**Literature**

*Offline Identification*

Offline identification is relatively the oldest techniquefor model prediction, it is easier to perform in terms of mathematical complexity than its counterpart, online identification. The online identification is essentially a modification of offline technique and is popular in modern plants that implement adaptive controllers. Not every plant needs an online identification, for example if a process is well known to be time-invariant then an offline is efficient.

The system investigated in this paper is a 2-input-2-output multivariable case both in open loop and closed loop. To facilitate the understanding of system identification in general a method for identification will be presented, particularly the least-square method. A general single-input-single-output linear system is described by

|  |  |  |
| --- | --- | --- |
|  |  | () |

Where y(t) is output and u(t) is the input. The difference equation difference equation above can be easily be transformed to either the time or the Laplace domain, this paper is making use of the difference equation for simulation and identification. Equation 1 can be written in matrix format as

|  |  |  |
| --- | --- | --- |
|  |  | () |

Where

|  |  |  |
| --- | --- | --- |
|  |  | () |

This paper will also employ the ARMAX model using shift operators, this model structure is written as

|  |  |  |
| --- | --- | --- |
|  |  | () |

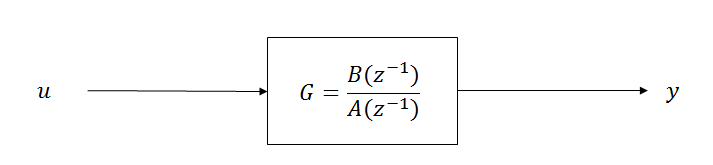
|  |  |  |
| --- | --- | --- |
|  |  | () |

Where

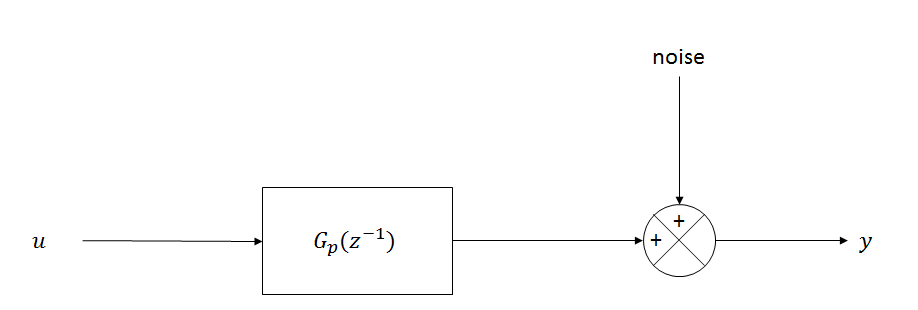
|  |  |  |
| --- | --- | --- |
|  |  | () |

In the equation above, z-1 denotes the backward shift operator so that

|  |  |  |
| --- | --- | --- |
|  |  | () |



Unlike the model, the real process contains disturbance elements such as noise that are neither correlated to the input nor the process output. The real process for SISO system is shown in the figure below.



The system above is assumed to be generating data for offline identification. This system is represented by

|  |  |  |
| --- | --- | --- |
|  |  | () |

**Least-Squares Estimator**

The least square method minimizes the error between the process output and the model output. By evaluating the gradient of the function that gives the residual error, it can be shown that the minimum error occurs at

|  |  |  |
| --- | --- | --- |
|  |  | () |

If we define a variable

|  |  |  |
| --- | --- | --- |
|  |  | () |

Then it follows that

|  |  |  |
| --- | --- | --- |
|  |  | () |

The algorithm above is known as the recursive identification by least-square method. This algorithm can be put into more useful form that eliminates matrix inversion, particularly the updating of P(t), the equation for P(t) is equivalent to

|  |  |  |
| --- | --- | --- |
|  |  | () |

Thus

|  |  |  |
| --- | --- | --- |
|  |  | () |

The algorithm above also applies to multivariable systems since multiple-input-multiple-output is merely an addition of multiple-input-single-output systems. It is as easy to identify a MISO system as is to identify as SISO system. The least-square method is, however, valid under certain condition, these conditions are

* The system should have a white noise for estimated parameters to converge to true values.
* For high-order systems, the deviation of the estimated parameters from true values is often more substantial than with low-order systems.
* The least-square method is sensitive to signal-to-noise ratio.

The process used in this paper has a white noise with a noise-to-signal ratio of < 0.1, this should be fine as the maximum signal-to-noise ratio is 0.2 according to the literature. The system is a 2x2 multivariable system which is relatively a low-order system. Different noise levels will be investigated also to determine the noise-to-signal ratio at which the least-square method breaks down.

*Initial Values*

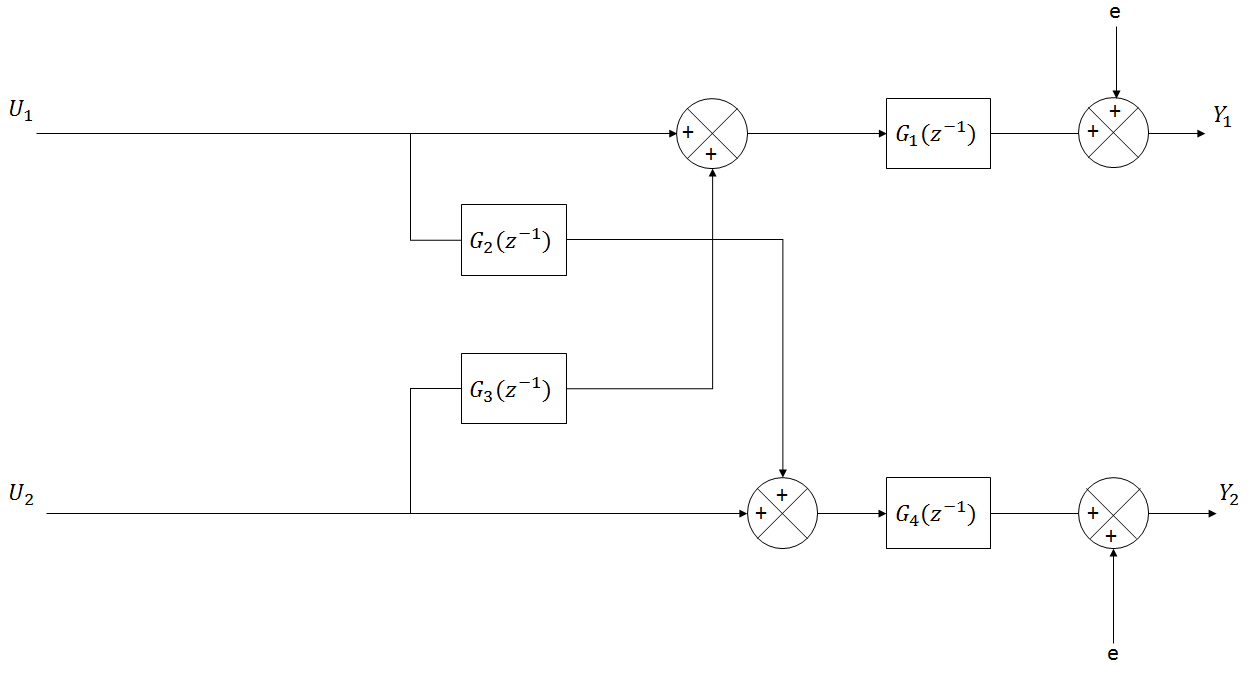
The algorithm from (12) to (13) requires initial values and. The parameters can be assumed to all start at zero but if the variable P(t) starts as a zeros matrix it will continue as a zeros matrix. The literature recommendation is that

|  |  |  |
| --- | --- | --- |
|  |  | () |

where σ is a very large scalar number. If P(0) is small then the parameters will converge slowly, this is evident from (13) since K(t) is proportional to P(t), small K(t) implies a small change in .

*Process Configuration*

The system to be investigated is presented in the configuration below, the controller will be implemented later in this paper.



The figure above represent the real process, Ui is the input, Yi is the output and e is noise. For simplicity, this system will be interpreted as a combination of two MISO systems where all the transfer functions, Gi(z-1), are first-order. This system can be mathematically modeled as

|  |  |  |
| --- | --- | --- |
|  |  | () |

In the matrix form, the above system above becomes

|  |  |  |
| --- | --- | --- |
|  |  | () |

Where

|  |  |  |
| --- | --- | --- |
|  |  | () |

The noise will treated as white, i.e. with zero mean and a variance of σ2 thus this favors the use of the least-square method.

The system above can also be modeled using the Laplace domain, but the configuration does not change and one can easily convert the transfer functions to be in the Laplace domain. The parameters are given in terms of the Laplace domain and the z-domain below.

|  |  |  |
| --- | --- | --- |
|  | **τ** | **K** |
| G1 | 1 | 1 |
| G2 | 5 | 0.5 |
| G3 | 6 | 0.2 |
| G4 | 2 | 1 |
|  | **a** | **b** |
| G1 | 0.6321 | 0.3679 |
| G2 | 0.0907 | 0.8187 |
| G3 | 0.0307 | 0.8465 |
| G4 | 0.3935 | 0.6065 |

Substituting the transfer functions into equation (15) results in

|  |  |  |
| --- | --- | --- |
|  |  | (18) |

Which can be written in a more useful way for simulation as

|  |  |  |
| --- | --- | --- |
|  |  | (19) |

Where

|  |  |  |
| --- | --- | --- |
|  |  | (20) |

The expression above appears to have twelve degrees of freedom (6 to each output) while the system in (18) has eight (4 to each output), the manipulation in (20) still has the same DOF as in (18), the “extra” parameters are just functions of the original parameters in (18) i.e. they are dependent. There is no way to reduce the expression in (20) to only eight independent parameters unless some parameters are equal.

*Model Assumption*

The assumed model has twelve degrees of freedom, this model is guaranteed to capture all the process behavior since it has extra four DOF, if this model is consistent, then the extra four parameters should be functions of the other eight parameters, for example if ,, and are successfully estimated, a consistency check can be done to see if the relationship in (20) holds. The assumed model is shown below

|  |  |  |
| --- | --- | --- |
|  |  | (21) |

**Inputs**

When identifying a process, the selection of the input signal is crucial in the convergence of the process parameters. Different inputs have different characteristics and will essentially cause a different reaction curve. It is a necessary condition that an input be independent of the process output for process identifiability, this condition is not met in the closed loop system because the input to the process depends on the output. This, however, does not mean that a closed loop system is not identifiable.

*Step function as an input*

A step input is not dependent on the process output and therefore will be a potential candidate for process identification. For low-order processes, a process reaction to a step change may reveal some information about the process such as the time constant and the process gain. For high-order and non-linear processes, this approach of identification may prove to be impossible.

To estimate process parameters and especially online, a persistently exciting signal is required to obtain good quality results. A step input is not exciting enough since it maintains a single value almost all the time. For offline identification it can be shown that a step input will give parameter estimates that converge to the true values as the number of samples increase and on a condition that the noise is white. On a closed loop system, the input to the process is not necessarily a step function, again, it will depend on the deviation of the process variable from the setpoint.

*An impulse as an input*

An impulse is also not dependent on the process variable. It is hard to reveal the process information using an impulse as an input, this is because an impulse does not excite the process sufficiently and it is equal to zero too often. This means that a reasonable model will not be obtained even in an offline identification with a very large sample, thus an impulse as an input will not be considered as a potential candidate for identification.

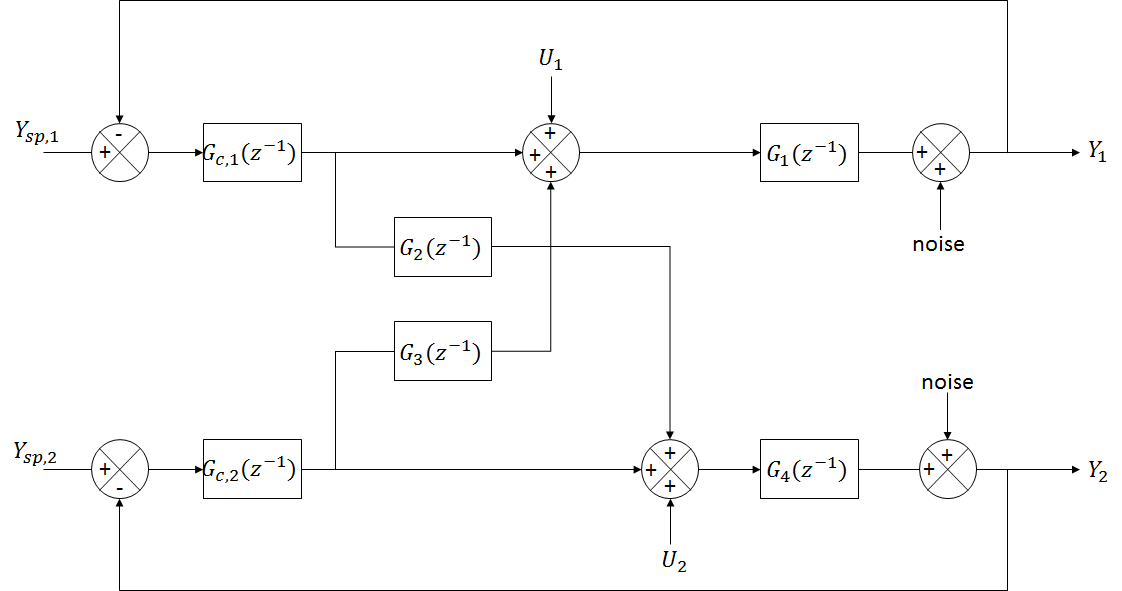
*A pseudorandom binary sequence (PRBS) as an input*

A PRBS signal shifts between two values such that its mean is zero. This signal is similar to a square wave except that its period is random. A PRBS signal can be shown to give consistent parameters in both online and offline identification. This is also apparent because this signal persistently excites the process and has no pattern at all. PRBS speed up identification i.e. faster convergence of the process parameters and hence this is an advantage during the start-up when the controller performance is a prime concern.

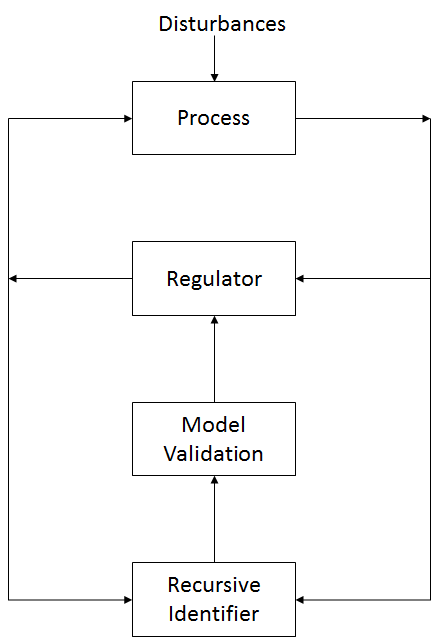
*Closed loop system.*

The closed loop system is relatively hard to identify, this is due to the input being dependent to the process variable as stated above. It is, however not impossible to identify a closed loop system. To solve the problem of the input being dependent on the output, a known disturbance is added to the controller output. This disturbance can be a step, an impulse or a PRBS signal dependent on the type of identification and the purpose of identification.

Most often, an online closed loop identification is done to find the optimum controller settings automatically, the quality of the identification in this case is not a prime concern. In this study, the main focus will be on the quality of the identification. The figure below represent the implementation of a discrete PID controller to the open loop process above.



The model above will be treated as an approximate to a real process where parameters may change with time, for example a process gain change that is due to fouling on the heat exchanger surface. The inputs to be sampled are; the first controller output, the second controller output and the two disturbances, the outputs will be sampled as well. The figure below illustrates the identification procedure that will typically be employed online in the industry.



*Model Validation*

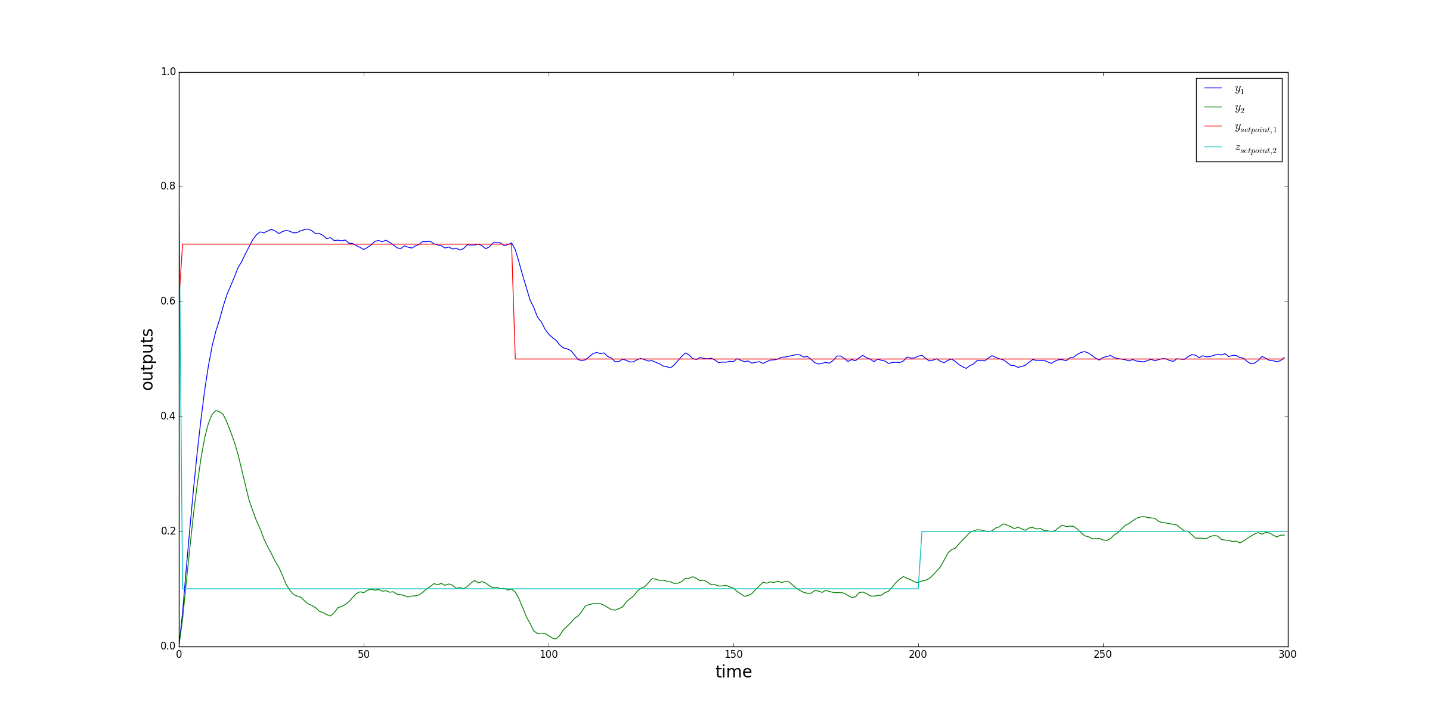
The subsequent stage after recursive identification is to decide if the model is adequate or not. A simple way would be to compare the estimated parameters to the true values but we would not have the true values in a real process. The only way we may validate the model is to compare the model output to the process output, an adequate model should explain all the patterns that are due to the input.

The model output will always deviate from the noise-free process output since the estimated parameters minimize the error between the model output and the process output.

*Controller*

The controller is not set to self-tuning mode, thus the controller parameters are constant throughout the whole run, this is because the process parameters are assumed to constant throughout. The controller parameters are summarized in the table below.

|  |  |  |
| --- | --- | --- |
|  | **τi** | **K** |
| GC,1(s) | 5 | 0.1 |
| GC,2(s) | 2 | 0.5 |



The figure shows the closed loop response to a step change in the setpoint, it can be seen that the controller keeps track of setpoint changes but this controller setting may not be the optimum setting. For closed loop simulations, this controller setting will be used.

*Dead Time*

Dead time is non-linear with respect to identification and therefore a linear identification cannot estimate the process dynamics and the dead time simultaneously and since this paper is restricted to linear identification, the dead time term will be intentionally omitted. Possible solutions to estimating the dead using linear identification involve:

* Identifying multiple process models, each with a different assumed value for dead-time and selecting the model with minimum error between the process output and model output.
* Employing a higher order model and then estimating the dead-time based on the significance of the identified parameters.

Dead time estimation hence introduces an addition step in process identification but does not really change the way at which the process dynamics are estimated.