BRAC University
MAT-215
Practice Sheet # 6

(Singularity & Residue Theorem)

1. Evaluate $\oint_C \frac{z^2}{2z^2+5z+2}dz$ using the residue at the poles, where C is the unit circle $|z|=1$.

2. Evaluate $\oint_C \frac{z^2+4}{z^3+2z^2+2z}dz$ using the residue at the poles, around the circle $|z|=3$.

3. Evaluate $\oint_C \frac{z\,e^{i\pi z}}{(z^2+2z+5)(z^2+1)^2}dz$ using the residue at the poles, where C is the upper half circle of the equation $|z|=2$.

4. Evaluate $\frac{1}{2\pi i}\oint_C \frac{z^2-z+2}{z^4+10z^2+9}dz$ using the residue at the poles, around the circle C with the equation $|z|=4$.

(Improper Real Integral)

1. Show that $\int_0^\infty \frac{\ln(x^2+1)}{x^2+1}dx = \pi \ln 2.$
2. Show that $\int_0^\infty \frac{1}{x^4+x^2+1}dx = \frac{\pi\sqrt{3}}{6}.$
3. Show that $\int_0^\infty \frac{1}{x^4+1}dx = \frac{\pi}{2\sqrt{2}}.$
4. Show that $\int_0^\infty \frac{\cos 2\pi x}{x^4+x^2+1}dx = \frac{-\pi}{2\sqrt{3}}e^{-\pi\sqrt{3}}.$