

# CSE 505 HW2 Report

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## Q1:

$p \text{ cnf } n \ 1$

$p_1 \ p_2 \ \dots \ p_n \ 0$

## Q2:

$p \text{ cnf } n \ n$

$\neg p_1 \ 0$

$\neg p_2 \ 0$

$\dots$

$\neg p_n \ 0$

## Q3:

$p \text{ cnf } n \ k$

$p_1 \ p_2 \ \dots \ p_{k-1} \ p_k \ 0$

$p_2 \ p_3 \ \dots \ p_k \ p_{k+1} \ 0$

$\dots$

$p_{n-k} \ p_{n-k+1} \ \dots \ p_{n-1} \ p_n \ 0$

## Q4:

see Q4.c

## Test:

3 6 8

1 2

2 3

3 4

4 5

5 6

6 1

2 5

3 6

**Output:** SATISFIABLE

## Q5:

1.

For all steps  $s \in [0, K]$ , integer  $\text{num}(s, i, j) \in [0, N \times N - 1]$  denoting which number or label the tile contains at step  $s$ .

2.

For all space  $(i, j)$ :  $\text{num}(0, i, j)$  is the number of the  $(i, j)$  in the initial configuration. The empty tile is numbered as 0.

3.

Integers  $d_i(s), d_j(s) \in \{-1, 0, 1\}$ , respectively denoting the vertical and horizontal movement of the empty tile during the transition from step  $s$  to step  $s + 1$ .

4.

For all steps  $s \in [0, K]$ , a Boolean  $goal(s)$  denoting whether the puzzle has been solved at step  $s$  or at an earlier step.

5.

Integers  $empty_i(s) \in [0, N - 1]$  and  $empty_j(s) \in [0, N - 1]$ , respectively denoting the vertical and horizontal coordinates of the empty tile at step  $s$ .

6.

$(num(s, i, j) = 0) \Leftrightarrow (empty_i(s) = i \wedge empty_j(s) = j)$ . This synchronizes the two ways of tracking the location of the empty tile.

7.

For all steps  $s \in [0, K] : (steps \leq s) \rightarrow (goal(s) = 1)$

8.

For all steps  $s \in [0, max - 1] : d_i(s) = 0 \vee d_j(s) = 0$ . This enforces that the player only makes orthogonal moves.

9.

$goal(s) = 1 \Leftrightarrow (d_i(s) = 0 \wedge d_j(s) = 0)$ . This enforces that no more moves are made once the puzzle has been solved.

10.

$(d_i(s) = 0 \wedge d_j(s) = 0) \rightarrow (empty_i(s) = empty_i(s + 1) \wedge empty_j(s) = empty_j(s + 1))$ . This enforces that, if one has not moved, the empty tile should not have moved.

11.

For the moves  $(i, j) \in \{(1, 0), (-1, 0), (0, 1), (0, -1)\} : (d_i(s) = i \wedge d_j(s) = j) \rightarrow (empty_i(s) + i \geq 0 \wedge empty_i(s) + i \leq N \wedge empty_j(s) + j \geq 0 \wedge empty_j(s) + j \leq N \wedge empty_i(s) + i = empty_i(s + 1) \wedge empty_j(s) + j = empty_j(s + 1))$ . This enforces that the player only makes valid moves and that the empty tile actually moves.

12.

$\neg((empty_i(s) = i \wedge empty_j(s) = j) \vee (empty_i(s + 1) = i \wedge empty_j(s + 1) = j)) \rightarrow (num(s, i, j) = num(s + 1, i, j))$ . This enforces that, if the tile was not the empty tile during either one of the two steps, its contents should be unchanged.

13.

For all steps  $s \in [1, K - 1] : d_i(s - 1) = 1 \rightarrow d_i(s) \neq -1, d_i(s - 1) = -1 \rightarrow d_i(s) \neq 1, d_j(s - 1) = 1 \rightarrow d_j(s) \neq -1, d_j(s - 1) = -1 \rightarrow d_j(s) \neq 1$ . The number of possible moves at every step reducing the size of the search space.

14.

See Q5.c

**Test:**

4 100

15 2 1 12

8 5 6 11

4 9 10 7

3 14 13 0

**Output:**

SATISFIABLE

