University of Zagreb
Faculty of Electrical Engineering and Computing

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# Machine Learning 1 – Homework 5

*The homework assignment contains **6 questions** and is worth a maximum of 6 points in total (scaled to 2 points for the course grade). A correct answer is worth 1 point. Unlike the exam scoring, incorrect answers do not carry negative points. Before you start solving these problems, we recommend that you complete all the exercises in the "Practice Problems" section for all learning units covered by this assignment.*

***Warning:*** *For every question you answer, you must also submit a handwritten solution. If you fail to do this for even a single question, you will forfeit all points accumulated through homework activity.*

***Note:*** *This is a personalized homework assignment. Each student receives a unique variant of the problems.*

## 17. Probabilistic graphical models

**17.1** (P) A Bayes network has five variables, with topological ordering $v, w, x, y, z$. With such an ordering, the following conditional independencies hold in the network:

$$\{v, w\} \perp y | x \qquad \{v, x\} \perp z | \{w, y\}$$

Using the d-separation algorithm, we examine the dependencies between pairs of variables. **Which of the following independence statements hold in this Bayes network?**

A $x \perp z | y$  B $w \perp y | x$  C $v \perp y | z$  D $v \perp y | w$

**17.2** (P) Consider a Bayes network with six nodes corresponding to the following factorization:

$$P(u, v, w, x, y, z) = P(u|x)P(v|x)P(w|u, x)P(x)P(y|v, x, z)P(z|u, w)$$

Use the d-separation method to examine the conditional independence of variables $x$ and $z$ given the remaining four variables. Determine which of the four remaining variables need to be observed and which need to remain unobserved for $x$ and $z$ to be d-separated. **For which of the four remaining variables does it not matter whether it is observed, provided that $x$ and $z$ are d-separated?**

A $v$  B $u$  C $y$  D $w$

**17.3** (P) A Bayes network has five variables, with a topological ordering $v, w, x, y, z$. All variables are binary, except for $v$ and $y$, which are ternary. Given this topological ordering, the following conditional independencies hold in the network:

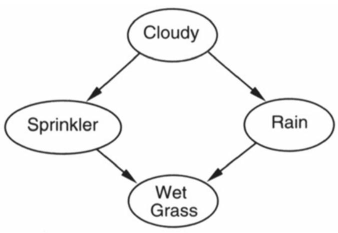
$$\{v, w\} \perp y | x \qquad \{v, x\} \perp z | \{w, y\}$$

Derive the factorization of the joint distribution corresponding to this Bayes network. **How many parameters does this Bayes network have?**

A 18  B 24  C 21  D 23

## 18. Probabilistic graphical models II

**18.1** (N) The image below shows the Bayes network for the sprinkler problem, which we used in lectures. The variables are: $C$ (cloudy), $S$ (sprinkler), $R$ (rain), and $W$ (wet grass). Conditional probability tables for each node are also provided.




| $C$ | $P(S=1\|C)$ |
|---|---|
| 0 | 0.5 |
| 1 | 0.4 |

| $C$ | $P(R=1\|C)$ |
|---|---|
| 0 | 0.2 |
| 1 | 0.8 |

| $S$ | $R$ | $P(W=1\|S,R)$ |
|---|---|---|
| 0 | 0 | 0.0 |
| 0 | 1 | 0.9 |
| 1 | 0 | 0.1 |
| 1 | 1 | 0.99 |

**Compute the posterior probability that the sprinkler is on given that the grass is not wet and it is cloudy.**

A 0.069 B 0.309 C 0.223 D 0.144

18.2 (N) We model constructs of positive psychology using a Bayes network with four variables. We use binary variables *Love* (L), *Happiness* (H), and *Anxiety* (A), with values 0 (absent) and 1 (present), as well as a ternary variable *Money* (M), with values 0 (none), 1 (little), and 2 (plenty). The structure of the Bayes network is defined to model the following assumed causal relationships: L causes H, and M causes both H and A. We then train the defined Bayes network on the following dataset of $N = 7$ examples:

| $L$ | $M$ | $H$ | $A$ | $L$ | $M$ | $H$ | $A$ |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| 0 | 2 | 0 | 1 | 1 | 2 | 1 | 1 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 2 | 1 | 0 | | | | |

We estimate the model parameters using the MAP estimator with $\alpha = \beta = 2$ (for binary variables) and $\alpha_k = 2$ (for the ternary variable), which is equivalent to Laplace smoothing of the MLE estimate. Finally, we are interested in the probability of a life with love, happiness, and plenty of money. Calculate the necessary MAP estimates of the parameters. **What is the joint probability $P(L = 1, H = 1, M = 2)$?**

A 0.148 B 0.813 C 0.074 D 0.023

18.3 (N) Consider a Bayes network corresponding to factorization $P(w, x, y, z) = P(w)P(x)P(y|w, x)P(z|x)$. All variables are binary. Let $P(w = 1) = 0.1$, $P(x = 1) = 0.2$, $P(z = 1|x = 0) = 0.9$, and $P(z = 1|x = 1) = 0.7$. The conditional probability table for node $y$ is as follows:

| $w$ | $x$ | $p(y = 1\|w, x)$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0.4 |
| 1 | 0 | 0.2 |
| 1 | 1 | 0.7 |

Using rejection sampling, we want to estimate the parameter $\mu$ of the conditional distribution $P(x = 0|y = 1, z = 0)$. We repeated the sampling a total of $N = 2000$ times, from which we had to discard some vectors, so our sample is smaller than $N$. Based on the obtained sample, we estimate the parameter $\mu$ with the MAP estimator with $\alpha = \beta = 2$. **What is the expected MAP estimate of the parameter $\mu$?**

A 0.2447 B 0.1146 C 0.0739 D 0.1765