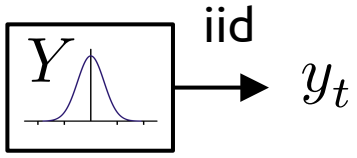
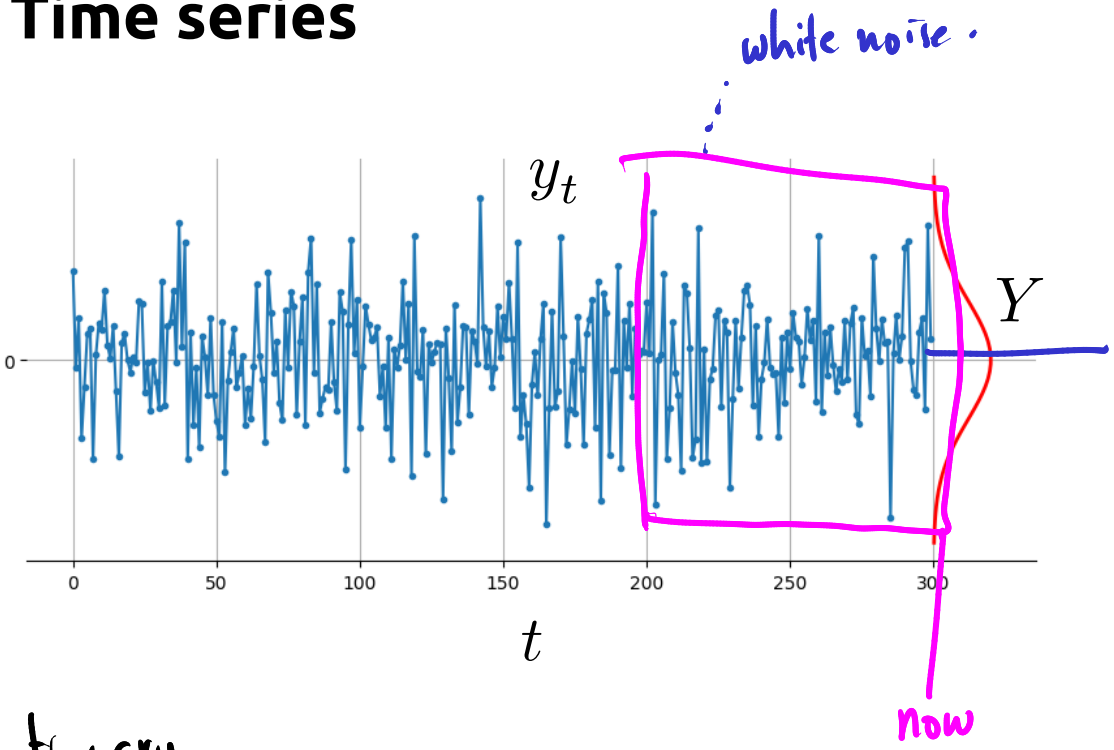
Statistics and Data Science for Engineers
E178 / ME276DS

Time series analysis
Part 1

Time series





white noise is stationary.

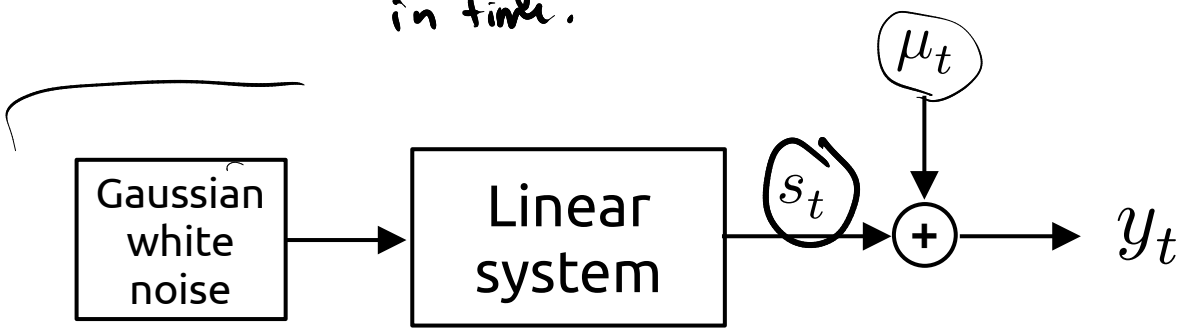
Time series decomposition

$$y_t = s_t + \mu_t$$

Zero-mean stationary stochastic signal

Time-varying deterministic signal

variance does not change in time.



stationary.

Time series forecasting

- **Classical approach**
  1. Model the deterministic signal: μ_t
  2. Model the stochastic signal: s_t
  3. Combine the two: $\hat{y}_t = \mu_t + s_t$

- **Neural networks**
  - Recurrent neural networks ... memory.
  - Attention networks ... not cover.

Forecasting the deterministic signal μ_t

$$\mu_t = \bar{\mu}_t + \tilde{\mu}_t$$

$\bar{\mu}_t$ — trend

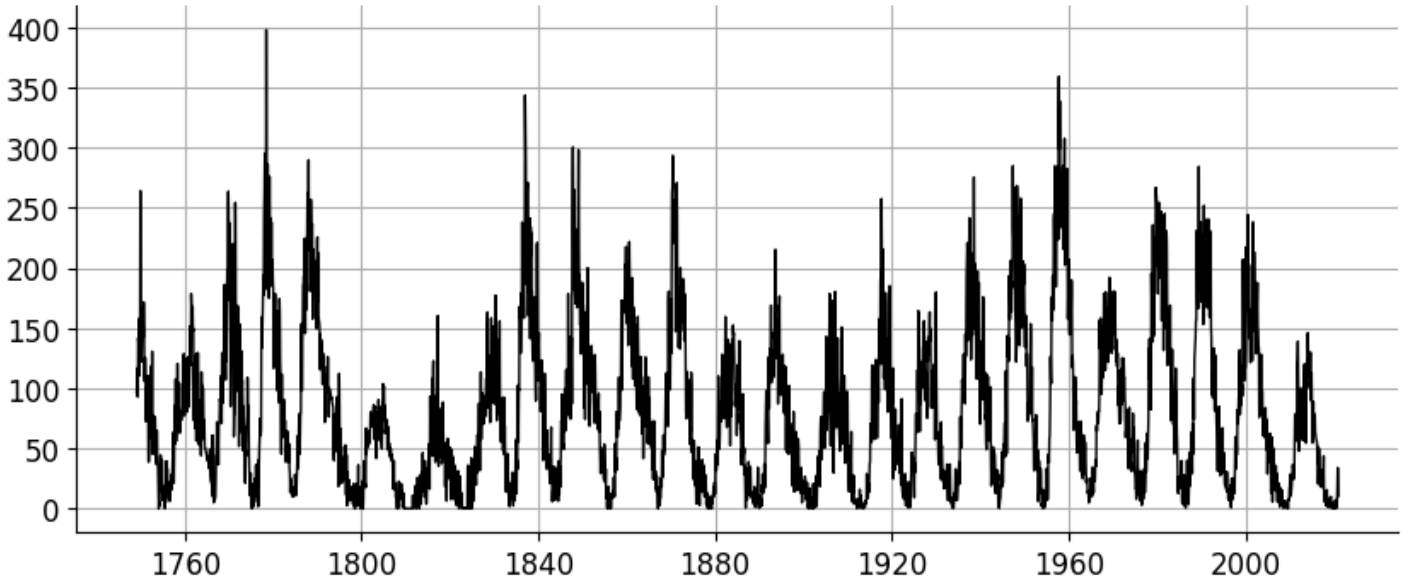
$\tilde{\mu}_t$ — seasonal.

1. Separate training and testing data.
2. Estimate the long-term trend $\bar{\mu}_t$ ... moving average / convolution kernel.
3. Estimate the seasonal component $\tilde{\mu}_t$
4. Combine the two, observe the residual ... it should be stationary.
5. Compute the forecast

Example: Forecasting the average yearly Sun spots

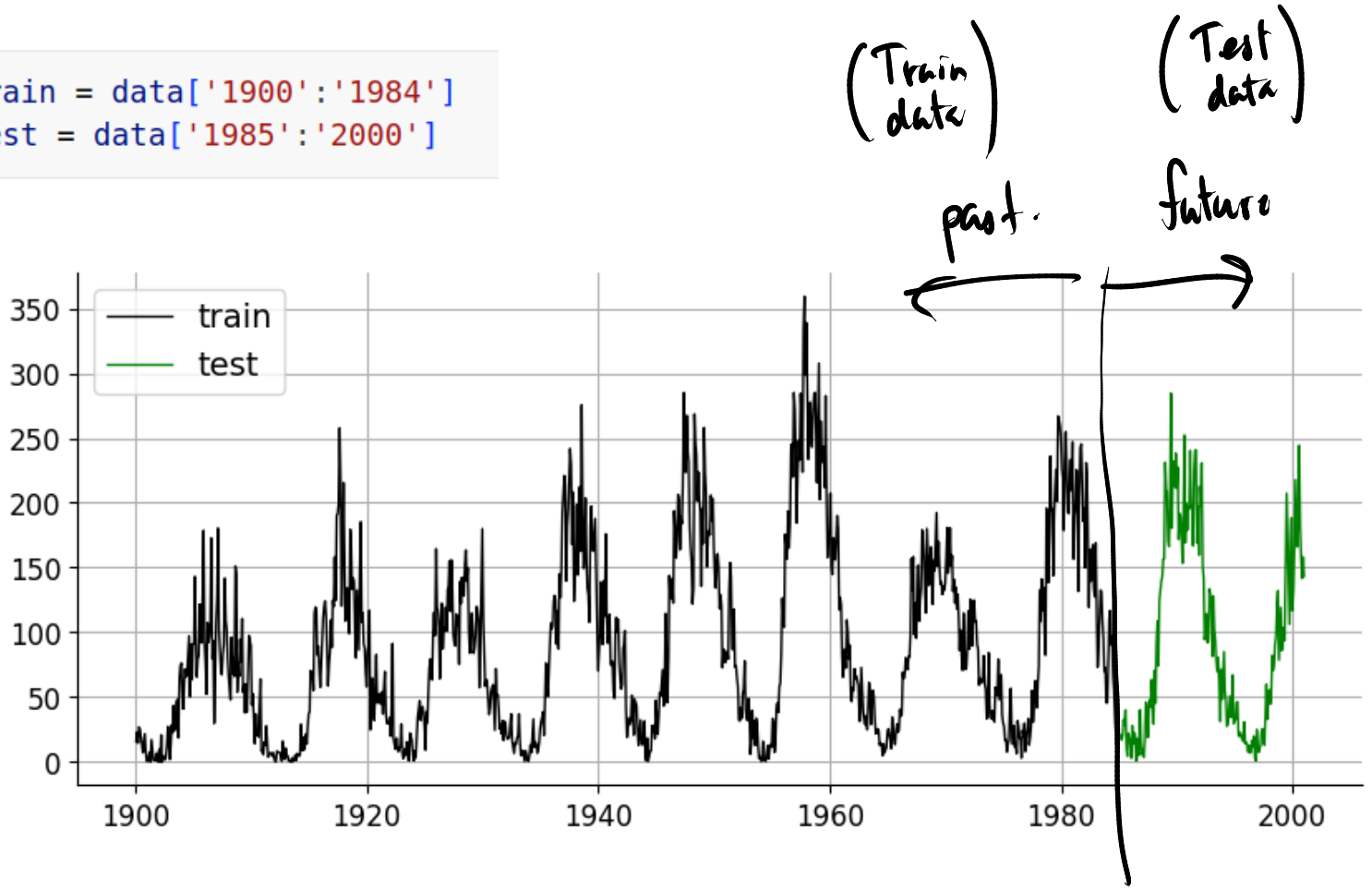
Date	avgspots
1749-01-31	96.7
1749-02-28	104.3
1749-03-31	116.7
1749-04-30	92.8
1749-05-31	141.7
...	...
2020-09-30	0.6
2020-10-31	14.4
2020-11-30	34.0
2020-12-31	21.8
2021-01-31	10.4

3265 rows × 1 columns



1. Separate training and testing data.

```
y_train = data['1900':'1984']
y_test = data['1985':'2000']
```



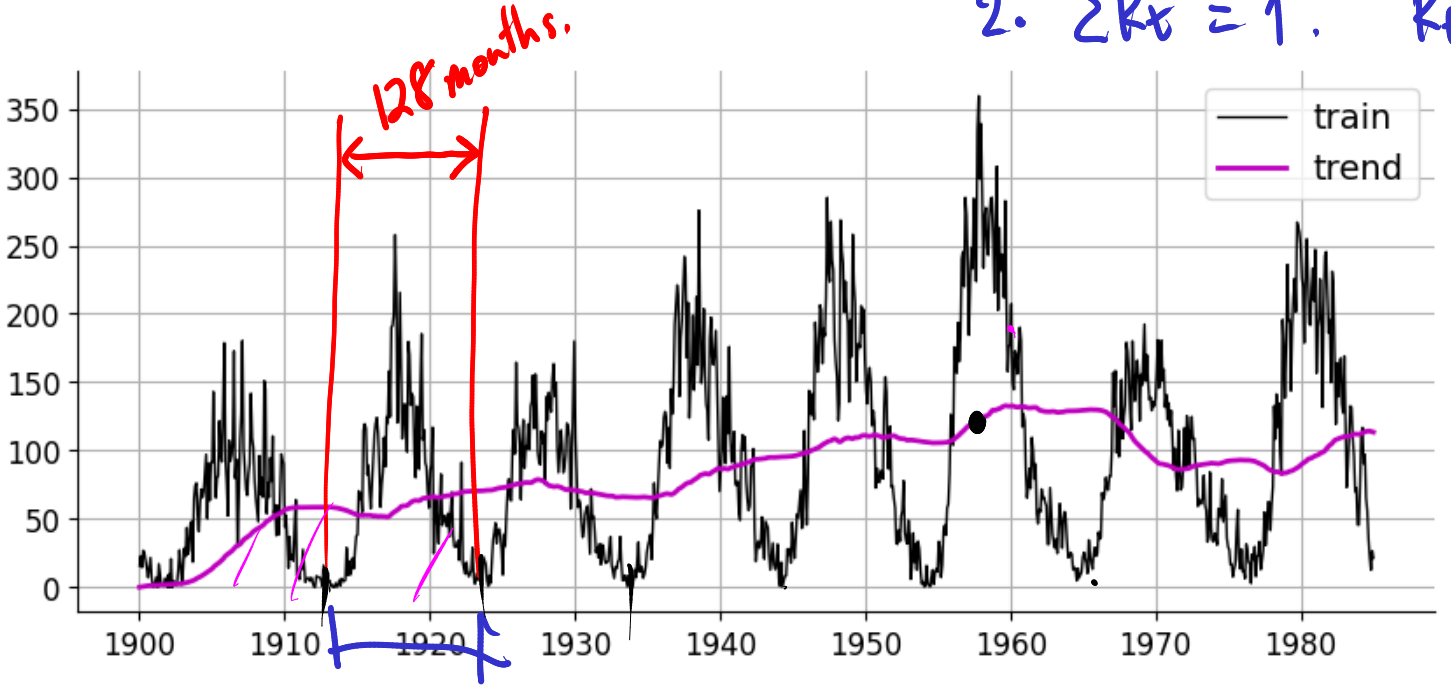
2. Estimate the long-term trend $\bar{\mu}_t$

1D convolution.

$$\bar{\mu}_t = y_t * k_t$$

k_t ... period-length smoothing kernel

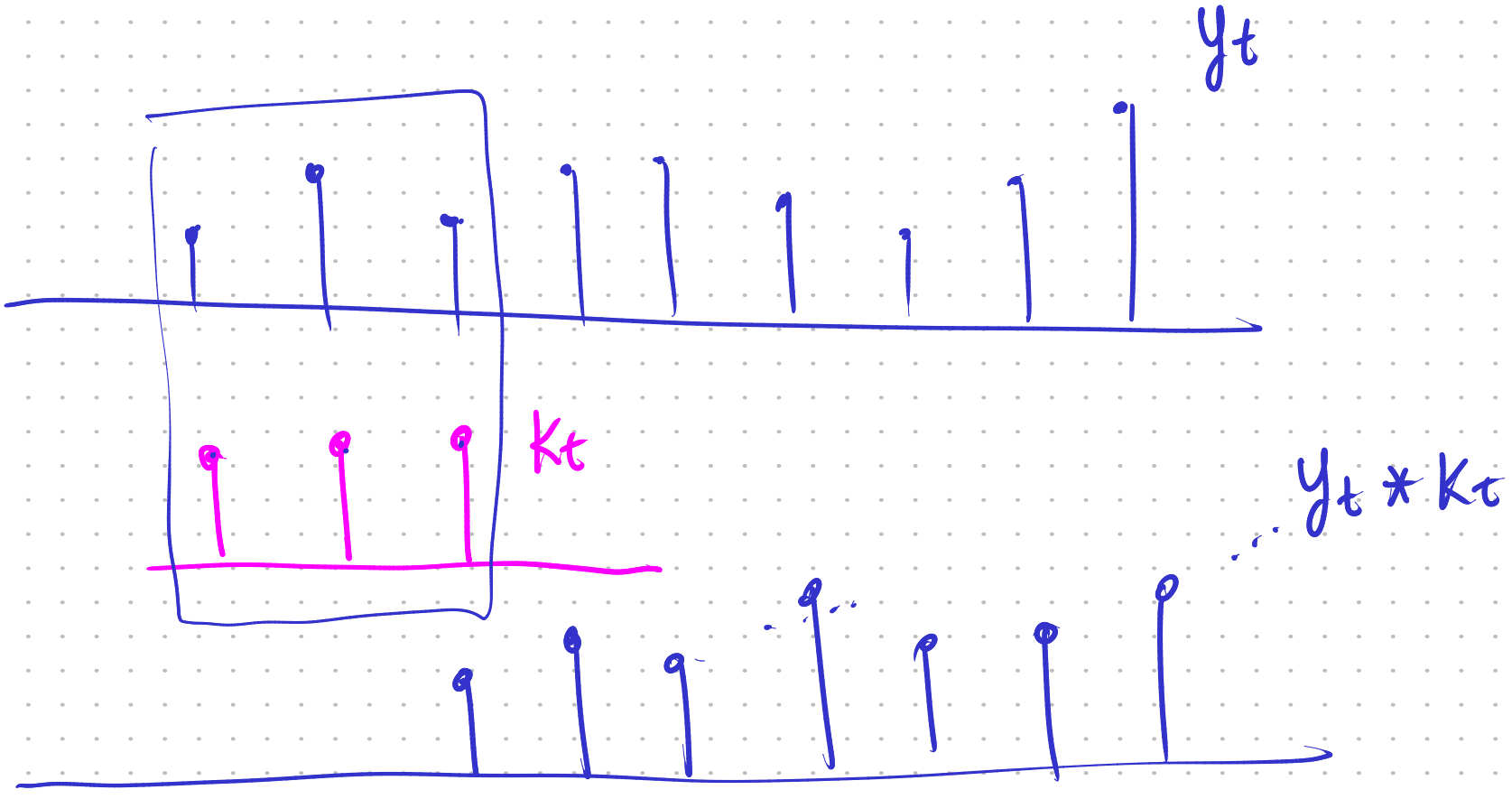
1. k_t should have length 128.
2. $\sum k_t = 1$. $k_t \geq 0$.



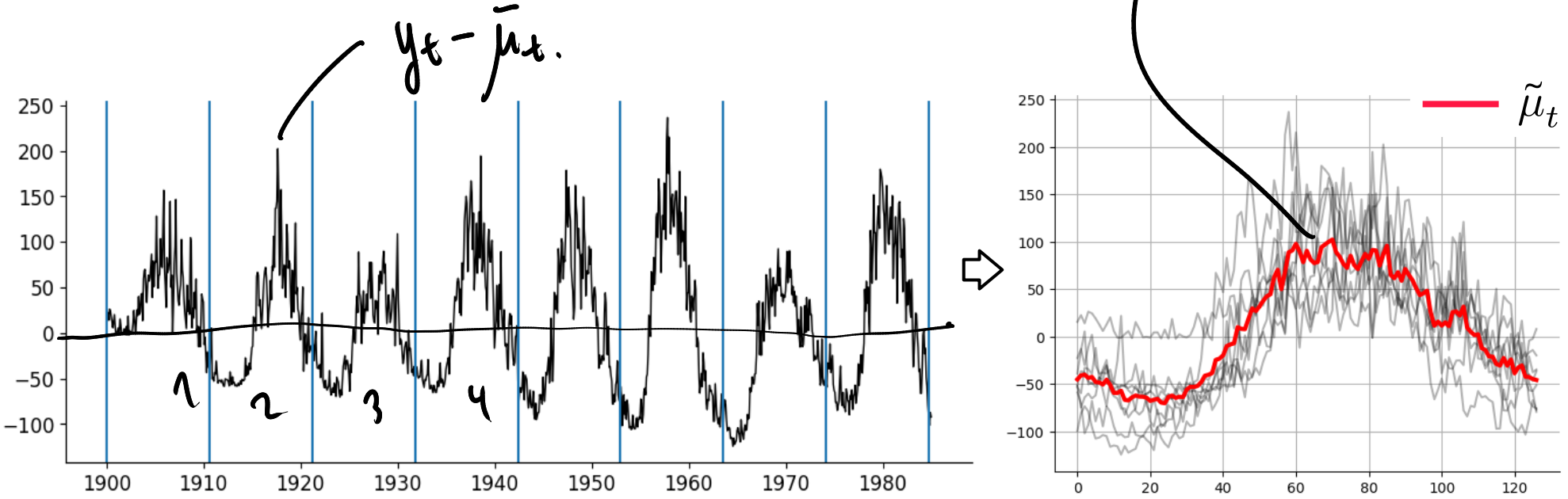
uniform values

period.

1D convolution

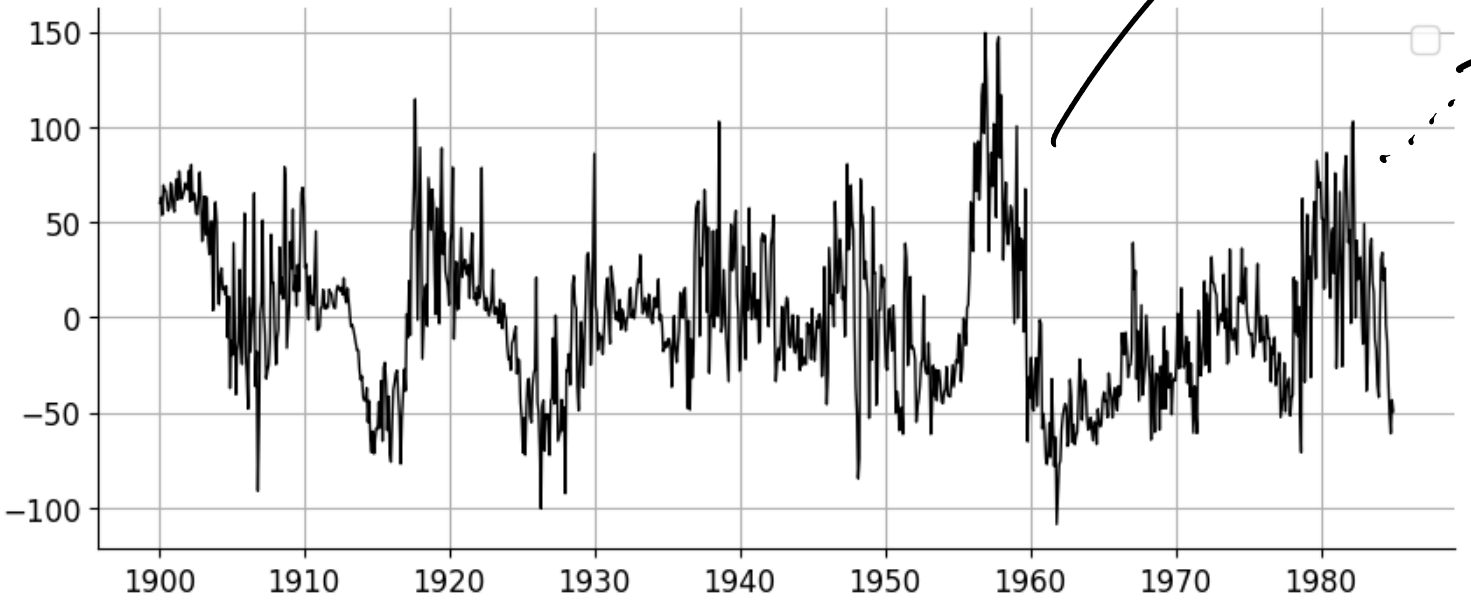


3. Estimate the seasonal component $\tilde{\mu}_t$



4. Combine the two, observe the residual

$$r_t = y_t - \bar{\mu}_t - \tilde{\mu}_t$$

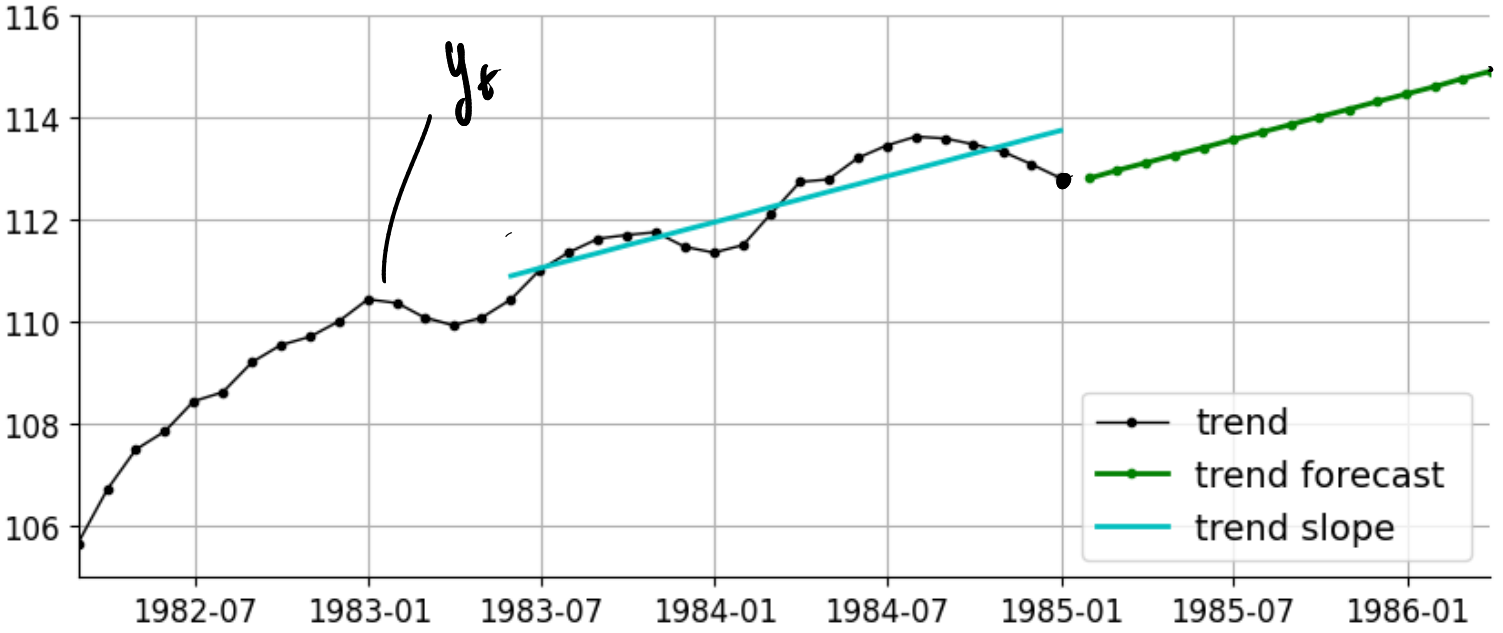


stationary ?

... Is the mean 0 ?

5. Compute the forecast

5.1. Forecast the long-term trend



5.2. Add in the seasonal component

$$\mu_t = \bar{\mu}_t + \tilde{\mu}_t$$