**10.009 The Digital World**

Term 3. 2018

Problem Set 8 (for Week 8)

Most recent updated: March 8, 2018

Due dates:

- **Cohort session problems** : Following week: Tuesday 11:59pm.

- **Homework problems** : Same as for the cohort session problems.

- **Exercises**: These are practice problems and will not be graded. You are encouraged to solve these to enhance your programming skills. Being able to solve these problems will likely help you prepare for the end term examination.


**Objectives**

1. Understand what is object-oriented programming (OOP).

2. Learn classes and methods.

**Note**: Solve the programming problems listed using your favorite text editor. Make sure you save your programs in files with suitably chosen names, **and try as much as possible to write your code with good style (see the style guide for python code)**. In each problem find out a way to test the correctness of your program. After writing each program, test it, debug it if the program is incorrect, correct it, and repeat this process until you have a fully working program. Show your working program to one of the cohort instructors.

**Problems: Cohort sessions**

1. *Classes and Methods: Make a Coordinate class.* Implement a class named `Coordinate` that represents a coordinate of a point in two dimensional space. The class has two attributes: $x$ and $y$. It also has several methods:

   - `__init__(self, x=0, y=0)` : This method takes in $x$ and $y$ to initialize the attributes. If $x$ and $y$ are not provided, the attributes are initialized to 0.

   - `magnitude(self)`: This method returns the magnitude, which is defined as $\sqrt{x^2 + y^2}$.

   - `translate(self, dx, dy)`: This method translates the coordinate by `dx` and `dy` so that it now represents the coordinate `(x + dx, y + dy)`.

   - `__eq__(self, other)`: This method takes another coordinate object and returns `True` or `False` depending on whether this coordinate specifies the same point in space as the other.

   A sample interactive session using class `Coordinate` is shown below.

   ```
   >>> p = Coordinate ()
   >>> print (p.x, p.y)
   0 0
   >>> print (p.magnitude ())
   0.0
   >>> p.x = 3
   >>> p.y = 4
   >>> print (p.magnitude ())
   5.0
   >>> q = Coordinate (3 ,4)
   >>> print (p == q)
   >>> True
   >>> q.translate (1, 2)
   >>> print (q.x)
   4
   >>> print (p == q)
   >>> False
   ```

2. *Clases and Methods: Make a Celsius class:* Implement a class named `Celsius` that represents a temperature in celsius. It has one attribute called `_temperature`. Note that by convention, an underscore prefix (like `_temperature`) designates a private attribute that should only be accessed within the class itself. Public attributes are those that are intended to be accessed anywhere. While Python does not prevent access to private attributes, it is a convention that is generally followed by Python programmers. The class has a property and the following methods:

   - `__init__(self,temperature)`: This method initializes the property `temperature` according to the argument `temperature`. If it is not specified, the property should be initialized to 0.

- `to_fahrenheit(self)`: This method returns the temperature in fahrenheit.

- `get_temperature(self)`: This method returns the temperature in celsius.

- `set_temperature(self, value)`: This method sets the attribute `_temperature` according to the argument `value`. If the `value` checked is less than -273, it will set the attribute to the minimum temperature, which is -273.

- `temperature`: This is a property that allows access to the `_temperature` attribute (See Python docs for property function). It functions similarly to a public attribute. Assign the above `get_temperature` and `set_temperature` as its getter and setter.

A sample interactive session using class `Coordinate` is shown below.

```
>>> c=Celsius()
>>> print(c.temperature)
0
>>> c.temperature=32
>>> c.to_fahrenheit()
89.6
>>> c.temperature=-300
>>> print(c.temperature)
-273
```

3. *Classes and Methods: Stopwatch:* Write a class named `StopWatch`. The class contains:

- Two instance attributes, i.e. `start_time` and `end_time`.

- During object instantiation, `start_time` should be initialized to the current time and `end_time` should be initialized to -1.

- A method named `start` that resets the `start_time` to the current time and `end_time` to -1.

- A method named `stop` that sets the `end_time` to the current time.

- A method named `elapsed_time` that returns the elapsed time for the stop watch in seconds as a float. Round the value of the elapsed time to one decimal place. If `end_time` is not valid (ie., it is -1), return `None`.

To test, run this script:

```
sw = StopWatch()
time.sleep(0.1)
sw.stop()
print(sw.elapsed_time())
sw.start()
time.sleep(0.2)
print(sw.elapsed_time())
sw.stop()
print(sw.elapsed_time())
```

It should output the following:

```
0.1
None
0.2
```

**Submission to Vocareum: Please submit your entire class with all the above methods implemented.**

4. *Classes and Methods:* Define a straight line class following the equation: $y = c_0 + c_1 x$. Make a class `Line` whose `__init__()` takes two parameters, $c_0$ and $c_1$. During object instantiation, the two parameters should initialize the coefficients $c_0$ and $c_1$ in the expression for the straight line: $y = c_0 + c_1 x$. The `__call__` method evaluates the function $y = c_0 + c_1 x$ as a float. The `table(L, R, n)` method samples the function at $n$ points for $L \leq x \leq R$ and creates a table of $x$ and $y$ values, with each value formatted to 2 decimal places in a field of width 10. Hint: Look up string formatting in Python docs to find out how to do the formatting required by the method `table(L,R,n)`. Refer to the following sample interactive session.

```
>>> line=Line(1,2)
>>> line(2)
5
>>>
>>> print(line.table(1,5,4))
      1.00         3.00
      2.33         5.67
      3.67         8.33
      5.00        11.00
```

For the below test cases, use the inputs in the same way they were used in the sample interactive session given above. Input 1 refers to the input $x$, $c_0$ and $c_1$ in `line(x)` and for instantiating the class LIne, output 1 refers tot he output after calling `line(x)`, input 2 refers to the input $L$, $R$, and $n$ in `line.table(L,R,n)` and output 2 refers to the output after printing `line.table(L,R,n)`.

**Test Cases:**

**Test case 1**

Input 1: x = 2, c0 = 1, c1 = 2

Output1: 5.0

Input 2: L = 1, R = 5, N = 4

Output 2:

| 1.00 | 3.00 |
|------|------|
| 2.33 | 5.67 |
| 3.67 | 8.33 |
| 5.00 | 11.00 |

**Test case 2**

Input 1: x = 2, c0 = -1, c1 = 2

Output 1: 3.0

Input 2: L = -1, R = 5, N = 10

Output 2:

| | |
|---|---|
| -1.00 | -3.00 |
| -0.33 | -1.67 |
| 0.33 | -0.33 |
| 1.00 | 1.00 |
| 1.67 | 2.33 |
| 2.33 | 3.67 |
| 3.00 | 5.00 |
| 3.67 | 6.33 |
| 4.33 | 7.67 |
| 5.00 | 9.00 |

**Test case 3**

Input 1: x = 2, c0 = 3, c1 = 4

Output 1: 11.0

Input 2: L = 1, R = 5, N = 15

Output 2:

| | |
|---|---|
| 1.00 | 7.00 |
| 1.29 | 8.14 |
| 1.57 | 9.29 |
| 1.86 | 10.43 |
| 2.14 | 11.57 |
| 2.43 | 12.71 |
| 2.71 | 13.86 |
| 3.00 | 15.00 |
| 3.29 | 16.14 |
| 3.57 | 17.29 |
| 3.86 | 18.43 |
| 4.14 | 19.57 |
| 4.43 | 20.71 |
| 4.71 | 21.86 |
| 5.00 | 23.00 |

**Test case 4**

Input 1: x = 2, c0 = 3, c1 = 4

Output 1: 11.0

Input 2: L = 1, R = 1, N = 15

Output 2:

1.00            7.00

**Test case 5**

Input 1: x = 2, c0 = 3, c1 = 4

Output 1: 11.0

Input 2: L = 1, R = 5, N = 0

Output 2:

Error in printing table

**Submission to Vocareum: Please submit your entire class with all the above methods implemented.**

**Problems: Homework**

1. *Classes and Methods: Time:* Write a class named `Time`. The class contains:

   - The instance attributes `_hours`, `_minutes`, and `_seconds` that represent a time.

   - During object instantiation of a `Time` object, it should initializes `_hours`, `_minutes`, and `_seconds` using the input parameters.

   - A property named `elapsed_time` with a getter and setter. The getter returns the total number of seconds that has elapsed since the time `00:00:00`, and the setter takes number of seconds elapsed and sets the `_hours`, `_minutes` and `_seconds` instance attributes accordingly.

   - Define the `__str__` method so that when you print a `Time` object you would get the following string representation: `''Time: H:M:S''`, where H is hour, M is minute, and S is seconds. For example, `''Time: 10:19:10''`

   Note that `_hours` can only go from 0 to 23, and `_minutes` and `_seconds` can only go from 0 to 59. You can assume that all our test cases will have valid values within those range. If the given number of elapsed seconds is so large that it overflows (ie. it is longer than one day), simply set the appropriate time in the different day. For example, if the elapsed time is 555550 seconds, `_hours` is 10, `_minutes` is 19, and `_seconds` is 10. An example of a test program is as follows:

   ```
   t = Time(10, 19, 10)
   print(t.elapsed_time)
   t.elapsed_time = 555550
   print(t.elapsed_time)
   print(t)
   ```

   It should output:

   ```
   37150
   37150
   Time: 10:19:10
   ```

   **Submission to Vocareum: Please submit your entire class with all the above methods implemented.**

2. *Classes and Methods: A bank account class:* Implement the concept of a bank account as a class named `Account`. The bank account has some data, namely the name of the account holder, the account number, and the current balance. The current balance can be a negative number. Three things we can do with an account is withdraw money, deposit money into the account, and print out the account information. These actions are modelled by methods inside the class. Implement the following methods:

`__init__(self, owner, account_number, amount), deposit(self, amount), withdraw(self, amou`

and `__str__(self)`. Name the attributes of your class as `_owner`, `_account_number`, and

`_balance`. Test the class can as follows:

```
>>> a1 = Account('John Olsson', '19371554951', 20000)
>>> a2 = Account('Liz Olsson', '19371564761', 20000)
>>> a1.deposit(1000)
>>> a1.withdraw(4000)
>>> a2.withdraw(10500)
>>> a1.withdraw(3500)
>>> print(a1)
John Olsson, 19371554951, balance: 13500
>>> print(a2)
Liz Olsson, 19371564761, balance: 9500
```

**Submission to Vocareum: Please submit your entire class with all the above methods implemented.**

3. *Classes and Methods: A class for numerical differentiation.* A widely used formula for numerical differentiation of a function $f(x)$ takes the form:

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}.$$

The goal of this exercise is to use the formula above to differentiate a mathematical function $f(x)$ implemented as a Python function `f(x)`.

Implement class `Diff` with two special methods. The `__init__()` method takes in a function object and also an optinal argument `h`. The default value of `h` is 1e-4. Implement also the special method `__call__` so that the instance is callable. This method returns the approximation of the derivative of the function using the formula above.

The following code shows you how this class is used.

```
def f(x):
    return 0.25*x**4

df = Diff(f)      # make function-like object df

# df(x) computes the derivative of f(x) approximately:
for x in range(1, 5, 10):
    df_value = df(x) # approx value of derivative of f at point x
    exact = x**3     # exact value of derivative
    print("f'({:d})={:g} (error={:.2E})" .format(x, df_value, exact-
        df_value))
```

**Test Cases:**

**Test case 1**

    Input:      x = 10.0, f = log, h = 0.1,

    Output:   (0.09950330853167877, 0.0004966914683212365) # derivative, approxi-

mation error

**Test case 2**

    Input:     x = 10.0, f = log, h = 0.5,

    Output:   (0.09758032833886343, 0.0024196716611365743)

**Test case 3**

    Input:     x = 10.0, f = log, h = 1.0E-5,

    Output:   (0.09999994996512383, 5.003487617283309e-08)

**Test case 4**

    Input:     x = 10.0, f = log, h = 1.0E-9,

    Output:   (0.1000000082740371, -8.274037094357922e-09)

**Test case 5**

    Input:     x = 10.0, f = log, h = 1.0E-11,

    Output:   (0.0999644811372491, 3.551886275091065e-05) **Submission to Vocareum:**

**Please submit your entire class with all the above methods implemented.**

4. *Polynomial class:* This exercise focuses on a class `Polynomial` for polynomials. The coefficients in the polynomial will be given as a list, and can be used as a parameter during object instantiation. Index number $i$ in this list represents the coefficients of the $x^i$ term in the polynomial. For example, writing `Polynomial([1,0,-1,2])` defines the polynomial:

$$1 + 0x - 1x^2 + 2x^3 = 1 - x^2 + 2x^3.$$

  (a) Polynomials can be added and subtracted(by adding/subtracting the coefficients). The class should implement the 'magic' `__add__` and `__sub__` methods.

  (b) Implement `__call__` so that we can evaluate the value of the polynomial expression given a certain value of $x$

  (c) Implement the `__mul__` method for polynomial multiplication. Let $p(x) = \sum_{i=0}^{M} c_i x^i$ and $q(x) = \sum_{j=0}^{N} d_j x^j$ be two polynomials. Their product is:

$$\sum_{i=0}^{M} \sum_{j=0}^{N} c_i d_j x^{i+j}.$$

  (d) Implement two different methods for differentiating the polynomial:

$$\frac{d}{dx} \sum_{i=0}^{n} c_i x^i = \sum_{i=1}^{n} i c_i x^{i-1}.$$

The first is `differentiate` which returns `None` but changes the coefficients of the polynomial instance on which it is called. The second is `derivative`, which returns a new `Polynomial` instance with coefficients corresponding to the derivative of `p`.

An interactive session for `Polynomial` is as follows:

```
>>> p1 = Polynomial([1, -1])
>>> p2 = Polynomial([0, 1, 0, 0, -6, -1])
>>> p3 = p1 + p2
>>> print(p3.coeff)
[1, 0, 0, 0, -6, -1]
>>> p4 = p1*p2
>>> print(p4.coeff)
[0, 1, -1, 0, -6, 5, 1]
>>> p5 = p2.derivative()
>>> print(p5.coeff)
[1, 0, 0, -24, -5]
>>> p = Polynomial([1, 2, 3])
>>> q = Polynomial([2, 3])
>>> r=p-q
>>> print(r.coeff)
[-1, -1, 3]
>>> r=q-p
>>> print(r.coeff)
[1, 1, -3]
>>>
```

**Test Cases:**

**Test case 1**
  Input:   [1, -1], [0, 1, 0, 0, -6, -1] # poly coeffs are added
  Output:  [1, 0, 0, 0, -6, -1]

**Test case 2**
  Input:   [1, -1], [0, 1, 0, 0, -6, -1], x = 3 # poly coeffs are subtracted and

evaluated at x = 3
  Output:  [1, -2, 0, 0, 6, 1, 724] # resultant poly coeff and evaluated value

**Test case 3**
  Input:   [1, 2, 3, 4], [1, 2, 3, 4] # multiplication
  Output:  [1, 4, 10, 20, 25, 24, 16]

**Test case 4**
  Input:   [1, 3, 5, 7, 9] # differentiation
  Output:  [3, 10, 21, 36]

**Test case 5**
  Input:   [2, 4, 6, 8, 10] # derivative - differentiation of polynomial copy
  Output:  [2, 4, 6, 8, 10]

**Submission to Vocareum: Please submit your entire class with all the above methods implemented.**

**Problems: Exercises**

1. *Classes and Methods: Make a function class:* Make a class named `F` that implements the function

$$f(x; a, w) = e^{-ax} sin(wx).$$

The ';' in `f(x;a,w)` separates teh variable of the function $x$ from constant parameters $a$ and $w$.

Class `F` implements `__call__` to compute the values of $f(x)$ for a given value of `x`; $a$ and $w$ are instance attributes. Test the class with the following program.

```
from math import *
f = F(a=1.0, w=0.1)
print(f(x=pi))
```

**Submission to Vocareum: Please submit your entire class with all the above methods implemented.**

**Test Cases:**

**Test case 1**

| | |
|---|---|
| Input: | a=1.0, w = 0.1, x=pi |
| Output: | 0.013353835137 |

**Test case 2**

| | |
|---|---|
| Input: | a = 3.0, w = 0.5, x=pi/2.0 |
| Output: | 0.00635214599841 |

**Test case 3**

| | |
|---|---|
| Input: | a = 5.0, w = 1.5 , x=pi/4.0 |
| Output: | 0.018203081084 |

**Test case 4**

| | |
|---|---|
| Input: | a = 5.0, w = 2.0, x=pi/6.0 |
| Output: | 11.8716456895 |

**Test case 5**

| | |
|---|---|
| Input: | a = 10.0, w = 3.0, x=pi/18.0 |
| Output: | 0.0872937481106 |

2. *Classes and Methods: Straight line class based on alternative definition:* Make a class `Line0` whose `__init__()` takes two points `p1` and `p2` (2- tuples or 2-lists) as input. The line passes through these two points. Implement `__call__` to take an `x` value and return the corresponding `y` value.

```
>>> line = Line0((0,-1), (2,4))
>>> print(line(0.5), line(0), line(1))
0.25 -1.0 1.5
```

**Test Cases:**

**Test case 1**
      Input:         p1 = (0,-1), p2 = (2,4), x = 0.5,
      Output:     0.25

**Test case 2**
      Input:         p1 = (0,-1), p2 = (2,4), x = 0,
      Output:     -1.0

**Test case 3**
      Input:         p1 = (0,-1), p2 = (2,4), x = 1,
      Output:     1.5

**Test case 4**
      Input:         p1 = (3,3), p2 = (8,8), x = -1,
      Output:     -1.0

**Test case 5**
      Input:         p1 = (3,3), p2 = (8,8), x = 4.3,
      Output:     4.3

**End of Problem Set 8.**