BRAC University
CSE230 : Discrete Mathematics
Midterm Examination

Duration : 100 minutes (4:30 pm - 6:10 pm)

Total Marks : 60 Set: A

[Answer all the questions from 1,2,3. Answer any 1 question from 4,5.]

ID: **Name:** **Sec:**

Q01: [CO1] [15 Points]

a) Prove whether $(p \land q) \lor (\neg p \land \neg q)$ and $p \leftrightarrow q$ are logically equivalent. (Truth table is one way to solve the problem.) **[5 points]**

b) Write the converse, inverse and contrapositive of the following statement: **[6 points]**

An integer has two factors only if it is prime.

c) Let L(x, y) be the statement "x loves y," where the domain for both x and y consists of all people in the world. Use quantifiers to express each of these statements. **[4 points]**

i) Everybody loves Ali.

ii) Nobody loves Babul.

iii) There is somebody whom Cindy loves.

iv) Didar loves nobody.

Q02: [CO4] [15 Points]

a) Using mathematical induction, prove the following statement for all positive integers n:

$$1 \times 2 + 2 \times 2^2 + 3 \times 2^3 + 4 \times 2^4 + \cdots + n \times 2^n = (n - 1) \cdot 2^{n+1} + 2$$

[8 points]

b) Prove by induction that $4^{2n+1} + 10 \times 11^{2n}$ is divisible by 7 for all non-negative integers n. **[7 points]**

Q03: [CO6] [15 Points]

a) In 2023, XYZ Bank completed its 12 year anniversary. On this occasion, it has announced a new account scheme. According to the scheme, at the end of each year, the account holders will receive a 5% interest on the currently saved amount and also an additional 500 taka will be added each year. Model the recurrence relation from the above scenario. **[5 points]**

b) Solve the following recurrence relation: **[8 points]**

$a_{n+1} + 40a_{n-2} = 3a_n + 18a_{n-1}$ where $a_1 = 16$, $a_3 = 322$, and $a_4 = 1010$

Also find the value of $a_9 - 18a_7$ **[2 points]**

Q04: [CO7] [15 Points]

a) Find the remainder when 5^{250} is divided by 227 using modular arithmetic. **[7 points]**

b) Determine the greatest common divisor of 1267 and 491 using the Euclidean algorithm. **[4 points]**

c) Show that if a, b and c be integers, where $a \neq 0$, $b \neq 0$, $a|b$ and $b|c$, then $ab \mid (4b^2 + 19bc)$ **[4 points]**

Or,

Q05: [CO7] [15 Points]

a) Solve the linear congruence $33x \equiv 4 \pmod{19}$ **[5 points]**

b) Prove that for any two positive integers a and b, the product of their GCD and LCM is equal to the product of the two numbers, i.e., GCD(a, b) * LCM(a, b) = a * b. **[5 points]**

c) Multiply the numbers $(34334)_5$ and $(41303)_5$ and give your answer in base-5. Show all necessary workings. **[5 points]**

Hint: You may convert the numbers to decimal to perform your desired operations