Enchan Field Notes v0.4.5

A Scalar-Field Description of Galaxy-Scale Acceleration Regularities

Mitsuhiro Kobayashi
Tokyo, Japan

Abstract

We present a compact theoretical framework that organizes several well-known galaxy-scale regularities into an effective scalar-field description. A dimensionless field S is introduced as a proxy for gravitational time-delay effects in the weak-field regime.

The framework is constructed to reproduce the Radial Acceleration Relation (RAR) and the Baryonic Tully–Fisher Relation (BTFR) through defect-like configurations of S anchored by baryonic structure. Environmental suppression in deep baryonic potentials is incorporated as an effective modulation of the acceleration scale, calibrated directly on observational data.

The purpose of these notes is not to modify General Relativity, but to provide a minimal and internally consistent parametrization of observed phenomenology within a controlled effective description.

Scope and Domain of Applicability

This document is restricted to the weak-field, galaxy-scale regime in which empirical acceleration regularities are defined. No claims are made regarding strong-field gravity, cosmology at early times, or precision Solar-System tests.

Contents

1 Overview

These notes present a compact effective description for organizing galaxy-scale acceleration regularities in rotationally supported systems. We introduce a dimensionless scalar field $S(x)$ and work in the weak-field regime where the relevant observables are the baryonic distribution, rotation curves, and derived radial accelerations.

1.1 Minimal definitions

We treat S as a dimensionless proxy field whose spatial gradients encode an effective gravitational response in the regime of interest. In the notation of this document, the weak-field response is summarized by

$$g_{\mathrm{tot}} \equiv c^2 \nabla S, \tag{1}$$

to be interpreted as an effective relation defining how S parametrizes the acceleration field used in the phenomenological analysis. No statement is made here about strong-field gravity or a fundamental completion.

1.2 Working assumptions (phenomenological)

The framework uses two working assumptions that are kept explicit throughout:

1. **Anchor ansatz.** The baryonic distribution provides an anchoring structure that selects characteristic scales for spatial variations of S in galaxies.

2. **Environmental modulation.** In deep baryonic potentials, the effective acceleration scale can be suppressed by an environmental proxy, enabling consistent fits across a heterogeneous galaxy sample.

These assumptions are treated as phenomenological inputs whose role is to organize the data in a controlled way.

1.3 Empirical targets

The empirical targets used for calibration and verification are: (i) the Radial Acceleration Relation (RAR), (ii) the Baryonic Tully–Fisher Relation (BTFR), and (iii) rotation-curve shape predictions under fixed-parameter rules. All comparisons are restricted to the weak-field, galaxy-scale domain.

1.4 Document structure

- **Chapter 2** introduces the minimal conceptual setup and notation.
- **Chapter 3** summarizes kinematic relations for rotation curves.
- **Chapter 4** states the field model and the effective equation used.
- **Chapter 5** discusses asymptotic behavior relevant for galactic dynamics.
- **Chapter 6** presents the derivations used to connect the model to an effective acceleration scale.
- **Chapter 7** confronts the framework with observational benchmarks.

Scope

This document is limited to a weak-field, galaxy-scale effective description. It does not address precision Solar-System tests, strong-field gravity, or early-universe cosmology.

2 Conceptual Setup and Effective Variables

This chapter introduces the minimal conceptual setup used throughout these notes. We work with a dimensionless scalar field $S(x)$ as an effective proxy variable in the weak-field regime and define the derived quantities used in later chapters.

2.1 Field variable

The scalar field $S(x)$ is taken to be dimensionless. In the intended regime of applicability, spatial gradients of S are used to parametrize an effective gravitational response, summarized by

$$g_{\mathrm{tot}} \equiv c^2 \nabla S. \tag{2}$$

This relation is used as a working definition for the phenomenological mapping between the field variable and the inferred acceleration field in galaxies.

2.2 Derived quantities

Two derived quantities are used repeatedly:

1. **Gradient magnitude.** We denote the magnitude of the spatial gradient by

   $$D(x) \equiv \|\nabla S\|. \tag{3}$$

   In the effective description, D serves as a compact measure of spatial variation in S.

2. **Kinetic invariant.** For covariant expressions we define

   $$\mathscr{X} \equiv \frac{\nabla_\mu S \nabla^\mu S}{a_0^2}, \tag{4}$$

   where a_0 is the characteristic acceleration scale introduced later. The function $\mu(\mathscr{X})$ used in the field equation is chosen so that the quasi-static limit reproduces the empirical interpolation employed in the observational benchmarks.

2.3 Effective source structure

The galaxy-scale description uses baryonic structure as the primary source input. We denote the baryonic mass density by ρ_{b} and adopt an anchoring ansatz in which baryonic distributions select characteristic scales for spatial variations of S. Environmental modulation is incorporated through an effective suppression of the acceleration scale in deep baryonic potentials, using a potential-depth proxy.

2.4 Notation and conventions

Unless stated otherwise, we work in the weak-field, nonrelativistic limit when connecting the field model to observational quantities. The field S is treated as an effective variable; no assumptions are made here about a unique fundamental completion beyond the regime studied in these notes.

3 Rotation and Kinematic Context

This chapter summarizes the role of rotation in the observational and theoretical context relevant for galaxy dynamics. Rotation curves provide direct access to the radial acceleration field inferred from circular motion, and therefore serve as a primary empirical interface for the effective description developed in later chapters.

3.1 Rotation curves as acceleration data

For an axisymmetric system with an observed circular velocity profile $v(r)$, the centripetal acceleration inferred from kinematics is

$$g_{\mathrm{obs}}(r) \equiv \frac{v^2(r)}{r}. \tag{5}$$

The baryonic contribution is computed from the luminous components under standard mass-to-light and geometric assumptions, yielding a baryonic acceleration proxy $g_{\mathrm{bar}}(r)$. The empirical relations discussed in these notes are formulated in terms of the pair $(g_{\mathrm{obs}}, g_{\mathrm{bar}})$ across radii and across galaxies.

3.2 Relativistic rotation effects (context only)

In General Relativity, rotating mass distributions admit frame-dependent effects associated with the gravitomagnetic sector of the metric (often referred to as frame dragging in the weak-field limit). In these notes, such effects are mentioned only as contextual motivation for why rotation can be a relevant structural feature of gravitational systems. No strong-field modeling or implementation-level interpretation is assumed or required.

3.3 Connection to the effective description

The scalar-field parametrization introduced in Chapters 2 and 4 is connected to galaxy kinematics through the effective acceleration field. In particular, the working definition

$$g_{\mathrm{tot}} \equiv c^2 \nabla S \tag{6}$$

is used to map the scalar field variable to an acceleration response in the weak-field, quasi-static regime. The remainder of this document focuses on reproducing the observed regularities in the kinematic data using baryonic inputs and an effective field equation.

Scope

This chapter provides kinematic definitions and limited theoretical context. It introduces no new assumptions beyond those stated in Chapters 1–2.

4 Minimal Field Model

This chapter states a minimal effective field model for a dimensionless scalar $S(x)$ and fixes the notation used in the remainder of the document. The model is used only as a weak-field, galaxy-scale effective description.

4.1 Field variable and conventions

We take $S(x)$ to be dimensionless. Indices are raised and lowered with the background metric; ∇_μ denotes the covariant derivative and $\Box \equiv \nabla_\mu \nabla^\mu$. In quasi-static applications we use the working relation

$$g_{\rm tot} \equiv c^2 \nabla S. \tag{7}$$

4.2 Minimal effective Lagrangian

A compact effective Lagrangian density is

$$\mathscr{L} = \frac{\sigma}{2} \nabla_\mu S \nabla^\mu S - V(S) + J(x)\, S, \tag{8}$$

where σ is a constant coefficient, $V(S)$ is an effective potential, and $J(x)$ denotes an external source term used to encode phenomenological anchoring by baryonic structure. No microscopic interpretation of σ, V, or J is assumed in these notes.

4.3 Equation of motion

Varying the action associated with Eq. (8) yields

$$\sigma \Box S = J(x) - V'(S). \tag{9}$$

For the galaxy-scale applications considered here, we primarily use the quasi-static limit in which time derivatives are negligible and sources vary slowly. In that limit, Eq. (9) reduces to a Poisson-like form,

$$\sigma \nabla^2 S \simeq J(x) - V'(S). \tag{10}$$

4.4 Nonlinear "vessel" form

To accommodate RAR-compatible phenomenology, it is convenient to generalize the kinetic sector to a nonlinear response function $\mu(\mathscr{X})$ with

$$\mathscr{X} \equiv \frac{\nabla_\mu S \nabla^\mu S}{a_0^2}, \tag{11}$$

leading to the effective field equation

$$\nabla_\mu \Big(\mu(\mathscr{X})\, \nabla^\mu S \Big) - V'(S) = -\kappa \rho_{\rm b} c^2, \tag{12}$$

where $\rho_{\rm b}$ is the baryonic rest-mass density used as the observational source input and κ is an effective coupling constant. The function $\mu(\mathscr{X})$ is chosen so that the quasi-static limit matches the empirical interpolation used for the observational benchmarks in Chapter 7.

Scope

Equations (9)–(12) are used as effective, weak-field relations for galaxy-scale dynamics only. No claims are made regarding strong-field gravity or precision Solar-System tests.

5 Asymptotic Structure and Galaxy-Scale Scaling Relations

This chapter summarizes the asymptotic behavior of the effective field description in the quasi-static regime and derives scaling relations relevant for galaxy rotation curves. The discussion is restricted to weak-field, large-radius behavior.

5.1 Quasi-static limit

In the quasi-static regime, the effective field equation stated in Chapter 4 admits a Poisson-like form. For the purpose of asymptotic scaling, we consider the outer-region limit in which source variations are slow and the solution is dominated by the field's large-radius behavior:

$$\nabla^2 S \simeq \mathscr{S}(S;\, \text{parameters}), \tag{13}$$

where $\mathscr{S}$ denotes the effective source/potential structure. The precise form used in later chapters is fixed by the phenomenological calibration.

5.2 Logarithmic asymptotic profile

A convenient and widely used asymptotic profile for producing flat rotation behavior is a logarithmic field configuration,

$$S(r) \simeq \eta_S \ln\left(\frac{r}{r_c}\right), \tag{14}$$

where η_S is a dimensionless amplitude and r_c is a characteristic scale. From Eq. (14) one obtains the radial gradient magnitude

$$|\nabla S| = \frac{\eta_S}{r}. \tag{15}$$

Using the working relation $g_{\mathrm{tot}} \equiv c^2 \nabla S$, Eq. (15) corresponds to an outer-region acceleration scaling $g_{\mathrm{tot}} \propto 1/r$.

5.3 Effective halo-like density scaling

For bookkeeping, it is useful to associate an effective density profile with the field-gradient sector by defining

$$\rho_{\mathrm{eff}}(r) \propto |\nabla S|^2. \tag{16}$$

Substituting Eq. (15) yields the characteristic scaling

$$\rho_{\mathrm{eff}}(r) \propto \frac{1}{r^2}. \tag{17}$$

This is the same radial scaling as the singular isothermal sphere profile used as a standard phenomenological representation of flat rotation curves.

5.4 Flat rotation scaling

If the outer-region dynamics are dominated by an effective density profile of the form $\rho_{\mathrm{eff}} \propto r^{-2}$, then the enclosed effective mass scales linearly with radius, $M_{\mathrm{eff}}(r) \propto r$. The corresponding circular velocity satisfies

$$v^2(r) \propto \frac{M_{\mathrm{eff}}(r)}{r} \simeq \text{const}, \tag{18}$$

which is the standard flat-rotation scaling.

Remark

This chapter isolates the asymptotic scaling structure used in later derivations. The connection between (η_S, r_c) and baryonic inputs, and the calibration of the effective response function used for observational benchmarks, are addressed in Chapters 6–7.

6 Derivation: the acceleration scale

6.1 Scope

This chapter fixes the minimal definitions and parametric dependencies used to introduce an effective acceleration scale a_0 within the field description.

6.2 Controlled assumptions

We work under the following assumptions:

- **Quasi-static limit:** $\partial_t S \simeq 0$ on galaxy scales.
- **Spherical profile (outer region):** the dominant large-radius behavior is treated as effectively spherical.
- **Anchor ansatz:** a defect-core energy-density scale is tied to a baryonic surface-density proxy evaluated at a characteristic radius $r = r_c$.

6.3 Core length scale

Let S be dimensionless and consider an effective Lagrangian with a constant kinetic coefficient σ and an effective core energy-density scale V_0. We define a characteristic core/anchor length scale

$$\ell_c \equiv \sqrt{\frac{\sigma}{2V_0}}. \tag{19}$$

6.4 Outer defect profile

Outside the core region, we adopt a logarithmic profile as a minimal asymptotic form,

$$S(r) \simeq S_0 + \eta_S \ln\left(\frac{r}{\ell_c}\right), \tag{20}$$

where η_S is a dimensionless amplitude and S_0 is an additive constant. This implies

$$|\nabla S| = \frac{\eta_S}{r}. \tag{21}$$

6.5 Definition of the acceleration scale

We define the effective acceleration scale by combining η_S and ℓ_c:

$$a_0 \equiv \frac{c^2\,\eta_S}{\ell_c} = c^2\,\eta_S\sqrt{\frac{2V_0}{\sigma}}. \tag{22}$$

6.6 Surface-density anchor (minimal form)

Let $\Sigma_{\rm b}(r_c)$ be a baryonic surface-density proxy evaluated at the characteristic radius r_c. We postulate a leading-order anchoring relation of the form

$$\frac{V_0(r_c)}{\eta_S} \simeq \frac{1}{\alpha_c} G\Sigma_{\rm b}(r_c)^2, \tag{23}$$

where α_c is a dimensionless efficiency parameter. Together with Eq. (22), this fixes the parametric dependence of a_0 on the surface-density proxy up to dimensionless factors.

6.7 Environmental suppression in deep potentials

We allow a minimal environmental suppression of the effective acceleration scale in deep baryonic potentials by defining

$$a_{0,\mathrm{eff}}(r_c) \equiv a_{0,\mathrm{free}}\big(\Sigma_\mathrm{b}(r_c)\big)\ \mathscr{S}_\Phi(|\Phi_\mathrm{bar}(r_c)|)\,, \tag{24}$$

with a suppression factor $0 < \mathscr{S}_\Phi \leq 1$. Operationally, we use the standard baryonic mass-model proxy

$$|\Phi_\mathrm{bar}(r_c)| \approx V_\mathrm{bar}(r_c)^2, \qquad V_\mathrm{bar}^2 \equiv V_\mathrm{gas}^2 + \Upsilon_d V_\mathrm{disk}^2 + \Upsilon_b V_\mathrm{bul}^2, \tag{25}$$

and adopt the minimal two-parameter form

$$\mathscr{S}_\Phi(|\Phi|) = \left[1 + \left(\frac{|\Phi|}{\Phi_c}\right)^n\right]^{-1}, \tag{26}$$

with global parameters (Φ_c, n) shared across the sample.

Connection to observational benchmarks

Chapter 7 confronts the model with observational benchmarks using the dependencies fixed in this chapter.

7 Observational anchors and benchmarks

Scope

This chapter defines a minimal set of observational benchmarks that the effective description must reproduce in the weak-field, galaxy-scale regime. Only definitions are fixed here; no implementation details, code structure, or procedural details are included.

7.1 Benchmark A: RAR / MDAR (multi-point relation)

For each radial point in each galaxy, define the observed centripetal acceleration

$$g_{\mathrm{obs}}(r) \equiv \frac{V_{\mathrm{obs}}(r)^2}{r}, \tag{27}$$

and the baryonic acceleration proxy from the standard mass-model decomposition

$$g_{\mathrm{bar}}(r) \equiv \frac{V_{\mathrm{gas}}(r)^2 + \Upsilon_d V_{\mathrm{disk}}(r)^2 + \Upsilon_b V_{\mathrm{bul}}(r)^2}{r}, \tag{28}$$

with consistent unit conversion.

The benchmark statement is the existence of a tight empirical mapping between g_{bar} and g_{obs} across many radii and many galaxies, commonly summarized by a one-parameter interpolation curve involving an acceleration scale a_0.

7.2 Benchmark B: BTFR (one galaxy = one point)

Define a galaxy-level relation between baryonic mass M_{b} (stars + gas) and a characteristic outer/flat rotation speed V_{f}. In log form,

$$x \equiv \log_{10}(V_{\mathrm{f}}/\mathrm{km\,s^{-1}}), \qquad y \equiv \log_{10}(M_{\mathrm{b}}/M_{\odot}), \tag{29}$$

and the benchmark is that a near-linear relation holds:

$$y = a + bx, \tag{30}$$

with small intrinsic scatter.

A commonly used acceleration-scale diagnostic associated with BTFR is

$$a_{0,\mathrm{BTFR}} \equiv \frac{V_{\mathrm{f}}^4}{G M_{\mathrm{b}}}, \tag{31}$$

which provides a per-galaxy summary scale for comparison with the RAR-scale normalization.

7.3 Benchmark C: Rotation-curve prediction (shape test)

Using the baryonic mass-model components at each radius, compute $g_{\mathrm{bar}}(r)$. Apply a fixed (globally shared) mapping to obtain a predicted response $g_{\mathrm{pred}}(r)$, and define

$$V_{\mathrm{pred}}(r) = \sqrt{g_{\mathrm{pred}}(r)\,r}. \tag{32}$$

The benchmark requirement is that a single fixed rule, without per-galaxy tuning, produces nontrivial rotation-curve shapes across a large heterogeneous sample, and that deviations exhibit interpretable systematics.

7.4 Interpretation

These benchmarks do not by themselves select a unique fundamental theory. They serve as externally defined targets: an Enchan-derived forward model must reproduce (A)–(C) within a unified mechanism and a consistent parameterization.

References

[1] S. S. McGaugh, F. Lelli, and J. M. Schombert, “Radial Acceleration Relation in Rotationally Supported Galaxies,” *Phys. Rev. Lett.* 117, 201101 (2016).

[2] F. Lelli et al., “The Baryonic Tully–Fisher Relation for Different Galaxy Types,” *Mon. Not. R. Astron. Soc.* 484, 3267 (2019).

[3] T. W. B. Kibble, “Topology of cosmic domains and strings,” *J. Phys. A* 9, 1387 (1976).

[4] M. Barriola and A. Vilenkin, “Gravitational Field of a Global Monopole,” *Phys. Rev. Lett.* 63, 341 (1989).