$K=4$

$K=8$

$=8$

$=2$

$=2$

$0{,}028125$

$=4$

$K=8$

$,25 = 0{,}01171875$

$\approx 0{,}70588$

1875

4

$= \boxed{0{,}7407}$

$0{,}00703$

$\boxed{B}$

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Homework 4.

14.2 $D = \{-10, 5, 6, 20\}$ $\mathcal{N}(\mu, \sigma^2)$ $n=4$.

$$\hat{\mu}_{MLE} = \frac{1}{N}\sum_{i=1}^{N} x^i = \frac{1}{4}(-10+5+6+20) = 5{,}25$$

$$\hat{\sigma}^2_{MLE} = \frac{1}{N}\sum_{i=1}^{N}(x^i - \hat{\mu}_{MLE})^2 = \frac{1}{4}\left((-10-5{,}25)^2 + (5-5{,}25)^2 + (6-5{,}25)^2 + (20-5{,}25)^2\right) = 112{,}6875$$

unbias estimator

$$E[\hat{\sigma}^2] = \frac{4}{3} \times 112{,}6875 = 150{,}25$$

$$\ln \mathcal{L}(\hat{\mu}_{MLE}, \hat{\sigma}^2 \mid D) = -\frac{N}{2}\ln(2\pi\sigma^2) - \frac{1}{2\sigma^2}\sum_i (x^i - \mu)^2$$

$$= -\frac{4}{2}\ln(2\pi \times 150{,}25) - \frac{1}{2(150{,}25)} \times 450{,}75 = -15{,}1987 \approx \boxed{-15{,}200} = \boxed{B}$$

15.2 $n=3$ $K=2$

$$\hat{\Sigma} = 0{,}1 \times \hat{\Sigma}_1 + 0{,}9 \times \hat{\Sigma}_2 = \begin{pmatrix} 5 & 2\times10^{-4} & 4\times10^{-4} \\ 2\times10^{-4} & 3 & 3\times10^{-4} \\ 4\times10^{-4} & 3\times10^{-4} & 1 \end{pmatrix}$$

$$\Sigma^{-1} = \begin{pmatrix} 0{,}2 & 0 & 0 \\ 0 & 0{,}33 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad |\Sigma| = 15$$

$\ln p(x \mid y=1)$

$$h_1(x) = -\frac{n}{2}\ln 2\pi - \frac{1}{2}\ln|\Sigma| - \frac{1}{2}(x-\hat{\mu}_1)^T \Sigma^{-1}(x-\hat{\mu}_1)$$

$$= -\frac{3}{2}\ln(2\pi) - \frac{1}{2}\ln 15 - \frac{1}{2}\left(\begin{pmatrix}0\\0\\3\end{pmatrix} \times \begin{pmatrix}0\\0\\3\end{pmatrix}\right)$$

$$(x-\mu)^T = (1,0,1) - (1,0,-2) = \begin{pmatrix}0\\0\\3\end{pmatrix}$$

$$= -8{,}6108 + \ln(p(y=j))$$

$$= -8{,}6108 + \ln(0{,}1) = \boxed{-10{,}913} = \boxed{D}$$

$$\Sigma^{-1}(x-\hat{\mu}_1)^T = \begin{pmatrix} 0{,}2 & 0 & 0 \\ 0 & 0{,}33 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix}0\\0\\3\end{pmatrix} = \begin{pmatrix}0\\0\\3\end{pmatrix}$$

16.1 $N=8$ $y=1 \to 6$ examples $P(y=1) = 0{,}75$

$y=0 \to 2$ examples $p(y=0) = 0{,}25$.

$x = (\text{Italy}, \text{yes}, \text{winter}, \text{master}, \text{no})$

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16.1 next

$P(\text{Italy, yes} \mid y=1) = \frac{1+1}{6+4} = \frac{2}{10} = \frac{1}{5} \leftarrow K=4$

$P(\text{Italy, yes} \mid y=1) = 2/14 = 0,1428 \leftarrow K=8$

$P(\text{winter} \mid y=1) = \frac{3+1}{6+2} = \frac{4}{8} = \frac{1}{2} \leftarrow K=2$

$P(\text{master} \mid y=1) = \frac{5+1}{6+2} = \frac{6}{8} \leftarrow K=2$

$P(\text{no} \mid y=1) = \frac{3+1}{6+2} = \frac{4}{8} = \frac{1}{2} \leftarrow K=2$

$S_1 \to y=1 = 0,75 \times 1/5 \times 1/2 \times 6/8 \times 1/2 = 0,028125$

$P(\text{italy, yes} \mid y=0) = \frac{1}{6} \leftarrow K=4$

$P(\text{italy, yes} \mid y=0) = 1/10 = 0,1 \leftarrow K=8$

$P(\text{winter} \mid y=0) = \frac{3}{4}$

$P(\text{master} \mid y=0) \ \frac{2}{4}$

$P(\text{no} \mid y=0) = 3/4$

$S_0 \to y=0 = 1/6 \times 3/4 \times 2/4 \times 3/4 \times 0,25 = 0,01171875$

$P(y=1 \mid x) = \frac{S_1}{S_1+S_0} = \frac{0,028125}{0,028125+0,01171875} \approx 0,70588$

using $K=8$ for P(Italy, yes) instead of $K=4$

we have $\frac{0,02008}{0,02008+0,00703} = \boxed{0,7407}$

$\boxed{B}$

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Homework 4.

14.2 $D = \{-4 …$

$\hat{\mu}_{MLE} = \frac{1}{N}\sum_{i=1}^{N} x …$

$\hat{\sigma}^2_{MLE} = \frac{1}{N}\sum_{i=1}^{N} (x …$

un bias estimator

$E[\hat{\sigma}^2] = \frac{3}{4} …$

$\ln L(\hat{\mu}_{MLE} …$

$= -\frac{4}{2}\ln(2\pi …$

15.2 $n = 3$

$\hat{\Sigma}_1 = 0,1 …$

$\Sigma_1^{-1} = (…$

$\ln p(x \mid y=1)$

$h_1(x) = - …$

$= -\frac{3}{2} …$

$(x-\mu)^T = (1,0,1) - (1,0,- …$

$= \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix}$

$\Sigma_1^{-1}(x-\hat{\mu}_1)^T = \begin{pmatrix} 0,2 & 0 & … \\ 0 & 0,… \\ 0 & 0 \end{pmatrix}$

16.1 $N = 8$

$x = (\text{Italy} …$

$\alpha + m_1 = 16$
$\beta^{t}(N_1 - m_1) = 68$

$\mu_1^{*} = 0{,}072 = \boxed{C}$

~~$I(X,Y) = \sum P(x,y) \ln \frac{P(x,y)}{P(x)P(y)}$~~

~~group x_1 and x_2~~

~~group x_1 and x_3~~

$\mu_0 = \frac{1}{3}\begin{pmatrix} -1+0+1 \\ -2+0+2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$1 + 0 + 0 + 1 = 2$

$\begin{pmatrix} \ & 0 \\ \ & 2{,}4 \end{pmatrix}$ $|\Sigma_i| = 0{,}8 \times 2{,}4 = 1{,}92$

$-\frac{1}{2}(x-\mu_0)^T \Sigma_i^{-1}(x-\mu_0)$
$10 = -13{,}01$

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<u>16.2</u> $n = 3$

$P(x_1 = 0) = 0{,}3 + 0{,}1 + 0 + 0{,}1 = 0{,}5$
$P(x_1 = 1) = 0{,}2 + 0 + 0{,}2 + 0{,}1 = 0{,}5$
$P(x_2 = 0) = 0{,}3 + 0{,}2 + 0 + 0{,}2 = 0{,}7$
$P(x_2 = 1) = 0{,}1 + 0{,}0 + 0{,}1 + 0{,}1 = 0{,}3$
$P(x_3 = 0) = 0{,}3 + 0{,}1 + 0{,}2 + 0 = 0{,}6$
$P(x_3 = 1) = 0{,}0 + 0{,}1 + 0{,}2 + 0{,}1 = 0{,}4$

Compute each pair $I(x_1, x_2)$ $I(x_1, x_3)$ $I(x_2, x_3)$

$$I(x_1, x_2) = 0{,}3 \ln \frac{0{,}3}{0{,}5 \times 0{,}4} + 0{,}2 \ln \frac{0{,}2}{0{,}5 \times 0{,}3} + 0{,}4 \ln \frac{0{,}4}{0{,}5 + 0{,}7}$$
$$+ 0{,}1 \ln \frac{0{,}1}{0{,}5 + 0{,}3} \simeq 0{,}024 > 0{,}01$$ x_1 and x_2 needs to be grouped.

$$I(x_1, x_3) = 0{,}4 \ln \frac{0{,}4}{0{,}5 \times 0{,}6} + 0{,}1 \ln \frac{0{,}1}{0{,}5 \times 0{,}4} + 0{,}2 \ln \frac{0{,}2}{0{,}5 \times 0{,}6} + 0{,}3 \ln \frac{0{,}3}{0{,}5 \times 0{,}4}$$

$0{,}2 \ln(\dots)$

$\simeq 0{,}082 > 0{,}01$ x_1 and x_3 grouped.

~~$I(x_2, x_3)$~~ Then we combine the 3 variables
$P(y, x_1, x_2, x_3) = P(y) \cdot P(x_1, x_2, x_3 \mid y)$
$\boxed{B}$

~~$P(x_1 = 0) = 0{,}3 + 0{,}1 + 0{,}0 + 0{,}1 = 0{,}5$~~
~~$P(x_2 = 0) = 0{,}3 + 0{,}2 + 0{,}0 + 0{,}2 = 0{,}7$~~
~~$P(x_3 = 0) = 0{,}3 + 0{,}1 + 0{,}2 + 0{,}0 = 0{,}6$~~
~~$I(X,Y) = \sum P(x,y) \ln \frac{P(x,y)}{P(x)P(y)}$~~
~~(x_1, x_2) $P(0,0) = 0{,}3$ $I(x_1, x_2) \simeq 0{,}024 > 0{,}01$ so we group x_1 and x_2~~
~~(x_1, x_3) $P(0,0) = 0{,}4$ $I(x_1, x_3) \simeq 0{,}082 > 0{,}01$ so we group x_1 and x_3~~

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14.1 ① $\alpha = \beta = 2$ $\quad N_1 = 80$ $\quad m_1 = 14$ $\quad D_1 = \alpha' = \alpha + m_1 = 16$

$\beta' = \beta'(N_1 - m_1) = 68$

$$\hat{\mu}^*_{MAP} = \frac{\alpha' - 1}{\alpha' + \beta' - 2} = \frac{16-1}{16+68-2} = 0,1829$$

② $N_2 = 20$ $\quad m_2 = 11$ $\quad D_2 = \alpha'' = 16 + 11 = 27$

$\beta'' = 68 + (20-11) = 77$

$$\hat{\mu}^*_{MAP} = \frac{27-1}{27+77+2} = 0,2549 \qquad \mu^*_2 - \mu^*_1 = 0,072 = \boxed{C}$$

~~16.2~~ $\cancel{P(x_1=0) = 0,3+0,1+0,0+0,1 = 0,5}$

$\cancel{P(x_2=0) = 0,3+0,2+0,0+0,2 = 0,7}$ $\quad \cancel{I(X,Y) = \sum P(x,y) \ln \frac{P(x,y)}{P(x)P(y)}}$

$\cancel{P(x_3=0) = 0,3+0,1+0,2+0,0 = 0,6}$

$\cancel{(x_1,x_2) \quad P(x_1=0, x_2=0) = 0,3+0,0 = 0,3}$

~~$I(x_1,x_2) = 0,024 > 0,01 \to$ group x_1 and x_2~~

$\cancel{(x_1,x_3) \quad P(x_1=0, x_3=0) = 0,3+0,1 = 0,4}$

~~$I(x_1,x_3) = 0,082 > 0,01 \to$ group x_1 and x_3.~~

$\cancel{P(y)P(x_1,x_2,x_3 \mid y)}$.

15.1 $x_1 = (-1,2)$ $\quad x_2 = (0,0)$ $\quad x_3 = 1,2)$ $\quad \mu_0 = \frac{1}{3}\begin{pmatrix} -1+0+1 \\ -2+0+2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$P(y=0) = 3/7$.

x_1: Class 0 $= -1^2 + 0^2 + 1^2 = 2$

class 1 $= (3-4)^2 + (4-4)^2 + (5-4)^2 = 1+0+0+1 = 2$

$\sigma_1^2 = \frac{2+2}{7-2} = 4/5 = 0,8$

x_2: class 0 $: (-2)^2 + 0 + 2^2 = 8$

class 1 $: -1^2 + (-1)^2 + 1^2 + 1^2 = 4$

$\sigma_2^2 = \frac{8+4}{7-2} = \frac{12}{5} = 2,4$ $\qquad \Sigma_i = \begin{pmatrix} 0,8 & 0 \\ 0 & 2,4 \end{pmatrix}$ $\qquad |\Sigma_i| = 0,8 \times 2,4 = 1,92$

$$h_0(x) = \ln P(y=0) - \frac{n}{2} \ln(2\pi) - \frac{1}{2} \ln|\Sigma_i| - \frac{1}{2}(x-\mu_0)^T \Sigma_i^{-1} (x-\mu_0)$$

$$= -0,8473 - 1,8379 - 0,3261 - 10 = -13,01$$

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16.2 $\quad n = 3$

$P(x_1 =$

$P(x_1 =$

$P(x_2 =$

$P(x_2 =$

$P(x_3 =$

$P(x_3 =$

Compute each pair

$I(x_1, x_2) = 0,3 \ln \frac{0,3}{0,5 \times}$

$+ 0,1 \ln \frac{0,1}{0,3 + 0,}$

needs to be groupe

$I(x_1, x_3) = 0,4 \ln \frac{0,4}{0,5 \times}$

$\simeq 0,082 > 0$

Then

$P(y, x_1,$

$\boxed{B}$