BRAC University

CSE230 : Discrete Mathematics

Duration : 1 hour 15 minutes (4:45 pm - 6:00 pm)

Total Marks : 50 Set: A

[Answer any 5 out of 6 questions. Answer all the sub-parts of a question together. Please start each question in a new page]

Student Name:	**Student ID:**

Q01: [CO2] [10 Marks]

(a) Draw a Venn diagram using 3 sets Q, T and P. None of these 3 sets are pairwise disjoint. Moreover, $P \cap Q \cap T \neq \phi$. How many disjoint regions are there? Indicate which regions fall under $(T \cap Q)^c - P$. **[1+1=2 marks]**

(b) Find the domain of $f(x) = \sqrt{\frac{3-x}{x^2+2x}}$. Show the domain in a number line. **[4 marks]**

(c) Find the range of $g(x) = \frac{3x+7}{2-5x}$. What should be the domain of g(x)? **[3+1=4 marks]**

Q02: [CO2] [10 Marks]

A,B,C,D,E,F,G and H are eight students of CSE230 Fall 2022. They want to take pictures.

a) In how many ways can they stand in a straight line to take the picture? **[2 marks]**

b) Now consider B,C,D and E are close friends and they want to stay together in the group picture. How many ways can they take pictures keeping the "close" friends together? **[4 marks]**

c) Well, D has a "thing" for H. So he(D) wants to stay beside her(H). But he also wants to stay with his friends(B,C, and E). For example, A**BCDE**FG**H** is not allowed as D is not beside H although BCDE are together, but A**CEBDH**FG is allowed as D is with his friends and beside H too. Another allowed arrangement can be A**HDCBE**FG. In how many ways can they stand for taking the picture keeping all these scenarios into consideration? **[4 marks]**

Q03: [CO1] [10 Marks]

a) How many arrangements of the word "**tiamaria**" are possible so that no two consonants are side-by-side? How many of them start and end with the same letter? **[3+3=6 marks]**

b) Adnan and Binti are playing a game in which Binti chooses k numbers from the set {1,2,...,20}. If Adnan can find 2 numbers from Binti's chosen numbers whose sum is divisible by 10, then Adnan wins. What is the minimum value of k so that Adnan always wins? **[2 marks]**

c) In how many ways can you arrange the letters of the word "**normalize**" so that z always comes after m? For example, "o**me**z**r**anli" is acceptable, whereas "**z**arle**m**oni" is not. **[2 marks]**

Q04: [CO3] [10 Marks]

Read the following equations.

1) ${}^{7}C_{a} = {}^{7}C_{4}$, a is an integer and $0 < a < 7$

2) $(x + y)^{n} = \sum_{r=0}^{n} {}^{n}C_{r} x^{n-r} y^{r}$, n is a non-negative integer.

Now, answer the following questions.

a) Find out all the possible values of a from the equation (1). **[2 marks]**

b) If $a < 4$, show that, the $(a + 2)$-th term in the expansion of $(5x + \frac{1}{ax})^{8}$ is a constant. [Use value of a from eq. No.1] **[4 marks]**

c) If $a \geq 4$, find out the coefficient of $x^{a-1} y^{-1-a}$ in the expansion of $(3x + \frac{1}{ay})^{8}$ [Use value of a from eq. No.1] **[4 marks]**

Q05: [CO4] [10 Marks]

A deck of DIEZ Cards has 4 different colors which are Red, Green, Yellow and Blue. Each color has 1 Wild Card, 1 Reverse Card, 1 Block Card and 7 Normal Cards numbered from 1-7.

a) Find the probability of picking a normal card from the deck. **[2 marks]**

b) If 3 cards are picked at random from the deck, what is the probability of picking at least 1 Wild card? **[4 marks]**

c) Now, imagine one card is lost from the deck at random. If we pick a card from the deck, what is the probability of that card being a Reverse Card? **[4 marks]**

Q06: [CO4] [10 Marks]

The Graduate Record Examinations Test (GRE) is a requirement for all applicants of Msc Programs.

Suppose that a survey of GRE students reveals that among GRE scorers above 310, 52% took Magoosh (An Online Education Company) paid subscription, whereas among GRE scorers of less than 310 only 23% took the subscription. An applicant thinks that in order to get into a certain university he needs more than 310. The chance of obtaining more than 310 is $x\%$

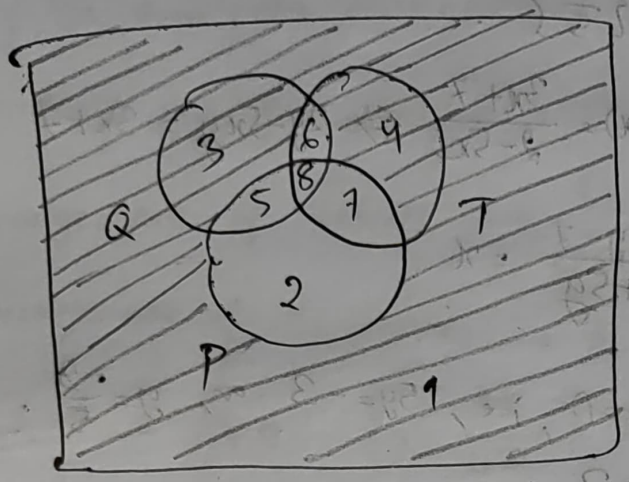
a) Suppose $x = 10\%$. Given that he took Magoosh subscription, What is the probability of getting more than 310? **[6 marks]**

b) If the probability of getting more than 310 given that he has taken the Magoosh Subscription is 50%. Then what is x? **[4 marks]**

Set-A

[1+1]

1(a)



For every region,

P	Q	T	
var x	var x	var x	
2	× 2	× 2	= 8

disjoint regions

$T \cap Q \rightarrow 6, 8$; $(T \cap Q)^c \rightarrow 1, 2, 3, 4, 5, 7$

$P \rightarrow 2, 5, 7, 8$; $(T \cap Q)^c - P \rightarrow 1, 3, 4.$

Alternately, $(T \cap Q)^c - P = (T \cap Q)^c \cap P^c = \left((T \cap Q) \cup P\right)^c$ [De morgan's law]

$(T \cap Q) \cup P \rightarrow 2, 5, 6, 7, 8.$ $\therefore \left((T \cap Q) \cup P\right)^c \rightarrow 1, 3, 4$

[4]

1(b) $\frac{3-x}{x^2+2x} \geq 0 \Leftrightarrow (3-x)(x^2+2x) \geq 0 \Leftrightarrow (x-3)x(x+2) \leq 0$

[x both sides by $(x^2+2x)^2$]

$\Leftrightarrow (x-3)x(x-(-2)) \leq 0$ [Also, we want $x^2+2x \neq 0$, i.e., $x(x+2) \neq 0$. Hence, $x \neq 0, x \neq -2$]

x	sign of $x-(-2)$	sign of x	sign of $x-3$	sign of $(x-3)x(x+2)$
✓ $x<-2$	−	−	−	−
$-2<x<0$	+	−	−	+
✓ $0<x<3$	+	+	−	−
$x>3$	+	+	+	+

We cannot take $x=0$ or $x=-2$, but $x=3$ gives $(x-3)x(x+2)=0$, hence it is acceptable.

$\therefore$ dom $f = (-\infty, -2) \cup (0, 3]$.

[3+1]

1(c) We don't want $2-5x=0$, i.e., $x=\frac{2}{5}$.

$\therefore \text{dom } g = \mathbb{R} - \{\frac{2}{5}\}$.

For the range, consider $y = g(x) = \frac{3x+7}{2-5x} \Leftrightarrow 2y - 5xy = 3x+7$

$\Leftrightarrow 2y - 7 = 3x + 5xy \Leftrightarrow \frac{2y-7}{3+5y} = x$

Now, we don't want $3+5y=0$, i.e., $5y=-3$ or, $y=\frac{-3}{5}$.

$\therefore \text{range } g = \mathbb{R} - \{-\frac{3}{5}\}$.

[2]

2(a) $8!$

[4]

2(b) Considering B, C, D, E as a unit (say BCDE), 5! ways.

Now B, C, D, E can rearrange among themselves in 4! ways.

$\therefore 5! \times 4!$ ways.

[4]

2(c) Considering B,C,D,E,H as a unit (say BCDEH), 4! ways.

Arrangement of unit BCDEH: Considering DH as a unit (say DH), 4! ways. Now 'DH' can be rearranged in 2! ways.

$$\therefore Total\ ways = 4! \times 4! \times 2!$$

[3+3]

3(a) $\_ V \_ V \_ V \_ V \_ V \_$

Placing the vowels (V) as shown above, the consonants (C) can be placed in any 3 of the 6 available positions, so placing C $\rightarrow \binom{6}{3}$ ways.

Arrangement of vowels $\rightarrow \frac{5!}{3!2!}$ (3 a's, 2 i's)

Arrangement of Consonants $\rightarrow 3!$ (t, m, r)

$\therefore$ total ways $= \binom{6}{3} \frac{5!}{3!2!} \times 3!$

Start & end with same letter: The same letter must be 'i' or 'a'.

$\underset{\times}{\_} V \_ V \_ V \_ V \_ V \underset{\times}{\_}$

Placing C $\rightarrow \binom{4}{3}$ ways

Arrangement of consonants $\rightarrow 3!$ (t, m, r)

Arrangement of vowels
- start & end with 'i' $\rightarrow \frac{3!}{3!} = 1$ (a, a, a)
- start and end with 'a' $\rightarrow \frac{3!}{2!} = 3$ (i, i, a)

$\Sigma = 4$ ways

$\therefore$ Total ways $= \binom{4}{3} \times 3! \times 4$

[2]

3(b)

10, 5, 20, 15	1, 9, 11, 19	2, 8, 12, 18	3, 7, 13, 17	4, 6, 14, 16

Note that choosing ~~at least~~ 3 numbers from any box ensures that Adnan wins. Hence, by Pigeonhole Principle, if $(2 \times 5) + 1 = 11$ numbers are chosen (choice: 2, box: 5) — Ans.

then there will be 3 numbers from a box, so Adnan will win.

10 numbers so that Adnan doesn't win $\rightarrow \{5, 10, 1, 11, 2, 12, 3, 13, 4, 14\}$

[2]

3(c) Replace z and m with a common character 'v'.

Now you have $\frac{9!}{2!}$ rearrangements. In the first 'v', place 'm' and in the 2nd 'v', place 'z' to get the desired arrangements. $\therefore \frac{9!}{2!}$ ways.

[2]

4(a) ${}^{7}C_a = {}^{7}C_4 \Rightarrow a = 4$ or $a = 7-4 = 3$.

$\therefore a \in \{3, 4\}$.

[4]

4(b) $a < 4 \Rightarrow a = 3$. Now $(a+2)^{th}$ or 5^{th} term in the expansion will be: ${}^{8}C_4 (5x)^4 \left(\frac{1}{3x^2}\right)^4 = \binom{8}{4} 5^4 . x^4 . \frac{1}{3^4 x^4} = \binom{8}{4}\left(\frac{5}{3}\right)^4$, a constant.

[$\because$ index from 0 to 8]

[4]

4(c) $a \geq 4 \Rightarrow a = 4$. We need to find the co-efficient of $x^{4-1} y^{-1-4}$ or $x^3 y^{-5}$. Now, for ~~some constant~~ the desired co-efficient k, there is an r so that

$$k x^3 y^{-5} = \binom{8}{r} (3x)^r \left(\frac{1}{4y}\right)^{8-r} = \binom{8}{r} 3^r x^r 4^{r-8} y^{r-8}$$

Clearly we must have $r = 3$.

Now, $k x^3 y^{-5} = \binom{8}{3} 3^3 x^3 4^{-5} y^{-5}$.

$\therefore$ Co-efficient, $k = \binom{8}{3} 3^3 4^{-5} = \binom{8}{3} \frac{3^3}{4^5}$.

[2]

5(a) Cards of each color = $1+1+1+7=10$

$\therefore$ Total cards $= 4\times 10 = 40$. [$\because$ 4 colors]

Normal cards $= 4\times 7 = 28$

$\therefore P(\text{pick a normal card}) = \frac{28}{40} = \frac{7}{10}$.

Alternately, if C be the event that a card of some color is picked, then

$C=S$, and $P(\text{normal card} \mid C) = \frac{7}{10}$.

Now, $P(\text{normal card}) = P(C \cap \text{normal card}) = P(C)\,P(\text{normal card} \mid C)$

$= 1 \times \frac{7}{10}$.

[4]

5(b) Wild cards $= 4$

Not wild cards $= 40-4 = 36$

$P(\text{at least 1 wild card}) = 1 - P(\text{no wild card}) = 1 - \frac{\binom{36}{3}}{\binom{40}{3}}$

[4]

5(c) Define events $L \rightarrow$ lost card is reverse card

$P \rightarrow$ reverse card is picked

Reverse cards $= 4$. $\therefore P(L) = \frac{4}{40} = \frac{1}{10}$ and $P(L^c) = 1 - \frac{1}{10} = \frac{9}{10}$.

Desired probability, $P(P) = P(L)\,P(P \mid L) + P(L^c)\,P(P \mid L^c)$

$= \frac{1}{10} \times \frac{3}{39} + \frac{9}{10} \times \frac{4}{39} = \frac{39}{10\times 39} = \frac{1}{10}$.

Alternately, we could think of a card being lost as analogous to picking a card. Then the 2nd picked card has to be reverse card. If we pick the 2nd card at first [which keeps the event set unchanged], then $P(\text{2nd card is reverse}) = \frac{4}{40} = \frac{1}{10}$.

[6]

6(a) Define events $M \rightarrow$ have a Mugoosh subscription

$G \rightarrow$ have a gre score ≥ 310

$P(G) = x = 10\%$, $P(M|G) = 52\%$, so $P(M^c|G) = 48\%$ [100% − 52%]

$\therefore P(G^c) = 90\%$

and, $P(M|G^c) = 23\%$, so $P(M^c|G^c) = 77\%$.

[100% − 23%]

• Now, desired probability,

$$P(G|M) = \frac{P(GM)}{P(M)} = \frac{P(G)P(M|G)}{P(G)P(M|G) + P(G^c)P(M|G^c)}$$

$$= \frac{10\% \times 52\%}{10\% \times 52\% + 90\% \times 23\%} \Rightarrow \frac{520\%^2}{2590\%^2} \quad \left[\% = \frac{1}{100}\right]$$

$$= \frac{520}{2590} \approx 0.2 .$$

Hence, chance is 20%.

[4]

6(b) $P(G|M) = 50\% =$ [from 6(a)] $\frac{P(G)P(M|G)}{P(G)P(M|G) + P(G^c)P(M|G^c)}$

$$\Rightarrow 50\% = \frac{x \times 52\%}{x \times 52\% + (1-x) \times 23\%} = \frac{52x}{52x + 23(1-x)}$$

$$\Rightarrow \frac{50}{100} = \frac{52x}{23 + 29x} \Rightarrow 5200x = 1150 + 1450x$$

$$\Rightarrow 3750x = 1150 \Rightarrow x \approx 0.307 \text{ or } 30.7\%.$$