Game theory Questions

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1 Part 1

1. What is the difference between decision under certainty and under uncertainty?
2. What are the three properties the preference relation is required to satisfy?
3. When are we indifference between ω and ω'? How do we write this relationship?
4. Is preferences or choices observable?
5. A utility function $u : \Omega \to \mathbb{R}$ is said to represent a preference relation $\succeq$ iff we have?
6. **Theorem:** write done the theorem of preference relation and utility function.
7. Someone said that he chooses ω over ω' because $u(\omega) > u(\omega')$, is this correct?
8. What does Σ represent?
9. Write the definition of the **outcome function**.
10. Write done the expression of the definition of **Decision making under certainty**.
11. For Decision making under certainty, what is the objective function of the decision maker select the strategy σ^*?
12. What is a lottery over a non empty set S?
13. What is the formula of expected utility $EU(l)$ of a lottery l?
14. **Theorem:** What 4 conditions are required to be satisfied for a preference relation $\succeq \subseteq \text{Lott}(\{\mathcal{W}, \mathcal{L}\}) \times \text{Lott}(\{\mathcal{W}, \mathcal{L}\})$ over win-lose lotteries iff there exists a utility function $u : \mathcal{W}, \mathcal{L} \to R$ such that $l_1 \succeq l_2$ iff () ?
15. List the 4 axioms introduced by **Von Neumann and Morgensterm's Axioms**, and results of each axiom.
16. Write done the **Von Neumann and Morgensterm's Theorem**.
17. Proof of Von Neumann and Morgensterm's Theorem.
18. Write done the expression of the definition of **Decision making under uncertainty**.
19. For Decision making under uncertainty, what is the objective function of the decision maker select the strategy σ^*?
20. What is the The Allais Paradox, ie. Paradoxes of Expected Utility Theory?
21. List three important classes of preference relation and their contents.

22. Write down the general expression of **Boolean Domains**.

23. What are the basic three properties of the propositional formula representation γ of **Dichotomous Boolean Domains**?

24. For utility functions $u : 2^N \to R$, the naive representation involves listing all $2^{\|N\|}$ representations, with **weighted formula representation**, what does rule base R used to define utility function is like, and how does the utility function look like?

25. What are the three theorems of **Weighted formula representations** related to its ability of reducing the size of utility function?

26. What are the two theorems of **Weighted formula representations** related to its complexity?

27. What is the utility function like for each player i?

28. For **normal form games**, ie. strategic form non-cooperative games, write done its game forms in 3 points: players, strategies, utility function.

29. Write down the definition of a **strategy profile**.

30. Write down the notation of replacing a strategy σ_i with $sigma'_i$.

31. Definition of dominant strategies.

32. What is a **dominant strategy equilibrium**?

33. What is a pure strategy **Nash equilibrium**?

34. What is the **best response** for a player to a strategy profile $\vec{\sigma}$? For each player i with a best response function $BR_i : \Sigma_1, ..., \Sigma_n \to 2^{\Sigma_i}$, ie. set of all subsets of Σ_i, what is expression of the best response $BR_i(\vec{\sigma})$?

35. What is the expression of the **Best Response function of the game** $BR(\vec{\sigma})$?

36. $s \in S$ is a fixed point of a function $f : S \to S$ if (?). $s \in S$ is a fixed point of a function $f : S \to \mathbf{2}^S$ if (?).

37. **Lemma:** $\vec{\sigma} \in NE(G)$ iff (?) What is the relationship between a game and its fixed points?

38. **Lemma:** What is the relationship between the dominant strategy and the best responses?

39. What is the definition of **Pareto efficient** of a strategy profile $\vec{\sigma}$?

40. What is the expression of a strategy profile $\vec{\sigma^*}$ that maximise **Utilitarian social welfare**?

41. What is the expression of a strategy profile $\vec{\sigma^*}$ that maximise **Egalitarian social welfare**?

42. What is the relationship between dominant strategy equilibrium and NE?

43. What is the relationship between dominant strategy equilibrium, NE and Pareto efficient, utilitarian and egalitarian social welfare?

44. What is the relationship between the outcome that max utilitarian social welfare and Pareto efficient?

45. Write down the payoff matrix of **Prisoner's Dilemma**. Which is the dominant strategy equilibrium, which is NE? which is Pareto optimal, which maximise utilitarian social welfare?

46. Write down the payoff matrix of **Game of Chicken**. Which is the dominant strategy equilibrium, which is NE? which is Pareto optimal, which maximise utilitarian social welfare?

47. Write down the payoff matrix of **Coordination Game**. What is Focal point? What is evolutionary approach?

48. Write down the payoff matrix of The Stag Hunt.

49. Write down the payoff matrix of The Hawk-Dove.

50. Write down the payoff matrix of the **Matching Pennies**.

51. What is the definition of **dominance-solvable**?

52. **Lemma:** If a game G is dominance-solvable, then ?

53. Are there classes of games in which pure NE are guaranteed to exist?

54. A game $\langle N, \Sigma_1, ..., \Sigma_n, u_1, ..., u_n \rangle$ is a **Potential Games(exact)** if (?) for all players, for all strategy profiles, for all strategies $\sigma_i \in \Sigma_i$ and $\sigma_i' \in \Sigma_i$, we have: (?)

55. **Theorem:** What is the relationship between potential game and NE?

56. Computational Considerations

57. **Theorem: Nash theorem**: Every finite game has (?)

58. What is the definition of the **support** of a mixed strategy $\mu_i : \Sigma_i \to [0, 1]$? What is the definition of a fully mixed mixed strategy μ_i?

59. Suppose $(p, q) \in (0, 1)^2$ is a pair of mixed strategies, write down $EU_1(T, q), EU_1(B, q), EU_2(L, p), EU_2(R, p)$

60. **Theorem: Indifference Principle of 2 players** If $(p, q) \in (0, 1)^2$ is a mixed strategy Nash equilibrium in the generic 2×2 game then (?)

61. **Theorem: Indifference Principle** If mixed strategy profile $\vec{\mu} = (\mu_1, \ldots, \mu_n)$ is a NE then:

    - for all $i \in N$, and
    - for all $\sigma_1, \sigma_2 \in \text{supp}(\mu_i)$

    we have?

62. The Support Enumeration Method (SEM)

63. What are the best response function and the best response of player i be like in mixed strategies?

64. What are the two properties of Zermelo's algorithm?

65. **Theorem:**What are the three properties of **Extensive Form Games**?

66. What is the definition of **Subgame Perfection Nash Equilibrium(SPNE)**?

67. What are the two properties of SPNE related to Zermelo's algorithm?

68. A game is a **Imperfect Information Game** if?

69. What is the strategy in Imperfect Information Game like?

70. What are the two ways of randomize in extensive form game?

71. What is the definition of **Behaviour Strategies**?

72. What are the two points include in **Kuhn's Theorem** that relates mixed strategies and behaviour strategies?

73. The value of **infinite run** $\omega_1, ..., \omega_k, ...$ to player i is? Does this value guarantee to converge? If not, when does this always converge?

74. Describe the **Finite State Machines** strategies for the iterated Prisoner's dilemma: 1) ALLD; 2) ALLC; 3) GRIM; 4) TIT-FOR-TAT; 5) TAT-FOR-TIT.

75. **Theorem:** Finite machine strategies playing against each other will generate a run in a form:

76. If players play automata strategies, what will the value of the infinite run be then?

77. In a game, what is the **security value** of player i?

78. **Theorem: Nash Folk Theorem** In an infinitely repeated game, what can be Nash Equilibrium?

79. What is used in **Discounted Sum** for computing utility of infinite runs. What does the value of infinite run become then?

80. boolean

81. In zero sum games strictly competitive?

82. In zero sum game, how does player 1 (row player) choose his strategy $\bar{v}$, and how does player 2 (column player) choose his strategy $\underline{v}$?

83. In zero sum game, why player one choose to maximise the minimum?

84. **Theorem: Minimax Theorem (Pure Strategies)** If we have a two player zero-sum game, in which (σ_1, σ_2) is a NE, then?

85. Formulate the linear programming for player 1 and player 2, given utility of player 1 as $u_1(\sigma_1, \sigma_2)$

86. **Theorem:** What can Zermelo's algorithm result in the Extensive form Win-Lose Game?

87. What is the complexity of determining whether a given player has a winning strategy in a finite extensive form win-lose game?

88. What are the Complexities of the following Win-Lose Extensive Form Games?

    (a) Explicitly represented games (game tree in input)

    (b) Games guaranteed to end after a small number of moves

    (c) Games that can go on for exponentially many moves

    (d) Games that can go on for a long time and require memory.

    (e) Games that go on for ever

2 Part 2

1. What is the definition of **congestion game** wrt players, set of resources, cost/latency functions, and strategies for each player.
2. For a selected joint strategy $(P_1, \ldots, P_n) \in S_1 \times \cdots \times S_n$, the cost (or disutility) of player i is?
3. What does the tuple $(n, E, (c_e)_{e \in E}, (S_i)_{i \in [n]})$
4. In congestion game, what defines the cost of using a resource?
5. What is the definition of **exact potential game**?
6. **Proposition:** A joint strategy is a local minimum of the exact potential iff?
7. Every exact potential game has a pure NE, true or False?
8. Every congestion game is a potential game or the reverse?
9. Every congestion game has (?) pure Nash Equilibrium?
10. What is the formula of **Price of Anarchy** of a cost-minimization game?
11. What is the definition of linear congestion game?
12. What is the Price of Anarchy of linear congestion games?
    - Upper bound
    - Lower bound
13. What is the relationship between **routing games** and congestion games? What do they have in common and what does routing games have as a special?
14. What is the Price of Anarchy of routing games with positive linear latency functions $c_e(f_e) = a_e f_e + b_e$?
15. What is the definition of **load balancing game**?
16. What is the objective of load balancing games?
17. What is the PoA of pure Nash equilibrium of the class of load balancing games of m identical machines, when the social cost is the makespan? (Makespan is the maximum load among the machines after all players have chosen their strategies)
18. Give the proof of the lower bound of the PoA of load balancing game.
19. What is the relationship between **Price of Stability** and Price of Anarchy?
20. Write down the formula of **Price of Stability**.
21. We study the PoS of the class of fair cost allocation games, with what cost function $c_e(k)$?
22. What is the unique NE in the game?
23. What is the optimal solution of the game?
24. **Theorem:** What is PoS of fair cost allocation game for $\epsilon \approx 0$?
25. **Theorem:** What is the PoS for linear congestion game with non-negative coefficients?

3 True False Questions

1. If a player i has a dominant strategy in a game, then in every Nash equilibrium of that game, player i will choose a dominant strategy.
2. A NE will never include **strictly dominated strategy** and **every dominant strategy equilibrium** is a NE.
3. If the game has a strictly dominant strategy equilibrium, then it is unique and it is also the NE.
4. If a game has a strictly dominant strategy equilibrium, then it is unique: the game has no other dominant strategy equilibrium.
5. Every player i can have only 1 weakly dominant strategy.
6. Every dominant strategy equilibrium of a game is a Nash equilibrium.
7. Every Nash equilibrium of a game is a dominant strategy equilibrium.
8. If a game outcome σ^* maximises utilitarian social welfare, then σ^* is Pareto efficient.
9. If a game outcome σ^* is Pareto efficient, then it maximises utilitarian social welfare.
10. If all utilities in a game are positive, then any outcome that maximises the product of utilities of players is Pareto efficient.
11. If all utilities in a game are positive, then any Pareto efficient outcome of the game will maximise the product of utilities of players.