Department of Computer Science & Engineering

University of Asia Pacific (UAP)

Program: B.Sc. in Computer Science and Engineering

Final Examination	**Fall 2020**	**4^{th} Year 1^{st} Semester**
Course Code: CSE 401	**Course Title: Mathematics for Computer Science**	**Credits: 3**
Full Marks: 120* (Written)		**Duration: 2 Hours**

* Total Marks of Final Examination: 150 (Written: 120 + Viva: 30)

Instructions:

1. There are **Four (4)** Questions. Answer all of them. All questions are of equal value. Part marks are shown in the margins.
2. Programmable calculators are not allowed.

1. **a)** Suppose, you are playing a game of "Ludo" where you cannot do anything until you roll the dice and the outcome is "6". Let, you are using a Random Variable ***X*** to store the number of times you need to roll the dice to get the first "6". 3+8+4

i) What is the type of random variable ***X***?

ii) What is the probability that you will need to roll the dice ***i*** times to get the first "6"?

iii) How many times are you expecting to roll the dice in order to get the first "6"?

Here, ***i*** **= (last 3 digits of your id mod 7) + 2**

b) Suppose, the rule of "Ludo" has changed and now initially you will roll the dice ***N*** times, and you have to get a "6" exactly ***i*** times. Let, you are using a Random Variable ***Y*** to store the number of times you get "6". 3+8+4

i) What is the type of random variable ***Y***?

ii) What is the probability that if you roll the dice ***N*** times, you will get "6" ***i*** times?

iii) How many times are you expecting to get "6", if you roll the dice ***N*** times?

Here, ***N*** **= (last 3 digits of your id mod 6) + 5**

Here, ***i*** **= (last 3 digits of your id mod 4) + 3**

2. **a)** Suppose, there are 3 possible states to classify a Covid-19 patient: Asymptotic (A), Moderate (M) and Critical (C). If a patient is Asymptotic today, the probabilities that he/she will be in A or M state the next day are 0.74 and 0.24 respectively. If the patient is in Moderate (M) state today, the probabilities that he/she will be in A or C state the next day are 0.58 and 0.13 respectively. Lastly, if the patient is in Critical (C) state today, the probabilities that he/she will be in M or C state the next day are 0.37 and 0.17 respectively. 4

Let, you want to model this scenario using Markov Chain. Write down the transition matrix for this.

b) Assume that, Asymptotic is state 0, Moderate is state 1, and Critical is state 2. 13

Now using the transition matrix from (a), find out if a patient is in state ***i*** today, what is the probability that he will be in state ***j*** after ***N*** days?
Here, ***i*** = **last 3 digits of your id mod 3**
j = **(last 3 digits of your id + 2) mod 3**
N = (**(last 3 digits of your id) mod 4)+ 3**

c) What is the probability that a patient will be in state ***i*** after 100 days? 13
Here, ***i*** = **last 2 digits of your id mod 3**

3. **a)** Suppose, there are 3 manufacturing companies that produce PPE. If company A_1, A_2 and A_3 produces PPE where there is 20%, 12% and 18% chances to be defective respectively. What is the probability that it was made by company A_i? 15
Here, ***i*** = **(last 3 digits of your id mod 3) + 1**

b) Corona test is 70% effective in detecting Covid-19 when it is positive (+ive). However, the test also shows a **False Positive** result for 5% of the healthy people. If ***n***% of the population actually has Corona virus, then what is the probability a person has actually Corona virus given that his test result is positive? 15
Here, ***n*** = **(last 3 digits of your id mod 3) + 4**

4. **a)** Suppose, you are working with the Josephus problem in which every third person is eliminated, instead of every second. Find out the solution for J (N) by using the general solution of Josephus problem. 20
Where N = ((**Last 3 digit of your id +2**) mod 10) + 20

b) Draw the lines in plane problem for n = 5 and manually number the disjoint areas. Verify the correctness of your answer using the derived solution during our class. 10

OR

a) Let, we have a recursive equation $\mathbf{a_n T_n = b_n T_{n-1} + c_n}$ 20
Where, $\mathbf{a_n}$ **= (Last 3 digits of your id) mod 4 + 1**
$\mathbf{b_n}$ **= (Last 3 digits of your id +1) mod 4 + 1**
$\mathbf{c_n}$ **= (Last 3 digits of your id +2) mod 4 + 1**

Simplify the aforementioned recursive equation by multiplying with a suitable summation factor to find the sum-recurrence form and finally solve the recurrence.

b) Find the Josephus problem solution for n = (**Last 3 digits of your id**) using the odd-even recursive equation. 10