Practice Sheet 1

Part A

1. Perform each of the indicated operations:

i) $(i-2)\{2(1+i)-3(i-1)\}$ ii) $\dfrac{(2+i)(3-2i)(1-i)}{(1-i)^2}$ iii) $(2i-1)^2\left(\dfrac{4}{1-i}+\dfrac{2-i}{1+i}\right)$

iv) $3\left(\dfrac{1+i}{1-i}\right)^2-2\left(\dfrac{1-i}{1+i}\right)^3$ v) $\dfrac{3i^{10}-i^{19}}{2i-1}$ vi) $\dfrac{i^4-i^9+i^{16}}{2-i^5+i^{10}-i^{15}}$

2. Show that, $(5+3i)+\{(-1+2i)+(7-5i)\}=\{(5+3i)+(-1+2i)\}+(7-5i)$ illustrates the associative law of addition.

3. If $z_1=1-i,\ z_2=-2+4i,\ z_3=\sqrt{3}-2i,$ evaluate each of the following:

(i) $|2z_2-3z_1|^2$ (ii) $\left|\dfrac{z_1+z_2+1}{z_1-z_2+i}\right|$ (iii) $\overline{(z_2+z_3)(z_1-z_3)}$ (iv) $\text{Re}\{2z_1^3+3z_2^3-5z_3^2\}$

(v) $\text{Im}\left\{\dfrac{z_1z_2}{z_3}\right\}$ (vi) $z_1^2+2z_1-3$ (vii) $|z_1\overline{z_2}+z_2\overline{z_1}|$ (viii) $\dfrac{1}{2}\left(\dfrac{z_3}{\overline{z_3}}+\dfrac{\overline{z_3}}{z_3}\right)$

4. Express each of the following complex number in polar form and show them graphically.

(i) $2+2\sqrt{3}i$ (ii) $2\sqrt{2}+2\sqrt{2}i$ (iii) $-2\sqrt{3}-2i$ (iv) $-1+\sqrt{3}i$

5. Prove that: (i) $\overline{z_1+z_2}=\overline{z_1}+\overline{z_2}$ (ii) $|z_1z_2|=|z_1||z_2|$ (iii) $|z_1\pm z_2|\le|z_1|+|z_2|$
(iv) $|z_1\pm z_2|\ge|z_1|-|z_2|$.

6. State and prove De Moivre's Theorem.

7. Evaluate each of the following by De Moivre's Theorem**:**

i) $\dfrac{(8\text{ cis }40°)^3}{(2\text{ cis }60°)^4}$ ii) $\dfrac{(3e^{i\pi/6})(2e^{-i5\pi/4})(6e^{5i\pi/3})}{(4e^{2i\pi/3})^2}$ iii) $(5\text{ cis }20°)(3\text{ cis }40°)$ iv) $(2\ c\text{is }50°)^6$

8. Find all the roots of the following equations.

i) $z^3=(-1+i)$ ii) $z^5=(-4+4i)$ iii) $z^4=-16i$ iv) $z^6=64i$ v) $z^4+z^2+1=0$

Part B

1. Perform the indicated operations analytically and graphically.
   i) $(2+3i)+(4-5i)$ ii) $(7+i)-(4-2i)$

2. Describe geometrically the set of points z satisfying the following conditions:
   i) $\mathrm{Re}(z) > 1$ ii) $|2z+3| > 4$
   iii) $1 < |z-2i| < 2$ iv) $|z+1-i| \leq |z-1+i|$
   v) $\mathrm{Re}(z) \geq 0$ vi) $\mathrm{Im}(z) \geq 0$
   vii) $|z-4| \geq |z|$ viii) $|z-2| \leq |z+2|$
   ix) $\mathrm{Re}(1/z) \leq 1/2$ x) $\pi/2 < \arg z < 3\pi/2,\ |z| > 2$

3. Using the properties of conjugate and modulus, show that:
   i. $\overline{\bar{z}+3i} = z-3i$
   ii. $\overline{iz} = -i\bar{z}$
   iii. $\left|(2\bar{z}+5)\left(\sqrt{2}-i\right)\right| = \sqrt{3}\,|2z+5|$
   iv. $|2z+3\bar{z}| \leq 4|\mathrm{Re}(z)| + |z|$
4. Find the modulus and argument of the following complex numbers:
   i) $\dfrac{2-i}{2+i}$ ii) $\dfrac{\sqrt{3}+i}{\sqrt{3}-i}$
   iii) $\left(\dfrac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^2$

5. Prove that $|z-i| = |z+i|$ represents a straight line.
6. Prove that $|z+2i| + |z-2i| = 6$ represents an ellipse.
7. Find an equation of a circle center at (2,3) with radius 3.
8. Sketch the region in xy-plane represented by the following set of points:
   i) $\mathrm{Re}(\bar{z}-1) = 2$ ii) $\mathrm{Im}(z^2) = 4$
   iii) $\left|\dfrac{2z-3}{2z+3}\right| = 1$ iv) $\mathrm{Re}(z) + \mathrm{Im}(z) = 0$