BRAC University

CSE230 : Discrete Mathematics

Midterm Examination

Duration : 100 minutes (4:30 pm - 6:10 pm)

Total Marks : 60 Set: B

[Answer all the questions from 1,2,3. Answer any 1 question from 4,5.]

ID: **Name:** **Sec:**

Q01: [CO1] [15 Points]

a) Prove whether $(\neg p \lor q) \land (p \lor \neg q)$ and $p \leftrightarrow q$ are logically equivalent. (Truth table is one way to solve the problem.) **[5 points]**

b) Write the converse, inverse and contrapositive of the following statement: **[6 points]**

An integer is prime only if it has two factors.

c) Let L(x, y) be the statement "x loves y," where the domain for both x and y consists of all people in the world. Use quantifiers to express each of these statements. **[4 points]**

i) Everybody loves Cindy.
ii) Ali loves nobody.
iii) There is nobody whom Babul loves.
iv) Somebody loves Didar.

Q02: [CO4] [15 Points]

a) Using mathematical induction, prove the following statement for all positive integers n:

$$1 \times 3 + 2 \times 3^2 + 3 \times 3^3 + 4 \times 3^4 + \cdots + n \times 3^n = \frac{3}{4}\left[(2n - 1)3^n + 1\right]$$

[8 points]

b) Prove by induction that $5^{2n+1} - 12 \times 16^{2n}$ is divisible by 7 for all non-negative integers n. **[7 points]**

Q03: [CO6] [15 Points]

a) In 2023, XYZ Bank completed its 12 year anniversary. On this occasion, it has announced a new account scheme. According to the scheme, at the end of each year, the account holders will receive a 10% interest on the currently saved amount and also an additional 1000 taka will be added each year. Model the recurrence relation from the above scenario. **[5 points]**

b) Solve the following recurrence relation: **[8 points]**

$a_{n+1} + 3a_n = 18a_{n-1} + 40a_{n-2}$ where $a_0 = -2$, $a_2 = -38$, and $a_4 = -1010$

Also find the value of $a_9 - 18a_7$ **[2 points]**

Q04: [CO7] [15 Points]

a) Find the remainder when 5^{254} is divided by 229 using modular arithmetic. **[7 points]**

b) Determine the greatest common divisor of 1189 and 570 using the Euclidean algorithm. **[4 points]**

c) Show that if a, b and c be integers, where $a \neq 0$, $b \neq 0$, $a|b$ and $b|c$, then $a^2 \mid (7b^2 + 24ac)$ **[4 points]**

Or,

Q05: [CO7] [15 Points]

a) Solve the linear congruence $24x \equiv 7 \pmod{19}$ **[5 points]**

b) Prove that for any two positive integers x and y, the product of their GCD and LCM is equal to the product of the two numbers, i.e., GCD(x, y) * LCM(x, y) = x * y. **[5 points]**

c) Multiply the numbers $(34334)_7$ and $(41303)_7$ and give your answer in base-7. Show all necessary workings. **[5 points]**

Hint: You may convert the numbers to decimal to perform your desired operations.