

**BRAC UNIVERSITY**  
**Department of Computer Science and Engineering**

Examination: Mid Semester Exam  
 Duration: 1 Hour 20 Minutes

Semester: Spring 2024  
 Full Marks: 35

**CSE 221: Algorithms**

Answer the following questions.  
 Figures in the right margin indicate marks.

|       |     |          |
|-------|-----|----------|
| Name: | ID: | Section: |
|-------|-----|----------|

- 1 a. Consider the following functions. 03  
 CO2

$$f_1(n) = \log(n!)$$

$$f_2(n) = n^2 \log_n(n^n)$$

$$f_3(n) = n^4 + 10n^2$$

$$f_4(n) = 17.13^n$$

$$f_5(n) = n \log n$$

$$f_6(n) = n * \sqrt[5]{n^2}$$

**Write** a correct asymptotic upper bound for each of the above and sort the functions in ascending order of their growth rate.

- b. **Write** the asymptotic time complexity of the following code snippet. Show your works/reasoning. 03  
 CO2

```
1. for i in range (1,n)
2.     j= 1
3.     while j < i*i
4.         j= j-1
```

- c. **Express** the following (either one of the two) running time  $T(n)$  with an asymptotic bound. 04  
 CO2

$$T(n) = 8T\left(\frac{n}{4}\right) + n\sqrt{n} \quad \text{Or} \quad T(n) = 2T(n-2) + 1$$

Any method is acceptable as long as you show calculations.

- 2 CO1 i. By showing necessary math, **explain** how Karatsuba's Fast Multiplication algorithm converts an 03  
 N-digit multiplication to three  $N/2$ -digit multiplications.  
 ii. Can we modify the algorithm to multiply two N-bit binary numbers? **Explain** how or why not. 02
- 3 a. **Write** the worst case time complexity of quick sort? **Illustrate** an array where the worst case of 04  
 CO1 quick sort occurs if the last element is chosen as pivot.

- b. Consider an array containing  $N$  unique values where for some index  $i$ , the values are in increasing order from index  $0$  to  $(i-1)$ , and then again from  $i$  to  $(N-1)$ . Moreover, it is guaranteed that all the values from index  $0$  to  $(i-1)$  are greater than all the values from  $i$  to  $(N-1)$ .

An example array is given below.

|       |   |    |    |   |   |   |   |   |
|-------|---|----|----|---|---|---|---|---|
| index | 0 | 1  | 2  | 3 | 4 | 5 | 6 | 7 |
| value | 9 | 12 | 15 | 2 | 4 | 5 | 7 | 8 |

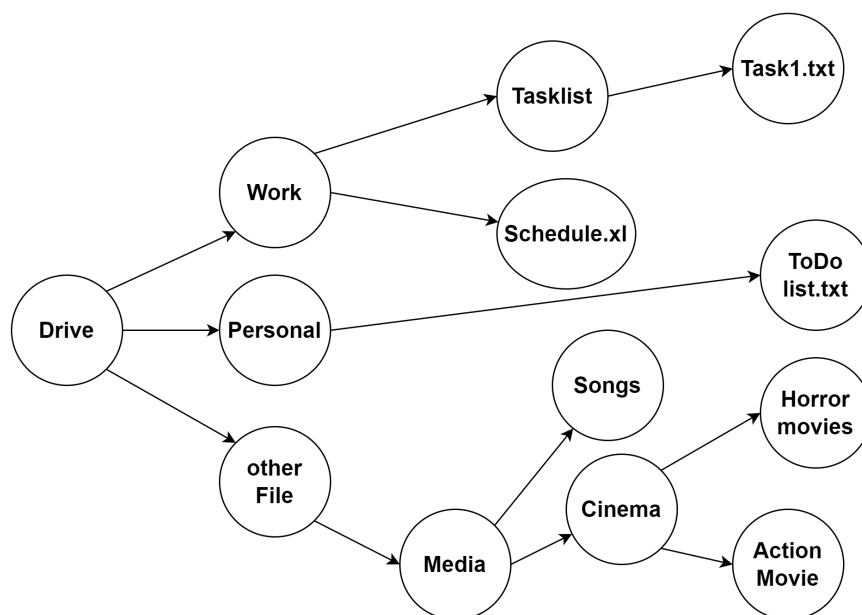
Here  $i=3$ , it means the values are in increasing order from index  $0$  to  $2$ , and then again from  $3$  to  $7$ . Also, all values from index  $0$  to  $2$  are greater than all values from  $3$  to  $7$  (it is guaranteed, no checking required).

Given such an array, propose an algorithm to find the index  $i$ .

- CO3 i) **Write** your algorithm with a code/pseudocode/flowchart/step-by-step instructions. 04  
CO2 ii) **Write** the time complexity of your algorithm. 02

- 4 CO1 You are a computer science student and you are given a file structure encoded in a graph. You want to make a navigator system where if a person wants to find a file they can enter the name of the file and your program will return the exact location of the file.

For example, consider the following graph:



If the user wants to find task1.txt; your program will give the following output:  
Drive->Work->Tasklist->task1.txt

If the user wants to find Horror Movies; your program will give the following output:  
Drive->other file->Media->Cinema->Horror Movies

Now answer the following questions:

- Give** the adjacency list representation of this graph. You can use either the whole name or a shorter version of each node. 03
- Write** the name of your preferred algorithm to solve the above mentioned problem (a navigator system). Explain your reasoning in brief. 02
- Show** a simulation of your presented solution with proper use of data structure and other necessary details to give the desired answers as the sample input shows. 05

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 CO2

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$$T(n) = 4T\left(\frac{n}{4}\right) + \sqrt{n} \quad \text{Or} \quad T(n) = T(n-3) + n$$

Any method is acceptable as long as you show calculations.

- 2 CO1 i. By showing necessary math, **explain** how Karatsuba's Fast Multiplication algorithm converts an N-digit multiplication to three N/2-digit multiplications. 03  
 ii. Can we modify the algorithm to multiply two N-digit hexadecimal numbers? **Explain** how or why not. 02

3 a. **Write** the worst case time complexity of quick sort? **Illustrate** an array where the worst case of quick sort occurs if the first element is chosen as pivot. **04**  
CO1

b. Consider an array containing  $N$  unique values where for some index  $i$ , the values are in decreasing order from index  $0$  to  $(i-1)$ , and then again from  $i$  to  $(N-1)$ . Moreover, it is guaranteed that all the values from index  $0$  to  $(i-1)$  are smaller than all the values from  $i$  to  $(N-1)$ .

An example array is given below.

|       |   |   |   |    |    |   |   |   |
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| index | 0 | 1 | 2 | 3  | 4  | 5 | 6 | 7 |
| value | 5 | 4 | 1 | 12 | 10 | 9 | 7 | 6 |

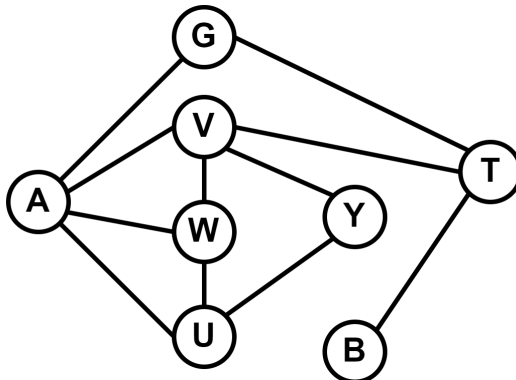
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Given such an array, propose an algorithm to find the index  $i$ .

CO3 i) **Write** your algorithm with a code/pseudocode/flowchart/step-by-step instructions. **04**

CO2 ii) **Write** the time complexity of your algorithm. **02**

4 CO1



- Give** the adjacency matrix representation of this graph. **03**
- Determine** whether the Graph is Bipartite/Bicolorable. Show a valid grouping/coloring of the vertices. **02**
- Show** a simulation of BFS algorithm with proper use of data structure and other necessary details to find the shortest path from A to B. **05**