Mid Term Exam

* Indicates required question

1. Email *

There are 2 parts in the exam.
part 1 contains 13 questions.
part 2 contains 5 questions.
You need to answer any 12 out of these 18 questions.
However you may answer more than 12 questions as well, in which case best 12 will be counted.
Each question carries 1 mark.

2. Enter your CSE230 section number *

3. Enter your student ID *

part 1

Read the instructions carefully.

There are 3 types of questions -- MCQ, MSQ & NV.

For Multiple **choice** Questions (**MCQ**)
Each question should have exactly one correct answer.
If you select the correct answer, you will get 1 mark.
Otherwise you will get 0 mark.

For Multiple **selection** Questions (**MSQ**)
Each question may have one or multiple correct answer(s). Say, we have *t* correct answers.
If you select any of the incorrect answers, you will get 0 mark.
Otherwise your mark will be *p*/*t* upon selecting *p* correct answers.

For Numerical Value questions (**NVQ**)
There might be multiple correct answers, however you **must** submit exactly one of them.
If your answer is correct, you will get 1.
Otherwise you will get 0.

Each question, be it **MCQ**, **MSQ** or **NVQ**, carries 1 mark.

4. MCQ

Let p, q be two propositions. Then find the truth value in "??" .

$p \vee q$	$p \wedge q$	$p \oplus q$
T	F	??

Mark only one oval.

- True
- False
- Insufficient information is given

5. MSQ

Let x, y, z be the last three digits of your Student ID.
[For example, if your student ID is 12345678, then $x = 6$, $y = 7$, $z = 8$]
Also consider the following propositions:
p**:** $(y + z)$ is **even**
q**:** $(y \times z)$ is **odd**
Which of the following proposition(s) is/are true?

Check all that apply.

- [ ] $p \rightarrow q$
- [ ] converse of $p \rightarrow q$
- [ ] contrapositive of $p \rightarrow q$
- [ ] inverse of $p \rightarrow q$

6. MSQ

Let the domain of x consists of all the people in the world.
Consider the following predicates:
P(x**):** x is miser
Q(x**):** x is rich
Which of the following statements represent the **negation** of: "Every rich person is miser"?

Check all that apply.

- [ ] $\neg\forall x\ (P(x) \land Q(x))$
- [ ] $\neg\forall x\ (P(x) \rightarrow Q(x))$
- [ ] $\neg\exists x\ (P(x) \land Q(x))$
- [ ] $\exists x\ (P(x) \land Q(x))$
- [ ] $\exists x\ (P(x) \land \neg Q(x))$

7. NVQ

How many values amount to **False (F)** in the last column of the following truth table?

p	q	r	$(p \rightarrow (r \oplus q))$
F	F	F	
F	F	T	
F	T	F	
F	T	T	
T	F	F	
T	F	T	
T	T	F	
T	T	T	

8. NVQ

Let x, y, z be the last three digits of your Student ID.
[For example, if your student ID is 12345678, then $x = 6, y = 7, z = 8$]
Let $w = 100 \times x + 10 \times y + z + 999$

$$(w)_{230} = (?)_{10}$$

9. NVQ

The following information is given about three integers p, q and t:

- $GCD(p, q) = 12$
- $GCD(q, t) = 16$
- $LCM(p, q) = 336$
- $LCM(q, t) = 240$
- $p \times t = 6720$

Find the value of q.

10. MSQ

Let x, y, z be the last three digits of your Student ID.
[For example, if your student ID is 12345678, then $x = 6, y = 7, z = 8$]
Let a, b, c be three positive integers and let $a|b$ and $b|c$. Then which of the following is/are true?

a. $a \mid ax - by + cz$
b. $a^2 \mid bc$
c. $ab \mid 3c^2 - bcy$
d. $c^2 \mid ax + by + cz$

Check all that apply.

- [ ] (a)
- [ ] (b)
- [ ] (c)
- [ ] (d)

11. NVQ

Let x, y, z be the last three digits of your Student ID.
[For example, if your student ID is 12345678, then $x = 6, y = 7, z = 8$]
Let $v = 40 + z$
Then find the value of $11^v (mod\ 13)$.

12. MSQ

Let x, y, z be the last three digits of your Student ID.
[For example, if your student ID is 12345678, then $x = 6, y = 7, z = 8$]
Let $w = 10 \times y + z + 99$
What is/are the possible remainder(s) when $\left(w^{300} - w^{100} + w\right)$ is divided by 3?

Check all that apply.

- [ ] 0
- [ ] 1
- [ ] 2
- [ ] 3

13. NVQ

Let x, y, z be the last three digits of your Student ID.
[For example, if your student ID is 12345678, then $x = 6$, $y = 7$, $z = 8$]
What is the degree of the following recurrence relation?

$$5a_{n+x+1} + 6a_{n+y} = 3a_n$$

14. MSQ

Which of the following recurrence relations is/are equivalent to $a_n = a_{n-1} + 2^n$?

a. $a_n = a_{n-2} + 2^{n+1}$

b. $a_n = a_{n-2} + 2^n$

c. $a_n = a_{n-2} + 2^{n-1}$

d. $a_n = a_{n-2} + 2^n + 2^{n-1}$

Check all that apply.

- [ ] (a)
- [ ] (b)
- [ ] (c)
- [ ] (d)

15. MSQ

Let x, y, z be the last three digits of your Student ID.
[For example, if your student ID is 12345678, then $x = 6, y = 7, z = 8$]
Which statement(s) is/are correct about the following recurrence relation?

$$12a_{n+4} + 2a_{n+2} - z \times n^{y} = 12a_{n+1} - 38a_{n+3}$$

a. The relation is non-homogeneous
b. The relation is non-linear
c. Homogeneous part of the relation has 3 characteristics roots
d. 3 is a characteristics root of the homogeneous part of the relation

Check all that apply.

- [ ] (a)
- [ ] (b)
- [ ] (c)
- [ ] (d)

16. NVQ

Let x, y, z be the last three digits of your Student ID.
[For example, if your student ID is 12345678, then $x = 6, y = 7, z = 8$]
Let $a_n = 2a_{n-1} + 3$ and $a_0 = 3$
Find the value of a_{z+17}

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part 2

Read the instructions carefully.

There are 3 types of questions -- MCQ, MSQ & NV.

For multiple **choice** questions (**MCQ**)
Each question should have exactly one correct answer.
If you select the correct answer, you will get 1 mark.
Otherwise you will get 0 mark.

For multiple **selection** questions (**MSQ**)
Each question may have one or multiple correct answer(s). Say, we have t correct answers.
If you select any of the incorrect answers, you will get 0 mark.
Otherwise your mark will be p/t upon selecting p correct answers.

For numerical value questions (**NVQ**)
There might be multiple correct answers, however you **must** submit exactly one of them.
If your answer is correct, you will get 1.
Otherwise you will get 0.

Each question, be it **MCQ**, **MSQ** or **NVQ**, carries 1 mark.

17. MSQ

In the Basis step, There is/are error(s) in:

Prove by induction that,

$P(n)$: $1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + \cdots\cdots\cdots + n \times 2^n = 2 + (n-1) \times 2^{n+1}$

Proof:

Basis Step:

To prove the base, we should start with $n = 2$ ________________(**Line 1**)

for $n = 2$, L.H.S of $P(n = 2) = 2 \times 2^2 = 8$ ________________(**Line 2**)

for $n = 2$, R.H.S of $P(n = 2) = (2 - 1) \times 2^{2+1} = 8$ ________________(**Line 3**)

$\Rightarrow P(n = 2)$ is true.

Inductive Step:

Let's assume that, $P(n = m)$ is true for some $m \in R$ ________________(**Line 4**)

$\Rightarrow 1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + \cdots + m \times 2^m = 2 + (m-1) \times 2^{m+1}$ ....**(i)**

Using **(i)**, we have to show that, $p(n = m + \square)$is also true ________________(**Line 5**)

$\Rightarrow 1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + \ldots\ldots + m \times 2^m + (m+1) \times 2^{m+1} = 2 + (m+1) \times 2^{m+2}$ ......**(ii)** __(**Line 6**)

Now,

L.H.S of **(ii)** $= 1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + \ldots\ldots + m \times 2^m + (m+1) \times 2^{m+1}$

L.H.S of **(ii)** $= 2 + (m-1) \times 2^{m+1} + (m+1) \times 2^{m+1}$ [from **(i)**] ____________(**Line 7**)

L.H.S of **(ii)** $= 2 + 2^{m+1}(m - 1 + m + 1)$ ________________(**Line 8**)

L.H.S of **(ii)** $= 2 + (m+1) \times 2^{m+2}$ ________________(**Line 9**)

L.H.S of **(ii)** = R.H.S of **(ii)**

So, Our claim is true for all positive integers n.

Check all that apply.

- [ ] Line 1
- [ ] Line 2
- [ ] Line 3

18. NVQ

In line 5, determine the numerical value in the empty box.

Prove by induction that,

$P(n)$: $1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + \cdots\cdots\cdots + n \times 2^n = 2 + (n-1) \times 2^{n+1}$

Proof:

Basis Step:

To prove the base, we should start with $n = 2$ ________________ (**Line 1**)

for $n = 2$, L.H.S of $P(n = 2) = 2 \times 2^2 = 8$ ________________ (**Line 2**)

for $n = 2$, R.H.S of $P(n = 2) = (2 - 1) \times 2^{2+1} = 8$ ________________ (**Line 3**)

$\Rightarrow P(n = 2)$ is true.

Inductive Step:

Let's assume that, $P(n = m)$ is true for some $m \in R$ ________________ (**Line 4**)

$\Rightarrow 1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + \cdots + m \times 2^m = 2 + (m-1) \times 2^{m+1}$ ....**(i)**

Using **(i)**, we have to show that , $p(n = m + \square)$ is also true ________________ (**Line 5**)

$\Rightarrow 1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + \ldots\ldots + m \times 2^m + (m+1) \times 2^{m+1} = 2 + (m+1) \times 2^{m+2}$ ......**(ii)** __(**Line 6**)

Now,

L.H.S of **(ii)** $= 1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + \ldots\ldots + m \times 2^m + (m+1) \times 2^{m+1}$

L.H.S of **(ii)** $= 2 + (m-1) \times 2^{m+1} + (m+1) \times 2^{m+1}$ [from **(i)**] ____________ (**Line 7**)

L.H.S of **(ii)** $= 2 + 2^{m+1}(m - 1 + m + 1)$ ________________ (**Line 8**)

L.H.S of **(ii)** $= 2 + (m+1) \times 2^{m+2}$ ________________ (**Line 9**)

L.H.S of **(ii)** = R.H.S of **(ii)**

So, Our claim is true for all positive integers n.

19. MCQ

In the Inductive step, between Line 4 and Line 6:

Prove by induction that,

$P(n)$: $1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + \cdots\cdots\cdots + n \times 2^n = 2 + (n-1) \times 2^{n+1}$

Proof:

Basis Step:

To prove the base, we should start with $n = 2$ ________________ (**Line 1**)

for $n = 2$, L.H.S of $P(n = 2) = 2 \times 2^2 = 8$ ________________ (**Line 2**)

for $n = 2$, R.H.S of $P(n = 2) = (2 - 1) \times 2^{2+1} = 8$ ________________ (**Line 3**)

$\Rightarrow P(n = 2)$ is true.

Inductive Step:

Let's assume that, $P(n = m)$ is true for some $m \in R$ ________________ (**Line 4**)

$\Rightarrow 1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + \cdots + m \times 2^m = 2 + (m - 1) \times 2^{m+1}$ ……………..**(i)**

Using **(i)**, we have to show that , $p(n = m + \square)$is also true ________________ (**Line 5**)

$\Rightarrow 1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + \ldots\ldots + m \times 2^m + (m+1) \times 2^{m+1} = 2 + (m+1) \times 2^{m+2}$ ……**(ii)** __**(Line 6)**

Now,

L.H.S of **(ii)** $= 1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + \ldots\ldots + m \times 2^m + (m+1) \times 2^{m+1}$

L.H.S of **(ii)** $= 2 + (m-1) \times 2^{m+1} + (m+1) \times 2^{m+1}$ [from **(i)**] ___________ (**Line 7**)

L.H.S of **(ii)** $= 2 + 2^{m+1}(m - 1 + m + 1)$ ________________ (**Line 8**)

L.H.S of **(ii)** $= 2 + (m+1) \times 2^{m+2}$ ________________ (**Line 9**)

L.H.S of **(ii)** = R.H.S of **(ii)**

So, Our claim is true for all positive integers n.

Mark only one oval.

- Line 4 has error, Line 6 does not
- Line 6 has error, Line 4 does not
- Both Line 4 and Line 6 have errors
- Both Line 4 and Line 6 are correct

20. MCQ

Is there any error in Line 7?

Prove by induction that,

$P(n)$: $1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + \cdots\cdots\cdots + n \times 2^n = 2 + (n-1) \times 2^{n+1}$

Proof:

Basis Step:

To prove the base, we should start with $n = 2$ ________________(**Line 1**)

for $n = 2$, L.H.S of $P(n = 2) = 2 \times 2^2 = 8$ ________________(**Line 2**)

for $n = 2$, R.H.S of $P(n = 2) = (2-1) \times 2^{2+1} = 8$ ________________(**Line 3**)

$\Rightarrow P(n = 2)$ is true.

Inductive Step:

Let's assume that, $P(n = m)$ is true for some $m \in R$ ________________(**Line 4**)

$\Rightarrow 1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + \cdots + m \times 2^m = 2 + (m-1) \times 2^{m+1}$ ....**(i)**

Using **(i)**, we have to show that , $p(n = m + \square)$is also true ________________(**Line 5**)

$\Rightarrow 1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + \ldots\ldots + m \times 2^m + (m+1) \times 2^{m+1} = 2 + (m+1) \times 2^{m+2}$ ......**(ii)** __**(Line 6)**

Now,

L.H.S of **(ii)** $= 1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + \ldots\ldots + m \times 2^m + (m+1) \times 2^{m+1}$

L.H.S of **(ii)** $= 2 + (m-1) \times 2^{m+1} + (m+1) \times 2^{m+1}$ [from **(i)**] ____________(**Line 7**)

L.H.S of **(ii)** $= 2 + 2^{m+1}(m - 1 + m + 1)$ ________________(**Line 8**)

L.H.S of **(ii)** $= 2 + (m+1) \times 2^{m+2}$ ________________(**Line 9**)

L.H.S of **(ii)** = R.H.S of **(ii)**

So, Our claim is true for all positive integers n.

Mark only one oval.

- Yes
- No

21. MCQ

Choose the correct answer after comparing the right hand sides of Line 8 and Line 9:

Prove by induction that,

$P(n)$: $1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + \cdots\cdots\cdots + n \times 2^n = 2 + (n-1) \times 2^{n+1}$

Proof:

Basis Step:

To prove the base, we should start with $n = 2$ ________________ **(Line 1)**

for $n = 2$, L.H.S of $P(n = 2) = 2 \times 2^2 = 8$ ________________ **(Line 2)**

for $n = 2$, R.H.S of $P(n = 2) = (2 - 1) \times 2^{2+1} = 8$ ________________ **(Line 3)**

$\Rightarrow P(n = 2)$ is true.

Inductive Step:

Let's assume that, $P(n = m)$ is true for some $m \in R$ ________________ **(Line 4)**

$\Rightarrow 1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + \cdots + m \times 2^m = 2 + (m-1) \times 2^{m+1}$ ……………….**(i)**

Using **(i)**, we have to show that , $p(n = m + \square)$is also true ________________ **(Line 5)**

$\Rightarrow 1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + \ldots\ldots + m \times 2^m + (m+1) \times 2^{m+1} = 2 + (m+1) \times 2^{m+2}$ ……**(ii)** __**(Line 6)**

Now,

L.H.S of **(ii)** $= 1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + \ldots\ldots + m \times 2^m + (m+1) \times 2^{m+1}$

L.H.S of **(ii)** $= 2 + (m-1) \times 2^{m+1} + (m+1) \times 2^{m+1}$ [from **(i)**] ____________ **(Line 7)**

L.H.S of **(ii)** $= 2 + 2^{m+1}(m - 1 + m + 1)$ ________________ **(Line 8)**

L.H.S of **(ii)** $= 2 + (m+1) \times 2^{m+2}$ ________________ **(Line 9)**

L.H.S of **(ii)** $=$ R.H.S of **(ii)**

So, Our claim is true for all positive integers n.

Mark only one oval.

- (R.H.S. of Line 8) > (R.H.S. of Line 9)
- (R.H.S. of Line 8) = (R.H.S. of Line 9)
- (R.H.S. of Line 8) ≥ (R.H.S. of Line 9)
- (R.H.S. of Line 8) < (R.H.S. of Line 9)