1) Axiomas de la probabilidad

1) $\mathbb{P} = a_1 \mathbb{P}_1 + a_2 \mathbb{P}_2$; $a_1 + a_2 = 1$

y $a_1, a_2 \in \mathbb{R}^+ \therefore a_1, a_2 \geq 0$

• ¿$\mathbb{P}(\Omega) = 1$?

$$\mathbb{P}(\Omega) = a_1 \mathbb{P}_1(\Omega) + a_2 \mathbb{P}_2(\Omega)$$

como $\mathbb{P}_1$ y $\mathbb{P}_2$ son medidas de probabilidad, $\mathbb{P}_1(\Omega) = \mathbb{P}_2(\Omega) = 1$, así:

$$\boxed{\mathbb{P}(\Omega) = a_1 + a_2 = 1}$$

• ¿$\mathbb{P}(A) \geq 0$? Sea $A \in \mathfrak{F}$, $\mathbb{P}_1(A) \geq 0$ y $\mathbb{P}_2(A) \geq 0$, pues $\mathbb{P}_1$ y $\mathbb{P}_2$ son medidas de probabilidad

$$\mathbb{P}(A) = a_1 \mathbb{P}_1(A) + a_2 \mathbb{P}(A) \Rightarrow \boxed{\mathbb{P}(A) \geq 0}$$

$$\left.\begin{array}{l} a_1, \mathbb{P}_1(A) \geq 0 \therefore a_1 \mathbb{P}_1(A) \geq 0 \\ a_2, \mathbb{P}_2(A) \geq 0 \therefore a_2 \mathbb{P}_2(A) \geq 0 \end{array}\right] \Rightarrow a_1 \mathbb{P}(A) + a_2 \mathbb{P}(A) \geq 0$$

Sean A_1 y $A_2 \in \mathcal{F}$

Como $\mathbb{P}_1$ y $\mathbb{P}_2$ son medidas de probabilidad,

$$\mathbb{P}_1(A_1 \cup A_2) = \mathbb{P}_1(A_1) + \mathbb{P}_1(A_2)$$

$$\mathbb{P}_2(A_1 \cup A_2) = \mathbb{P}_2(A_1) + \mathbb{P}_2(A_2)$$

- ¿$\mathbb{P}(A_1 \cup A_2) = \mathbb{P}(A_1) + \mathbb{P}(A_2)$?

$$\mathbb{P}(A_1) = a_1 \mathbb{P}_1(A_1) + a_2 \mathbb{P}_2(A_1)$$

$$\mathbb{P}(A_2) = a_1 \mathbb{P}_1(A_2) + a_2 \mathbb{P}_2(A_2)$$

$$\mathbb{P}(A_1 \cup A_2) = a_1 \mathbb{P}_1(A_1 \cup A_2) + a_2 \mathbb{P}_2(A_1 \cup A_2)$$

$$\mathbb{P}(A_1 \cup A_2) = a_1 \left(\mathbb{P}_1(A_1) + \mathbb{P}(A_2) \right) + a_2 \left(\mathbb{P}_2(A_1) + \mathbb{P}_2(A_2) \right)$$

$$\mathbb{P}(A_1 \cup A_2) = \underline{a_1 \mathbb{P}_1(A_1) + a_2 \mathbb{P}_2(A_1)} + \underline{a_1 \mathbb{P}_1(A_2) + a_2 \mathbb{P}_2(A_2)}$$

$$\boxed{\mathbb{P}(A_1 \cup A_2) = \mathbb{P}(A_1) + \mathbb{P}(A_2)}$$

2) $\Omega = \{1,2\}$, $\mathcal{F} = \sigma(\Omega)$ y $\mathbb{P}$ tal que

$$\mathbb{P}(A) = \begin{cases} 0 & \text{Si } A = \{\emptyset\} \\ 1/3 & \text{si } A = \{1\} \\ 2/3 & \text{si } A = \{2\} \\ 1 & \text{Si } A = \{1,2\} \end{cases}$$

- ¿ $\mathbb{P}(\Omega) = 1$?

$$\Omega = \{1,2\} \therefore \mathbb{P}(\Omega) = \mathbb{P}(\{1,2\}) = 1$$

- ¿ $\mathbb{P}(A_i) \geq 0$?

$$\Omega = \{1,2\} \Rightarrow A_1 = \{1\},\ A_2 = \{2\}$$

$$\mathbb{P}(A_1) = \frac{1}{3} > 0$$

$$\mathbb{P}(A_2) = \frac{2}{3} > 0$$

- ¿$\mathbb{P}(A_1 \cup A_2) = \mathbb{P}(A_1) \cup \mathbb{P}(A_2)$?

$A_1 = \{1\}, \quad A_2 = \{2\} \Rightarrow A_1 \cup A_2 = \{1,2\}$

$$\mathbb{P}(\{1,2\}) = \mathbb{P}(\{1\}) + \mathbb{P}(\{2\})$$

$$1 = \frac{1}{3} + \frac{2}{3}$$

$$1 = 1$$

3) $(\Omega, \xi, \mathbb{P})$ es un espacio de probabilidad

a) $\mathbb{P}(\emptyset) = 0$

$$A \cup \emptyset = A$$

$$\mathbb{P}(A \cup \emptyset) = \mathbb{P}(A) + \mathbb{P}(\emptyset)$$

$$\mathbb{P}(A) = \mathbb{P}(A) + \mathbb{P}(\emptyset)$$

$$\mathbb{P}(A) - \mathbb{P}(A) = \mathbb{P}(\emptyset)$$

$$0 = \mathbb{P}(\emptyset)$$

b) $\mathbb{P}(A^c) = 1 - \mathbb{P}(A)$

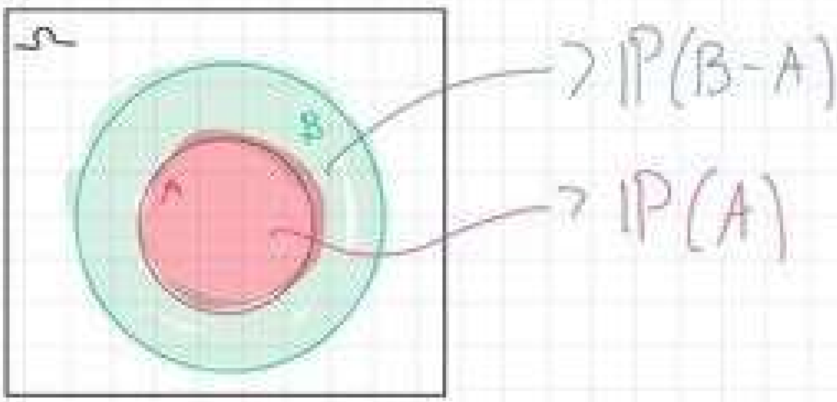
$A \cup A^c = \Omega$

$\mathbb{P}(\Omega) = \mathbb{P}(A \cup A^c)$

$1 = \mathbb{P}(A) + \mathbb{P}(A^c)$

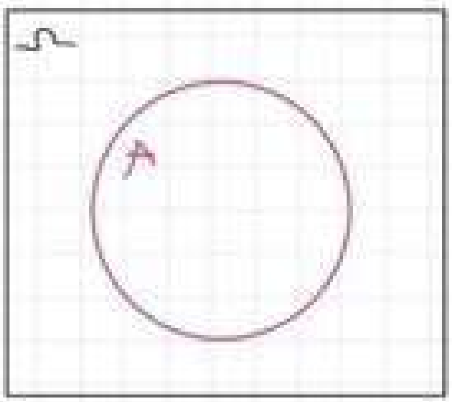
$\ulcorner 1 - \mathbb{P}(A) = \mathbb{P}(A^c) \lrcorner$

c) $A \subset B$, entonces $\mathbb{P}(B) = \mathbb{P}(A) + \mathbb{P}(B-A)$



$\mathbb{P}(B) = \mathbb{P}(A) + \mathbb{P}(B-A)$

d) $\mathbb{P}(A) \leq 1 \ \forall A \in \mathfrak{F}$



$\mathbb{P}(\Omega) = \mathbb{P}(A) + \mathbb{P}(\Omega - A)$

$\mathbb{P}(A) = 1 - \mathbb{P}(\Omega - A)$

Como $\mathbb{P}(\Omega - A) \geq 0 \Rightarrow \ulcorner \mathbb{P}(A) \leq 1 \lrcorner$

2) Probabilidad condicional y total

1)

Total personas : 1000

hombres con gafas : 185

" sin " : 415

mujeres con gafas : 115

mujeres sin gafas : 285

a) probabilidad de ser hombre:

$$\mathbb{P}(H) = \frac{185 + 415}{1000} = \frac{6\cancel{00}}{10\cancel{00}} = \frac{6}{10} = \frac{3}{5}$$

b) Probabilidad de ser mujer:

$$\mathbb{P}(M) = \frac{115 + 285}{1000} = \frac{4\cancel{00}}{10\cancel{00}} = \frac{4}{10} = \frac{2}{5}$$

c) Probabilidad de usar gafas:

$$\mathbb{P}(G) = \frac{185 + 115}{1000} = \frac{3\cancel{00}}{10\cancel{00}} = \frac{3}{10}$$

d) Probabilidad de que lleve gafas si sabemos que es mujer:

$$\mathbb{P}(G/M) = \frac{\mathbb{P}(G \cap M)}{\mathbb{P}(M)} = \frac{\frac{115}{1000}}{\frac{2}{5}} = \frac{\frac{23}{200}}{\frac{2}{5}}$$

$$= \frac{23 \times \cancel{5}}{\cancel{200}_{40} \times 2} = \frac{23}{80}$$

2)

a) $\mathbb{P}(R) = \mathbb{P}(1,2)\mathbb{P}(R_1) + \mathbb{P}(3,6)\mathbb{P}(R_2)$

$\mathbb{P}(1,2)$: probabilidad de que el dado de 6 caras saque 1 o 2

$\mathbb{P}(3,6)$: probabilidad de que el dado de 6 caras saque 3, 4, 5 o 6

$\mathbb{P}(R_1)$: Probabilidad de que se saque una bola roja de la urna 1

$\mathbb{P}(R_2)$: Probabilidad de que se saque una bola roja de la urna 2

$\mathbb{P}(1,2) = \frac{2}{6} = \frac{1}{3}$

$\mathbb{P}(3,6) = \frac{4}{6} = \frac{2}{3}$

$\mathbb{P}(R_1) = \frac{3}{10}$

$\mathbb{P}(R_2) = \frac{6}{10} = \frac{3}{5}$

$\mathbb{P}(R) = \frac{1}{\cancel{3}} \cdot \frac{\cancel{3}}{10} + \frac{2}{\cancel{3}} \cdot \frac{\cancel{3}}{5}$

$\mathbb{P}(R) = \frac{1}{10} + \frac{2}{5} = \frac{1+4}{10} = \frac{5}{10}$

$\mathbb{P}(R)\ \frac{1}{2}$

b) $\mathbb{P}(N) = \mathbb{P}(1,2)\mathbb{P}(N_1) + \mathbb{P}(3,6)\mathbb{P}(N_2)$

$\mathbb{P}(1,2)$: probabilidad de que el dado de 6 caras saque 1 o 2

$\mathbb{P}(3,6)$: probabilidad de que el dado de 6 caras saque 3, 4, 5 o 6

$\mathbb{P}(N_1)$: probabilidad de que se saque una bola negra de la urna 1

$\mathbb{P}(N_2)$: probabilidad de que se saque una bola negra de la urna 2

$$\mathbb{P}(1,2) = \frac{2}{6} = \frac{1}{3}$$

$$\mathbb{P}(3,6) = \frac{4}{6} = \frac{2}{3}$$

$$\mathbb{P}(N_1) = \frac{1}{10}$$

$$\mathbb{P}(N_2) = \frac{2}{10} = \frac{1}{5}$$

$$\mathbb{P}(N) = \frac{1}{3} \cdot \frac{1}{10} + \frac{2}{3} \cdot \frac{1}{5}$$

$$\mathbb{P}(N) = \frac{1}{30} + \frac{2}{15} = \frac{1+4}{30}$$

$$\mathbb{P}(N) = \frac{5}{30} = \frac{1}{6}$$

c) $\mathbb{P}(1/N) = \dfrac{\mathbb{P}(1,2)\,\mathbb{P}(N_2)}{\mathbb{P}(N)} = \dfrac{\frac{1}{3}\cdot\frac{1}{10}}{\frac{1}{6}} = \dfrac{\frac{1}{30}}{\frac{1}{6}}$

$\mathbb{P}(1/N) = \dfrac{6}{30} = \dfrac{1}{5}$

d) $\mathbb{P}(2/N) = \dfrac{\mathbb{P}(3,6)\,\mathbb{P}(N_2)}{\mathbb{P}(N)} = \dfrac{\frac{2}{3}\cdot\frac{1}{5}}{\frac{1}{6}} = \dfrac{\frac{2}{15}}{\frac{1}{6}}$

$\mathbb{P}(2/N) = \dfrac{12}{15} = \dfrac{4}{5}$

3) $\mathbb{P}(F \cap F) = \mathbb{P}(F_1) \cdot \mathbb{P}(F_2)$

$\mathbb{P}(F_1)$: Probabilidad de sacar dulce de fresa la primera vez

$\mathbb{P}(F_2)$: Probabilidad de sacar dulce de fresa la segunda vez

$\mathbb{P}(F \cap F) = \dfrac{3}{5} \cdot \dfrac{2}{4} = \dfrac{3}{5} \cdot \dfrac{1}{2} = \dfrac{3}{10}$

3) Teorema de Bayes

1)

$$40\% \text{ fuman} \begin{cases} \to 75\% \text{ hombres} \\ \to 25\% \text{ mujeres} \end{cases} \qquad 60\% \text{ No fuman} \begin{cases} \to 40\% \text{ hombres} \\ \to 60\% \text{ mujeres} \end{cases}$$

a) $\mathbb{P}(M) = \mathbb{P}(F \cap M) + \mathbb{P}(NF \cap M)$

$$\mathbb{P}(M) = \frac{\cancel{2}^{1}}{5} \cdot \frac{1}{\cancel{4}_{2}} + \frac{3}{5} \cdot \frac{3}{5}$$

$$\mathbb{P}(M) = \frac{1}{10} + \frac{9}{25} = \frac{25+90}{250} = \frac{115}{250} = \frac{23}{50}$$

b) $\mathbb{P}(F \cap H) = \frac{\cancel{2}^{1}}{5} \cdot \frac{3}{\cancel{4}_{2}} = \frac{3}{10}$

c) $\mathbb{P}(F/M) = \frac{\mathbb{P}(F \cap M)}{\mathbb{P}(M)} = \frac{\frac{\cancel{2}^{1}}{5} \cdot \frac{1}{\cancel{4}_{2}}}{\frac{23}{50}} = \frac{\frac{1}{10}}{\frac{23}{50}}$

$$\mathbb{P}(F/M) = \frac{5\cancel{0}}{1\cancel{0} \cdot 23} = \frac{5}{23}$$

5) Generales de probabilidad

1) $\mathbb{P}(A) = \frac{3}{36} = \frac{1}{12}$

$\mathbb{P}(B) = \frac{3}{6} = \frac{1}{2}$

$\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)$

$\mathbb{P}(A \cup B) = \frac{3}{36} + \frac{18}{36} - \frac{2}{36}$

$\mathbb{P}(A \cap B) = \frac{2}{36} = \frac{1}{18}$

$\mathbb{P}(A \cup B) = \frac{19}{36}$

$\mathbb{P}(A^c) = 1 - \mathbb{P}(A) \Rightarrow 1 - \frac{1}{12} = \frac{11}{12}$

2)

A = Al menos un celular sale defectuoso

A^c = Ningún celular sale defectuoso

$$\mathbb{P}(A^c) = \frac{\frac{48!}{5!(43)!}}{\frac{50!}{5!(45)!}} = \frac{48! \cdot 45! \cdot \cancel{5!}}{50! \cdot 43! \cdot \cancel{5!}} = \frac{\cancel{48!} \cdot 45 \cdot 44 \cdot \cancel{43!}}{50 \cdot 49 \cdot \cancel{48!} \cdot \cancel{43!}}$$

$$\frac{\overset{9}{\cancel{45}}\cdot 44}{\underset{10}{\cancel{50}}\cdot 49} = \frac{396}{490} = \boxed{\frac{198}{245}}$$

$$\mathbb{P}(A) = 1 - \mathbb{P}(A^c)$$

$$\mathbb{P}(A) = 1 - \frac{198}{245} \Rightarrow \frac{245}{245} - \frac{198}{245} = \boxed{\frac{47}{245}}$$

3)

A = Estar suscrito en el diario

B = Estar suscrito al cable

$\mathbb{P}(A) = 0,6$; $\mathbb{P}(B) = 0,8$; $\mathbb{P}(A \cap B) = 0,5$

a) $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)$

$$\mathbb{P}(A \cup B) = 0,6 + 0,8 - 0,5$$

$$\boxed{\mathbb{P}(A \cup B) = 1,4 - 0,5 = 0,9}$$

b) $\ulcorner \mathbb{P}(A \cup B) - \mathbb{P}(A \cap B)$

$0{,}9 - 0{,}5 = 0{,}4 \lrcorner$

5)

A = La Suma de los 2 lanzamientos da 8

B = El segundo lanzamiento es impar

$\mathbb{P}(A) = \frac{5}{36}$; $\mathbb{P}(B) = \frac{3}{6} = \frac{1}{2}$

$\mathbb{P}(A \cap B) \Rightarrow (5,3), (3,5)$

$\ulcorner \mathbb{P}(A \cap B) = \frac{2}{36} = \frac{1}{18} \lrcorner$

$\ulcorner \mathbb{P}(A) \cdot \mathbb{P}(B) = \frac{5}{36} \cdot \frac{1}{2} = \frac{5}{72} \lrcorner$

$\frac{5}{72} > \frac{1}{18} \therefore \mathbb{P}(A) \cdot \mathbb{P}(B) > \mathbb{P}(A \cap B)$

6)

A = probabilidad de obtener un par al lanzar 3 dados de 6 caras

$$(\underbrace{1, 1}_{6}, \underbrace{1}_{5}) \Rightarrow \frac{(6\times 5)3}{6^3} = \frac{90}{216} = \frac{45}{108} = \frac{5}{12}$$

$$\mathbb{P}(A) = \frac{5}{12}$$

7)

A = probabilidad de obtener un par al lanzar 5 dados de 6 caras

B = probabilidad de obtener dos pares distintos al lanzar 5 dados de 6 caras

C = probabilidad de obtener 4 lanzamientos iguales al lanzar 5 dados de 6 caras

$$(\underbrace{1, 1}_{6}, \underbrace{1}_{5}, \underbrace{1}_{4}, \underbrace{1}_{3}) \Rightarrow \frac{(6\times 5\times 4\times 3)10}{6^5}$$

$$= \frac{6\times 60\times 10}{6^5} = \frac{600}{6^4} = \frac{6\times 100}{6^4} = \frac{100}{6^3} = \frac{100}{216}$$

$$= \frac{50}{108} = \frac{25}{54} \qquad \mathbb{P}(A) = \frac{25}{54}$$

$$(\underbrace{1,1}_{6},\underbrace{1,1}_{5},\underbrace{1}_{4}) \Rightarrow \frac{(\cancel{6}\times5\times4)(3\times5)}{6^{\cancel{5}}}$$

$$= \frac{20\times15}{6^4} = \frac{300}{1296} = \boxed{\frac{25}{108}} \quad \therefore \boxed{\mathbb{P}(B) = \frac{25}{108}}$$

$$(\underbrace{1,1,1,1}_{6},\underbrace{1}_{5}) \Rightarrow \frac{\cancel{6}\times5\times5}{6^{\cancel{5}}} = \boxed{\frac{25}{1296}}$$

$$\therefore \boxed{\mathbb{P}(C) = \frac{25}{1296}}$$

7) Muestreo

d)

Definiciones:

Media: $\bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i$

Varianza: $Var(y) = \mathbb{E}[(y-\mathbb{E}[y])^2]$

Covarianza: $Cov(x,y) = \mathbb{E}[(x-\mathbb{E}[x])(y-\mathbb{E}[y])]$

Propiedades:

$Var(ay) = a^2 Var(y)$; $a \in \mathbb{R}$

$$Var(\bar{x}) = Var\left(\frac{1}{N}\sum_{i=1}^{N} X_i\right) = \frac{1}{N^2} Var\left(\sum_{i=1}^{N} X_i\right)$$

$$= \frac{1}{N^2}\left(\mathbb{E}\left[\left(\sum_{i=1}^{N} X_i - \mathbb{E}\left[\sum_{i=1}^{N} X_i\right]\right)^2\right]\right)$$

$$= \frac{1}{N^2}\left(\mathbb{E}\left[\left(\sum_{i=1}^{N} X_i - \sum_{i=1}^{N}\mathbb{E}[X_i]\right)^2\right]\right)$$

$$= \frac{1}{N^2}\left(\mathbb{E}\left[\left(\sum_{i=1}^{N}(X_i - \mathbb{E}[X_i])\right)^2\right]\right)$$

$$= \frac{1}{N^2}\left(\mathbb{E}\left[\sum_{i=1}^{N}(X_i - \mathbb{E}[X_i])\cdot\sum_{j=1}^{N}(X_j - \mathbb{E}[X_j])\right]\right)$$

$$= \frac{1}{N^2}\left(\mathbb{E}\left[\sum_{i=1}^{N}\sum_{j=1}^{N} (x_i - \mathbb{E}[x_i])(x_j - \mathbb{E}[x_j]) \right] \right)$$

Se separa en $i=j$ y $i \neq j$:

$$= \frac{1}{N^2}\left(\mathbb{E}\left[\sum_{i=1}^{N} (x_i - \mathbb{E}[x_i])^2 + \sum_{i=1}^{N}\sum_{j=1}^{N} (x_i - \mathbb{E}[x_i])(x_j - \mathbb{E}[x_j]) \right] \right)$$

Se tiene que: $i,j = j,i$ $\therefore$

$$= \frac{1}{N^2}\left(\mathbb{E}\left[\sum_{i=1}^{N} (x_i - \mathbb{E}[x_i])^2 + 2\sum_{i=1}^{N}\sum_{j=i+1}^{N} (x_i - \mathbb{E}[x_i])(x_j - \mathbb{E}[x_j]) \right] \right)$$

$$= \frac{1}{N^2}\left(\sum_{i=1}^{N} \mathbb{E}\left[(x_i - \mathbb{E}[x_i])^2\right] + 2\sum_{i=1}^{N}\sum_{j=i+1}^{N} \mathbb{E}\left[(x_i - \mathbb{E}[x_i])(x_j - \mathbb{E}[x_j])\right] \right)$$

Analizando los términos:

$$= \frac{1}{N^2}\left(\sum_{i=1}^{N} \mathrm{Var}(x_i) + 2\sum_{i=1}^{N}\sum_{j=i+1}^{N} \mathrm{Cov}(x_i, x_j) \right)$$

$$= \frac{1}{N^2}\sum_{i=1}^{N} \mathrm{Var}(x_i) + \frac{2}{N^2}\sum_{i=1}^{N}\sum_{j=i+1}^{N} \mathrm{Cov}(x_i, x_j)$$

12) $N = n_0 + n_1$; $E = \varepsilon_0 n_0 + \varepsilon_1 n_1$

$S(N, n_0) = k_B \ln(\Omega)$; $\ln(N!) \approx N\ln(N) - N$

a) Se deben <u>Combinar</u> N partículas en 2 posibles estados (n_0 y n_1); en el que no importa el orden ni repetición:

$$C(N, n_0) = \frac{N!}{n_0!(N-n_0)!} = \frac{N!}{n_0!\, n_1!} = \Omega(N, n_0)$$

b) $$S(N, n_0, n_1) = k_B \ln\left(\frac{N!}{n_0!\, n_1!}\right) = k_B\left[\ln(N!) - \ln(n_0!) - \ln(n_1!)\right]$$

$$= k_B\left[N\ln(N) - \cancel{N} - n_0\ln(n_0) + \cancel{n_0} - n_1\ln(n_1) + \cancel{n_1}\right]$$

$$= k_B\left[N\ln(N) - n_0\ln(n_0) - n_1\ln(n_1)\right]$$

$$= k_B\left[N\ln(N) - \sum_{i=0}^{1} n_i\ln(n_i)\right]$$

c) $X = \frac{n_1}{N} \rightarrow XN = n_1$; $n_0 = N(1-X)$

$$S(N,X) = K_B\left[N\ln(N) - N(1-X)\ln(N(1-X)) - NX\ln(NX)\right]$$

$$= K_B N\left[\ln(N) - (1-X)\left[\ln(N) + \ln(1-X)\right] - X\left[\ln(N) + \ln(X)\right]\right]$$

$$= K_B N\left[\cancel{\ln(N)} - \cancel{\ln(N)}\left[(1-\cancel{X}) + \cancel{X}\right] - (1-X)\ln(1-X) - X\ln(X)\right]$$

$$= K_B N\left[-(1-X)\ln(1-X) - X\ln(X)\right]$$

$$= \boxed{-K_B N\left[X\ln(X) + (1-X)\ln(1-X)\right]}$$

d) $\frac{1}{T} = \left(\frac{\partial S}{\partial E}\right)_N = \left(\frac{\partial S}{\partial X}\right)_N \left(\frac{\partial X}{\partial E}\right)_N$; $\Delta E = \varepsilon_1 - \varepsilon_0$

$$X = \frac{E - N\varepsilon_0}{N(\varepsilon_1 - \varepsilon_0)} \rightarrow \frac{\partial X}{\partial E} = \frac{1}{N(\varepsilon_1 - \varepsilon_0)} = \frac{1}{N\Delta E},$$

$$S(N,X) = -K_B N\left[X\ln(X) + (1-X)\ln(1-X)\right]$$

$$\frac{\partial S}{\partial X} = -K_B N\left[\cancel{X}\cdot\frac{1}{\cancel{X}} + \ln(X) + (1-X)\frac{1}{1-X}(-1) + (-1)\ln(1-X)\right]$$

$$\frac{\partial S}{\partial x} = -k_B N \left(\cancel{1} + \ln(x) - \cancel{1} - \ln(1-x) \right)$$

$$\frac{\partial S}{\partial x} = k_B N \left(\ln(1-x) - \ln(x) \right) = k_B N \left(\ln\left(\frac{1-x}{x} \right) \right)$$

$$\frac{1}{T} = k_B \cancel{N} \ln\left(\frac{1-x}{x} \right) \cdot \frac{1}{\cancel{N} \Delta E}$$

$$\frac{\Delta E}{k_B T} = \ln\left(\frac{1-x}{x} \right) \Rightarrow e^{\Delta E / k_B T} = \frac{1-x}{x}$$

$$x e^{\Delta E / k_B T} = 1 - x \Rightarrow x \left(e^{\Delta E / k_B T} + 1 \right) = 1$$

$$\left[X(T) = \frac{1}{1 + e^{\Delta E / k_B T}} \right]$$

f)

$$\left[\lim_{T \to 0} X(T) = \lim_{T \to 0} \frac{1}{1 + e^{\Delta E / k_B T}} = \frac{1}{\infty} = 0 \right]$$

$$\left[\lim_{T \to \infty} X(T) = \lim_{T \to \infty} \frac{1}{1 + e^{\Delta E / k_B T}} = \frac{1}{1 + e^0} = \frac{1}{2} \right]$$

$$S(t) = S(X(t))$$

$$\lim_{T \to \infty} S(t) = \lim_{T \to \infty} S(X(t)) = S\left(\frac{1}{2}\right)$$

$$S\left(N, \frac{1}{2}\right) = -K_B N \left[\frac{1}{2} \ln\left(\frac{1}{2}\right) + \frac{1}{2} \ln\left(\frac{1}{2}\right) \right]$$

$$S\left(N, \frac{1}{2}\right) = -K_B N \left[\ln\left(\frac{1}{2}\right) \right] = -K_B N \left[\cancel{\ln(1)}^{0} - \ln(2) \right]$$

$$S\left(N, \frac{1}{2}\right) = K_B N \ln(2) \, J$$

g) $T = cte$; $V_1 = V$; $V_2 = 2V$

$$\Delta S = \int \frac{dQ}{T} = \frac{1}{T} \int \cancel{dU}^{0} + dW$$

$$dU = dQ - dW \Rightarrow dU + dW = dQ$$

$$\Delta S = \frac{1}{T} \int_{V_1}^{V_2} P dV$$

$PV = nRT$; $n = \frac{N}{N_A}$

$$PV = \frac{N}{N_A} R T$$

$$PV = N K_B T$$

$$\Delta S = \frac{1}{T}\int_{V_1}^{V_2} p\,dV \qquad p = \frac{K_B N T}{V}$$

$$\Delta S = \frac{1}{\cancel{T}}\int_{V_1}^{V_2} \frac{K_B N \cancel{T}}{V}\,dV = K_B N \ln\left(\frac{V_2}{V_1}\right)$$

$$= K_B N \ln\left(\frac{2\cancel{V}}{\cancel{V}}\right) = K_B N \ln(2)$$

Los resultados son iguales