PROOF FOR ANDERSON LOCALIZATION

...

This paper is dedicated to our advisors.

Abstract. This paper gives a new proof of localization for one dimension Anderson model.

1. Introduction

The Anderson model is given by a class of discrete analogs of Schrödinger operators H_ω with real *i.i.d* potetials $\{V_\omega(n)\}$:

$$(H_\omega \Psi)(n) = \Psi(n+1) + \Psi(n-1) + V_\omega(n)\Psi(n), \tag{1.1}$$

where $\omega = \{\omega_n\}_{n\in\mathbb{Z}} \in \Omega = S^{\mathbb{Z}}$, $S = supp\{\mu\} \subset \mathbb{R}$ is assumed to be compact and contains at least two points, μ is a borel probability on $\mathbb{R}$. *i.e.* for each $n \in \mathbb{Z}$, $V_\omega(n)$ is *i.i.d.* random variables depending on ω_n in (S, μ), but we will consider V_ω in the product probability space $(S^{\mathbb{Z}}, \mu^{\mathbb{Z}})$ as a whole instead.

We say that H_ω exhibits the pectral localization property in an interval I if for $a.e.\omega$, H_ω has only pure point spectrum in I and its eigenfunction $\Psi(n)$ decays exponentially in n. We are gonna give a new proof for Anderson model based on the large deviation estimates and subharmonicity of Lyapunov exponents.

2. proof abstract

The proof is following LANA's idea.

First of all,

Definition 2.1 (*g.e.v.*)**.** E is a generalized eigenvalue (denote as *g.e.v.*), if there exists a nonzero polynomially bounded function $\Psi(n)$ such that $H\Psi = E\Psi$. We call $\Psi(n)$ generalized eigenfunction.

Then due to the fact from [1] that: *spectrally almost surely,*

$$\sigma(H) = \overline{\{E : E \text{ is } g.e.v.\}},$$

We only need to show:

Theorem 2.2. *For a.e. ω, $\forall$ g.e.v. E, the corresponding generalized eigenfunction $\Psi_{\omega,E}(n)$ decays exponentially in n.*

Since then, E is a pure point. [1] [2] [3] [4]

Date: July 23, 2017.

Key words and phrases. Anderson Localization, spectrum.

References

[1] Barry Simon. Schrödinger semigroups. *Bulletin of the American Mathematical Society*, 7(3):447–526, 1982.

[2] Henrique von Dreifus and Abel Klein. A new proof of localization in the anderson tight binding model. *Communications in Mathematical Physics*, 124(2):285–299, 1989.

[3] Svetlana Ya Jitomirskaya. Metal-insulator transition for the almost mathieu operator. *Annals of Mathematics*, 150(3):1159–1175, 1999.

[4] Hans L Cycon, Richard G Froese, Werner Kirsch, and Barry Simon. *Schrödinger operators: With application to quantum mechanics and global geometry*. Springer, 2009.

Department of Mathematics, University of California, Irvine, Rowland 410C