prod_{i=1}^{n} P(x^{(i)}<0 \mid Y=-1) = 0.5^{\frac{n}{2}}$

- for last $n/2$ points, we know there can be two cases for error equal to zero

for case i: ① $Y^{(i)}=+1 \Rightarrow$ $P(Y=+1)\cdot P(x<0 \mid Y=+1) = P(Y=+1)\cdot 0.5$

② $Y^{(i)}=-1 \Rightarrow$ $P(Y=-1)\cdot P(x>0 \mid Y=-1) = (1-P(x^{(i)}<0 \mid Y=-1)) = P(Y=-1)\cdot 0.5$

$\therefore P_r = (P(Y=+1)+P(Y=-1))\cdot 0.5 = 0.5$

$\therefore$ the whole probablity for all the cases is

$$\prod_{i=1}^{n/2} 0.5 = 0.5^{n/2}$$

$\therefore$ The whole probability is $0.5^{n/2}\cdot 0.5^{n/2} = 0.5^n$

- Since each dimension is independent of each other

$P(k \text{ dimensions achieves zero loss is}) = (0.5)^{kn}$

$P(k \text{ dimensions achives non zero loss is}) = (1-0.5^n)^{(D-k)}$

$Pr(\text{exactly } k \text{ out of } D) = \binom{D}{k} P(k \text{ dimension achieves zero loss}) \cdot P(k\text{-}D \text{ dimensions achives not zero loss})$

$= \binom{D}{k} (0.5^n)^k (1-0.5^n)^{D-k}$

it follow a Binomial distribution $B(D, 0.5^n)$, parameter is D & 0.5^n

- $Pr = 1 - Pr(\text{all of them not achieving zero-one loss})$

  $= 1-(1-0.5^N)^D$

- $\lim_{D\to\infty} Pr = 1$

  $O(1-(1-0.5^N)^D) = O(e^D)$