2. A die (six faces) has the number 1 painted on three of its faces, the number 2 painted on two of its faces, and the number 3 painted on one face. Assume that each face is equally likely to come up.

a. Find a sample space for this experiment.

b. Find P(odd number).

a. $\Omega = \{1, 2, 3\}$

b. $P(\text{odd number}) = P(\{1,3\})$

$= P(\{1\}) + P(\{3\})$

$= 3/6 + 1/6 = 2/3$

2-21. A digital scale that provides weights to the nearest gram is used.

(a) What is the sample space for this experiment?

Let A denote the event that a weight exceeds 11 grams, let B denote the event that a weight is less than or equal to 15 grams, and let C denote the event that a weight is greater than or equal to 8 grams and less than 12 grams.

Describe the following events.

(b) $A \cup B$ (c) $A \cap B$
(d) A' (e) $A \cup B \cup C$
(f) $A \cap B'$ (g) $A \cap B \cap C$
(h) $B' \cap C$ (i) $A \cup (B \cap C)$

(a) Ω = The natural numbers and 0.

$= \mathbb{N}_+$

Use "dot dot dot" notation for sequences of integers:

$A = [12 \ldots \infty)$ $B = [0 \ldots 15]$ $C = [8 \ldots 11]$

(b) $A \cup B = [12 \ldots \infty) \cup [0 \ldots 15] = [0, \infty)$

(c) $A \cap B = [12 \ldots \infty) \cap [0 \ldots 15] = [12 \ldots 15]$

(d) A' (aka A^c) $= \mathbb{N}_+ \setminus [12 \ldots \infty)$

$= [0 \ldots 11]$ set subtraction.

(e) $A \cup B \cup C = [12 \ldots \infty) \cup [0 \ldots 15] \cup [8 \ldots, 11]$

$= \mathbb{N}_+$

(f) $A \cap B' = [12 \ldots \infty) \cup [0 \ldots 15]'$

$= [12 \ldots \infty) \cup [16 \ldots \infty)$

$= [16 \ldots \infty)$

(g) $A \cap B \cap C = [12 \ldots \infty) \cap [0 \ldots 15] \cap [8 \ldots, 11]$

$= \phi$ (empty set)

(h) $B' \cap C = [0 \ldots 15]' \cap [8 \ldots, 11]$

$= [16 \ldots \infty) \cap [8 \ldots, 11] = \phi$

(i) $A \cup (B \cap C) = [12 \ldots \infty) \cup ([0 \ldots 15] \cap [8 \ldots, 11])$

$= [12 \ldots \infty) \cup [8 \ldots, 11]$

$= [8 \ldots \infty)$

7. According to a report by the Agency for Healthcare Research and Quality, the age distribution for people admitted to a hospital for an asthma-related illness wasasfollows.

Age (years)	Proportion
Less than 1	0.02
1–17	0.25
18–14 44	0.19
45–64	0.30
65–84	0.20
85 and up	0.04

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a. What is the probability that an asthma patient is between 18 and 64 years old?

b. What is the probability that an asthma patient is less than 85 years old?

a. $P([18, 64]) = P([18 \ldots 44]) + P([45 \ldots 64])$

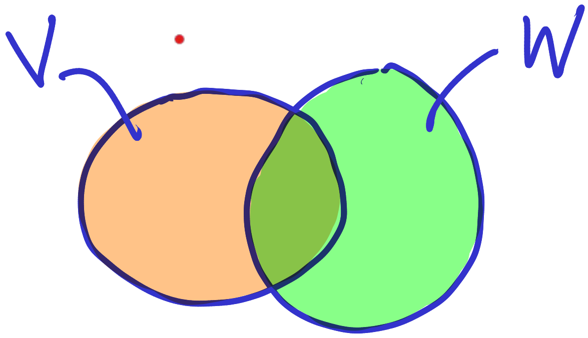
$= 0.19 + 0.3 = 0.49$

b. $P([0, \ldots 84]) = 1 - P(85 \text{ and up})$

$= 0.96$

12. Let V be the event that a computer contains a virus, and let W be the event that a computer contains a worm. Suppose $P(V) = 0.15$, $P(W) = 0.05$, and $P(V \cup W) = 0.17$.

a. Find the probability that the computer contains both a virus and a worm.
b. Find the probability that the computer contains neither a virus nor a worm.
c. Find the probability that the computer contains a virus but not a worm.



$$P(V \cup W) = P(V) + P(W) - P(V \cap W)$$

a) $P(V \cap W) = P(V) + P(W) - P(V \cup W)$

$= 0.15 + 0.05 - 0.17 = 0.03$

b) $P((V \cup W)') = 1 - P(V \cup W)$

$= 1 - 0.17 = 0.83$

c) $P(V \cap W') = P(V) - P(V \cap W)$ .... by observation of the picture.

orange region.

$= 0.15 - 0.03$

$= 0.12$

17. A system contains two components, A and B. The system will function only if both components function. The probability that A functions is 0.98, the probability that B functions is 0.95, and the probability that either A or B functions is 0.99. What is the probability that the system functions?

$P(A) = 0.98$

$P(B) = 0.95$

$P(A \cup B) = 0.99$

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$= 0.98 + 0.95 - 0.99 = 0.94.$$

Navidi Section 2.4

5. Four candidates are to be interviewed for a job. Two of them, numbered 1 and 2, are qualified, and the other two, numbered 3 and 4, are not. The candidates are interviewed at random, and the first qualified candidate interviewed will be hired. The outcomes are the sequences of candidates that are interviewed. So one outcome is 2, and another is 431.

   a. List all the possible outcomes.
   b. Let *A* be the event that only one candidate is interviewed. List the outcomes in *A*.
   c. Let *B* be the event that three candidates are interviewed. List the outcomes in *B*.
   d. Let *C* be the event that candidate 3 is interviewed. List the outcomes in *C*.
   e. Let *D* be the event that candidate 2 is not interviewed. List the outcomes in *D*.
   f. Let *E* be the event that candidate 4 is interviewed. Are *A* and *E* mutually exclusive? How about *B* and *E*, *C* and *E*, *D* and *E*?

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a. $\Omega = \{1, 2, 31, 32, 41, 42, 341, 342, 431, 432\}$.

b. $A = \{1, 2\}$.

c. $B = \{341, 342, 431, 432\}$.

d. $C = \{31, 32, 341, 342, 431, 432\}$.

e. $D = \{1, 31, 41, 341, 431\}$.

f. $E = \{41, 42, 341, 342, 431, 432\}$.

$A \cap E = \phi$ ... mutually exclusive

$B \cap E \neq \phi$, $C \cap E \neq \phi$, $D \cap E \neq \phi$

4. Let X represent the number of tires with low air pressure on a randomly chosen car.

a. Which of the three functions below is a possible probability mass function of X? Explain.

	x				
	0	**1**	**2**	**3**	**4**
$p_1(x)$	0.2	0.2	0.3	0.1	0.1
$p_2(x)$	0.1	0.3	0.3	0.2	0.2
$p_3(x)$	0.1	0.2	0.4	0.2	0.1

b. For the possible probability mass function, compute μ_X and .

a. • $\sum_{x=1}^{4} P_1(x) = 0.9$ … not a pdf (aka pmf when discrete)

• $\sum_{x=1}^{4} P_2(x) = 1.1$ … "

• $\sum_{x=1}^{4} P_3(x) = 1.0$ … yes a pdf.

b. $\mu_X = \sum_{x=1}^{4} x\, P_3(x) = 0 \cdot 0.1 + 1 \cdot 0.2 + 2 \cdot 0.4 + 3 \cdot 0.2 + 4 \cdot 0.1$

$= 0 + 0.2 + 0.8 + 0.6 + 0.4$

$= 2$

Navidi Section 2.4

2. Computer chips often contain surface imperfections. For a certain type of computer chip, the probability mass function of the number of defects X is presented in the following table.

x	0	1	2	3	4
$p(x)$	0.4	0.3	0.15	0.10	0.05

a. Find $P(X \leq 2)$.
b. Find $P(X > 1)$.
c. Find μ_X.
d. Find σ_X^2.

a. $P(X \leq 2)$ ... is alternative notation for $P_X((-\infty, 2])$

$= p(0) + p(1) + p(2)$

$= 0.4 + 0.3 + 0.15 = 0.85$

b. $P(X>1) = p(2) + p(3) + p(4)$

$= 0.15 + 0.1 + 0.05$

$= 0.3$

c. $\mu_X = 0(0.4) + 1(0.3) + 2(0.15) + 3(0.10) + 4\cdot(0.05)$

$= 0 + 0.3 + 0.3 + 0.3 + 0.2 = 1.1$

d. $\sigma_X^2 = E[(X-1.1)^2] = \sum_{x=0}^{4} P_X(x)(x-1.1)^2$

$= 0.4(0-1.1)^2 + 0.3(1-1.1)^2 + 0.15(2-1.1)^2 + 0.1(3-1.1)^2 + 0.05(4-1.1)^2$

$= 1.39$

Navidi Section 2.4

8. After manufacture, computer disks are tested for errors. Let X be the number of errors detected on a randomly chosen disk. The following table presents values of the cumulative distribution function Φ of X.

x	Φ
0	0.41
1	0.72
2	0.83
3	0.95
4	1.00

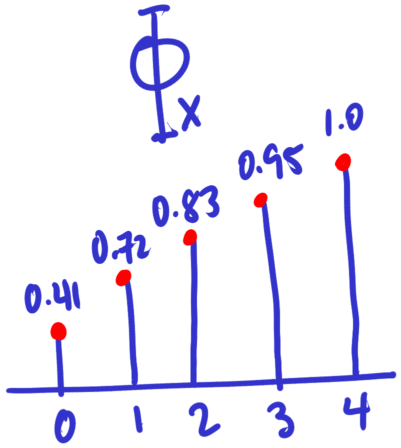
a. What is the probability that two or fewer errors are detected?

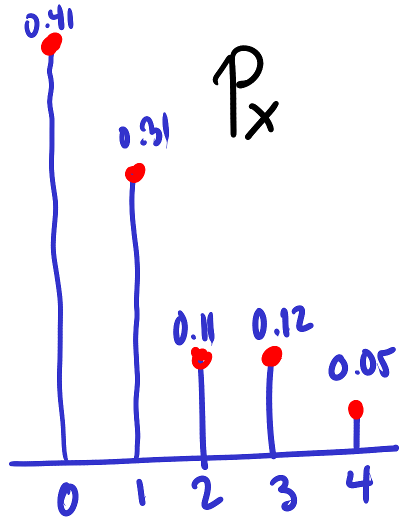
b. What is the probability that more than three errors are detected?

c. What is the probability that exactly one error is detected?

d. What is the probability that no errors are detected?

e. What is the most probable number of errors to be detected?





a. $P(X \leq 2) = \Phi(2) = 0.83$

b. $P(X > 3) = P(X = 4) = 0.05$

c. $P(X = 1) = 0.31$

d. $P(X = 0) = 0.41$

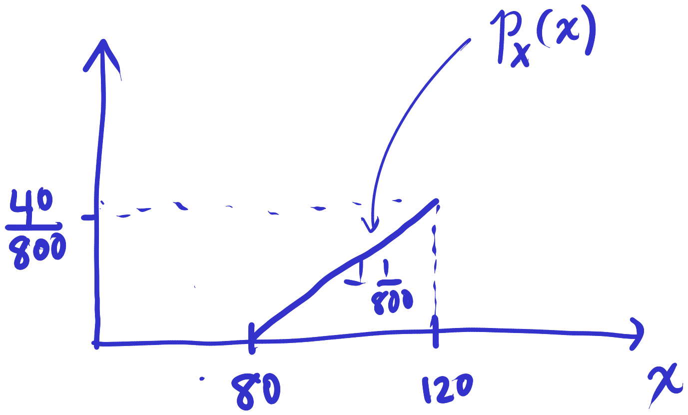
e. $\underset{x}{\operatorname{argmax}}\, P_X(x) = 0.$

13. Resistors labeled 100 Ω have true resistances that are between 80 Ω and 120 Ω. Let X be the resistance of a randomly chosen resistor. The probability density function of X is given by

$$p_X(x) = \begin{cases} \frac{x-80}{800} & 80 < x < 120 \\ 0 & \text{otherwise} \end{cases}$$

a. What proportion of resistors have resistances less than 90 Ω?
b. Find the mean resistance.
c. Find the standard deviation of the resistances.
d. Find the cumulative distribution function of the resistances.

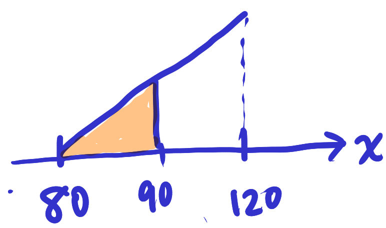
$$\Omega_X = [80, 120]$$



Check that it is a valid pdf:

$$P(\Omega) = \int_{80}^{120} p_X(x)\,dx = 40 \cdot \frac{40}{800} \cdot \frac{1}{2} = 1 \quad \checkmark$$

a)



$$P([80,90]) = \int_{80}^{90} p_X(x)\,dx$$

$$= \int_{80}^{90} \frac{x-80}{800} = \frac{1}{800} \cdot \frac{(x-80)^2}{2} \Big|_{80}^{90}$$

$$= \frac{1}{800} \cdot \frac{100}{2} = \frac{50}{800} = \frac{1}{16}$$

b) $$\mu_x = \int_{80}^{120} x \, P_x(x) \, dx$$

$$= \int_{80}^{120} x \, \frac{(x-80)}{800} \, dx$$

$$= \frac{1}{800} \int_{80}^{120} (x^2 - 80x) \, dx$$

$$= \frac{1}{800} \left(\frac{x^3}{3} - 40x^2 \right) \Bigg|_{80}^{120}$$

$$= \frac{1}{800} \left(\frac{120^3 - 80^3}{3} - 40(120^2 - 80^2) \right)$$

$$= 106.\hat{6}$$

c) $$\sigma_x^2 = \int_{80}^{120} P_x(x) (x - \mu_x)^2 \, dx$$

$$= \int_{80}^{120} \frac{x-80}{800} \cdot (x^2 - 2\mu_x x + \mu_x^2) \, dx$$

$$= \frac{1}{800} \int_{80}^{120} \Big(x^3 - \underbrace{(80 + 2\mu_x)}_{\alpha_2} x^2 + \underbrace{(\mu_x^2 + 160\mu_x)}_{\alpha_1} x \underbrace{- 80\mu_x^2}_{\alpha_0} \Big) \, dx$$

$$= \frac{1}{800} \left(\frac{120^4 - 80^4}{4} + \alpha_2 \frac{120^3 - 80^3}{3} + \alpha_1 \frac{120^2 - 80^2}{2} + \alpha_0 (120 - 80) \right)$$

$$= 88.\hat{8}$$

d. $\Phi(x) = \int_{80}^{x} p_x(\xi)\, d\xi = \int_{80}^{x} \frac{\xi - 80}{800}\, d\xi$

$$= \frac{1}{800} \cdot \frac{(\xi-80)^2}{2} \Big|_{80}^{x} = \frac{(x-80)^2}{1600}$$