Home work: 05

Mathematics III

Problems based on Taylor, Laurent's series and residue[1]

1. Expand each of the following functions in a Taylor series about the indicated points

(a) e^{-z} at $z = 0$

(b) $\cos z$ at $z = \dfrac{\pi}{2}$

(c) $z^3 - z^2 + 4z - 2$ at $z = 2$

(d) ze^z at $z = -1$.

2. Expand $f(z) = \dfrac{z}{(z-1)(2-z)}$ in a Laurent series valid for

(a) $|z| < 1$

(b) $1 < |z| < 2$

(c) $|z| > 2$

(d) $|z - 1| > 1$

(e) $0 < |z - 2| < 1$.

3. Evaluate $\displaystyle\oint_C \frac{z^2}{2z^2 + 5z + 2}\, dz$ using the residue at the poles, where C is the unit circle $|z| = 1$.

4. Evaluate $\displaystyle\oint_C \frac{z^2 + 4}{z^3 + 2z^2 + 2z}\, dz$ using the residue at the poles around the circle $|z| = 3$.

5. Evaluate $\dfrac{1}{2\pi i}\displaystyle\oint_C \frac{z^2 - z + 2}{z^4 + 10z^2 + 9}\, dz$ using the residue at the poles around the circle $|z| = 4$.

6. Evaluate $\displaystyle\oint_C \frac{ze^{i\pi z}}{(z^2 + 2z + 5)(z^2 + 1)^2}\, dz$ using the residue at the poles where C is the upper half of the circle $|z| = 2$.

[1] Follow the class lectures