

This is a Prolog programming assignment. You are expected to write a Prolog program that consists of predicates described below. You will place the definitions of all the predicates in a single file and submit it via Blackboard.

Warm-Up

1. **prefix:** Write a predicate `prefix(L1, L2)` that succeeds if and only if list `L2` is a *prefix* of list `L1`: i.e. all elements of `L2` occur, in the same order, at the beginning of `L1`. For instance:
 - `prefix([1,2,3], [])`, `prefix([1,2,3], [1])`, `prefix([1,2,3], [1,2])`, `prefix([1,2,3], [1,2,3])` all succeed.
 - `prefix([1,2,3], [2])`, `prefix([1,2,3], [1,4])`, `prefix([1,2], [1,2,3])` all fail.

For this question, your predicate definition cannot use any helper predicates.

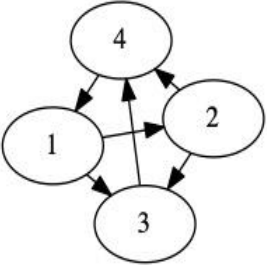
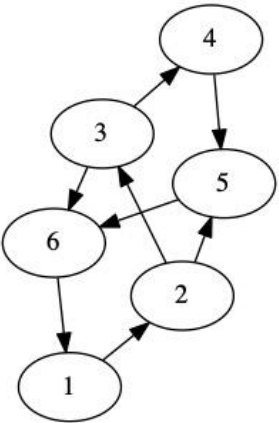
For full credit, when `L2` is not given, your predicate should return the possible values of `L2` by backtracking. For instance, `prefix([1,2,3], L2)` should return `L2 = []`, and upon backtracking, return `L2 = [1]`, `L2 = [1,2]` and `L2 = [1,2,3]`.

2. **increasing_subsequence:** Write a Prolog predicate `incsub` such that, given a list of integers `L1`, `incsub(L1, L2)` returns in `L2` an increasing subsequence of `L1`. (You may assume that `L1` contains distinct elements).
 L_2 is a subsequence of L_1 if all elements of L_2 also occur in L_1 , and if a occurs before b in L_2 then a occurs before b in L_1 .
 L is an increasing sequence if the elements are in ascending order: i.e. if a occurs before b in L , then $a < b$.
 For instance, `incsub([2,4,1,7,3,8], L2)` should succeed with answers `L2 = []`, `L2 = [2]`, `L2 = [4]`, `L2 = [2,4]`, `L2 = [1]`, ... (upon backtracking; a total of 26 distinct answers, not necessarily in this order).
3. **rotate:** Write a Prolog predicate `rotate(L1, L2)` that succeeds if and only if list `L2` is a *prefix* of list `L1`. A rotation of list $[a_1, a_2, \dots, a_n]$ is a list $[a_k, a_{k+1}, \dots, a_n, a_1, a_2, \dots, a_{k-1}]$ for some k in $[1, n]$. For example, `[2,3,4,5,1]` and `[4,5,1,2,3]` are rotations of `[1,2,3,4,5]`. For full credit, when `L2` is not given, your predicate should return the possible values of `L2` by backtracking. For instance, `rotate([1,2,3], L2)` should return `L2 = [1,2,3]`, and upon backtracking, return `L2 = [2,3,1]` and `L2 = [3,1,2]` (not necessarily in the same order).

Graph Problems:

The following set of problems are over graphs, and we will use the following encoding to represent **directed** graphs in Prolog.

Each graph is given an identifier (name or number), and we enumerate its edges as triples in a 3-ary **edge** relation. Each triple of the form **edge(*i*, *v1*, *v2*)** represents an edge in the graph *i* from vertex *v1* to vertex *v2*. Example graphs and their Prolog encoding are shown below:

a		<pre> edge(a, 1, 2). edge(a, 1, 3). edge(a, 2, 3). edge(a, 2, 4). edge(a, 3, 4). edge(a, 4, 1). </pre>
b		<pre> edge(b, 1, 2). edge(b, 2, 3). edge(b, 2, 5). edge(b, 3, 4). edge(b, 4, 5). edge(b, 5, 6). edge(b, 6, 1). edge(b, 2, 5). edge(b, 3, 6). </pre>

4. **exists_path**: Write a binary predicate **exists_path** such that, given a graph *I* and a list of vertices *P*, **exists_path(*I*, *P*)** succeeds if and only if vertices in *P* forms a path in the graph.

For the example graphs above, **exists_path(a, [1,3,4])**, **exists_path(a, [2,4,1,3,4,1,2])**, **exists_path(b, [1,2,3,4,5,6])** all succeed; **exists_path(a, [1,2,3])** and **exists_path(b, [1,2,3,5])** fail.

5. **exists_simple_path**: Write a binary predicate **exists_simple_path** such that, given a graph *I* and a list of vertices *P*, **exists_simple_path(*I*, *P*)** succeeds if and only if vertices in *P* forms a **simple (i.e cycle-free) path** in the graph.

For the example graphs above, **exists_simple_path(a, [1,3,4])** and **exists_simple_path(b, [1,2,3,4,5,6])** succeed; **exists_path(a, [1,2,3])**, **exists_path(a, [2,4,1,3,4,1,2])**, and **exists_path(b, [1,2,5,6,1,2,3])** fail.

6. **simple_path**: Write a 4-ary predicate **simple_path** such that, for queries of the form **simple_path(*i*, *v1*, *v2*, *P*)** where *i* is a graph id, *v1* and *v2* are vertices in graph *i*, succeeds by binding *P* to a list of vertices that lie on a simple path (i.e. cycle-free) path from *v1* to *v2*. For example, **simple_path(a, 1, 4, Y)** should succeed with *Y*=[1,2,4], and upon backtracking, *Y*=[1,3,4] and *Y*=[1,2,3,4]. The order of answers does not matter.

Analysis of Program Traces

For this problem, we consider *traces* of a procedural program's execution. We log four events in these traces:

- `malloc(p)`: Allocate a block of memory and assign its address to pointer *p*.
- `read(p)`: Read the value at the address in pointer *p*
- `write(p)`: Write a value to address in pointer *p*
- `free(p)`: Free (i.e. de-allocate) memory located at the address in pointer *p*.

A trace is a list of such events. (**Note:** this problem assumes no aliases: no two pointers refer to the same address in memory).

7. Write a predicate `invalid_access`, that, given a trace, succeeds if and only if the trace has a read/write event after that pointer has been freed, or before that pointer is malloced.
8. Write a predicate `memory_safe`, that, given a trace, succeeds if and only if all the memory operations are safe:
 - reads and writes are to previously malloced (and not yet freed) memory.
 - free refers to a previously malloced (but not yet freed) memory.
9. Write a predicate `leak`, that, given a trace, succeeds if and only there is a memory leak: a pointer is re-assigned with a malloc without freeing the earlier block.
10. Write a predicate `useful_writes`, that, given a trace, succeeds if and only if every value that is written to an address is read before the pointer is freed or re-allocated.

Your program should be in a **single** file (see submission instructions below).

Grading:

All problems are worth 3 points each. The grading rubric will be:

3	A correct solution or a solution with very few flaws
2	A solution that is substantially correct; some flaws but mostly correct
1	A solution that is substantially incorrect; many flaws with few correct parts
0	Incorrect or missing solution

Code documentation is expected unless the intent of the solution is obvious from its form. Partial credit may not be possible without clear documentation.

Submission:

Your submission will consist of a **single** Prolog program file (name of the file and the file extension does not matter for the submission). Submit the file as an attachment to the HW1 form on Blackboard.

Important: Work on the warm-up problems individually. You can work on Graph and Trace problems with 1 other team member. In that case, both have to submit complete solutions separately. Make sure that your submission has a clear marking of who your team mate is.

Errata:

- Sep 9, 9:30am: Added note in the "Submission" section on teams (in purple)