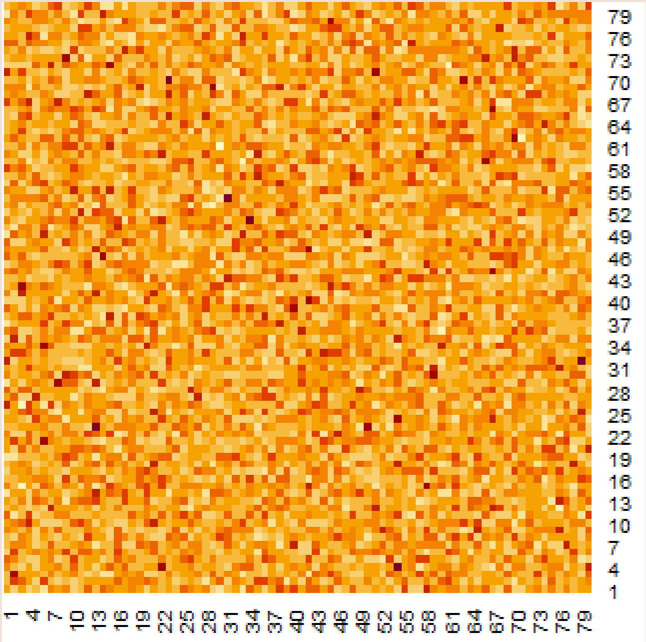
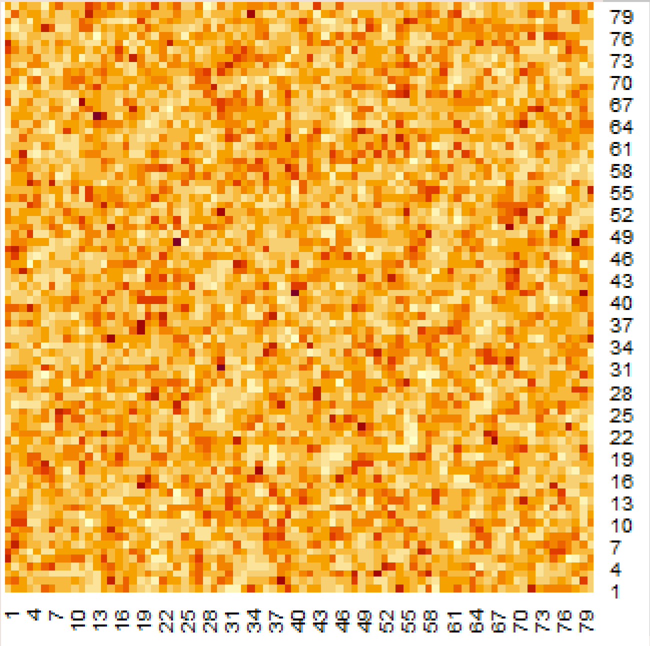
Gamma Random Field Simulation Using R

Result Presentation

Result
(2 Dimensional)

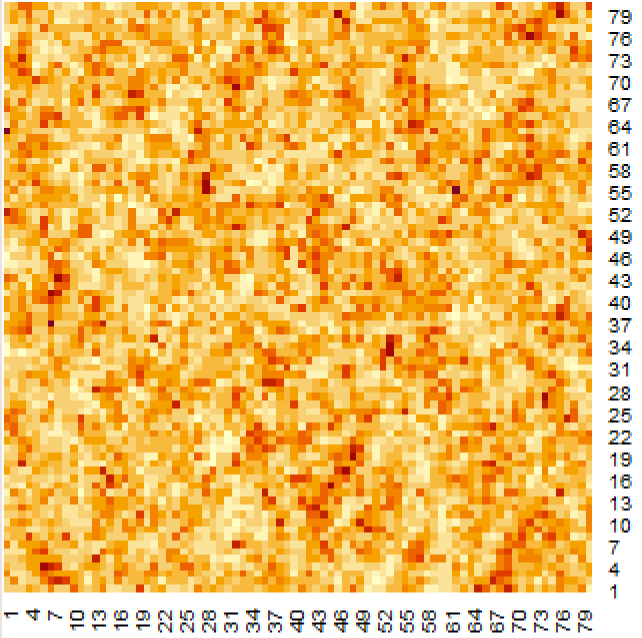


range=1

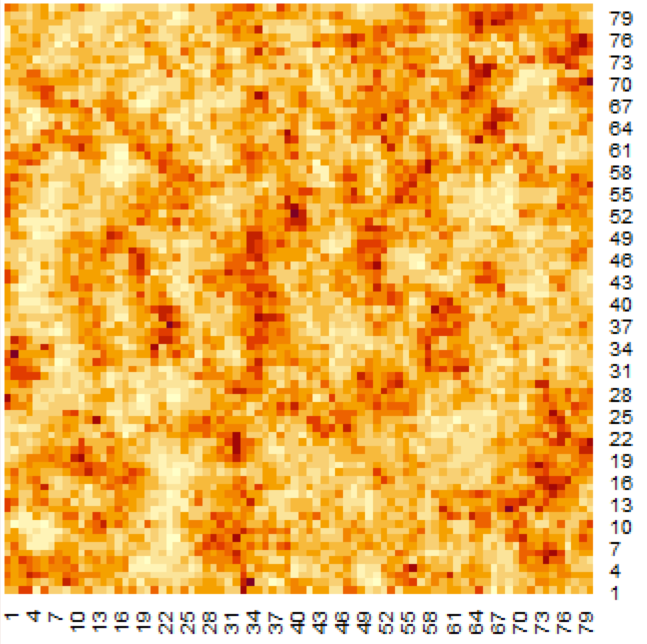


range=2

Result
(2 Dimensional)

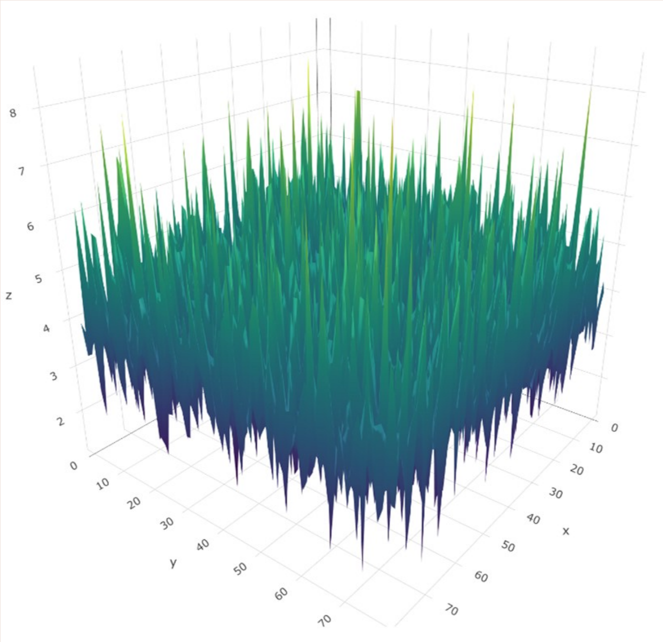


range=3

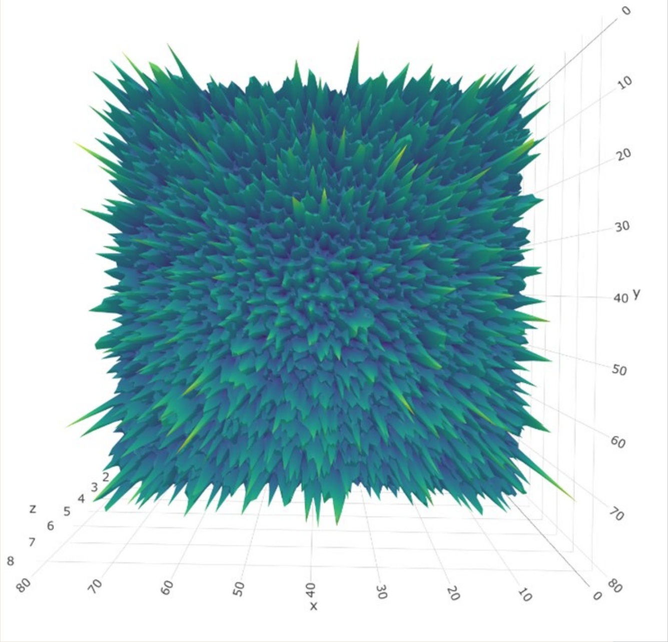


range=6

Result
(3 Dimensional)

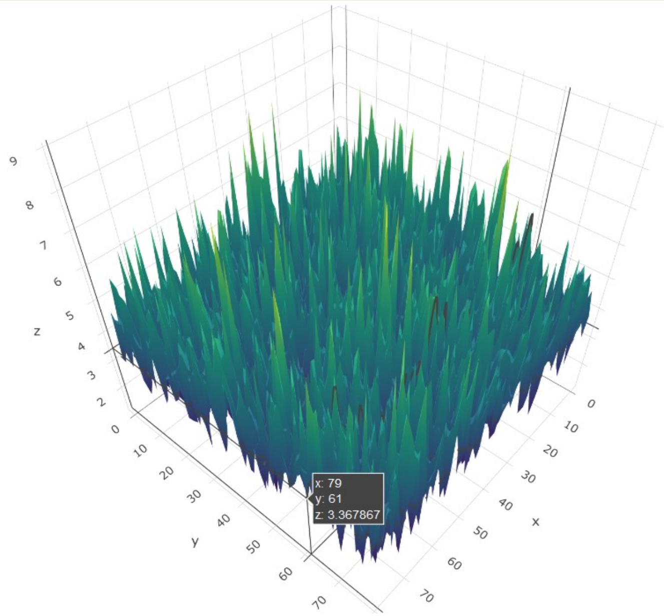


range=1
(側視圖)

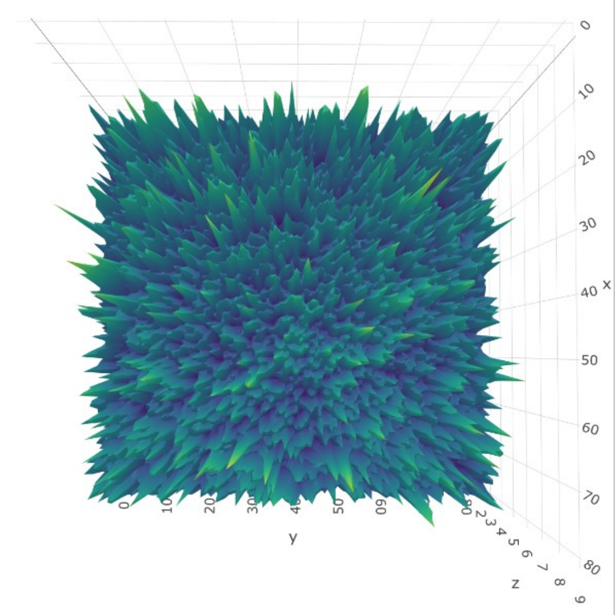


range=1
(俯視圖)

Result
(3 Dimensional)

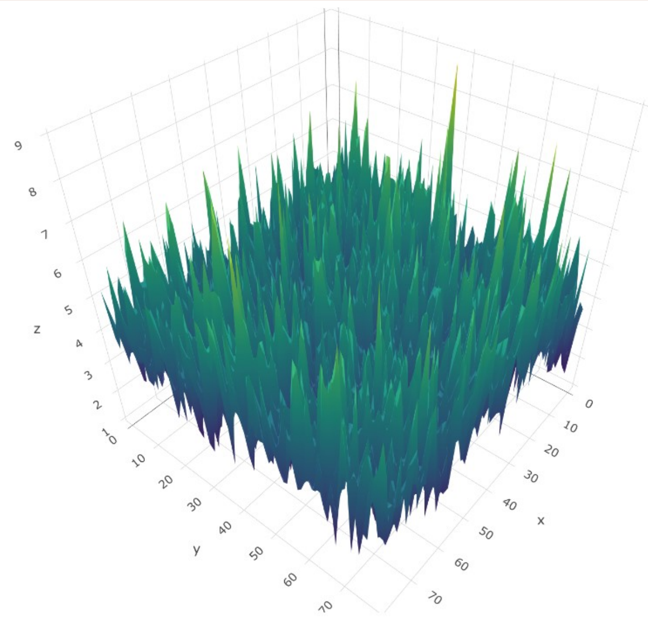


range=2
(側視圖)

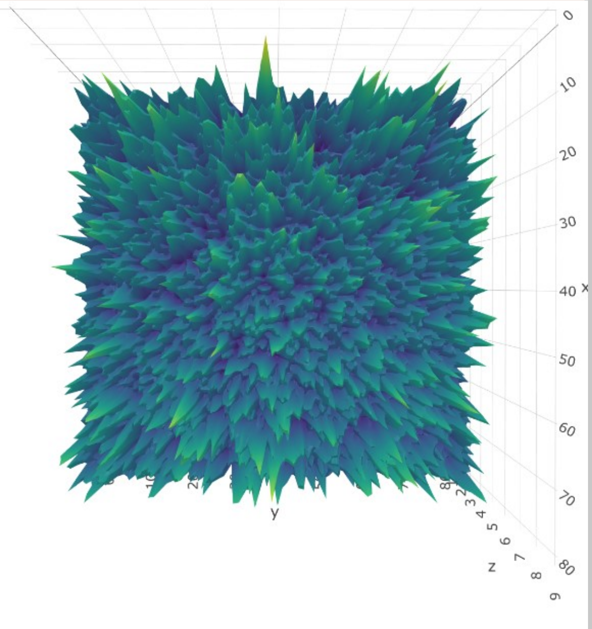


range=2
(俯視圖)

Result
(3 Dimensional)

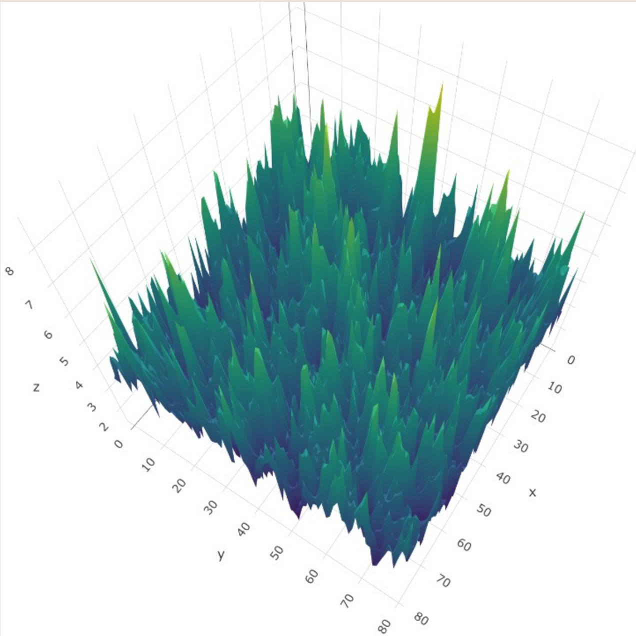


range=3
(側視圖)

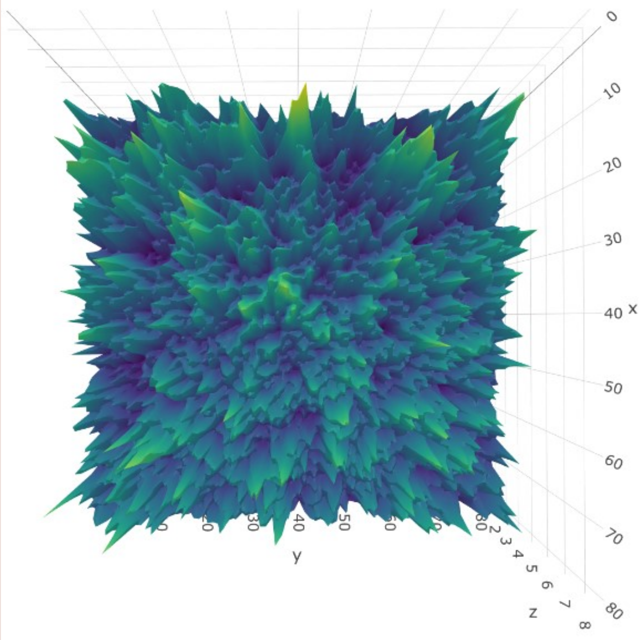


range=3
(俯視圖)

Result
(3 Dimensional)



range=6
(側視圖)



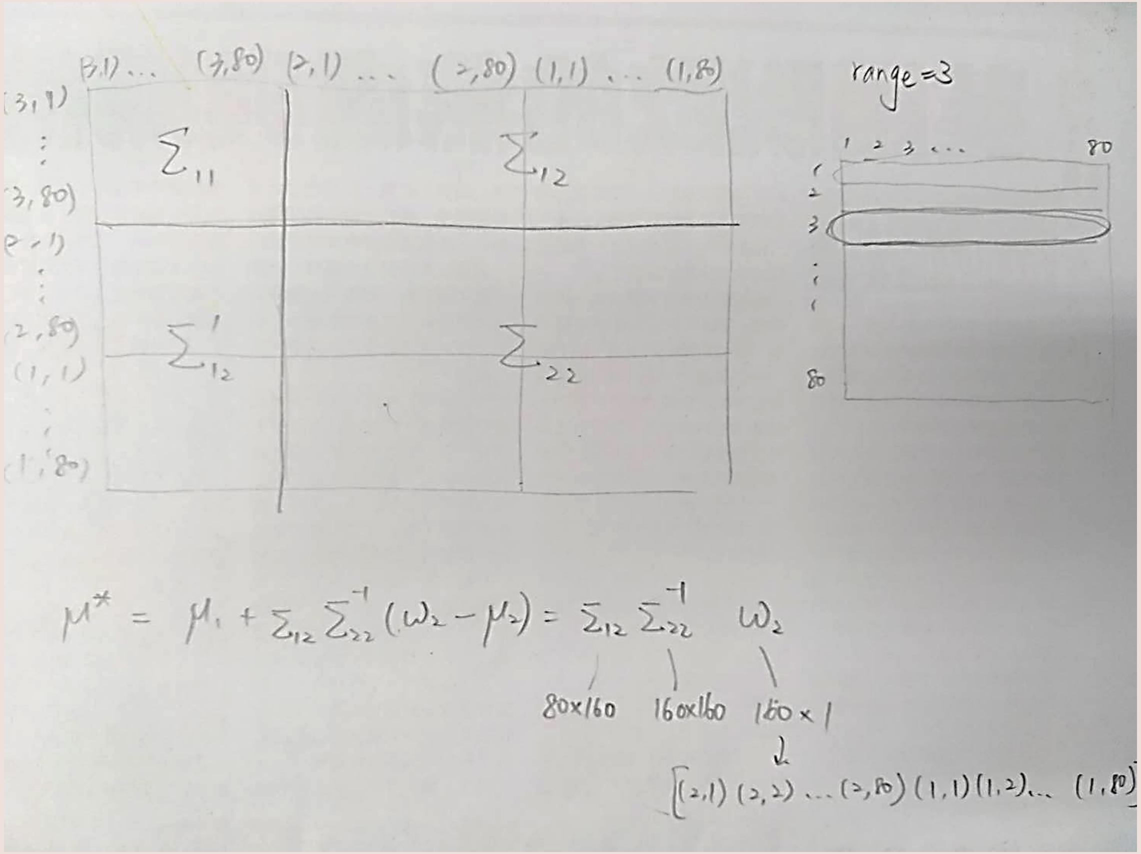
range=6
(俯視圖)

Simulation Procedure

Simulation Procedure

1. Generate standard Gaussian random number at (1,1)
2. Considering the range of random field, determine neighboring nodes. Use covariance function Cz(h) to establish the Gamma covariance matrix ∑Z
3. Transform Gamma ∑Z to Gaussian ∑W
4. Use conditional Gaussian density to generate target nodes
5. Repeat (2) - (4)
6. Conduct point-to-point Gaussian to Gamma transformation

Simulation Procedure



Step by Step Explanation

range = 3示範

步驟 0：參數設定

```
#參數設定
mu <- 4
r <- 0.5; r_x <- r; r_y <- r_x
alpha <- 16
lamda <- 4
s <- 1
size <- 80
range <- 3
```

Parameters of the gamma density and spherical semi-variogram model designated for random field stochastic simulation.

Scenario type	Gamma density parameters μ	γ	α	λ	Variogram parameters $\omega = \sigma^2$	a	Size of simulation†
I	0.67	2.985	0.449	0.67	1	1, 2, 3, 6	80×80 40×40
II	1	2	1	1	1	1, 2, 3, 6	80×80 40×40
III	2	1	4	2	1	1, 2, 3, 6	80×80 40×40
IV	4	0.5	16	4	1	1, 2, 3, 6	80×80 40×40

†Size of simulation represents the spatial domain Ω of the random field $Z(x)$.

步驟1:
第一列Gaussian Random Field 模擬(1)

```
#根據決定的range，找gamma random field中相對應的sigma (令為rho_xy1)
sigma_gamma_11 <- matrix(0, nrow = size, ncol = size)
for (i in 1:80){
  for (j in 1:80){
    if(abs(i-j)<range){
      sigma_gamma_11[i,j] <- (s*exp(-3*abs(i-j)/range))/s^2
    }else{
      sigma_gamma_11[i,j] <- 0
    }
  }
}
sigma_gamma_22<-sigma_gamma_11
```

$$\Sigma_W = \left[\begin{array}{c|cccc} C_{11} & C_{12} & \cdots & C_{1,1+q} \\ \hline C_{21} & C_{22} & \cdots & C_{2,1+q} \\ \vdots & \vdots & \ddots & \vdots \\ C_{1+q,1} & C_{1+q,2} & \cdots & C_{1+q,1+q} \end{array}\right] = \left[\begin{array}{c|c} \Sigma_{11} & \Sigma_{12} \\ \hline \Sigma_{12}' & \Sigma_{22} \end{array}\right]$$

步驟1:
第一列Gaussian Random Field 模擬(1)

```
#根據決定的range，找gamma random field中相對應的sigma (令為rho_xy1)
sigma_gamma_12<-matrix(NA,nrow=size,ncol=size)
for (i in 1:80){
  for (j in 1:80){
    if((1^2+(i-j)^2)^0.5>=range){
      sigma_gamma_12[i,j] <- 0
    }else{
      sigma_gamma_12[i,j] <-  s*exp(-3*(1^2+(i-j)^2)^0.5/range)/s^2
    }
  }
}
sigma_gamma_21<-sigma_gamma_12
sigma_gamma_23<-sigma_gamma_12
sigma_gamma_32<-sigma_gamma_12
```

步驟1:
第一列Gaussian Random Field 模擬(1)

```
#根據決定的range，找gamma random field中相對應的sigma (令為rho_xy1)
sigma_gamma_13<-matrix(NA,nrow=size,ncol=size)
for (i in 1:80){
  for (j in 1:80){
    if(sqrt(2^2+(i-j)^2) >= range){
      sigma_gamma_13[i,j] <- 0
    }else{
      sigma_gamma_13[i,j] <-  s*exp(-3*(sqrt(2^2+(i-j)^2))/range)/s^2
    }
  }
}
sigma_gamma_31<-sigma_gamma_13
```

步驟1:
第一列Gaussian Random Field 模擬(2)

```
# 建立Gamma的Sigma轉Gaussian的Sigma的function)
Ax <- 1+(r_x/6)^4;            Ay <- 1+(r_y/6)^4
Bx <- r_x/6-(r_x/6)^3;   By <- r_y/6-(r_y/6)^3
Cx <- ((r_x/6)^2)/3;           Cy <- ((r_y/6)^2)/3
f <- function(rho_uv, rho_xy, Ax, Bx, Cx, Ay, By, Cy){
    return((Ax*Ay-3*Ax*Cy-3*Cx*Ay+9*Cx*Cy)*rho_uv+2*Bx*By*rho_uv^2+6*Cx*Cy*rho_uv^3-rho_xy)
}
```

$$\rho_{XY} \approx (A_X A_Y - 3A_X C_Y - 3C_X A_Y + 9C_X C_Y)\rho_{UV} + 2B_X B_Y \rho_{UV}^2 + 6C_X C_Y \rho_{UV}^3$$

步驟1:
第一列Gaussian Random Field 模擬(3)

```
#將Gamma field的sigma利用前面建立的轉換function轉為Gaussian field的sigma
sigma_gaussian_11 <- matrix(0,80,80)
for (i in 1:80){
  for (j in 1:80){
    if(sigma_gamma_11[i,j]==1){
      sigma_gaussian_11[i,j] <- 1
    }
    else if(sigma_gamma_11[i,j]==0){
      sigma_gaussian_11[i,j] <- 0
    }
    else{
      sigma_gaussian_11[i,j] <- uniroot(f, c(-1,1), Ax=Ax, Bx=Bx,Cx=Cx, Ay=Ay, By=By, Cy=Cy, rho_xy=sigma_gamma_11[i,j])$root
    }
  }
};sigma_gaussian_22<-sigma_gaussian_11;sigma_gaussian_33<-sigma_gaussian_11
```

步驟1:
第一列Gaussian Random Field 模擬(3)

```
#將Gamma field的sigma利用前面建立的轉換function轉為Gaussian field的sigma
sigma_gaussian_12 <- matrix(0,80,80)
for (i in 1:80){
 for (j in 1:80){
  if(sigma_gamma_12[i,j]==1){
   sigma_gaussian_12[i,j] <- 1
  }
  else if(sigma_gamma_12[i,j]==0){
   sigma_gaussian_12[i,j] <- 0
  }
  else{
   sigma_gaussian_12[i,j] <- uniroot(f, c(-1,1), Ax=Ax, Bx=Bx,Cx=Cx, Ay=Ay, By=By, Cy=Cy, rho_xy=sigma_gamma_12[i,j])$root
  }
 }
}
sigma_gaussian_21<-sigma_gaussian_12;
sigma_gaussian_23<-sigma_gaussian_12
sigma_gaussian_32<-sigma_gaussian_12
```

步驟1:

第一列Gaussian Random Field 模擬(3)

```
#將Gamma field的sigma利用前面建立的轉換function轉為Gaussian field的sigma
sigma_gaussian_13 <- matrix(0,80,80)
for (i in 1:80){
  for (j in 1:80){
    if(sigma_gamma_13[i,j]==1){
      sigma_gaussian_13[i,j] <- 1
    }
    else if(sigma_gamma_13[i,j]==0){
      sigma_gaussian_13[i,j] <- 0
    }
    else{
      sigma_gaussian_13[i,j] <- uniroot(f, c(-1,1), Ax=Ax, Bx=Bx,Cx=Cx, Ay=Ay,
By=By, Cy=Cy, rho_xy=sigma_gamma_13[i,j])$root
    }
  }
}
sigma_gaussian_31<-sigma_gaussian_13
```

步驟2:

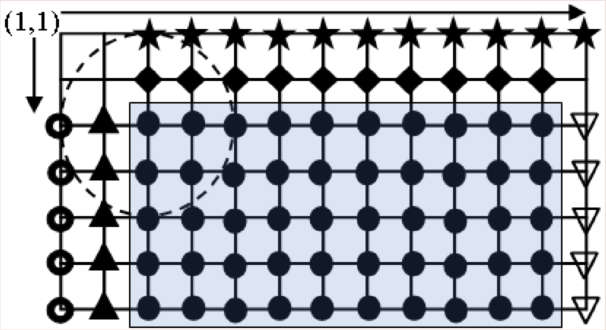
將模擬好的第一列Gaussian random field往下拓展79列(1)

```
#前兩列Gaussian Random Field模擬
 #library(mvtnorm)
 w <- matrix(NA, nrow = size, ncol = size)
 w[1,] <- rmvnorm(80,mean=c(0),sigma=diag(1))
 mu_star<- sigma_gaussian_21%*% solve(sigma_gaussian_11)%*% w[1,]
 w[2,]<-rmvnorm(1,mu_star,sigma_star)
```

$$f_{W_1|W_2}(w_1 \mid w_2) = \frac{1}{(2\pi)^{p/2} \mid \Sigma^* \mid^{1/2}} e^{-\frac{1}{2}(w_1-\mu^*)'(\Sigma^*)^{-1}(w_1-\mu^*)}$$

$$\mu^* = \mu_{W_1|w_2} = \mu_1 + \Sigma_{12}\Sigma_{22}^{-1}(w_2 - \mu_2)$$

$$\Sigma^* = \Sigma_{W_1|w_2} = \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma'_{12}$$



步驟2:

將模擬好的第一列Gaussian random field往下拓展78列(2)

```
#建立range=3時往下拓展78列所需要使用的sigma star
library(MASS)
sigma_star <- sigma_gaussian_33 -
cbind(sigma_gaussian_32,sigma_gaussian_31)%*%
ginv(rbind(cbind(sigma_gaussian_22,sigma_gaussian_21),cbind(sigma_gau
ssian_12,sigma_gaussian_11))) %*%
(rbind(sigma_gaussian_23,sigma_gaussian_13))
```

$$f_{W_1|W_2}(w_1 \mid w_2) = \frac{1}{(2\pi)^{p/2} \mid \Sigma^* \mid^{1/2}} e^{-\frac{1}{2}(w_1-\mu^*)'(\Sigma^*)^{-1}(w_1-\mu^*)}$$

$$\mu^* = \mu_{W_1|w_2} = \mu_1 + \Sigma_{12}\Sigma_{22}^{-1}(w_2 - \mu_2)$$

$$\Sigma^* = \Sigma_{W_1|w_2} = \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{12}'$$

步驟2:

將模擬好的第一列Gaussian random field往下拓展78列(3)

```
#建立range=3時往下拓展78列所需要使用的sigma star
library(MASS)
sigma_star <- sigma_gaussian_33 -
cbind(sigma_gaussian_32,sigma_gaussian_31)%*%
ginv(rbind(cbind(sigma_gaussian_22,sigma_gaussian_21),cbind(sigma_gau
ssian_12,sigma_gaussian_11))) %*%
(rbind(sigma_gaussian_23,sigma_gaussian_13))
```

$$f_{W_1|W_2}(w_1 \mid w_2) = \frac{1}{(2\pi)^{p/2} \mid \Sigma^* \mid^{1/2}} e^{-\frac{1}{2}(w_1-\mu^*)'(\Sigma^*)^{-1}(w_1-\mu^*)}$$

$$\mu^* = \mu_{W_1|w_2} = \mu_1 + \Sigma_{12}\Sigma_{22}^{-1}(w_2 - \mu_2)$$

$$\Sigma^* = \Sigma_{W_1|w_2} = \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma'_{12}$$

步驟3:

將模擬好的第一列Gaussian random field往下拓展78列(4)

```
#將模擬好的Gaussian random field轉回Gamma random field
 x <- matrix(NA, nrow = size, ncol = size)
 alpha <- 16
 lamda <- 4
 for (i in 1:80){
   for (j in 1:80){
     x[i,j] <- alpha/lamda*(1-1/9/alpha+w[i,j]*(1/9/alpha)^0.5)^3
   }
 }
```

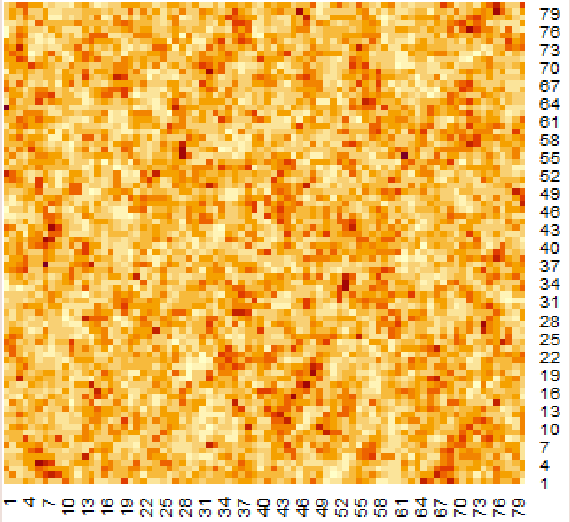
$$x = \frac{y}{2\lambda} \approx \frac{\alpha}{\lambda}\left\{1 - \frac{1}{9\alpha} + w\sqrt{\frac{1}{9\alpha}}\right\}^3$$

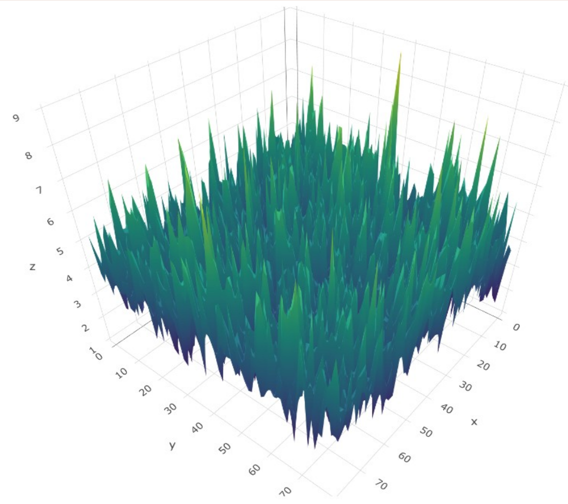
步驟4:

視覺化

```
#heatmap
heatmap(rg3[[1]], Colv = NA, Rowv = NA)

#3D plot
library(plotly)
s <- plot_ly(z = rg3[[1]], type = "surface")
s
```





Thanks for your attention