**BRAC University**
**MAT-215**
**Practice Sheet # 4**

1. Evaluate $\int_{(0,1)}^{(2,5)} (3x+y)dx+(2y-x)dy$ along

   (a) the curve $y=x^2+1$

   (b) the straight line joining $(0,1)$ and $(2,5)$

   (c) the straight lines from $(0,1)$ to $(0,5)$ and then from $(0,5)$ to $(2,5)$

   (d) the straight lines from $(0,1)$ to $(2,1)$ and then from $(2,1)$ to $(2,5)$.

2. Evaluate $\oint_C (x+2y)\,dx+(y-2x)dy$ around the ellipse $C$ defined by $x=4\cos\theta$, $y=3\sin\theta,\ 0\le\theta\le 2\pi$ if $C$ is described in a counterclockwise direction.

3. Evaluate $\int_C (x^2-iy^2)\,dz$ along

   (a) the parabola $y=2x^2$ from $(1,2)$ to $(2,8)$

   (b) the straight lines from $(1,2)$ to $(2,2)$ and then from $(2,2)$ to $(2,8)$

   (c) the straight line from $(1,2)$ to $(2,8)$.

4. Evaluate $\oint_C |z|^2 dz$ around the square with vertices at $(0,0), (1,0), (1,1), (0,1)$.

5. Evaluate $\int_C (z^2+3z)dz$

   (a) along the circle $|z|=2$ from $(2,0)$ to $(0,2)$ in a counter clockwise direction.

   (b) the straight line from $(2,0)$ to $(2,2)$ and then from $(2,2)$ to $(0,2)$.

6. Evaluate $\int_{i}^{2-i} (3xy+iy^2)dz$

   (a) along the straight line joining $z=i$ and $z=2-i$

   (b) along the parabola $x=2t-2,\ y=1+t-t^2$.

7. Evaluate $\oint_C (\bar{z})^2 dz$ around the circles (a) $|z|=1$ and (b) $|z-1|=1$.

8. Evaluate $\oint_C \frac{dz}{z-2}$ around (a) the circle $|z-2|=4$ (b) the circle $|z-1|=9$.

9. Evaluate $\oint_C (5z^4 - z^3 + 2)\,dz$ around the circle $|z|=1$.

10. (a) Show that the line integral $\int_C (6z^2 + 8i\,z)dz$ is independent of path where C is

the curve given by $y = x^3 - 3x^2 + 4x - 1$.

(b) Hence or otherwise evaluate the line integral in Question (a) along the curve C joining the points (3,4) and (4,-3).

11. (a) Show that the line integral $\int_C (12z^2 - 4i\,z)dz$ is independent of path where C is

the curve given by $y = x^3 - 3x^2 + 4x - 1$

(b) Hence or otherwise evaluate the line integral in Question (a) along the curve C joining the points (1,1) and (2,3).