A

BRAC UNIVERSITY
Department of Computer Science and Engineering

Examination: Mid Semester Exam
Duration: 1 Hour 20 Minutes

Semester: Summer 2024
Full Marks: 35

CSE 221: Algorithms

Answer the following questions.
Figures in the right margin indicate marks.

Name:	ID:	Section:

1 **a.** **CO2** Analyze the code snippet and answer the following questions: **05**

```
int func(int n) {
    if (n <= 1) {
        return 1;
    } else {
        int a = 0;
        for (int i = 0; i < 3; i++) {
            a += func(n / 2);
        }
        int b = func2(n);
        return a + b;
    }
}

int func2(int n) {
    int c = 0;
    for (int k = n; k >= 1; k -= 4) {
        for (int m = 0; m < n; m += 2) {
            c += k + m;
        }
    }
    return c;
}
```

i. Write the recurrence relation of the following function "`func`".

ii. Calculate the time complexity of the recurrence relation using any of the methods.

b. **CO2** For each of these pairs of functions $(f(n), g(n))$, identify whether $f(n) = O(g(n))$ or $f(n) = ⍰(g(n))$. **03**

Show your calculation logic in the script and **Tick only the most specific one** of the two options in each case in the question paper. ***Marks for this question depend on the calculation shown in your scripts.***

$f(n)$	$g(n)$	$f(n) = O(g(n))$	$f(n) = ⍰(g(n))$
3^n	$2^n . n^4$	□	□
$(\sqrt{n} + n)(30\sqrt{n})$	n^2	□	□

c. **CO2** Consider the recurrence relation $T(n) = T(\sqrt{n}) + O(n)$. Can we solve this using the Master Theorem? **Explain** whether the structure of this recurrence fits the requirements of the Master Theorem. **02**

2 **a.** Your friend gave you an integer (`key`) and asked you to find its integer square root without using built-in functions like `sqrt()` or exponentiation. For example, if the input is 25, the output should be 5 because $5^2 = 25$. If the square root is not an integer, return the floor value of the actual square root. For example, for an input of 10, the output should be 3.

Your friend's initial approach is to use linear search (time complexity $O(n)$) as shown below.

```
def LinearSearchToFindSquareRoot(key):
     result = -1
     for i in range(1, key, 1):
         if i * i <= key:
             result = i
     return result
```

Now to find the square root he is working on a possible solution **P,** which will result in time complexity $O(\sqrt{n})$. As an algorithm student, you propose another algorithm **R** within time complexity $O(log_2 N)$, which is much faster than $O(\sqrt{n})$.

CO2 i. Present the **solution your friend** was trying to achieve **(Algorithm P)** with pseudocode/programmable code/step-by-step logical instructions. **03**

CO3 ii. Present your proposed solution **(Algorithm R)** with pseudocode/programmable code/step-by-step logical instructions. **04**

3 **a.** **CO2** Imagine yourself helping Taylor Swift following her "Eras Tour first show." Taylor is curious about her performance and the groups of songs her audience connected most with them. She receives comments for every song on her setlist: some calm the audience down a bit (***and have a negative score***), while others get the crowd quite thrilled (***and have a positive score***).

Taylor can choose any song to start her performance, but she likes to keep the flow in order. So the next song she sings has to be the one right after she has chosen the starting song on the list, regardless of the public reactions**.**

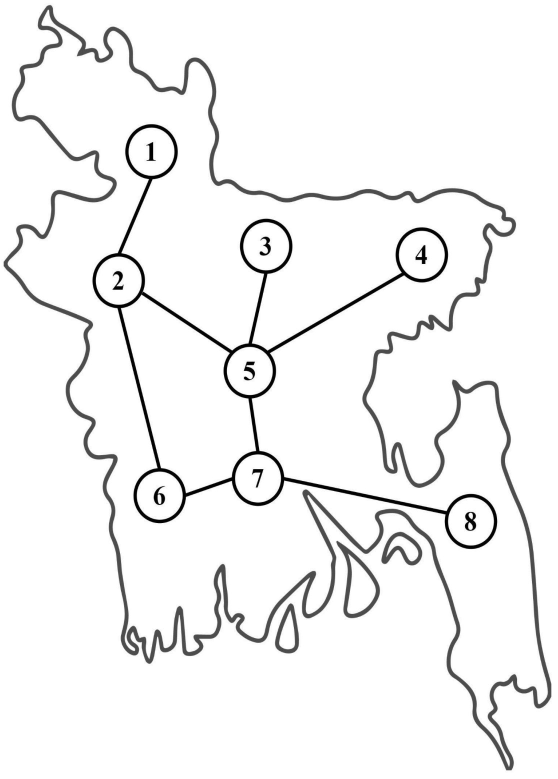
Given a list of scores for each song (indicating how much they boost or calm the crowd), can you:

[-3, 7, 12, -8, -2, 83, -7, 4]

i. Help Taylor determine which segment of consecutive songs she should perform at her next concerts to maximize audience interest. What is the total comment score of this segment? **Simulate** with proper steps. **05**

ii. **Explain** the time taken by your preferred method, particularly given the *"n"* numbers of songs Taylor has in her setlist. Ideally, your method should not exceed $O(nlgn)$. **03**

4 The "Road Transport and Highways Division" of Bangladesh is working on a project where they want to establish six-lane highways among the 8 divisions of Bangladesh. Currently, there are 8 highways connecting these 8 divisions as given in the map below.



MAP OF BANGLADESH

1	**Rangpur**	**5**	**Dhaka**
2	**Rajshahi**	**6**	**Khulna**
3	**Mymensingh**	**7**	**Barisal**
4	**Sylhet**	**8**	**Chattogram**

Fig 1: Graph For Question 4

CO1 i. **Create** a graph representation for the given map so that it does not take more than $O(|V| + |E|)$ space/memory. **02**

CO2 ii. **Determine** how many additional highways we need to make sure that every division is directly connected with each other. **02**

CO2 iii. In a meeting, the director looked at the map and wondered if the divisions of Bangladesh could be split into two groups (**A** and **B**) such that all highways directly connect locations in Group A to Group B, with no highways between locations within the same group. Using an appropriate data structure, simulate the algorithm that can determine if such a division is possible. **04**

CO2 iv. Based on the given graph in Fig 1, **explain** how you can find out if there is an odd number cycle present in the graph. An odd number cycle is a cycle that has an odd number of vertices**.** **02**

For your convenience, Figure 1 is repeated below:

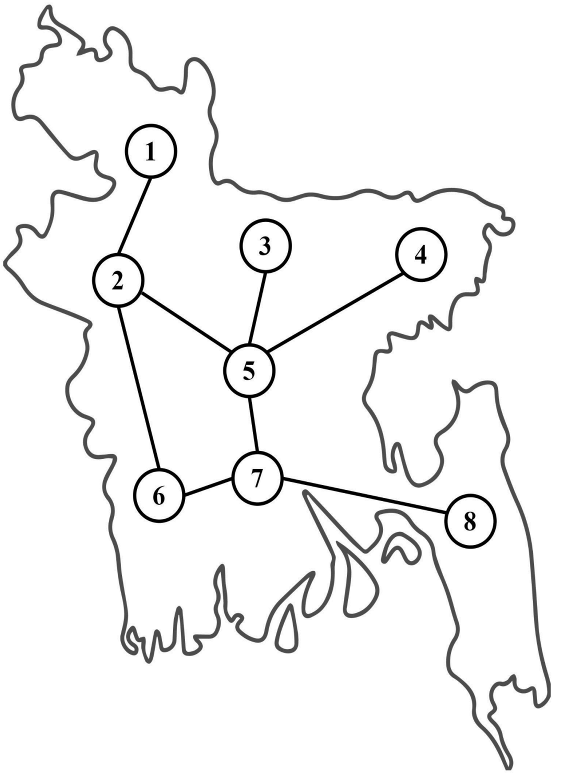


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Fig 1: Graph For Question 4

END OF QUESTIONS

B

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        int b = func2(n);
        return a + b;
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int func2(int n) {
    int c = 0;
    for (int k = n; k >= 1; k -= 3) {
        for (int m = 0; m < n; m += 2) {
            c += k + m;
        }
    }
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```

i. Write the recurrence relation of the following function "`func`".

ii. Calculate the time complexity of the recurrence relation using any of the methods.

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Show your calculation logic in the script and **Tick only the most specific one** of the two options in each case in the question paper. ***Marks for this question depend on the calculation shown in your scripts***.

$f(n)$	$g(n)$	$f(n) = O(g(n))$	$f(n) =$ ⍰$(g(n))$
3^n	$2^n.n^{10}$	□	□
$(\sqrt{n} + n)(30\sqrt{n})$	$\sqrt{n}$	□	□

c. **CO2** Consider the recurrence relation $T(n) = 4T(\sqrt[3]{n}) + O(n)$. Can we solve this using the Master Theorem? **Explain** whether the structure of this recurrence fits the requirements of the Master Theorem. **02**

2 **a.** Your friend gave you an integer (`key`) and asked you to find its integer square root without using built-in functions like `sqrt()` or exponentiation. For example, if the input is 49, the output should be 7 because $7^2 = 49$. If the square root is not an integer, return the floor value of the actual square root. For example, for an input of 26, the output should be 5.

Your friend's initial approach is to use linear search (time complexity $O(n)$) as shown below.

```
def LinearSearchToFindSquareRoot(key):
     result = -1
     for i in range(1, key, 1):
         if i * i <= key:
              result = i
     return result
```

Now to find the square root, he is working on a possible solution **P,** which will result in time complexity $O(\sqrt{n})$. As an algorithm student, you propose another algorithm **R** within time complexity $O(log_2 N)$, which is much faster than $O(\sqrt{n})$

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3 **a.** **CO2** Imagine you're helping Dr. Eleanor, who's studying old papers about a long-gone city. Each paper talks about a year in the city's life. Some events brought prosperity to the civilization (***represented by a positive number***), while others brought decline and challenges (***represented by a negative number***).

Dr. Eleanor believes it is essential to study events in a continuous sequence to fully understand the rise and fall of civilizations. She can start with any manuscript, but once she begins, she must analyze the remaining manuscripts sequentially to maintain the timeline,

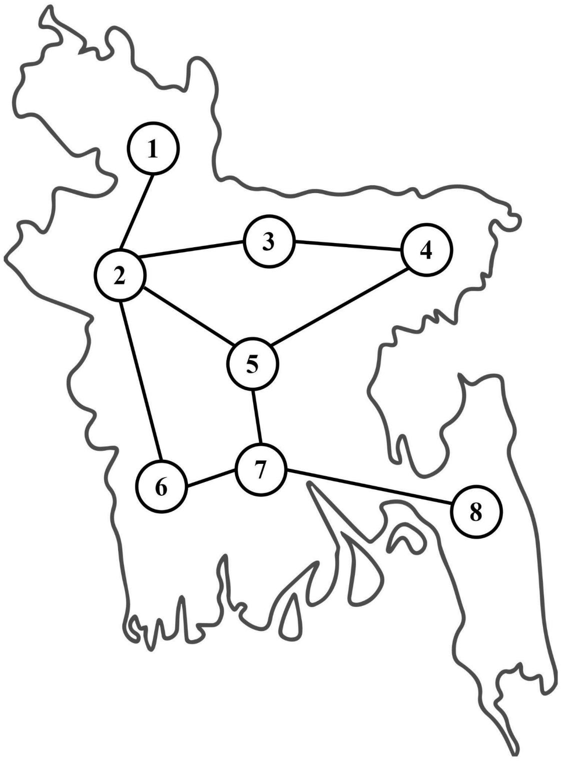
Given a list of numbers for each manuscript (indicating the prosperity or decline they brought), can you:

[2, -6, 8, 8, -9, -1, -2, 20]

a. Tell Dr. Eleanor which group of years she should read about to learn of the city's best times. How good were those years combined? Show in detail how you found this out. **05**

b. Explain the time taken by your preferred method, particularly given the "*n*" numbers of manuscripts Dr. Eleanor has in her studies. It should be an easy and quick method, as she has many papers to go through! **03**

4 The "Road Transport and Highways Division" of Bangladesh is working on a project where they want to establish six-lane highways among the 8 divisions of Bangladesh. Currently, there are 9 highways connecting these 8 divisions as given in the map below.



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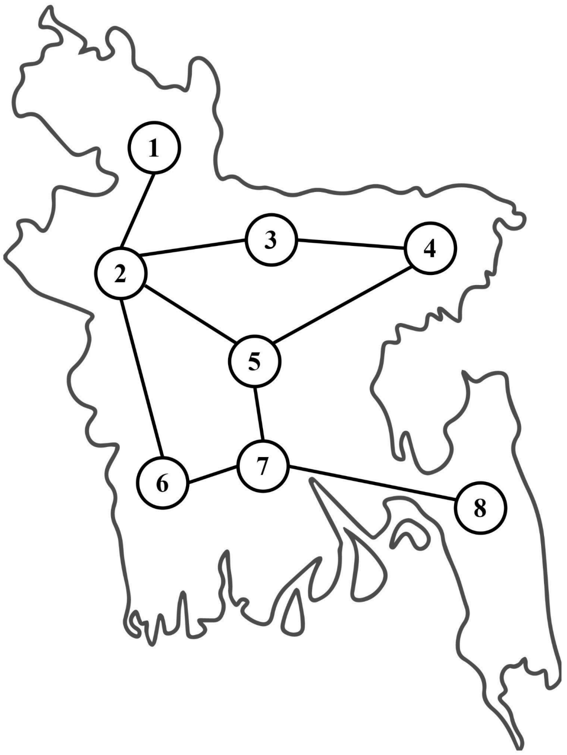

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