New Methods in EEG Source Localization based on EEG and Post-Mortem Pathology Data

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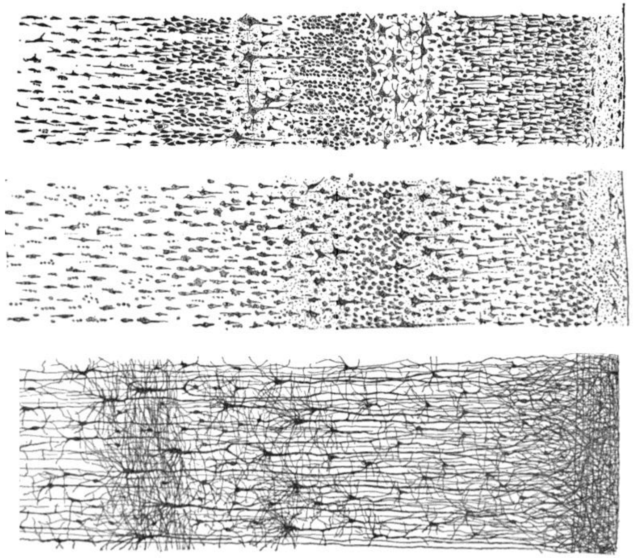
Feb 26, 2024

Motivation

Goal: Determine the location of the ‘sources’ of the electrical activity which we observe at the EEG sensors.

- Electrical recordings have high-res in time, but low-res in space.
- EEG symptoms are known for some conditions.
- For cognitive tasks, epileptogenesis, mild cognitive impairment, among others, it is relevant to know which brain areas are active.

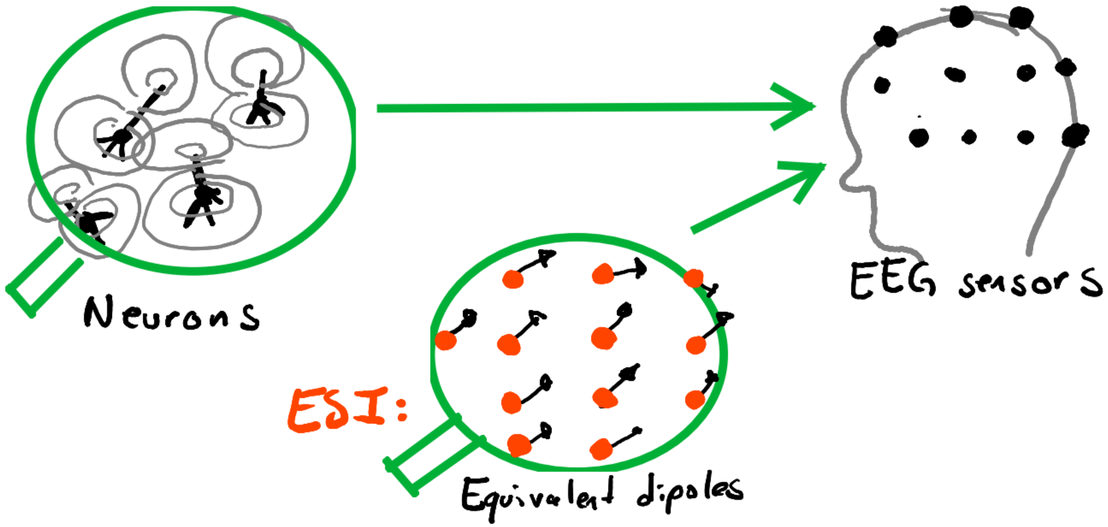
Electrical Source Imaging (ESI)

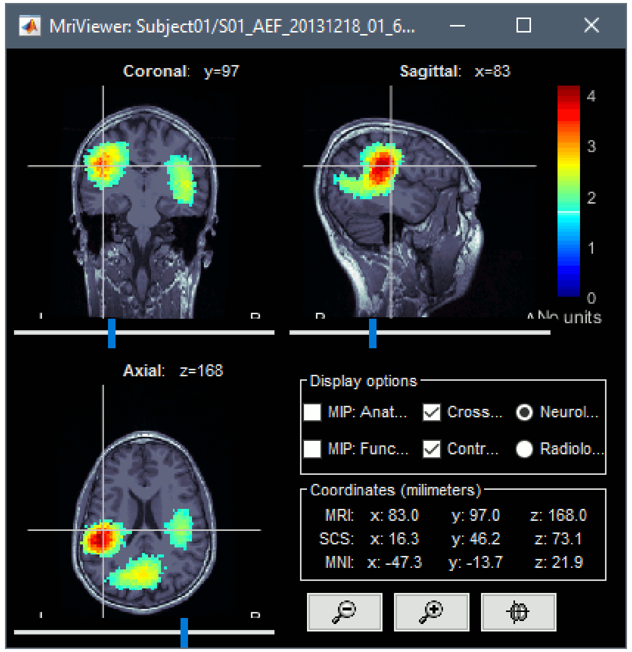


We want to reconstruct electric activity of neurons.

Forward problem: Physics modeling.

Inverse problem: Convex Optimization.





ESI: Forward and Inverse problem

We want: Magnitude of equivalent dipoles, **J**.

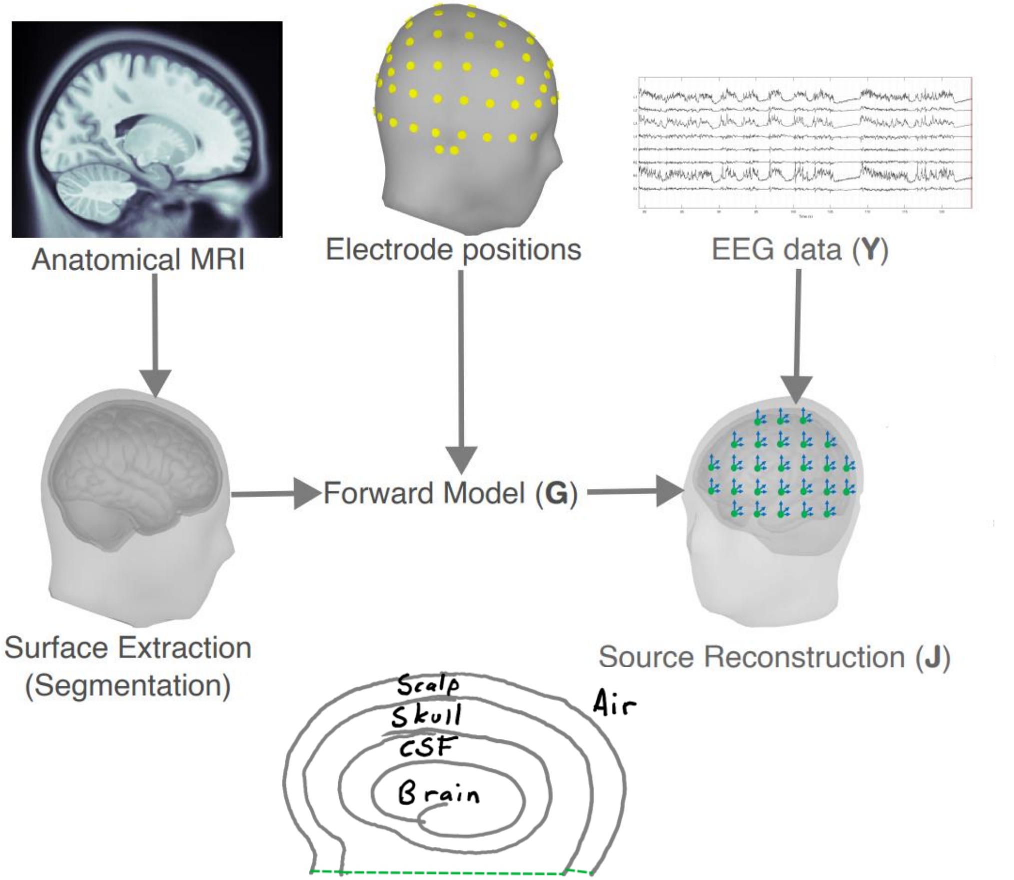
We have: Recordings from EEG sensors, **Y**.

$$\mathbf{Y} = \mathbf{G}\,\mathbf{J} + e$$

where **G** is the gain matrix, computed by solving a Maxwell's equation over the anatomical space.

Forward problem: Compute **G**. Easy ☺.

Inverse problem: Compute **J**. Ill-posed ☹.



$$\sigma \Delta V(\mathbf{r}) = -\delta\left(\mathbf{r} - \mathbf{r}_{dip_n} - \kappa\right) + \delta\left(\mathbf{r} - \mathbf{r}_{dip_n} + \kappa\right)$$

$$\mathbf{G}_{n,m} = V\left(\mathbf{r}_m; \mathbf{r}_{dip_n}, \mathbf{e}_n\right)$$

What is the worst that can happen? How to fix it?

The Max-Likelihood estimator

$$\hat{\mathbf{J}}_{ML} = \arg\min_{\mathbf{J}} \|\mathbf{Y} - \mathbf{G}\mathbf{J}\|_2^2$$

may not have a unique solution.

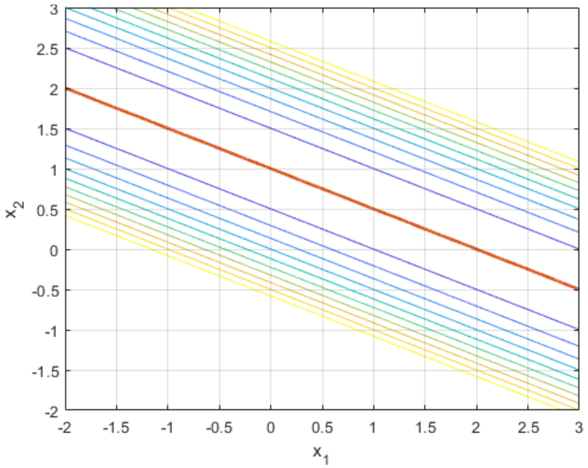
Enter the assumption of **Minimal-norm**: the most likely sol. is the one with smaller norm.

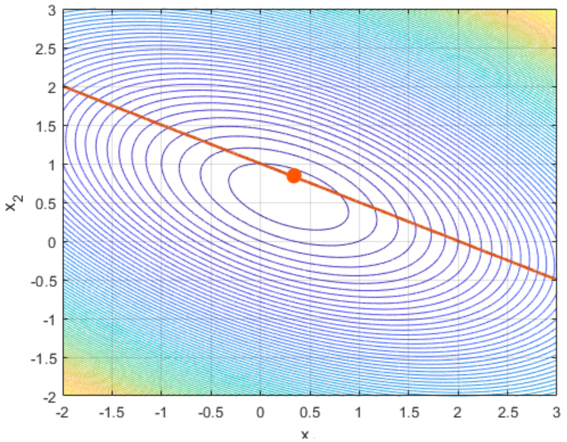
MNE estimator:

$$\hat{\mathbf{J}}_{MNE} = \arg\min_{\mathbf{J}} \|\mathbf{Y} - \mathbf{G}\mathbf{J}\|_2^2 + \lambda\|\mathbf{J}\|_2^2 = \mathbf{G}^T\left(\mathbf{G}\,\mathbf{G}^T + \lambda\mathbf{I}\right)^{-1}\mathbf{Y}$$

$$\hat{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{MLE} = \arg\min_{x_1,x_2} \| [2] - [1, 2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \|_2^2$$

Max-Likelihood: infinite solutions





$$\hat{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{MNE} = \arg\min_{x_1,x_2} \| [2] - [1, 2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \|_2^2 + 2\| \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \|_2^2$$

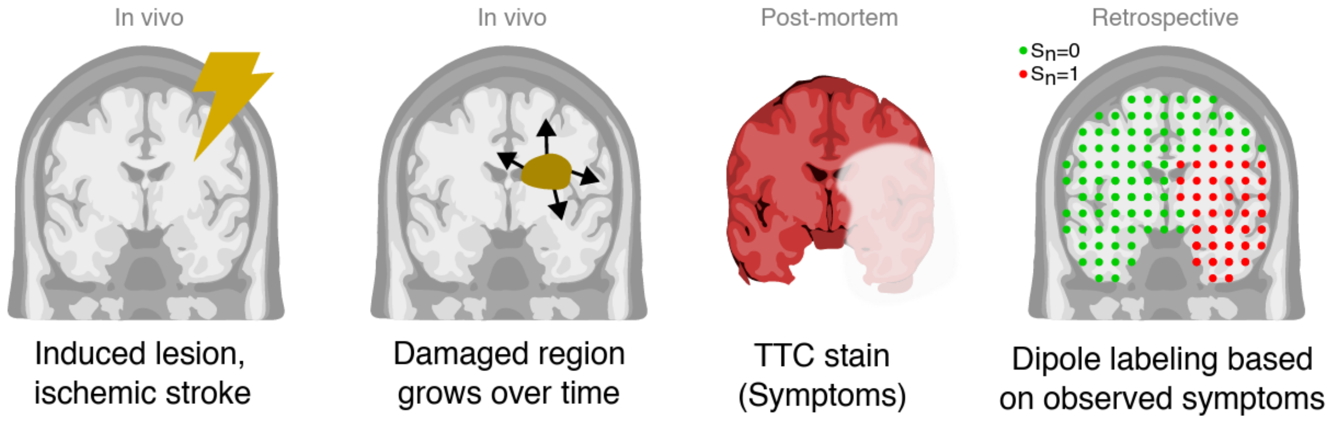
Min-Norm: unique solutions

Post-mortem Pathology Data

Project: stufy acute ischemic stroke on an animal model (pig) via an induced lesion on the Middle Central Artery.

Post-mortem, subject's brain is stained with triphenyltetrazolium (TTC) to identify tissues damaged by hypoxia; this data is referred to as **symptoms**.

EEG is recorded during the procedure.



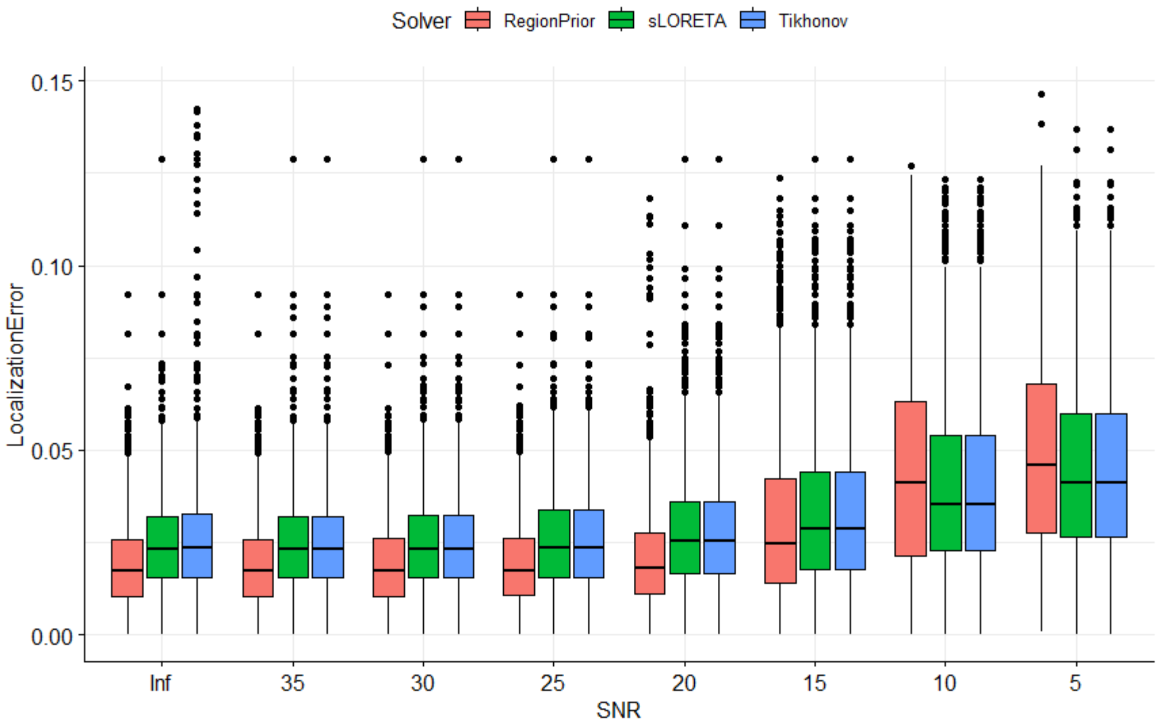
$$\hat{\mathbf{J}} = \arg\max_{\mathbf{J}} \frac{1}{\sigma^2}\|\mathbf{Y} - \mathbf{G}\left(L_S\mathbf{U} + \mathbf{N}\right)\|^2 + \frac{1}{\gamma_0^2}\|\mathbf{N}\|^2 + \frac{1}{\gamma_1^2}\|L_S\mathbf{U}\|^2$$

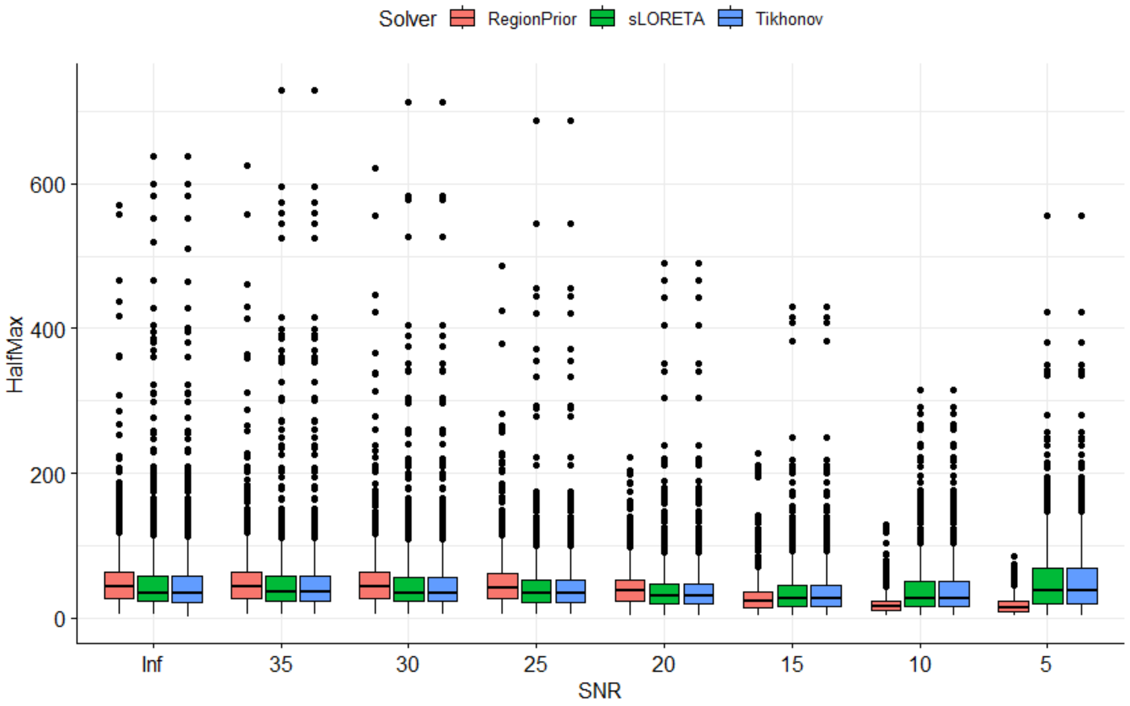
$$\mathbf{Y} = \mathbf{G}\mathbf{J} + \varepsilon$$
$$\mathbf{J} = L_S\mathbf{U} + \mathbf{N}$$

$$\hat{\mathbf{J}} = \left[\gamma_0\mathbf{I} + L_s\left(L_S^T L_S\right)L_S^T\right]\mathbf{M}\,\mathbf{G}^T\mathbf{Y},$$
$$\mathbf{M} = \left[\mathbf{G}\left(\gamma_0\mathbf{I} + L_s\left(L_S^T L_S\right)L_S^T\right)\mathbf{G}^T + \sigma^2\mathbf{I}\right]^{-1}$$

Is it worth? (Results)

Results from 1,000 simulations against two traditional estimators: sLORETA and WMNE (aka Tikhonov).





Time for questions ☺

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