

**BRAC University**  
**MAT-215**  
**Practice Sheet # 7**

1. Find the Laplace transformation of each of the following function:

$$(i) 3e^{-2t} \quad (ii) 4t^3 - e^{-t} \quad (iii) 7\sin 2t - 3\cos 2t \quad (iv) (t^2 + 1)^2 \quad (v) (4e^{2t} - 2)^3.$$

2. Evaluate each of the following:

$$(i) \Lambda \{t^3 e^{-3t}\} \quad (ii) \Lambda \{5e^{3t} \sin 4t\} \quad (iii) \Lambda \{(t+2)^2 e^t\} \quad (iv) \Lambda \{e^{-t} (3\sinh 2t - 5\cosh 2t)\} \\ (v) \Lambda \{e^{-4t} \cosh 2t\} \quad (vi) \Lambda \{e^{2t} (3\sin 4t - 4\cos 4t)\}.$$

3. Determine each of the following:

$$(i) \Lambda^{-1} \left\{ \frac{12}{4-3s} \right\} \quad (ii) \Lambda^{-1} \left\{ \frac{2s-5}{s^2-9} \right\} \quad (iii) \Lambda^{-1} \left\{ \frac{23s-15}{s^2+8} \right\} \quad (iv) \Lambda^{-1} \left\{ \frac{1}{s^{3/2}} \right\} \\ (v) \Lambda^{-1} \left\{ \frac{s+1}{s^{4/3}} \right\} \quad (vi) \Lambda^{-1} \left\{ \frac{1}{s^4} \right\} \quad (vii) \Lambda^{-1} \left\{ \frac{1}{\sqrt{2s+3}} \right\}.$$

4. Evaluate each of the following using partial fraction:

$$(i) \Lambda^{-1} \left\{ \frac{6s-4}{s^2-4s+20} \right\} \quad (ii) \Lambda^{-1} \left\{ \frac{4s+12}{s^2+8s+16} \right\} \quad (iii) \Lambda^{-1} \left\{ \frac{2s^2-4}{(s+1)(s-2)(s-3)} \right\} \\ (iv) \Lambda^{-1} \left\{ \frac{5s^2-15s-11}{(s+1)(s-2)^3} \right\} \quad (v) \Lambda^{-1} \left\{ \frac{3s+1}{(s^2+1)(s-1)} \right\} \quad (vi) \Lambda^{-1} \left\{ \frac{2s^2-4}{(s+1)(s-2)(s-3)} \right\} \\ (vii) \Lambda^{-1} \left\{ \frac{s^2+2s+3}{(s^2+2s+2)(s^2+2s+5)} \right\}.$$

5. Solve the given differential equation:

$$(i) Y'' - 3Y' + 2Y = 4e^{2t}, \quad Y(0) = -3, \quad Y'(0) = 5 \\ (ii) Y'' + 9Y = \cos 2t, \quad Y(0) = 1, \quad Y(\pi/2) = -1 \\ (iii) Y'' + 2Y' + 5Y = e^{-t} \sin t, \quad Y(0) = 0, \quad Y'(0) = 1 \\ (iv) Y''' - 3Y'' + 3Y' - Y = e^t t^2, \quad Y(0) = 0, \quad Y'(0) = 0, \quad Y''(0) = -2.$$

**Laplace Transformation of some elementary functions:**

	$F(t)$	$L\{F(t)\} = f(s)$
1.	1	$\frac{1}{s}, s > 0$
2.	$t$	$\frac{1}{s^2}, s > 0$
3.	$t^n$ $n = 0, 1, 2, \dots$	$\frac{n!}{s^{n+1}}, s > 0$
4.	$e^{at}$	$\frac{1}{s-a}, s > a$
5.	$\sin at$	$\frac{a}{s^2 + a^2}, s > 0$
6.	$\cos at$	$\frac{s}{s^2 + a^2}, s > 0$
7.	$\sinh at$	$\frac{a}{s^2 - a^2}, s >  a $
8.	$\cosh at$	$\frac{s}{s^2 - a^2}, s >  a $

## Inverse Laplace Transformation

	$f(s)$	$L^{-1}\{f(s)\} = F(t)$
1.	$\frac{1}{s}$	1
2.	$\frac{1}{s^2}$	t
3.	$\frac{1}{s^{n+1}}, \quad n = 0,1,2, \dots$	$\frac{t^n}{n!}$
4.	$\frac{1}{s-a}$	$e^{at}$
5.	$\frac{1}{s^2+a^2}$	$\frac{\sin at}{a}$
6.	$\frac{s}{s^2+a^2}$	$\cos at$
7.	$\frac{1}{s^2-a^2}$	$\frac{\sinh at}{a}$
8.	$\frac{s}{s^2+a^2}$	$\cosh at$
9.	$\frac{1}{s^{n+1}}, n > -1$	$\frac{t^n}{(n+1)!}, \quad n > -1$