

Established in collaboration with MIT

SINGAPORE UNIVERSITY OF TECHNOLOGY AND DESIGN

01.112 Machine Learning

HW₂

Barry Tee Wei Cong 1001549

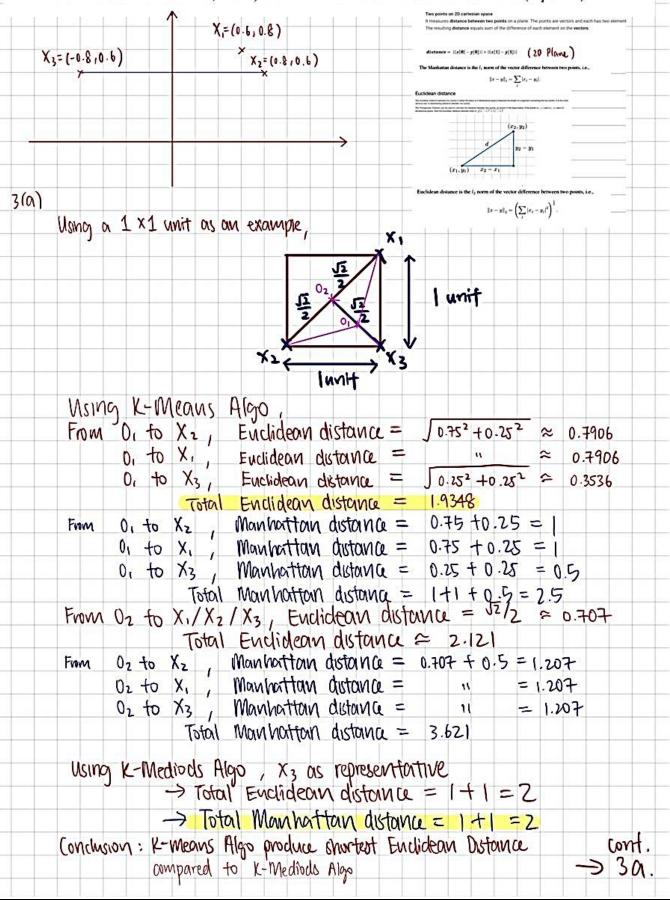
WRITTEN QUESTIONS Qn 1

affine function: y=Ax+c (does not need to fix é origin) (unlike linear functions)

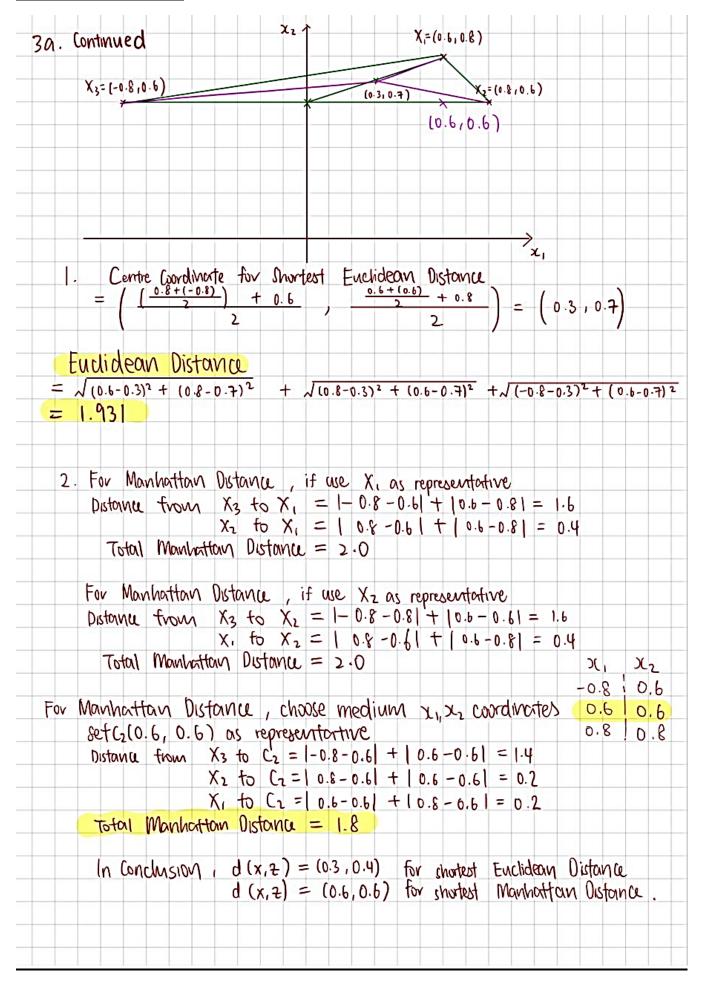
	(unlike linear functions)
Question 1. Assume a function $f: \mathbb{R}^n \to \mathbb{R}$ is continuous. As discussed in class, if it is a corner function, then it satisfies the following property:	
Property A: $f\left(\frac{x_1+x_2}{2}\right) \leq \frac{f(x_1)+f(x_2)}{2}$	
for any x_3 , $x_2 \in \mathbb{R}^n$. Show that the following statements are true (with formal peools):	
(a) If two functions f(x) and g(x) both satisfy Property A, then the following function also satisfies Property A:	
h(x) = f(x) + g(x) (5 points)	
(b) If two functions f(x) and g(x) both satisfy Property A, then the following function also satisfies Property A:	
$h(x) = \max(f(x), g(x))$ (5 points)	
a) dy dev-u (1)	d= x2-x1 (1)
u	
[u,v] = { u+ hd : he[o,1]} 129	[x1,x2] = {x1+ Ax2 : A 6[0,1] } (2)
Sate (1) into (2):	Sub (1) into (2)
7	
= { u+ \(v-u\) : \(\chi = [0.1]\)}	$= \{x_1 + \lambda(x_2 - x_1) : \lambda \in [0,1]\}$
= { (1-X)u + Xv : X €[0,1]}	= { (1-y)x1+ yx2 . y e[0,1]}
CERT is convex if Yx, yer, [x,y] CC	CERT is convex if Yziy ER, [x,y] EC
	1
()	
not couse's couse's	
Toko	(4(A) = (1 A) (A
Take x1, 22 EC 3 Dx, Dy E[0,1] &	$f(x) = (1 - \lambda x) x_1 + \lambda x_2$
York as	$g(x) = (1-\lambda_y)x_1 + \lambda_y x_2$
Proof	\$100 \$100 \$100 \$100 \$100 \$100 \$100 \$100
Take h(x) & [f(x), g(x)]. Then & A & CO.1] &	$t h(x) = (1-\lambda)x_1 + \lambda x_2$
	Prof. 1281
$h(x) = \alpha \left[f(x) \right] + \beta \left[g(x) \right]$	$(\alpha, \beta \geqslant 0)$ $(\alpha, \beta \geqslant 0)$ $(\alpha + \beta \Rightarrow 1)$
$= \alpha (1 - \lambda_x) x_1 + \alpha \lambda_x x_2 + \beta (1 - \lambda_y)$	$1x_1 + \beta \lambda_1 x_2$ $\alpha + \beta = 1$
$= (\alpha + \beta) x_i - (\alpha \lambda_x + \beta \lambda_y) x_i + (\alpha \lambda$	$\alpha \lambda_x + \beta \lambda_y \chi_2$, $\delta = \alpha \lambda_x + \beta \lambda_y$
$= 1 \cdot x_1 - \gamma x_1 +$	
$= (1-x)x_1 + x_2 \Rightarrow$	h(x) &C -> [x,x] &C
$= (1-\xi)x_1 + \delta x_2 \Rightarrow \frac{1}{n_{00}-v_{0}}$	WICK FC / LAIVAS TO
\rightarrow h(x) is also convex if f(x) and g(x) are low	
h(x) in line people of blu x and xz and hen	or in C where C is convex.
Aka intersection of 2 convex sets.	
$h((1-\lambda)x_1+\lambda x_2) = \max (f(1-\lambda x)x_1$	+ x=xx), 9((1-xu)x1+xux1))
	$(\lambda_x f(x_2)) + mox((1-\lambda_y)f(x_1), \lambda_y f(x_2))$
€ (1-λ) x, + λx2	
(a) Code, Input & Instructions Attached	
eters θ , θ_0) to use on the test set? (2 poi	
As shown in 2(a), mean squared emor thereafter. This means that the error increas ideal number of iterations for tuning the f tune, the weights based on the training	would decrease until \$5500 iteration and start to increase ses after that number of iterations. This means that is an parameters when setting the number of iterations to

Qn 3a

(a) Consider a set of points X = (0.6, 0.8), (0.8, 0.6), (-0.8, 0.6). Compute the value of z that minimizes $\sum_{x \in X} d(x, z)$ when d(x, z) is defined as follows respectively: 1) the Euclidean distance between x and z, and 2) the Manhattan distance between x and z. (5 points)



Qn 3a Continued



Qn 3b

(b) The following figures (points in the same cluster have the same color) are produced by the k-medoids algorithm for k=3 clusters using l_1 , l_2 , and l_∞ distance measures. Indicate which distance measure is used for each figure. (5 points)



It measures distance between two points on a plane. The points are vectors and each has two elements. The resulting distance equals sum of the difference of each element on the vectors.

distance = |(x(0) - y(0))| + |(x(1) - y(1))| (20 Plane)

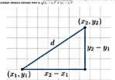
The Manhattan distance is the l_1 norm of the vector difference between two points, i.e.,

$$||x - y||_1 = \sum_i |x_j - y_i|.$$

Euclidean distance

X₁ (4, 4)

The Comment Comment of the Comment o



Euclidean distance is the l_2 norm of the vector difference between two points, i.e.,

$$||x - y||_2 = \left(\sum_j |x_j - y_j|^2\right)^{\frac{1}{2}}$$
.

The l_∞ distance is the maximum absolute element in the vector difference between two points, i.e.,

$$||x - y||_{\infty} = \max_{j} |x_j - y_j|.$$

• x, (-5, 2)

3(6)

For x3, Manhaffan Dist = 5+2=7

Euclidean Dist =
$$\int 5^2 + 2^2 = \int 25 + 4 = \int 29 \approx 5.39$$
 B.

 L_{∞} Dist = max (x1, x2) = max (5,2) = 5

Euclidean Dist = $\int 0^2 + 6^2 = \int 6^2 = \sqrt{36}$

x, (4, 4)

For x_i , Manhattan Dist = 4+4=8

Euclidean Dist = $\sqrt{4^2+4^2}$ = $\sqrt{16+16}$ = $\sqrt{32}$ ≈ 5.66

$$L_{\infty}$$
 Dist = max (x1, x2) = max (4,4) = 4

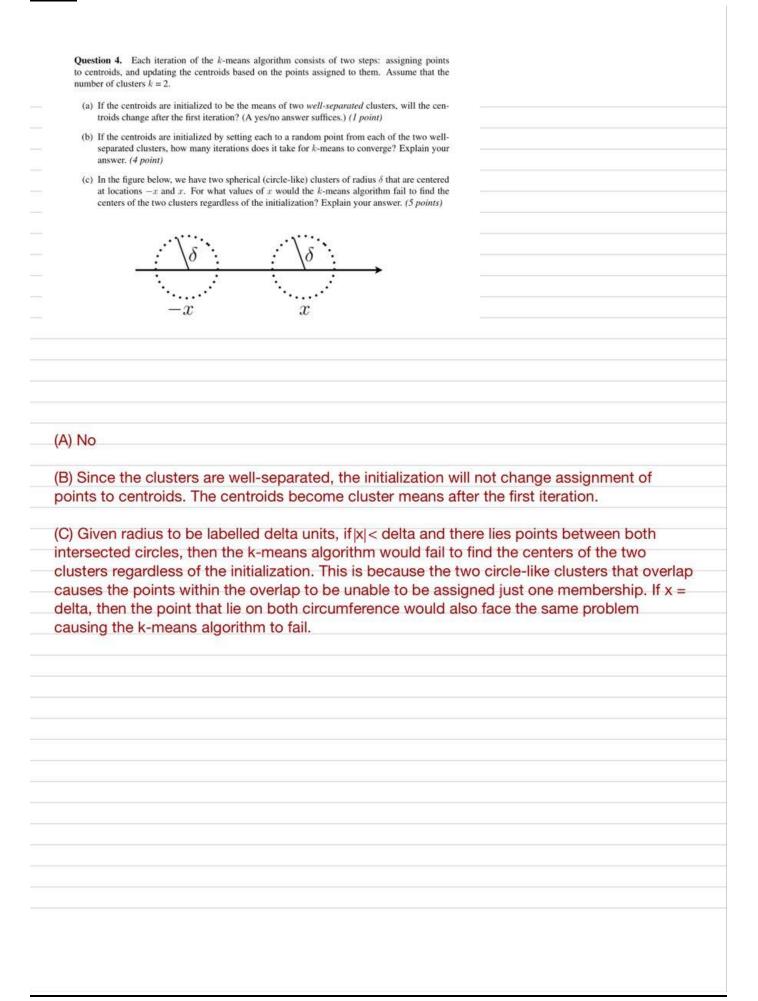
Rationale: Find the shortest distance from point to origin using the distance method. ie Euclidean Distance, $x_1 = 5.66$, $x_2 = 6$, $x_3 = 5.39$

Shortest Distance Hence B is labelled blue with ongon.

Distance Measure

A.

C.



CODING QUESTIONS Qn 2a

Instructions on How to Run the Code for Qn 2a

- 1) Download Anaconda. Highly recommended to download Anaconda's latest Python 3 version (currently Python 3.6).
- 2) Install the version of Anaconda which you downloaded, following the instructions on the download page. Please ensure that Anaconda installer is installed via *Run By Administrator*. This is to prevent an error in installing Jupyter Notebook and access the local directory.
- 3) Unzip the Folder named HW2_1001549_BarryTeeWeiCong
- 4) Open Jupyter Notebook and access the folder to open Homework2.2final.ipynb.
- 5) Press Play Button once to obtain the answer for Qn2a.
- 6) Feel free to call Barry Tee at +65 81393748 if there are any errors running the code.
- 7) Below is my code with the answers to each question

Qn 2a

```
import csv
     import os
 2
     import numpy as np
 3
      import pandas as pd
 4
 5
     from matplotlib import pyplot as plt
 6
 7
     cost train = []
     cost test = []
 8
     cost valid = []
 9
     cost_total = []
10
11
     theta = []
12
13
      def get_data_train(path): # path to read data from
14
        raw panda data = pd.read csv(path, header = None) #pandas
15
        raw panda data.insert(0, 'Ones', 1) # python, append a column of ones to the front of the data set for normalization
        num columns = raw panda data.shape[1]
                                                            # numpy, (num rows, num columns)
16
        panda X = raw panda data.iloc[:, 0:num columns-1]
                                                                 # pandas, [ slice of rows, slice of columns 0 to 21 ]
17
18
        panda_y = raw_panda_data.iloc[: , num_columns-1:num_columns] # pandas, [ slice_of_rows, slice_of_columns 21 to 22 ]
19
20
       X1 = np.matrix(panda X.values) # numpy, pandas.DataFrame -> numpy.ndarray -> numpy.matrix
21
        y1 = np.matrix(panda_y.values) # numpy, pandas.DataFrame -> numpy.ndarray -> numpy.matrix
22
        #print (X1)
23
        return X1, y1
24
25
      def get data test(path): # path to read data from
        raw panda data = pd.read csv(path, header = None) #pandas
26
27
        raw_panda_data.insert(0, 'Ones', 1) # python, append a column of ones to the front of the data set for normalization
28
        num columns = raw panda data.shape[1]
                                                            # numpy, (num rows, num columns)
29
        panda X = raw panda data.iloc[:,0:num columns-1]
                                                                # pandas, [ slice of rows, slice of columns 0 to 21 ]
```

```
30
        panda y = raw panda data.iloc[:,num columns-1:num columns] # pandas, [ slice of rows, slice of columns 21 to 22 ]
31
       X2 = np.matrix(panda X.values) # numpy, pandas.DataFrame -> numpy.ndarray -> numpy.matrix
32
33
       y2 = np.matrix(panda y.values) # numpy, pandas.DataFrame -> numpy.ndarray -> numpy.matrix
34
        return X2, y2
35
36
     def get data valid(path): # path to read data from
37
        raw panda data = pd.read csv(path, header = None) #pandas
38
        raw panda data.insert(0, 'Ones', 1) # python, append a column of ones to the front of the data set for normalization
        num columns = raw panda data.shape[1]
39
                                                            # numpy, (num_rows, num_columns)
        panda X = raw panda data.iloc[:,0:num columns-1]
                                                                # pandas, [ slice of rows, slice of columns 0 to 21 ]
40
        panda y = raw panda data.iloc[:,num columns-1:num columns] # pandas, [ slice of rows, slice of columns 21 to 22 ]
41
42
43
       X3 = np.matrix(panda X.values) # numpy, pandas.DataFrame -> numpy.ndarray -> numpy.matrix
        y3 = np.matrix(panda y.values) # numpy, pandas.DataFrame -> numpy.ndarray -> numpy.matrix
44
45
        return X3. v3
46
47
     def compute mean square error(X, y, theta):
48
        summands = np.dot(X * theta.T - y, 2) #numpy, (X*T -y)^2
49
        mse = np.sum(summands) / (2 * len(X))
50
        return mse
51
52
     def gradient descent(X1, y1, X2, y2, X3, y3, learning rate, num iterations):
53
        num sample = num iterations / 100
54
        num parameters = X1.shape[1]
                                                     # numpy, dim theta = 21 columns
55
        theta = np.matrix([0.0 for i1 in range(num parameters)]) # numpy, init theta matrix with 21 columns
56
57
        for i3 in range(num iterations):
58
          error_train = np.repeat((X1 * theta.T) - y1, num_parameters, axis=1)
59
          error derivative train = np.sum(np.multiply(error train, X1), axis=0)
60
          if i3 % 100 == 0:
```

```
if i3 == 0:
61
62
               print (")
63
             if i3 > 0:
64
               #print(i3)
65
               theta = theta - (learning rate / len(y1)) * error derivative train
66
               print (i3 , 'iteration, weights =' , theta )
67
               cost = [0.0 for i2 in range(num_iterations)]
               cost[i3] = abs(compute_mean_square_error(X1, y1, theta))
68
69
               cost train.append(cost[i3])
70
               cost[i3] = abs(compute_mean_square_error(X2, y2, theta))
71
               cost test.append(cost[i3])
72
               cost[i3] = abs(compute_mean_square_error(X3, y3, theta))
               cost_valid.append(cost[i3])
73
        cost total.append(cost train)
74
        cost_total.append(cost_test)
75
76
        cost_total.append(cost_valid)
77
        #print(cost_total)
78
        return theta, cost total
79
80
      def plot_line(cost_ref):
81
        x values = [i4 for i4 in range(len(cost train))] # plotting from 0 to 99
82
        #print(len(cost_train))
83
        #y1 values = cost total[0]
84
        #y2_values = cost_total[1]
85
        #y3_values = cost_total[2]
86
        #plt.plot(x_values, y1_values, 'b')
87
        #plt.plot(x values, y2 values, 'r')
88
        #plt.plot(x_values, y3_values, 'g')
89
90
        for i5 in range(len(cost total)):
91
          #print (i5)
```

```
92
            plt.plot( x_values, cost_total[i5] )
          plt.title("Graph for Training, Testing and Validation Set")
 93
          plt.xlabel("No. of Iterations")
 94
          plt.ylabel("Mean Square Errors")
 95
 96
          plt.legend(('error_train','error_test','error_valid'))
 97
          plt.show()
       X1, y1 = get_data_train(os.getcwd() + '/train_warfarin.csv')
 98
 99
       X2, y2 = get_data_test(os.getcwd() + '/test_warfarin.csv')
       X3, y3 = get_data_valid(os.getcwd() + '/validation_warfarin.csv')
100
101
       theta, cost = gradient_descent(X1, y1, X2, y2, X3, y3, 0.00015, 10000)
```

102

plot_line(cost_total)

Output for Python Code for Qn2a

```
100 iteration, weights = [[ 7.20000000e-05 1.24270242e-03 3.80250000e-
05 8.25750000e-05
          5.76300000e-05 1.95000000e-05
                                                                                                 2.55000000e-05
                                                                                                                                        1.05000000e-05
          1.65000000e-05 1.35000000e-05 1.05000000e-05 1.05000000e-05
          7.50000000e-06 1.05000000e-05 1.20000000e-05 7.50000000e-06
         1.95000000e-05 2.10000000e-05 1.20000000e-05 1.95000000e-05
          3.60000000e-05]]
200 iteration, weights = [[ 1.41292363e-04 2.44185514e-03 7.45780688e-
05 1.62029734e-04
                                                                                                 5.01977545e-05 2.04284515e-05
         1.13110794e-04
                                                 3.84299436e-05
         3.22362134e-05 2.66264718e-05 2.04289866e-05 2.07421590e-05
                                                2.05315157e-05
         1.46761522e-05
                                                                                                2.36112556e-05 1.46758222e-05
          3.84616228e-05 4.11128717e-05 2.34325146e-05 3.82853541e-05
          7.03804399e-05]]
300 iteration, weights = \begin{bmatrix} 2.07972118e-04 & 3.59898645e-03 & 1.09710866e- \end{bmatrix}
04 2.38473719e-04
         1.66517815e-04
                                                  5.68098368e-05
                                                                                                7.41214125e-05
                                                                                                                                         2.98054221e-05
         4.72354470e-05 3.93925201e-05 2.98070044e-05 3.07355299e-05
         2.15398251e-05 3.01109898e-05 3.48474070e-05 2.15388422e-05
          5.69037602e-05 6.03697510e-05 3.43174664e-05 5.63811408e-05
          1.03198162e-04]]
400 iteration, weights = \begin{bmatrix} 2.72130960e-04 & 4.71557100e-03 & 1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473286-1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473238e-1.43473286-1.43473286-1.43473286-1.43473286-1.43473286-1.43473286-1.43473286-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.43486-1.4386-1.43866-1.43866-1.43866-1.4386-1.43866-1.43866-1.438
04 3.12012628e-04
         2.17923849e-04 7.46589833e-05
                                                                                                9.72981348e-05
                                                                                                                                         3.86502751e-05
         6.15235668e-05 5.18107897e-05
                                                                                                3.86533949e-05 4.04888480e-05
         2.81019879e-05 3.92542878e-05
                                                                                                4.57216155e-05 2.81000363e-05
          7.48446412e-05 7.88006810e-05
                                                                                               4.46740789e-05 7.38115589e-05
          1.34508013e-04]]
500 iteration, weights = [[ 3.33857364e-04 5.79303169e-03 1.75913283e-
04 3.82748426e-04
          2.67399129e-04 9.19960095e-05 1.19754129e-04 4.69816942e-05
          7.51255311e-05 6.38934818e-05 4.69868205e-05 5.00105418e-05
         3.43732251e-05 4.79767187e-05 5.62465808e-05 3.43699956e-05
          9.23018550e-05 9.64346505e-05 5.45209011e-05 9.05999577e-05
          1.64362915e-04]]
600 iteration, weights = \begin{bmatrix} 3.93236703e-04 & 6.83274152e-03 & 2.07077409e-14 & 1.83274152e-14 & 1.83274164e-14 & 1.83274164e-14 & 1.8327416e-14 & 1.832746e-14 & 1.832746
04 4.50779501e-04
         3.15011423e-04 1.08838888e-04
                                                                                                1.41514685e-04
                                                                                                                                         5.48177076e-05
         8.80654223e-05
                                                    7.56523693e-05
                                                                                                 5.48252889e-05
                                                                                                                                            5.93087445e-05
         4.03637497e-05 5.62930540e-05 6.64345568e-05 4.03589395e-05
         1.09292374e-04 1.13299631e-04 6.38758310e-05 1.06768867e-04
          1.92813935e-04]]
700 iteration, weights = [[4.50351351e-04 7.83602525e-03 2.37010398e-
        5.16200786e-04
         3.60826118e-04 1.25204961e-04 1.62604201e-04 6.21757107e-05
         1.00366478e-04 8.70988120e-05 6.21861758e-05 6.83913033e-05
```

```
4.60834164e-05 6.42175470e-05 7.62973673e-05 4.60767290e-05
   1.25832573e-04 1.29422613e-04 7.27561383e-05 1.22340026e-04
   2.19910345e-0411
800 iteration, weights = [[ 5.05280793e-04 8.80416120e-03 2.65755459e-
04 5.79103884e-04
   4.04906311e-04 1.41110962e-04 1.83046223e-04 6.90724886e-05
   1.12051119e-04 9.82437709e-05 6.90862470e-05 7.72657905e-05
   5.15417338e-05 7.17639508e-05 8.58464214e-05 5.15328787e-05
   1.41938256e-04 1.44829638e-04 8.11784869e-05 1.37334412e-04
   2.45699689e-04]]
900 iteration, weights = [[ 5.58101725e-04 9.73838281e-03 2.93354285e-
   6.39577183e-04
   4.47312881e-04 1.56573037e-04
                                  2.02863469e-04 7.55242371e-05
   1.23140982e-04 1.09097823e-04 7.55416802e-05 8.59395123e-05
   5.67478771e-05 7.89455356e-05 9.50927275e-05 5.67365695e-05
   1.57624668e-04 1.59545836e-04 8.91589559e-05 1.51772265e-04
   2.70227843e-04]]
1000 iteration, weights = [[6.08888150e-04 1.06398802e-02 3.19847105e]
-04 6.97705970e-04
   4.88104571e-04 1.71606766e-04 2.22077859e-04 8.15465838e-05
   1.33656941e-04 1.19671173e-04 8.15680852e-05 9.44195188e-05
   6.17106993e-05 8.57751063e-05 1.04046908e-04 6.16966602e-05
   1.72906522e-04 1.73595454e-04 9.67130600e-05 1.65673114e-04
   2.93539072e-0411
1100 iteration, weights = [[ 6.57711476e-04 1.15098018e-02 3.45272739e
-04 7.53572537e-04
   5.27338066e-04 1.86227182e-04
                                  2.40710546e-04 8.71546079e-05
   1.43619141e-04 1.29973668e-04 8.71805240e-05 1.02712613e-04
   6.64387428e-05 9.22650182e-05 1.12719212e-04 6.64216985e-05
   1.87798015e-04 1.87001887e-04 1.03855770e-04 1.79055804e-04
   3.15676089e-04]]
1200 iteration, weights = [[ 7.04640603e-04 1.23492557e-02 3.69668641e
-04 8.07256290e-04
   5.65068056e-04 2.00448789e-04
                                  2.58781938e-04 9.23628595e-05
   1.53047016e-04 1.40014810e-04 9.23935300e-05 1.10825357e-04
   7.09402504e-05 9.84271930e-05 1.21119530e-04 7.09199323e-05
   2.02312842e-04 1.99787712e-04 1.10601529e-04 1.91938520e-04
   3.36680108e-04]]
1300 iteration, weights = \begin{bmatrix} 7.49742014e-04 & 1.31593111e-02 & 3.93070953e \end{bmatrix}
-04 8.58833847e-04
                 2.14285584e-04 2.76311730e-04
                                                 9.71853778e-05
   6.01347318e-04
   1.61959323e-04 1.49803767e-04 9.72211268e-05 1.18764087e-04
   7.52231753e-05 \qquad 1.04273134e-04 \qquad 1.29257404e-04 \qquad 7.51993200e-05
   2.16464222e-04 2.11974709e-04 1.16964278e-04 2.04338805e-04
   3.56590895e-04]]
-04 9.08379139e-04
```

```
6.36226771e-04 2.27751072e-04 2.93318922e-04 1.01635710e-04
      1.70374155e-04 1.59349385e-04 1.01676845e-04 1.26534912e-04
      7.92951918e-05 1.09813941e-04 1.37142043e-04 7.92675407e-05
      2.30264907e-04 2.23583899e-04 1.22957463e-04 2.16273589e-04
      3.75446821e-04]]
1500 iteration, weights = \begin{bmatrix} 8.34716037e-04 & 1.46953169e-02 & 4.37033077e & 4.370307e & 4.3707e 
-04 9.55963501e-04
      6.69755552e-04 2.40858285e-04 3.09821850e-04 1.05726926e-04
      1.78308975e-04 1.68660201e-04 1.05773743e-04 1.34143729e-04
      8.31637048e-05 1.15060325e-04 1.44782330e-04 8.31320041e-05
      2.43727203e-04 2.34635562e-04 1.28594063e-04 2.27759208e-04
       3.93284913e-04]]
1600 iteration, weights = [[ 8.74710274e-04 1.54232233e-02 4.57659005e
-04 1.00165576e-03
     7.01981071e-04 2.53619795e-04
                                                                3.25838207e-04 1.09471640e-04
      1.85780631e-04 1.77744451e-04 1.09524417e-04 1.41596229e-04
      8.68358595e-05 1.20022619e-04 1.52186839e-04 8.67998597e-05
      2.56862984e-04 2.45149267e-04 1.33886599e-04 2.38811424e-04
       4.10140895e-04]]
1700 iteration, weights = [[9.13120200e-04 1.61256458e-02 4.77423663e]
-04 1.04552235e-03
      7.32949074e-04 2.66047737e-04 3.41385062e-04 1.12882022e-04
      1.92805380e-04 1.86610082e-04 1.12941023e-04 1.48897901e-04
      9.03185502e-05 1.24710795e-04 1.59363842e-04 9.02780062e-05
      2.69683706e-04 2.55143896e-04 1.38847152e-04 2.49445446e-04
      4.26049239e-0411
1800 iteration, weights = [[ 9.50001425e-04 1.68034785e-02 4.96357281e
-04 1.08762733e-03
      7.62703702e-04 2.78153816e-04 3.56478888e-04 1.15969813e-04
      1.99398908e-04 1.95264763e-04 1.16035293e-04 1.56054044e-04
      9.36184292e-05 1.29134475e-04 1.66321321e-04 9.35731002e-05
      2.82200425e-04 2.64637669e-04 1.43487382e-04 2.59675949e-04
       4.41043208e-04]]
1900 iteration, weights = [[ 9.85407603e-04 1.74575846e-02 5.14489027e
-04 1.12803256e-03
       7.91287549e-04 2.89949328e-04 3.71135578e-04 1.18746346e-04
      2.05576351e-04 2.03715893e-04 1.18818542e-04 1.63069768e-04
      9.67419156e-05 1.33302943e-04 1.73066977e-04 9.66915649e-05
      2.94423807e-04 2.73648166e-04 1.47818537e-04 2.69517094e-04
      4.55154896e-04]]
2000 iteration, weights = [[ 1.01939051e-03 1.80887965e-02 5.31847045e
-04 1.16679770e-03
      8.18741712e-04 3.01445174e-04 3.85370469e-04 1.21222553e-04
      2.11352314e-04 2.11970611e-04 1.21301693e-04 1.69950006e-04
      9.96952030e-05 1.37225158e-04 1.79608241e-04 9.96395976e-05
      3.06364146e-04 2.82192351e-04 1.51851471e-04 2.78982543e-04
      4.68415272e-04]]
```

```
2100 iteration, weights = [[0.001052 \quad 0.01869792 \quad 0.00054846 \quad 0.00120398]
0.00084511 0.00031265
   0.0003992 0.00012341 0.00021674 0.00022004 0.0001235 0.0001767
   0.00010248 \quad 0.00014091 \quad 0.00018595 \quad 0.00010242 \quad 0.00031803 \quad 0.00029029
   0.0001556 0.00028809 0.00048085]]
2200 iteration, weights = [[ 0.00108328  0.01928572  0.00056435  0.00123964
0.00087042 0.00032358
   0.00041263 0.00012532 0.00022176 0.00022792 0.00012541 0.00018332
   0.00010511 \quad 0.00014437 \quad 0.00019211 \quad 0.00010505 \quad 0.00032944 \quad 0.00029795
   0.00015906 0.00029684 0.0004925 ]]
2300 iteration, weights = [[0.00111329 0.01985296 0.00057955 0.00127382]
0.00089472 0.00033424
   0.00042569 0.00012695 0.00022641 0.00023562 0.00012705 0.00018982
   0.00010759 0.0001476 0.00019808 0.00010752 0.00034058 0.00030519
   0.00016226 0.00030525 0.0005033811
2400 iteration, weights = [[ 0.00114206  0.02040035  0.00059407  0.00130658
0.00091803 0.00034464
   0.00043838 0.00012833 0.00023072 0.00024316 0.00012844 0.00019621
   0.00010992 \quad 0.00015062 \quad 0.00020387 \quad 0.00010984 \quad 0.00035149 \quad 0.00031202
   0.00016521 0.00031334 0.00051353]]
2500 iteration, weights = [[ 0.00116964  0.0209286  0.00060795  0.00133797
0.00094041 0.00035478
   0.00045072 \quad 0.00012946 \quad 0.00023469 \quad 0.00025053 \quad 0.00012957 \quad 0.00020248
   0.00011211 \quad 0.00015343 \quad 0.00020949 \quad 0.00011203 \quad 0.00036216 \quad 0.00031847
   0.0001679 0.00032112 0.00052296]]
2600 iteration, weights = [[ 0.00119608  0.02143836  0.0006212  0.00136803
0.00096187 0.00036469
   0.00046272 \quad 0.00013034 \quad 0.00023833 \quad 0.00025774 \quad 0.00013046 \quad 0.00020864
   0.00011416 \quad 0.00015605 \quad 0.00021495 \quad 0.00011407 \quad 0.00037259 \quad 0.00032454
   0.00017035 0.00032859 0.0005317 11
0.00098245 0.00037436
   0.00047438 \quad 0.00013099 \quad 0.00024167 \quad 0.0002648 \quad 0.00013112 \quad 0.0002147
   0.00011608 \quad 0.00015847 \quad 0.00022025 \quad 0.00011598 \quad 0.00038281 \quad 0.00033025
   0.00017257 0.00033577 0.0005397811
2800 iteration, weights = [[0.00124566 \ 0.02240504 \ 0.00064592 \ 0.00142438]
0.00100218 0.00038381
   0.00048574 \quad 0.00013141 \quad 0.0002447 \quad \quad 0.00027172 \quad 0.00013155 \quad 0.00022065
   0.00011787 \quad 0.00016071 \quad 0.00022539 \quad 0.00011776 \quad 0.00039282 \quad 0.00033561
   0.00017456 0.00034267 0.00054723]]
2900 iteration, weights = [[0.00126888 \ 0.02286317 \ 0.00065743 \ 0.00145074]
0.0010211 0.00039304
   0.00049678 \quad 0.00013162 \quad 0.00024744 \quad 0.00027849 \quad 0.00013177 \quad 0.00022651
   0.00011954 \quad 0.00016277 \quad 0.00023039 \quad 0.00011942 \quad 0.00040262 \quad 0.00034063
   0.00017634 0.0003493 0.00055405]]
3000 iteration, weights = [[ 0.00129111  0.02330529  0.0006684  0.00147596
0.00103922 0.00040206
```

```
0.00050753 \quad 0.00013161 \quad 0.0002499 \quad 0.00028512 \quad 0.00013177 \quad 0.00023227
   0.00017791 0.00035566 0.0005602811
3100 iteration, weights = [[ 0.00131238  0.02373195  0.00067885  0.00150008
0.00105658 0.00041088
   0.000518
               0.00013141 0.00025209 0.00029162 0.00013158 0.00023794
   0.00012252 \quad 0.00016639 \quad 0.00023995 \quad 0.00012239 \quad 0.00042163 \quad 0.0003497
   0.00017928 0.00036177 0.0005659311
3200 iteration, weights = [[ 0.00133272  0.0241437  0.00068879  0.00152312
0.00107321 0.0004195
   0.00052819 \quad 0.00013101 \quad 0.00025402 \quad 0.00029799 \quad 0.00013118 \quad 0.00024352
   0.00012384 \quad 0.00016795 \quad 0.00024454 \quad 0.0001237 \quad \quad 0.00043086 \quad 0.00035377
   0.00018045 0.00036763 0.00057103]]
3300 iteration, weights = [[0.00135216 \ 0.02454106 \ 0.00069824 \ 0.00154513]
0.00108913 0.00042794
   0.00053812 \quad 0.00013042 \quad 0.00025569 \quad 0.00030424 \quad 0.0001306 \quad 0.00024902
   0.00012505 \quad 0.00016935 \quad 0.00024899 \quad 0.00012491 \quad 0.00043991 \quad 0.00035755
   0.00018144 0.00037326 0.0005756 ]]
3400 iteration, weights = [[0.00137074 0.02492454 0.00070723 0.00156615]
0.00110436 0.00043619
   0.00054779 \quad 0.00012964 \quad 0.00025712 \quad 0.00031037 \quad 0.00012984 \quad 0.00025443
   0.00012617 \quad 0.00017061 \quad 0.00025332 \quad 0.00012601 \quad 0.00044879 \quad 0.00036105
   0.00018225 0.00037866 0.0005796511
3500 iteration, weights = [[0.00138849 \ 0.02529462 \ 0.00071576 \ 0.00158621
0.00111893 0.00044427
   0.00055721 0.00012869 0.00025832 0.00031638 0.0001289 0.00025977
   0.00012718 \quad 0.00017173 \quad 0.00025753 \quad 0.00012702 \quad 0.0004575 \quad 0.00036428
   0.00018288 0.00038384 0.0005832 ]]
3600 iteration, weights = [[0.00140544 \ 0.02565178 \ 0.00072386 \ 0.00160535]
0.00113286 0.00045218
   0.00056639 \quad 0.00012758 \quad 0.00025929 \quad 0.00032228 \quad 0.00012779 \quad 0.00026503
   0.00012809 \quad 0.0001727 \quad 0.00026162 \quad 0.00012793 \quad 0.00046605 \quad 0.00036724
   0.00018334 0.00038881 0.00058627]]
3700 iteration, weights = [[0.00142161 \ 0.02599646 \ 0.00073153 \ 0.00162359]
0.00114618 0.00045993
   0.00057535 0.0001263 0.00026004 0.00032808 0.00012652 0.00027021
   0.00012892 \quad 0.00017354 \quad 0.00026561 \quad 0.00012874 \quad 0.00047445 \quad 0.00036995
   0.00018365 0.00039357 0.00058888]]
3800 iteration, weights = [[ 0.00143704  0.0263291  0.0007388  0.00164097
0.0011589 0.00046751
   0.00058408 \quad 0.00012486 \quad 0.00026058 \quad 0.00033377 \quad 0.00012509 \quad 0.00027533
   0.00012965 0.00017425 0.00026948 0.00012947 0.0004827 0.00037241
   0.00018379 0.00039814 0.00059104]]
3900 iteration, weights = [[0.00145174 \ 0.02665013 \ 0.00074568 \ 0.00165752]
0.00117105 0.00047495
   0.0005926 \qquad 0.00012327 \quad 0.00026092 \quad 0.00033937 \quad 0.00012351 \quad 0.00028037
   0.0001303 0.00017484 0.00027325 0.0001301 0.0004908 0.00037464
```

```
0.00018378 0.00040251 0.00059276]]
4000 iteration, weights = [[0.00146574 \ 0.02695996 \ 0.00075218 \ 0.00167326]
0.00118265 0.00048224
   0.00060092 \quad 0.00012153 \quad 0.00026106 \quad 0.00034486 \quad 0.00012178 \quad 0.00028535
   0.00013086 0.0001753 0.00027692 0.00013066 0.00049877 0.00037664
   0.00018363 0.0004067 0.00059407]]
4100 iteration, weights = [[ 0.00147907  0.02725897  0.00075831  0.00168823
0.00119371 0.00048939
   0.00060903 \quad 0.00011965 \quad 0.00026101 \quad 0.00035027 \quad 0.00011991 \quad 0.00029027
   0.00013135 0.00017565 0.0002805 0.00013113 0.0005066 0.00037842
   0.00018334 0.00041072 0.00059498]]
4200 iteration, weights = [[0.00149176 \ 0.02754755 \ 0.00076409 \ 0.00170245]
0.00120426 0.0004964
   0.00061695 0.00011763 0.00026078 0.00035558 0.0001179 0.00029512
   0.00013176 0.00017589 0.00028398 0.00013153 0.00051431 0.00037998
   0.00018291 0.00041456 0.0005955 ]]
4300 iteration, weights = [[0.00150382 \ 0.02782607 \ 0.00076953 \ 0.00171596]
0.00121432 0.00050328
   0.00062469 0.00011548 0.00026037 0.00036081 0.00011576 0.00029991
   0.00013209 0.00017602 0.00028737 0.00013185 0.00052189 0.00038134
   0.00018235 0.00041824 0.00059565]]
4400 iteration, weights = [[ 0.00151527  0.02809487  0.00077464  0.00172876
0.00122389 0.00051003
   0.00063224 0.00011321 0.00025979 0.00036595 0.0001135 0.00030465
   0.00013235 \quad 0.00017604 \quad 0.00029068 \quad 0.0001321 \quad 0.00052935 \quad 0.00038251
   0.00018167 0.00042175 0.00059544]]
4500 iteration, weights = [[ 0.00152615  0.0283543  0.00077943  0.0017409
0.001233 0.00051666
   0.00063963 \quad 0.00011081 \quad 0.00025905 \quad 0.00037102 \quad 0.00011111 \quad 0.00030933
   0.00013254 0.00017597 0.0002939 0.00013228 0.00053669 0.00038348
   0.00018086 0.00042512 0.00059487]]
4600 iteration, weights = [[0.00153646 \ 0.02860468 \ 0.00078392 \ 0.00175239]
0.00124167 0.00052317
   0.00064685 0.00010829 0.00025815 0.000376
                                                  0.0001086 0.00031396
   0.00013266 0.00017579 0.00029704 0.0001324 0.00054392 0.00038427
   0.00017993 0.00042834 0.00059398]]
4700 iteration, weights = [[ 0.00154622  0.02884634  0.00078812  0.00176325
0.0012499 0.00052957
   0.0006539 0.00010566 0.00025709 0.00038092 0.00010598 0.00031853
   0.00013272 \quad 0.00017553 \quad 0.0003001 \quad 0.00013244 \quad 0.00055104 \quad 0.00038488
   0.00017889 0.00043141 0.00059275]]
4800 iteration, weights = [[0.00155547 \ 0.02907958 \ 0.00079202 \ 0.00177351]
0.00125772 0.00053585
   0.0006608 \quad 0.00010292 \quad 0.00025589 \quad 0.00038576 \quad 0.00010325 \quad 0.00032306
   0.00013271 0.00017517 0.00030309 0.00013243 0.00055806 0.00038532
   0.00017774 0.00043434 0.00059122]]
```

```
4900 iteration, weights = [[ 0.00156421  0.02930469  0.00079566  0.00178319
0.00126514 0.00054203
   0.00066755 \quad 0.00010007 \quad 0.00025455 \quad 0.00039053 \quad 0.00010041 \quad 0.00032754
   0.00013265 \quad 0.00017472 \quad 0.00030601 \quad 0.00013235 \quad 0.00056498 \quad 0.00038559
   0.00017649 0.00043714 0.00058938]]
5000 iteration, weights = \begin{bmatrix} 1.57246093e-03 & 2.95219643e-02 & 7.99030208e \end{bmatrix}
-04 1.79230455e-03
   1.27217470e-03 5.48112793e-04 6.74161959e-04 9.71213751e-05
   2.53064805e-04 3.95227788e-04 9.74674712e-05 3.31971213e-04
   1.32527587e-04 1.74194030e-04 3.08851972e-04 1.32220870e-04
   5.71805682e-04 3.85706110e-04 1.75133070e-04 4.39816070e-04
    5.87248676e-0411
5100 iteration, weights = \begin{bmatrix} 1.58024357e-03 & 2.97316736e-02 & 8.02144970e \end{bmatrix}
-04 1.80088021e-03
   1.27883439e-03 5.54091798e-04 6.80629559e-04 9.40719402e-05
   2.51450277e-04 3.99865136e-04 9.44277161e-05 3.36358632e-04
   1.32348618e-04 1.73583237e-04 3.11629451e-04 1.32030785e-04
   5.78534417e-04 3.85666399e-04 1.73678299e-04 4.42364458e-04
    5.84838006e-0411
5200 iteration, weights = \begin{bmatrix} 1.58757265e-03 & 2.99340840e-02 & 8.05013155e \end{bmatrix}
-04 1.80893331e-03
   1.28513413e-03 5.59975282e-04 6.86962599e-04 9.09269544e-05
   2.49707814e-04 4.04439797e-04 9.12924047e-05 3.40702943e-04
   1.32115455e-04 1.72893975e-04 3.14341727e-04 1.31786348e-04
   5.85172887e-04 3.85478096e-04 1.72128604e-04 4.44793062e-04
    5.82156058e-0411
5300 iteration, weights = [[1.59446408e-03 3.01294515e-02 8.07643421e]
-04 1.81648222e-03
   1.29108658e-03 5.65766599e-04 6.93165797e-04 8.76897811e-05
   2.47841908e-04 4.08953968e-04 8.80648963e-05 3.45005664e-04
   1.31830004e-04 1.72129002e-04 3.16991085e-04 1.31489465e-04
   5.91724258e-04 3.85146418e-04 1.70487324e-04 4.47106084e-04
    5.79212358e-04]]
5400 iteration, weights = [[1.60093325e-03 3.03180234e-02 8.10044122e]
-04 1.82354465e-03
   1.29670394e-03 5.71468982e-04 6.99243705e-04 8.43636656e-05
   2.45856894e-04 4.13409768e-04 8.47484326e-05 3.49268258e-04
   1.31494104e-04 1.71290975e-04 3.19579733e-04 1.31141976e-04
   5.98191586e-04 3.84676400e-04 1.68757681e-04 4.49307580e-04
   5.76016099e-04]]
5500 iteration, weights = [[1.60699497e-03 3.05000381e-02 8.12223319e]
-04 1.83013769e-03
   1.30199797e-03 5.77085555e-04 7.05200716e-04 8.09517391e-05
   2.43756956e-04 4.17809243e-04 8.13461413e-05 3.53492139e-04
   1.31109529e-04 1.70382461e-04 3.22109798e-04 1.30745655e-04
   6.04577818e-04 3.84072902e-04 1.66942783e-04 4.51401464e-04
   5.72576149e-04]]
```

```
5600 iteration, weights = [[ 1.61266355e-03 3.06757257e-02 8.14188790e
-04 1.83627782e-03
   1.30698003e-03 5.82619330e-04
                                    7.11041068e-04
                                                   7.74570232e-05
   2.41546130e-04 4.22154365e-04 7.78610406e-05
                                                   3.57678669e-04
   1.30677990e-04 1.69405936e-04 3.24583334e-04 1.30302216e-04
   6.10885798e-04 3.83340611e-04 1.65045631e-04 4.53391510e-04
    5.68901069e-04]]
5700 iteration, weights = \begin{bmatrix} 1.61795280e-03 & 3.08453083e-02 & 8.15948041e \end{bmatrix}
-04 1.84198095e-03
   1.31166108e-03 5.88073212e-04
                                    7.16768850e-04 7.38824334e-05
   2.39228309e-04 4.26447038e-04 7.42960424e-05 3.61829163e-04
   1.30201139e-04 1.68363788e-04 3.27002322e-04 1.29813312e-04
   6.17118272e-04 3.82484050e-04 1.63069118e-04 4.55281364e-04
   5.64999115e-0411
5800 iteration, weights = [[ 1.62287605e-03 3.10090001e-02 8.17508313e
-04 1.84726245e-03
   1.31605171e-03 5.93450006e-04
                                    7.22388009e-04
                                                    7.02307827e-05
   2.36807253e-04 4.30689100e-04 7.06539569e-05 3.65944891e-04
   1.29680570e-04 1.67258321e-04 3.29368676e-04 1.29280536e-04
   6.23277888e-04 3.81507584e-04 1.61016038e-04 4.57074540e-04
   5.60878257e-04]]
5900 iteration, weights = \begin{bmatrix} 1.62744614e-03 & 3.11670078e-02 & 8.18876594e \end{bmatrix}
-04 1.85213711e-03
   1.32016210e-03
                  5.98752419e-04
                                    7.27902352e-04
                                                    6.65047855e-05
   2.34286586e-04 4.34882322e-04 6.69374953e-05 3.70027077e-04
   1.29117819e-04 1.66091762e-04 3.31684239e-04 1.28705428e-04
   6.29367201e-04 3.80415425e-04 1.58889085e-04 4.58774431e-04
   5.56546182e-04]]
6000 iteration, weights = \begin{bmatrix} 1.63167548e-03 & 3.13195309e-02 & 8.20059627e \end{bmatrix}
-04 1.85661925e-03
   1.32400210e-03 6.03983061e-04 7.33315552e-04 6.27070611e-05
   2.31669809e-04 4.39028416e-04 6.31492738e-05 3.74076902e-04
   1.28514372e-04 1.64866254e-04 3.33950792e-04 1.28089473e-04
   6.35388677e-04 3.79211637e-04 1.56690860e-04 4.60384309e-04
   5.52010310e-0411
6100 iteration, weights = [[ 1.63557604e-03 3.14667618e-02 8.21063917e
-04 1.86072264e-03
   1.32758122e-03 6.09144452e-04 7.38631156e-04 5.88401366e-05
   2.28960296e-04 4.43129031e-04 5.92918168e-05 3.78095508e-04
   1.27871660e-04 1.63583870e-04 3.36170054e-04 1.27434102e-04
   6.41344695e-04 3.77900141e-04 1.54423871e-04 4.61907334e-04
   5.47277800e-0411
6200 iteration, weights = [[ 1.63915936e-03 3.16088863e-02 8.21895742e
-04 1.86446060e-03
   1.33090862e-03 6.14239024e-04 7.43852584e-04 5.49064506e-05
   2.26161305e-04 4.47185760e-04 5.53675600e-05 3.82083993e-04
   1.27191064e-04 1.62246606e-04 3.38343680e-04 1.26740699e-04
```

```
6.47237551e-04 3.76484720e-04 1.52090539e-04 4.63346553e-04
    5.42355558e-04]]
6300 iteration, weights = \begin{bmatrix} 1.64243659e-03 & 3.17460836e-02 & 8.22561158e \end{bmatrix}
-04 1.86784596e-03
   1.33399315e-03 6.19269121e-04 7.48983135e-04 5.09083560e-05
   2.23275980e-04 4.51200140e-04 5.13788538e-05 3.86043419e-04
   1.26473917e-04 1.60856392e-04 3.40473272e-04 1.26010596e-04
   6.53069460e-04 3.74969025e-04 1.49693202e-04 4.64704905e-04
   5.37250250e-0411
6400 iteration, weights = [[ 1.64541848e-03 3.18785265e-02 8.23066012e
-04 1.87089112e-03
   1.33684334e-03 6.24237007e-04
                                    7.54025995e-04 4.68481234e-05
   2.20307353e-04 4.55173654e-04 4.73279660e-05 3.89974811e-04
   1.25721506e-04 1.59415087e-04 3.42560372e-04 1.25245082e-04
   6.58842559e-04 3.73356578e-04 1.47234111e-04 4.65985230e-04
   5.31968306e-04]]
6500 iteration, weights = \begin{bmatrix} 1.64811540e-03 & 3.20063820e-02 & 8.23415942e \end{bmatrix}
-04 1.87360804e-03
   1.33946743e-03 6.29144867e-04 7.58984237e-04 4.27279433e-05
   2.17258350e-04 4.59107731e-04 4.32170849e-05 3.93879157e-04
   1.24935071e-04 1.57924487e-04 3.44606469e-04 1.24445398e-04
   6.64558910e-04 3.71650780e-04 1.44715443e-04 4.67190264e-04
   5.26515932e-0411
6600 iteration, weights = [[ 1.65053736e-03 3.21298109e-02 8.23616390e
-04 1.87600825e-03
   1.34187335e-03 6.33994807e-04 7.63860826e-04 3.85499296e-05
   2.14131797e-04 4.63003753e-04 3.90483218e-05 3.97757410e-04
   1.24115809e-04 1.56386323e-04 3.46612999e-04 1.23612742e-04
   6.70220505e-04 3.69854909e-04 1.42139296e-04 4.68322649e-04
   5.20899116e-0411
6700 iteration, weights = [[ 1.65269402e-03 3.22489687e-02 8.23672607e
-04 1.87810288e-03
   1.34406879e-03 6.38788861e-04 7.68658623e-04 3.43161220e-05
   2.10930416e-04 4.66863052e-04 3.48237141e-05 4.01610491e-04
   1.23264878e-04 1.54802267e-04 3.48581350e-04 1.22748271e-04
   6.75829262e-04 3.67972130e-04 1.39507695e-04 4.69384935e-04
   5.15123635e-04]]
6800 iteration, weights = [[1.65459471e-03 3.23640053e-02 8.23589658e]
-04 1.87990267e-03
   1.34606113e-03 6.43528991e-04 7.73380388e-04 3.00284885e-05
   2.07656838e-04 4.70686912e-04 3.05452278e-05 4.05439287e-04
   1.22383390e-04 1.53173931e-04 3.50512860e-04 1.21853099e-04
   6.81387035e-04 3.66005497e-04 1.36822593e-04 4.70379580e-04
   5.09195063e-04]]
6900 iteration, weights = [[1.65624840e-03 3.24750652e-02 8.23372434e]
-04 1.88141799e-03
   1.34785752e-03 6.48217091e-04 7.78028786e-04 2.56889283e-05
```

```
2.04313598e-04 4.74476573e-04 2.62147595e-05 4.09244655e-04
   1.21472423e-04 1.51502871e-04 3.52408818e-04 1.20928304e-04
   6.86895612e-04 3.63957956e-04 1.34085877e-04 4.71308958e-04
   5.03118780e-04]]
7000 iteration, weights = \begin{bmatrix} 1.65766379e-03 & 3.25822879e-02 & 8.23025650e \end{bmatrix}
-04 1.88265883e-03
   1.34946484e-03 6.52854986e-04 7.82606387e-04 2.12992736e-05
   2.00903145e-04 4.78233233e-04 2.18341397e-05 4.13027421e-04
   1.20533014e-04 1.49790589e-04 3.54270470e-04 1.19974925e-04
   6.92356717e-04 3.61832351e-04 1.31299365e-04 4.72175359e-04
   4.96899976e-04]]
7100 iteration, weights = [[ 1.65884924e-03 3.26858083e-02 8.22553859e
-04 1.88363485e-03
   1.35088974e-03 6.57444440e-04 7.87115671e-04 1.68612925e-05
   1.97427841e-04 4.81958046e-04 1.74051345e-05 4.16788384e-04
   1.19566165e-04 1.48038533e-04 3.56099018e-04 1.18993963e-04
   6.97772015e-04 3.59631424e-04 1.28464812e-04 4.72980993e-04
   4.90543659e-04]]
7200 iteration, weights = [[ 1.65981284e-03 3.27857561e-02 8.21961452e
-04 1.88435535e-03
   1.35213863e-03 6.61987153e-04 7.91559030e-04 1.23766910e-05
   1.93889965e-04 4.85652126e-04 1.29294478e-05 4.20528313e-04
   1.18572842e-04 1.46248101e-04 3.57895623e-04 1.17986386e-04
   7.03143112e-04 3.57357823e-04 1.25583913e-04 4.73727991e-04
   4.84054662e-0411
7300 iteration, weights = [[ 1.66056237e-03 3.28822567e-02 8.21252667e
-04 1.88482932e-03
   1.35321770e-03 6.66484767e-04 7.95938775e-04 7.84711532e-06
   1.90291714e-04 4.89316548e-04 8.40872399e-06 4.24247951e-04
   1.17553977e-04 1.44420643e-04 3.59661402e-04 1.16953126e-04
   7.08471558e-04 3.55014101e-04 1.22658301e-04 4.74418411e-04
   4.77437647e-04]]
7400 iteration, weights = \begin{bmatrix} 1.66110536e-03 & 3.29754312e-02 & 8.20431592e \end{bmatrix}
-04 1.88506544e-03
   1.35413293e-03 6.70938864e-04 8.00257132e-04 3.27415381e-06
   1.86635213e-04 4.92952350e-04 3.84454967e-06 4.27948013e-04
   1.16510472e-04 1.42557459e-04 3.61397435e-04 1.15895085e-04
   7.13758848e-04 3.52602723e-04 1.19689554e-04 4.75054238e-04
    4.70697114e-04]]
7500 iteration, weights = \begin{bmatrix} 1.66144907e-03 & 3.30653962e-02 & 8.19502174e \end{bmatrix}
-04 1.88507205e-03
   1.35489006e-03 6.75350974e-04 8.04516252e-04 -1.34066090e-06
   1.82922507e-04 4.96560532e-04 -7.61544269e-07 4.31629192e-04
   1.15443193e-04 1.40659805e-04 3.63104764e-04 1.14813131e-04
   7.19006425e-04 3.50126067e-04 1.16679193e-04 4.75637387e-04
   4.63837403e-04]]
```

```
7600 iteration, weights = [[ 1.66160050e-03 3.31522644e-02 8.18468220e
-04 1.88485723e-03
   1.35549467e-03 6.79722570e-04 8.08718210e-04 -5.99584996e-06
   1.79155572e-04 5.00142059e-04 -5.40808065e-06 4.35292154e-04
   1.14352979e-04 1.38728893e-04 3.64784395e-04 1.13708102e-04
   7.24215681e-04 3.47586429e-04 1.13628685e-04 4.76169707e-04
    4.56862705e-04]]
7700 iteration, weights = \begin{bmatrix} 1.66156641e-03 & 3.32361444e-02 & 8.17333402e \end{bmatrix}
-04 1.88442878e-03
   1.35595211e-03 6.84055075e-04 8.12865007e-04 -1.06899864e-05
   1.75336314e-04 5.03697865e-04 -1.00936341e-05 4.38937544e-04
   1.13240639e-04 1.36765893e-04 3.66437297e-04 1.12580807e-04
   7.29387960e-04 3.44986020e-04 1.10539449e-04 4.76652980e-04
   4.49777060e-0411
7800 iteration, weights = [[ 1.66135332e-03 3.33171411e-02 8.16101265e
-04 1.88379420e-03
   1.35626756e-03 6.88349861e-04 8.16958574e-04 -1.54216934e-05
   1.71466574e-04 5.07228847e-04 -1.48168294e-05 4.42565982e-04
   1.12106951e-04 1.34771932e-04 3.68064406e-04 1.11432027e-04
   7.34524558e-04 3.42326979e-04 1.07412850e-04 4.77088928e-04
   4.42584369e-04]]
7900 iteration, weights = \begin{bmatrix} 1.66096751e-03 & 3.33953557e-02 & 8.14775230e \end{bmatrix}
-04 1.88296075e-03
   1.35644602e-03 6.92608253e-04 8.21000775e-04 -2.01896423e-05
   1.67548125e-04 5.10735874e-04 -1.95763394e-05 4.46178068e-04
   1.10952669e-04 1.32748098e-04 3.69666626e-04 1.10262515e-04
   7.39626725e-04 3.39611366e-04 1.04250208e-04 4.77479211e-04
   4.35288396e-04]]
8000 iteration, weights = \begin{bmatrix} 1.66041507e-03 & 3.34708858e-02 & 8.13358596e \end{bmatrix}
-04 1.88193541e-03
   1.35649230e-03 6.96831529e-04 8.24993408e-04 -2.49925512e-05
   1.63582679e-04 5.14219783e-04 -2.43708836e-05 4.49774380e-04
   1.09778519e-04 1.30695444e-04 3.71244826e-04 1.09072997e-04
   7.44695669e-04 3.36841170e-04 1.01052796e-04 4.77825430e-04
   4.27892770e-0411
8100 iteration, weights = [[1.65970184e-03 3.35438256e-02 8.11854548e]
-04 1.88072495e-03
   1.35641105e-03 7.01020922e-04 8.28938209e-04 -2.98291832e-05
   1.59571891e-04 5.17681382e-04 -2.91992265e-05 4.53355476e-04
   1.08585201e-04 1.28614981e-04 3.72799850e-04 1.07864175e-04
   7.49732554e-04 3.34018311e-04 9.78218409e-05 4.78129133e-04
   4.20400997e-04]]
8200 iteration, weights = \begin{bmatrix} 1.65883348e-03 & 3.36142660e-02 & 8.10266159e \end{bmatrix}
-04 1.87933588e-03
   1.35620676e-03 7.05177622e-04 8.32836851e-04 -3.46983447e-05
   1.55517352e-04 5.21121449e-04 -3.40601759e-05 4.56921895e-04
   1.07373393e-04 1.26507688e-04 3.74332506e-04 1.06636726e-04
```

```
7.54738503e-04 3.31144640e-04 9.45585282e-05 4.78391809e-04
    4.12816456e-04]]
8300 iteration, weights = \begin{bmatrix} 1.65781545e-03 & 3.36822947e-02 & 8.08596392e \end{bmatrix}
-04 1.87777447e-03
   1.35588376e-03 7.09302778e-04 8.36690950e-04 -3.95988841e-05
   1.51420603e-04 5.24540737e-04 -3.89525814e-05 4.60474155e-04
   1.06143745e-04 1.24374509e-04 3.75843580e-04 1.05391302e-04
   7.59714601e-04 3.28221945e-04 9.12640013e-05 4.78614899e-04
   4.05142411e-04]]
8400 iteration, weights = [[1.65665300e-03 3.37479964e-02 8.06848109e]
-04 1.87604680e-03
   1.35544623e-03 7.13397496e-04 8.40502064e-04 -4.45296902e-05
   1.47283128e-04 5.27939972e-04 -4.38753331e-05 4.64012759e-04
   1.04896889e-04 1.22216353e-04 3.77333825e-04 1.04128533e-04
   7.64661894e-04 3.25251949e-04 8.79393633e-05 4.78799792e-04
    3.97382008e-04]]
8500 iteration, weights = \begin{bmatrix} 1.65535121e-03 & 3.38114527e-02 & 8.05024070e \end{bmatrix}
-04 1.87415872e-03
   1.35489820e-03 7.17462846e-04 8.44271699e-04 -4.94896906e-05
   1.43106358e-04 5.31319855e-04 -4.88273599e-05 4.67538191e-04
   1.03633430e-04 1.20034100e-04 3.78803970e-04 1.02849026e-04
   7.69581391e-04 3.22236316e-04 8.45856787e-05 4.78947826e-04
   3.89538284e-0411
8600 iteration, weights = \begin{bmatrix} 1.65391499e-03 & 3.38727424e-02 & 8.03126939e \end{bmatrix}
-04 1.87211587e-03
   1.35424355e-03 7.21499859e-04 8.48001304e-04 -5.44778509e-05
   1.38891676e-04 5.34681059e-04 -5.38076284e-05 4.71050915e-04
   1.02353956e-04 1.17828598e-04 3.80254720e-04 1.01553369e-04
   7.74474066e-04 3.19176651e-04 8.12039746e-05 4.79060297e-04
   3.81614172e-0411
8700 iteration, weights = [[ 1.65234906e-03 3.39319416e-02 8.01159285e]
-04 1.86992370e-03
   1.35348603e-03 7.25509531e-04 8.51692281e-04 -5.94931726e-05
   1.34640416e-04 5.38024238e-04 -5.88151414e-05 4.74551384e-04
   1.01059032e-04 1.15600663e-04 3.81686751e-04 1.00242128e-04
   7.79340860e-04 3.16074504e-04 7.77952422e-05 4.79138450e-04
   3.73612497e-04]]
8800 iteration, weights = \begin{bmatrix} 1.65065797e-03 & 3.39891235e-02 & 7.99123587e \end{bmatrix}
-04 1.86758747e-03
   1.35262928e-03 7.29492822e-04 8.55345980e-04 -6.45346926e-05
   1.30353865e-04 5.41350021e-04 -6.38489367e-05 4.78040032e-04
   9.97492031e-05 1.13351087e-04 3.83100720e-04 9.89158477e-05
   7.84182679e-04 3.12931369e-04 7.43604379e-05 4.79183489e-04
    3.65535990e-0411
8900 iteration, weights = \begin{bmatrix} 1.64884615e-03 & 3.40443590e-02 & 7.97022240e \end{bmatrix}
-04 1.86511226e-03
   1.35167678e-03 7.33450659e-04 8.58963706e-04 -6.96014813e-05
```

```
1.26033263e-04 5.44659015e-04 -6.89080860e-05 4.81517278e-04
   9.84249964e-05 1.11080629e-04 3.84497258e-04 9.75750559e-05
   7.89000397e-04 3.09748687e-04 7.09004842e-05 4.79196577e-04
   3.57387281e-04]]
9000 iteration, weights = \begin{bmatrix} 1.64691782e-03 & 3.40977164e-02 & 7.94857552e \end{bmatrix}
-04 1.86250294e-03
   1.35063191e-03 7.37383935e-04 8.62546716e-04 -7.46926419e-05
   1.21679809e-04 5.47951804e-04 -7.39916933e-05 4.84983526e-04
   9.70869200e-05 1.08790026e-04 3.85876974e-04 9.62202607e-05
   7.93794860e-04 3.06527851e-04 6.74162716e-05 4.79178835e-04
   3.49168913e-04]]
9100 iteration, weights = [[ 1.64487709e-03 3.41492616e-02 7.92631749e
-04 1.85976426e-03
   1.34949791e-03 7.41293513e-04 8.66096225e-04 -7.98073091e-05
   1.17294660e-04 5.51228955e-04 -7.90988943e-05 4.88439167e-04
   9.57354639e-05 1.06479986e-04 3.87240458e-04 9.48519527e-05
   7.98566882e-04 3.03270204e-04 6.39086593e-05 4.79131343e-04
   3.40883336e-04]]
9200 iteration, weights = \begin{bmatrix} 1.64272791e-03 & 3.41990582e-02 & 7.90346983e \end{bmatrix}
-04 1.85690076e-03
   1.34827793e-03 7.45180226e-04 8.69613403e-04 -8.49446478e-05
   1.12878929e-04 5.54491013e-04 -8.42288549e-05 4.91884578e-04
   9.43711013e-05 1.04151192e-04 3.88588275e-04 9.34706053e-05
   8.03317249e-04 2.99977040e-04 6.03784762e-05 4.79055145e-04
   3.32532916e-04]]
9300 iteration, weights = [[ 1.64047410e-03 3.42471675e-02 7.88005327e
-04 1.85391684e-03
   1.34697499e-03 7.49044877e-04 8.73099381e-04 -9.01038523e-05
   1.08433695e-04 5.57738504e-04 -8.93807703e-05 4.95320122e-04
   9.29942886e-05 1.01804307e-04 3.89920975e-04 9.20766752e-05
   8.08046720e-04 2.96649610e-04 5.68265222e-05 4.78951248e-04
   3.24119934e-04]]
9400 iteration, weights = [[ 1.63811934e-03 3.42936488e-02 7.85608782e
-04 1.85081675e-03
   1.34559202e-03 7.52888241e-04 8.76555249e-04 -9.52841453e-05
   1.03959995e-04 5.60971935e-04 -9.45538640e-05 4.98746150e-04
   9.16054662e-05 9.94399656e-05 3.91239085e-04 9.06706032e-05
   8.12756027e-04 2.93289120e-04 5.32535691e-05 4.78820624e-04
    3.15646592e-04]]
9500 iteration, weights = [[ 1.63566718e-03 3.43385592e-02 7.83159279e
-04 1.84760458e-03
   1.34413184e-03 7.56711066e-04 8.79982060e-04 -1.00484777e-04
   9.94588316e-05 5.64191796e-04 -9.97473867e-05 5.02163000e-04
   9.02050590e-05 9.70587835e-05 3.92543115e-04 8.92528143e-05
   8.17445875e-04 2.89896735e-04 4.96603616e-05 4.78664210e-04
   3.07115017e-04]]
```

```
9600 iteration, weights = [[ 1.63312105e-03 3.43819537e-02 7.80658683e
-04 1.84428427e-03
   1.34259717e-03 7.60514073e-04
                                      8.83380829e-04 -1.05705022e-04
    9.49311713e-05
                   5.67398561e-04 -1.04960615e-04
                                                      5.05570998e-04
    8.87934771e-05 9.46613536e-05 3.93833558e-04 8.78237191e-05
    8.22116946e-04 2.86473576e-04
                                     4.60476184e-05 4.78482911e-04
    2.98527257e-04]]
9700 iteration, weights = \begin{bmatrix} 1.63048426e-03 & 3.44238857e-02 & 7.78108790e \end{bmatrix}
-04 1.84085965e-03
   1.34099062e-03 7.64297960e-04 8.86752535e-04 -1.10944184e-04
    9.03779474e-05 5.70592685e-04 -1.10192852e-04
                                                      5.08970461e-04
    8.73711162e-05 9.22482482e-05 3.95110887e-04 8.63837133e-05
    8.26769898e-04 2.83020728e-04
                                      4.24160328e-05
                                                       4.78277600e-04
    2.89885291e-0411
9800 iteration, weights = [[1.62775999e-03] 3.44644064e-02
                                                                7.75511336e
-04 1.83733438e-03
                   7.68063396e-04 8.90098125e-04 -1.16201588e-04
    1.33931475e-03
    8.58000600e-05 5.73774608e-04 -1.15443425e-04 5.12361691e-04
    8.59383580e-05 8.98200194e-05
                                     3.96375562e-04
                                                       8.49331790e-05
    8.31405365e-04 2.79539237e-04 3.87662738e-05 4.78049118e-04
    2.81191029e-0411
9900 iteration, weights = \begin{bmatrix} 1.62495134e-03 & 3.45035653e-02 & 7.72867994e \end{bmatrix}
-04 1.83371203e-03
    1.33757199e-03
                    7.71811032e-04
                                      8.93418509e-04 -1.21476584e-04
    8.11983779e-05 5.76944755e-04 -1.20711682e-04 5.15744982e-04
    8.44955709e-05
                   8.73772000e-05
                                     3.97628024e-04
                                                       8.34724849e-05
   8.36023959e-04 2.76030111e-04
                                     3.50989870e-05 4.77798279e-04
    2.72446312e-04]]
            Graph for Training, Testing and Validation Set
   0.5
                                                     error train
                                                     error test
   0.4
                                                     error valid
Mean Square Errors
   0.3
   0.2
   0.1
```

0.0

0

20

40

No. of Iterations

60

80

100