Propositional Logic Practice Problems

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1. Let p denote “He is rich” and let q denote “He is happy.” Write each statement in symbolic form using p and q. Note that “He is poor” and “He is unhappy” are equivalent to ¬p and ¬q, respectively.
   (a) If he is rich, then he is unhappy.
   (c) It is necessary to be poor in order to be happy.
   (d) He is neither rich nor happy.
   (d) To be poor is to be unhappy

2. Let p and q be the propositions “Swimming at the New Jersey shore is allowed” and “Sharks have been spotted near the shore,” respectively. Express each of these compound propositions as an English sentence.

a) $\neg q$	g) $p \iff \neg q$
b) $p \wedge q$	h) $\neg p \wedge (p \vee \neg q)$
c) $\neg p \vee q$	i) $p \oplus q$
d) $p \rightarrow \neg q$	
e) $\neg q \rightarrow p$	
f) $\neg p \rightarrow \neg q$	

3. Let p and q be the propositions
   $p:$ You drive over 65 miles per hour.
   $q:$ You get a speeding ticket.
   Write these propositions using p and q and logical connectives (including negations).
   a) You do not drive over 65 miles per hour.
   b) You drive over 65 miles per hour, but you do not get a speeding ticket.
   c) You will get a speeding ticket if you drive over 65 miles per hour.
   d) If you do not drive over 65 miles per hour, then you will not get a speeding ticket.
   e) Driving over 65 miles per hour is sufficient for getting a speeding ticket.
   f) You get a speeding ticket, but you do not drive over 65 miles per hour.
   g) Whenever you get a speeding ticket, you are driving over 65 miles per hour.

4. State the converse, contrapositive, and inverse of each of these conditional statements.
   a) If it snows today, I will ski tomorrow.
   b) I come to class whenever there is going to be a quiz.
   c) A positive integer is a prime only if it has no divisors other than 1 and itself.

5. Show that $\neg(p \wedge q) \equiv \neg p \vee \neg q$.

6. Show that the proposition is tautology $p \rightarrow (p \wedge (q \rightarrow p))$

7. Construct the truth table for $(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$

8. Construct the truth table for $(p \oplus q) \rightarrow (p \oplus \neg q)$

9. Show that ”In the FIFA World Cup either Germany will reach the final or England and Argentina will reach the final” and ”Germany or England wil reach the final, and Germany or Argentina will reach the final” are logically equivalent.

10. Let P(x) be the statement "x spends more than five hours every weekday in class," where the domain for x consists of all students. Express each of these quantifications in English.
a) $\exists x P(x)$
b) $\forall x P(x)$
c) $\exists x \neg P(x)$
d) $\forall x \neg P(x)$

11. Translate these statements into English, where C(x) is "x is a comedian" and F(x) is "x is funny" and the domain consists of all people.
a) $\forall x(C(x) \to F(x))$
b) $\forall x(C(x) \wedge F(x)$
c) $\exists x(C(x) \to F(x)$
d) $\exists x(C(x) \wedge F(x)$

12. Translate these statements into English, where
$A(x)$: x is teaching CSE230
$T(x)$: x is taking STA201
$F(x)$: x has a Facebook page
$C(x)$: x likes to cook
a) $\forall x(T(x) \to F(x))$
b) $\exists x(T(x) \wedge A(x)$
c) $\neg\forall x(T(x) \to (F(x) \vee C(x))$

13. Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives.
a) No one is perfect.
b) Not everyone is perfect.
c) All your friends are perfect.
d) At least one of your friends is perfect.
e) Everyone is your friend and is perfect.
f) Not everybody is your friend or someone is not perfect.

14. Let P(x) be the statement "x can speak English" and let Q(x) be the statement "x knows programming language Python." Express each of these sentences in terms of P(x), Q(x), quantifiers, and logical connectives. The domain for quantifiers consists of all students at your school.
(a) There is a student at your school who can speak English and who knows Python.
(b) There is a student at your school who can speak English but who doesn't know Python.
(c) Every student at your school either can speak English or knows Python.
(d) No student at your school can speak English or knows Python.

15. Let P (x), Q(x), and R(x) be the statements "x is a professor," "x is ignorant," and "x is vain," respectively. Express each of these statements using quantifiers; logical connectives; and P (x), Q(x), and R(x), where the domain consists of all people.
a) No professors are ignorant.
b) All ignorant people are vain.
c) No professors are vain.