Machine learning homework

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1 Q1

1.1

Without knowing any node, There are four path from X_2 to x_6:

The first path is close because one of the gate is closed
$\rightarrow X_3(white) \rightarrow$ (open gate)
$\rightarrow X_4(white) \rightarrow$ (open gate)
$\rightarrow X_5(white) \rightarrow$ (open gate)
$\rightarrow X_7(white) \rightarrow$ (open gate)
$\rightarrow X_9(white) \leftarrow$ (closed gate)

The second path is closed because one of the gate is closed
$\rightarrow X_3(white) \rightarrow$ (open gate)
$\rightarrow X_4(white) \rightarrow$ (open gate)
$\rightarrow X_5(white) \rightarrow$ (open gate)
$\rightarrow X_8(white) \rightarrow$ (open gate)
$\rightarrow X_9(white) \leftarrow$ (closed gate)

The Third path is closed because one of the gate is closed
$\rightarrow X_3(white) \rightarrow$ (open gate)
$\rightarrow X_4(white) \rightarrow$ (open gate)
$\rightarrow X_5(white) \rightarrow$ (open gate)
$\rightarrow X_8(white) \rightarrow$ (open gate)
$\rightarrow X_9(white) \rightarrow$ (open gate)
$\rightarrow X_{10}(white) \rightarrow$ (open gate)
$\rightarrow X_{11}(white) \leftarrow$ (closed gate)
$\rightarrow X_{10}(white) \rightarrow$ (open gate)
$\rightarrow X_9(white) \rightarrow$ (open gate)

The fourth path is closed becasue one of the gate is closed :
$\rightarrow X_3(white) \rightarrow$ (open gate)
$\rightarrow X_4(white) \rightarrow$ (open gate)
$\rightarrow X_5(white) \rightarrow$ (open gate)

$\rightarrow X_7(white) \rightarrow$ (open gate)
$\rightarrow X_9(white) \rightarrow$ (open gate)
$\rightarrow X_{10}(white) \rightarrow$ (open gate)
$\rightarrow X_{11}(white) \leftarrow$ (closed gate)
$\rightarrow X_{10}(white) \rightarrow$ (open gate)
$\rightarrow X_9(white) \rightarrow$ (open gate)

Since all the path from X_4 to X_6 are closed, X_4 and X_6 are independent.

1.2

Without knowing any node, There are four path from X_2 to x_6:

The first path is close because one of the gate is closed
$\rightarrow X_3(white) \rightarrow$ (open gate)
$\rightarrow X_4(white) \rightarrow$ (open gate)
$\rightarrow X_5(white) \rightarrow$ (open gate)
$\rightarrow X_7(black) \rightarrow$ (closed gate)
$\rightarrow X_9(white) \leftarrow$ (closed gate)

The second path is closed because one of the gate is closed
$\rightarrow X_3(white) \rightarrow$ (open gate)
$\rightarrow X_4(white) \rightarrow$ (open gate)
$\rightarrow X_5(white) \rightarrow$ (open gate)
$\rightarrow X_8(white) \rightarrow$ (open gate)
$\rightarrow X_9(white) \leftarrow$ (closed gate)

The Third path is open because all the gates are open because
$\rightarrow X_3(white) \rightarrow$ (open gate)
$\rightarrow X_4(white) \rightarrow$ (open gate)
$\rightarrow X_5(white) \rightarrow$ (open gate)
$\rightarrow X_8(white) \rightarrow$ (open gate)
$\rightarrow X_9(white) \rightarrow$ (open gate)
$\rightarrow X_{10}(white) \rightarrow$ (open gate)
$\rightarrow X_{11}(black) \leftarrow$ (open gate)
$\rightarrow X_{10}(white) \rightarrow$ (open gate)
$\rightarrow X_9(white) \rightarrow$ (open gate)

The fourth path is closed because one of the gate is closed
$\rightarrow X_3(white) \rightarrow$ (open gate)
$\rightarrow X_4(white) \rightarrow$ (open gate)
$\rightarrow X_5(white) \rightarrow$ (open gate)
$\rightarrow X_7(black) \rightarrow$ (closed gate)
$\rightarrow X_9(white) \rightarrow$ (open gate)
$\rightarrow X_{10}(white) \rightarrow$ (open gate)

$\rightarrow X_{11}(black) \leftarrow$ (open gate)
$\rightarrow X_{10}(white) \rightarrow$ (open gate)
$\rightarrow X_9(white) \rightarrow$ (open gate)

There four there is an open path from X_4 to X_6, therefore, they are dependent.

2 Q2

Following equation: $FreeParameters = transtionFrom * (transitionTo - 1)$

2.1

The number of free parameters for the every state can take two states 1,2 are:
X_1 and X_6 have starting transition, each have two parameters, but they only have 1 free parameters
X_{11} have ending transition have two parameters. But since they will definitely go to last state, so there is not any free parameters needed.
For the other states, although each state transition matrix have 4 parameters, it only have 2 free parameters, and the other 2 are determined.
X_9 have 8 free parameters, because it have transition from X_6, X_7, X_8
Therefore, the whole model have $2 * (11 - 1) - 1 - 1 + 8 = 26$ free parameters.

2.2

X_i	Number Of Free Paramters
X_1	1 * (5-1) = 4
X_2	5 * (5-1) = 20
X_3	5 * (3-1) = 10
X_4	3 * (5-1) = 12
X_5	5 * (5-1) = 20
X_6	1 * (5-1) = 4
X_7	5 * (5-1) = 20
X_8	5 * (5-1) = 20
X_9	(5 * 5 * 5) * (3-1) = 250
X_10	3 * (5-1) = 12
X_11	5 * (5-1) = 20
STOP	0

Therefore, Adding all of this up will result in 392 free parameters.

3 Q3

3.1 a

We can know that $X_4 = 1$ have two possibility, transition from $X_3 = 1$ or $X_3 = 2$
the probability on X_3, X_4 are not dependent on all the probabilities behind the X_4 , So $P(X_3 = 1|X_4 = 1) = \frac{P(X_3=1,X_4=1)}{P(X_4=1)}$
Since the probability for $P(X_3)$ is transitioned from $P(X_2)$, therefore, we assume $P(X_2 = 1) = p_1, P(X_2 = 2) = p_2$

$P(X_3 = 1) = \sum_i P(X_3 = 1|X_2 = i) * P(X_2 = i) = 0.3 * p_1 + 0.3 * p_2 = 0.3(p_1 + p_2) = 0.3$
$P(X_3 = 2) = \sum_i P(X_3 = 2|X_2 = i) * P(X_2 = i) = 0.7 * p_1 + 0.7 * p_2 = 0.3(p_1 + p_2) = 0.7$

$P(X_4 = 1, X_3 = 1) = P(X_3 = 1)P(X_4 = 1|X_3 = 1) = 0.3 * 0.1 = 0.03$
$P(X_4 = 1, X_3 = 2) = P(X_3 = 2)P(X_4 = 1|X_3 = 2) = 0.7 * 0.5 = 0.35$
$P(X_3 = 1|X_4 = 1) = \frac{P(X_4=1,X_3=1)}{P(X_4=1,X_3=1)+P(X_4=1,X_3=2)} = 0.0789$

3.2 b

$$P(X_5 = 2|X_3 = 2, X_{11} = 2, X_1 = 2) = \frac{P(X_5 = 2, X_3 = 2, X_{11} = 2, X_1 = 2)}{\sum_i P(X_5 = i, X_3 = 2, X_{11} = 2, X_1 = 2)} \tag{1}$$

$P(X_5 = i, X_3 = 2, X_{11} = 2, X_1 = 2) = \sum_j P(X_3 = 2|X_1 = 2) * P(X_5 = i|X_3 = 2) * P(X_{10} = j|X_5 = i) * P(X_{11} = 2|X_{10} = j)$

$P(X_{10} = 1|X_5 = i) = 0.8, P(X_{10} = 2|X_5 = i) = 0.2$
therefore we can eliminate $P(X_{11} = 2|X_{10} = 5)$ from the two equation.

since we have proved in the first question that $P(X_3 = 1) = 0.3$ so the equation will become:
$P(X_5 = 2|X_3 = 2, X_{11} = 2, X_1 = 2) = \frac{P(X_5=2,X_3=2)}{\sum_i P(X_5=i,X_3=2)}$

$P(X_3 = 2, X_5 = i) = (0.3, 0.7) * [(0.1, 0.9), (0.5, 0.5)] * [(0.5, 0.5, (0.6, 0.4)] = 0.0441, 0.0539$
$P(X_5 = 2|X_3 = 2, X_{11} = 2, X_1 = 2) = 0.45$