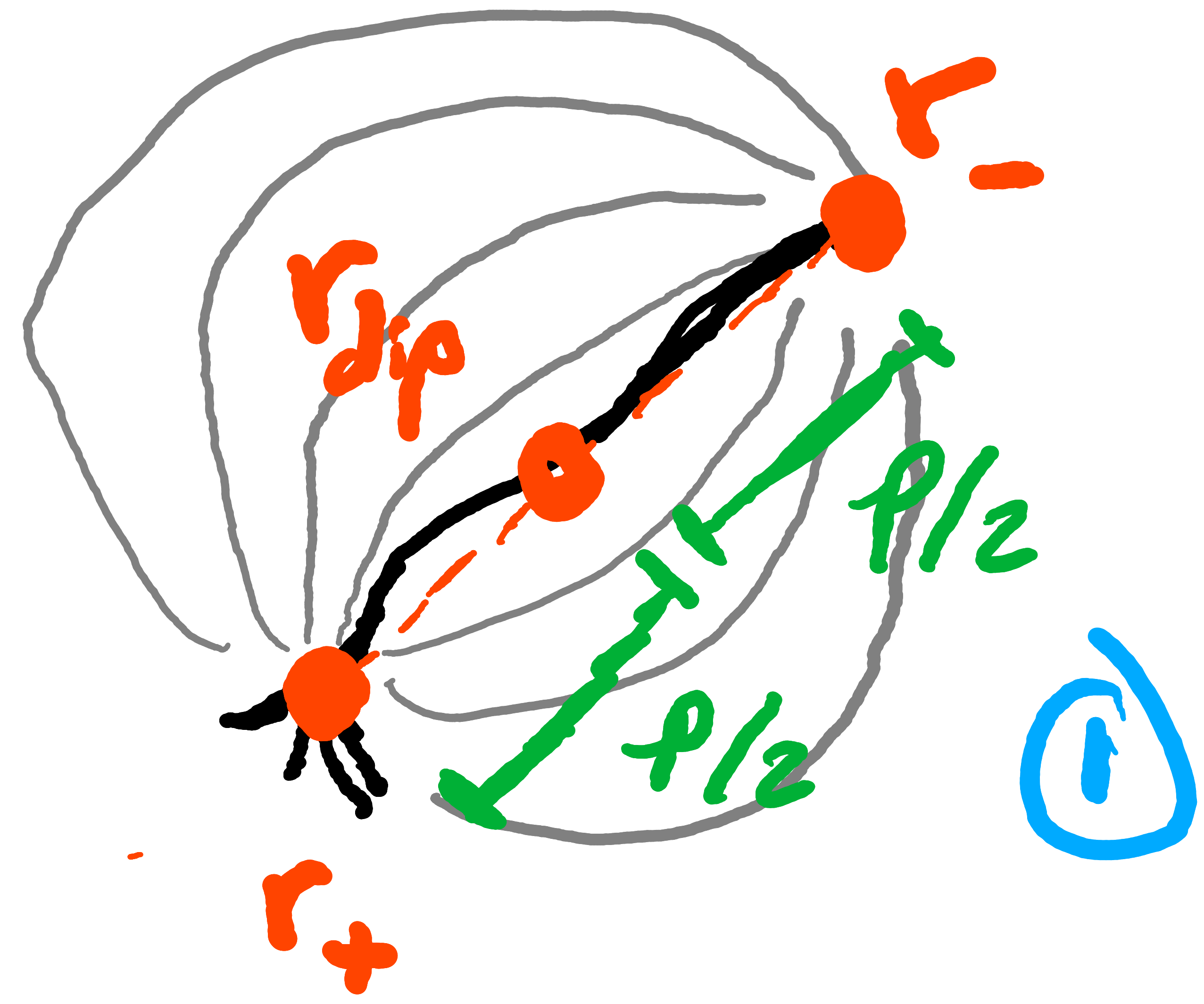
Model derivation

(1 single dipole)



$K: \mathbb{R}^3 \to \mathbb{R}$

Current density field

$$\nabla \cdot K(r) = \lim_{R(r) \to \{r\}} \frac{1}{\mathrm{Vol}\, R(r)} \oint_{\partial R(r)} K(r)\, dS$$

$G(r) \subseteq \mathbb{R}^3$ st $r \in G(r)$

$$\nabla \cdot K(r) = \begin{cases} m, & r = r_+ \\ -m, & r = r_- \\ \emptyset, & \text{otherwise} \end{cases} = m\,\delta(r_+) - m\,\delta(r_-)$$

$E: \mathbb{R}^3 \to \mathbb{R}^3$ Electric field ; $\sigma: \mathbb{R}^3 \to \mathbb{R}^{3\times 3}$ Position-dependent conductivity tensor

$$K = \sigma E$$

$$\nabla \cdot \left(\sigma(r)\, \nabla V(r)\right) = -J\,\delta(r_+) + J\,\delta(r_-)$$

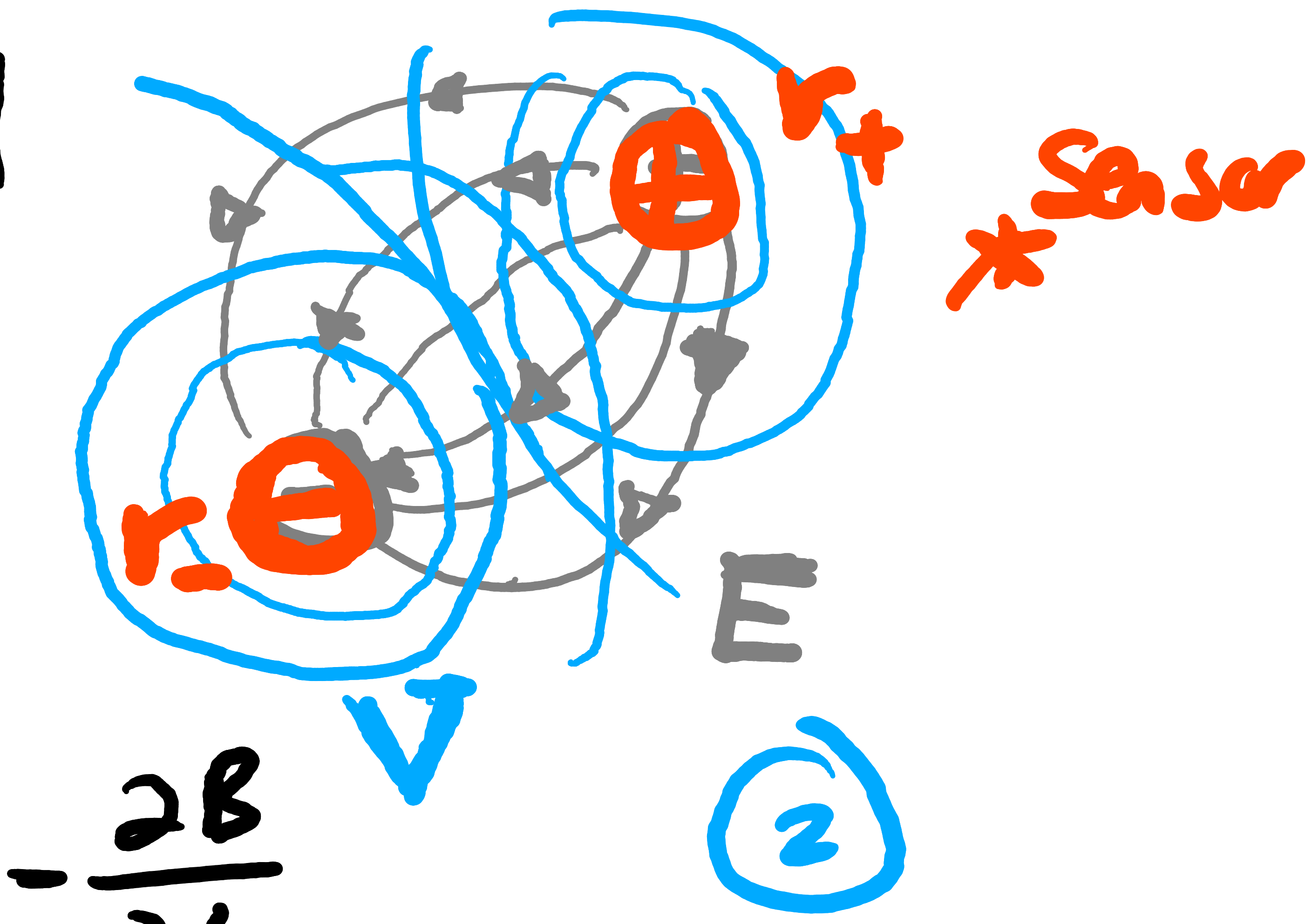
Quasi-static Maxwell eq ($\nabla \times E = \emptyset$)

$$E = -\nabla V$$

$V: \mathbb{R}^3 \to \mathbb{R}$ scalar potential field.

1. Scalar potential field

$$V : \mathbb{R}^3 \to \mathbb{R}$$



2. Maxwell eqn.

Elec field

$$\nabla \cdot E = \rho / \varepsilon_0 \qquad \nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \cdot B = \emptyset \qquad \nabla \times B = \mu_0 \left(J + \varepsilon_0 \frac{\partial E}{\partial t} \right)$$

E electric field
B magnetic field
J current density
ρ electric charge density
ε_0 vaccum permittivity
μ_0 vaccuum permeability

We know

magnetic vector potential

$$E = -\nabla V - \frac{\partial}{\partial t} A$$

$$B = \nabla \times A$$

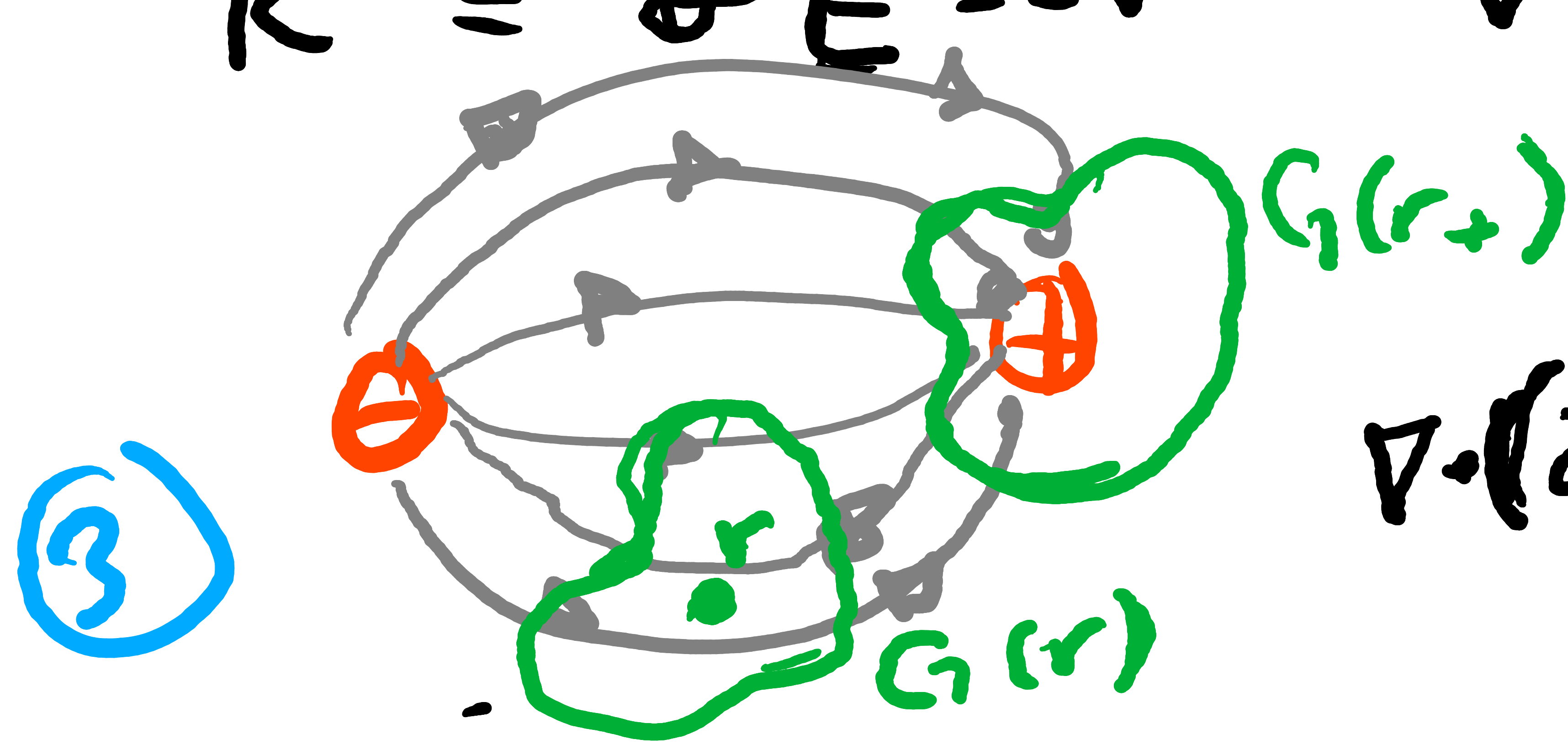
3. Quasistatic Maxwell eq. If $\frac{\partial}{\partial t} A = \emptyset$

$$E = -\nabla V$$

$$K = \sigma E = -\sigma \nabla V$$

4. Current density field

$$K = \sigma E = -\nabla \qquad \nabla \cdot K = \lim_{G(r) \to \{r\}} \oint_{\partial G(r)} K(r)\, dS$$



$$\nabla \cdot (\sigma E) = \begin{cases} +m, & r = r_+ \\ -m, & r = r_- \\ \emptyset, & \text{otherwise} \end{cases}$$

$$= J\delta(r_+) - J\delta(r_-)$$

$$\therefore\quad \nabla\cdot(\sigma\nabla V) = J\,\delta(r_+) - J\,\delta(r_-) \qquad (*)$$

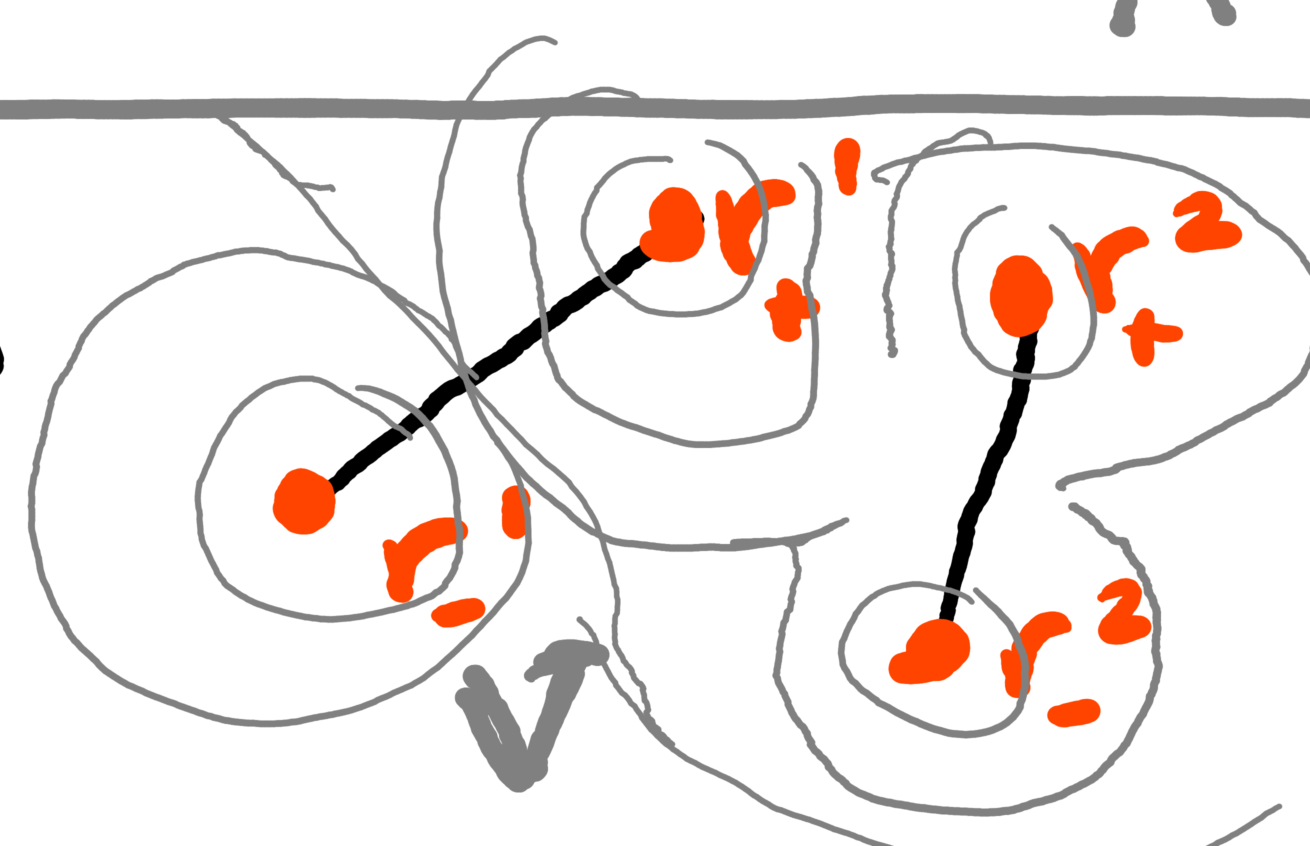
If $\sigma(r) = \kappa\, Id$ for all r

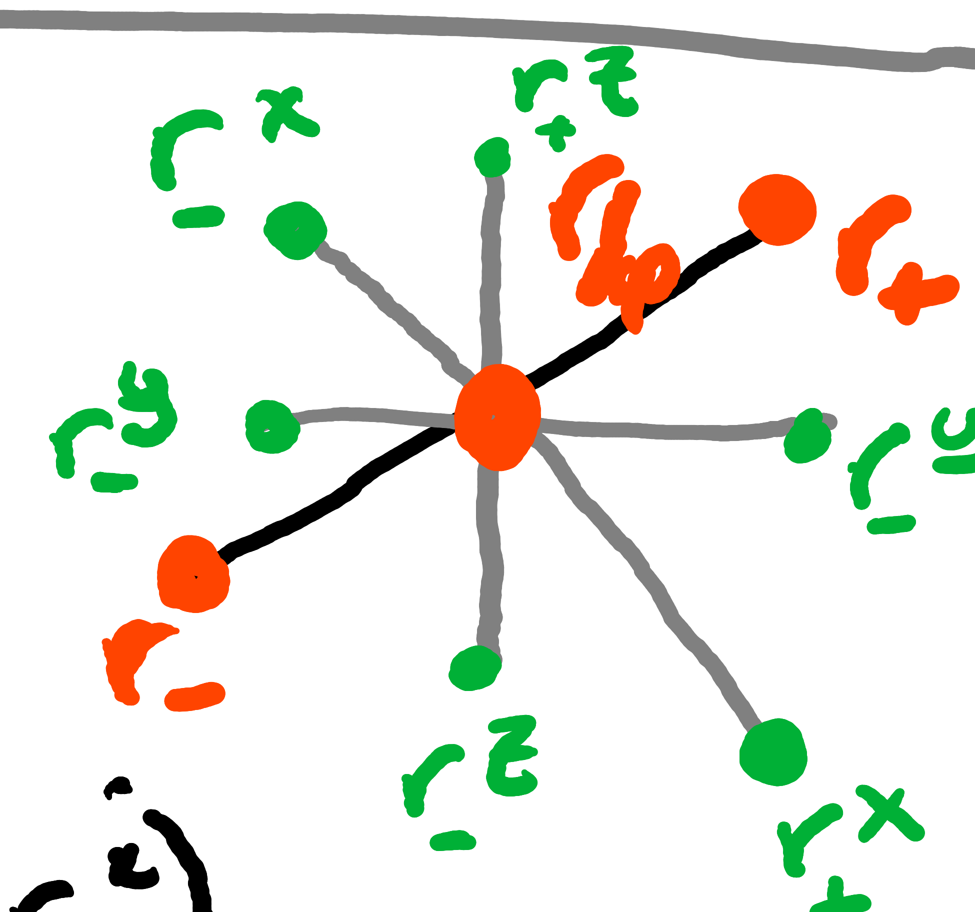
i.e. isotropic case w constant conductance

$$\kappa\,\Delta V = J\,\delta(r_+) - J\,\delta(r_-)$$

We want $V(r_{sensor})$

Two dipoles
non-interaction





Assume

$$\kappa\,\Delta V^i = J_i\,\delta(r_+^i) - J_i\,\delta(r_-^i)$$

$$\kappa\,\Delta\left(\Sigma_i V^i\right) = \sum_i \left(J_i\,\delta(r_+^i) - J_i\,\delta(r_-^i)\right)$$

$$g(r, r_{dip}) = \left[\begin{array}{l} \tilde{V}(r),\ \text{with } \tilde{V} \text{ st} \\ \kappa\,\Delta\tilde{V}(r) = \delta(r_+ - r_{dip}) - \\ \qquad - \delta(r_- - r_{dip}) \end{array}\right]$$

$d = 1/m$

So, overall

$$V(r) = \sum_i J_i\, g(r, r_i)$$

$$= \sum_i J_i^x\, g(r, e_x^i) + J_i^y\, g(r, e_y^i) + J_i^z\, g(r, e_z^i)$$

$$V(r) = \sum_i J_i \, g(r, r_{dip}^i)$$

$$= \left[g(r, r_{dip}^1) \cdots g(r, r_{dip}^N) \right] \begin{bmatrix} J_1 \\ \vdots \\ J_N \end{bmatrix}$$

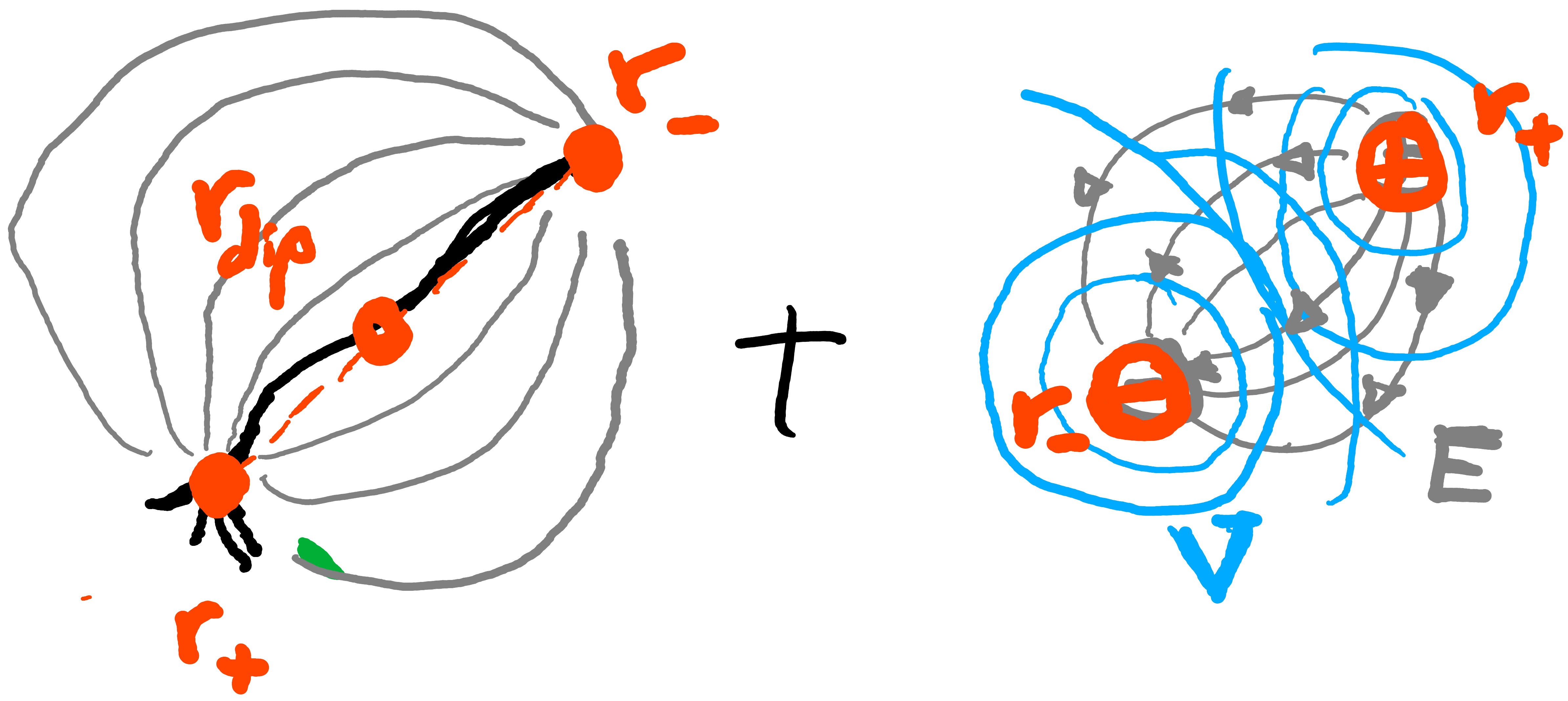
After staking m sensors @ single timepoint

$$\begin{bmatrix} V(r_{sens1}) \\ \vdots \\ V(r_{sensM}) \end{bmatrix} = \begin{bmatrix} g(r_{sens1}, r_{dip1}) & \cdots & g(r_{sens1}, dipN) \\ \vdots & \ddots & \vdots \\ g(r_{sensM}, r_{dip1}) & \cdots & g(r_{sensM}, dipN) \end{bmatrix}$$

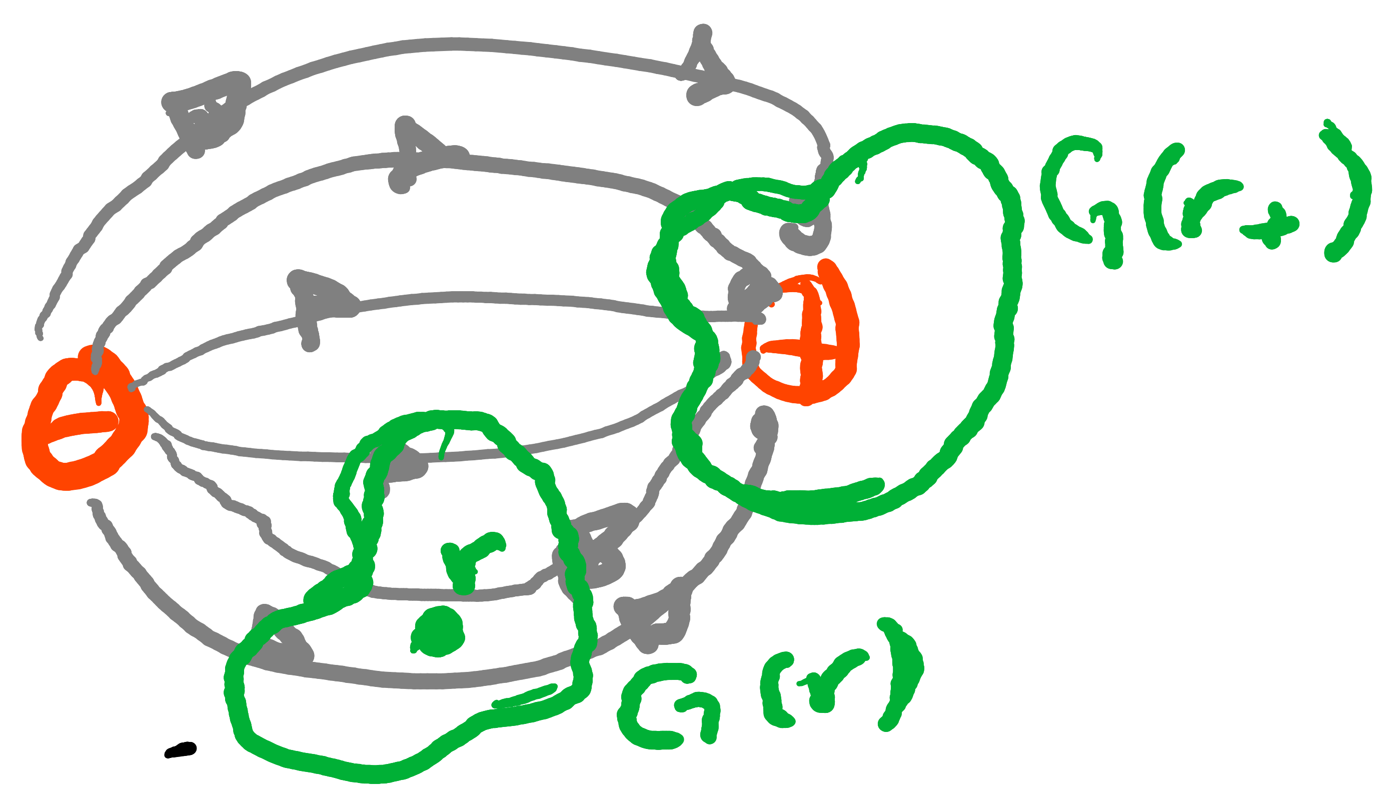
$$* \left[J_1 \cdots J_N \right]^T$$

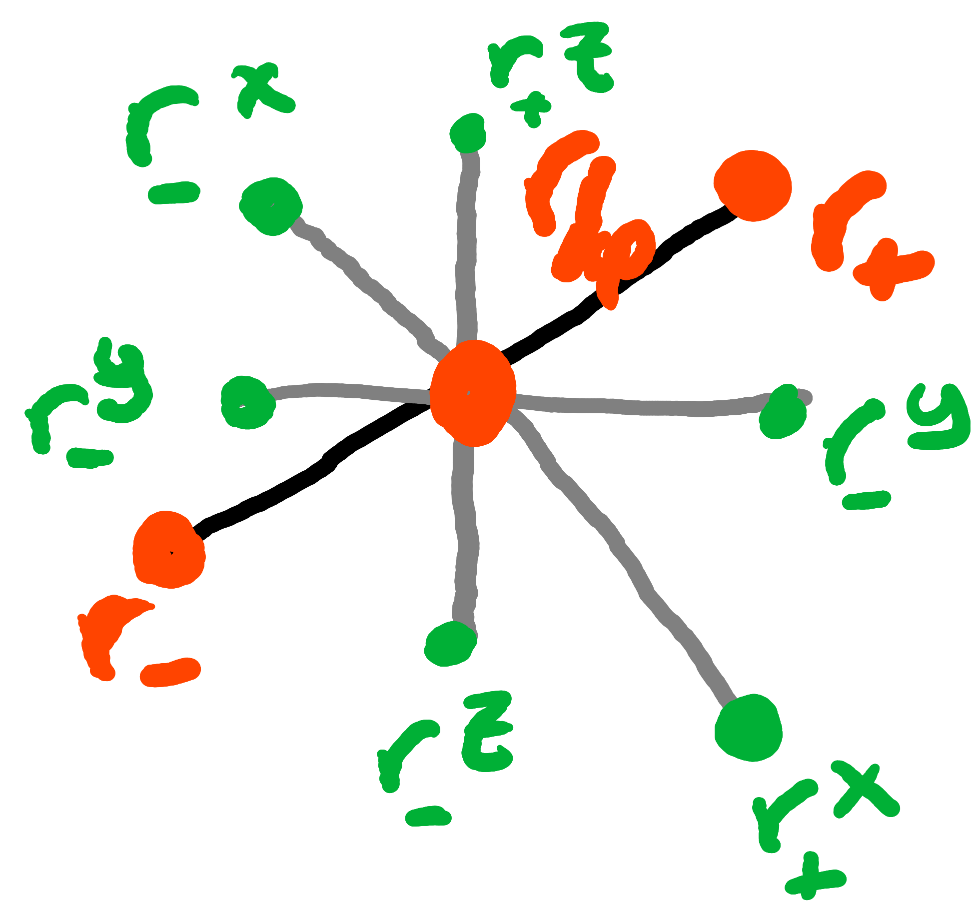
$V(r_{sens\,m})$ = EEG sensor m-th

Replace m by j


r_-
r_dip
r_+
+
r_+
r_-
E
V

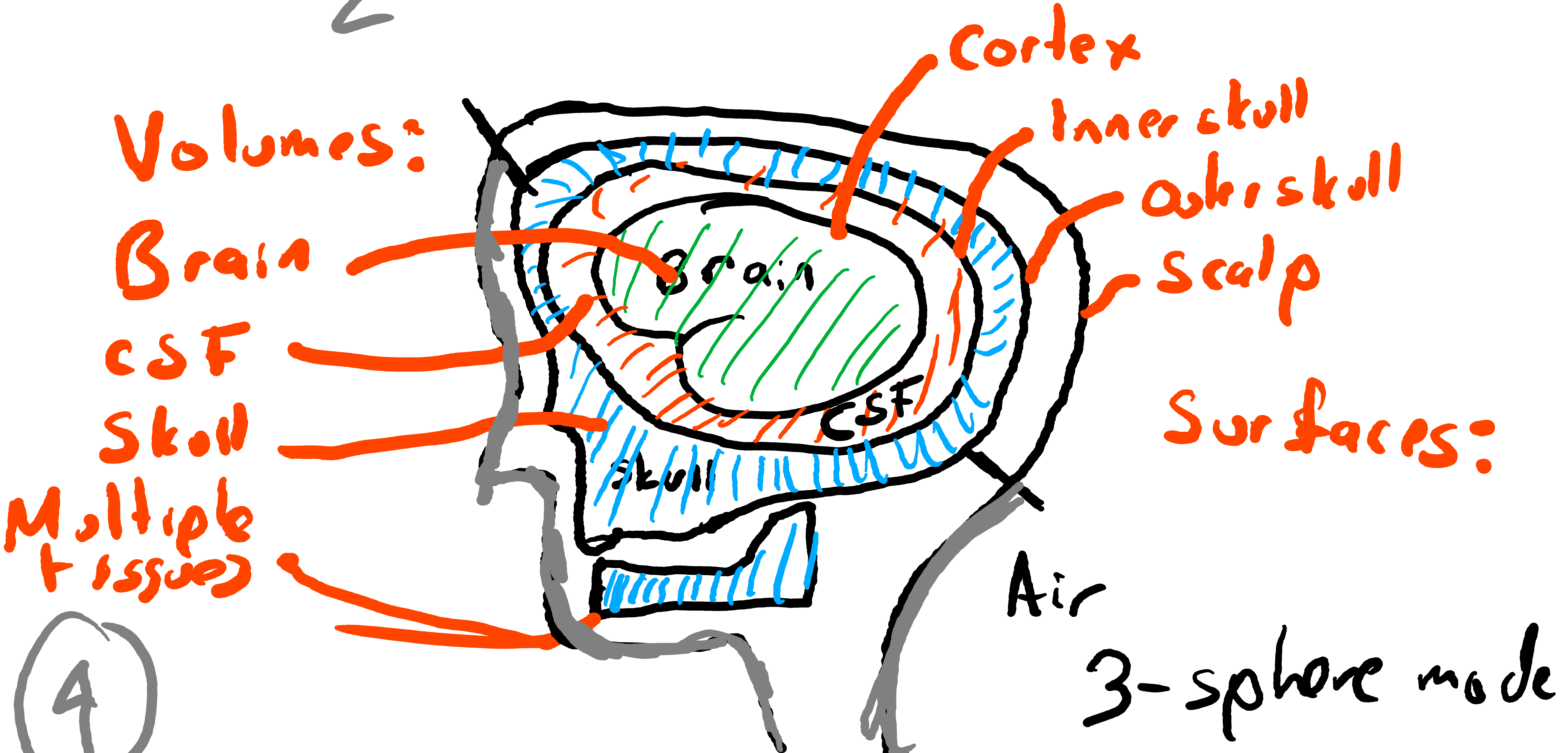
1


G(r_+)
r
G(r)


r^x_-
r^z_+
r_dip
r_+
r^y_-
r^y_-
r_-
r^z_-
r^x_+

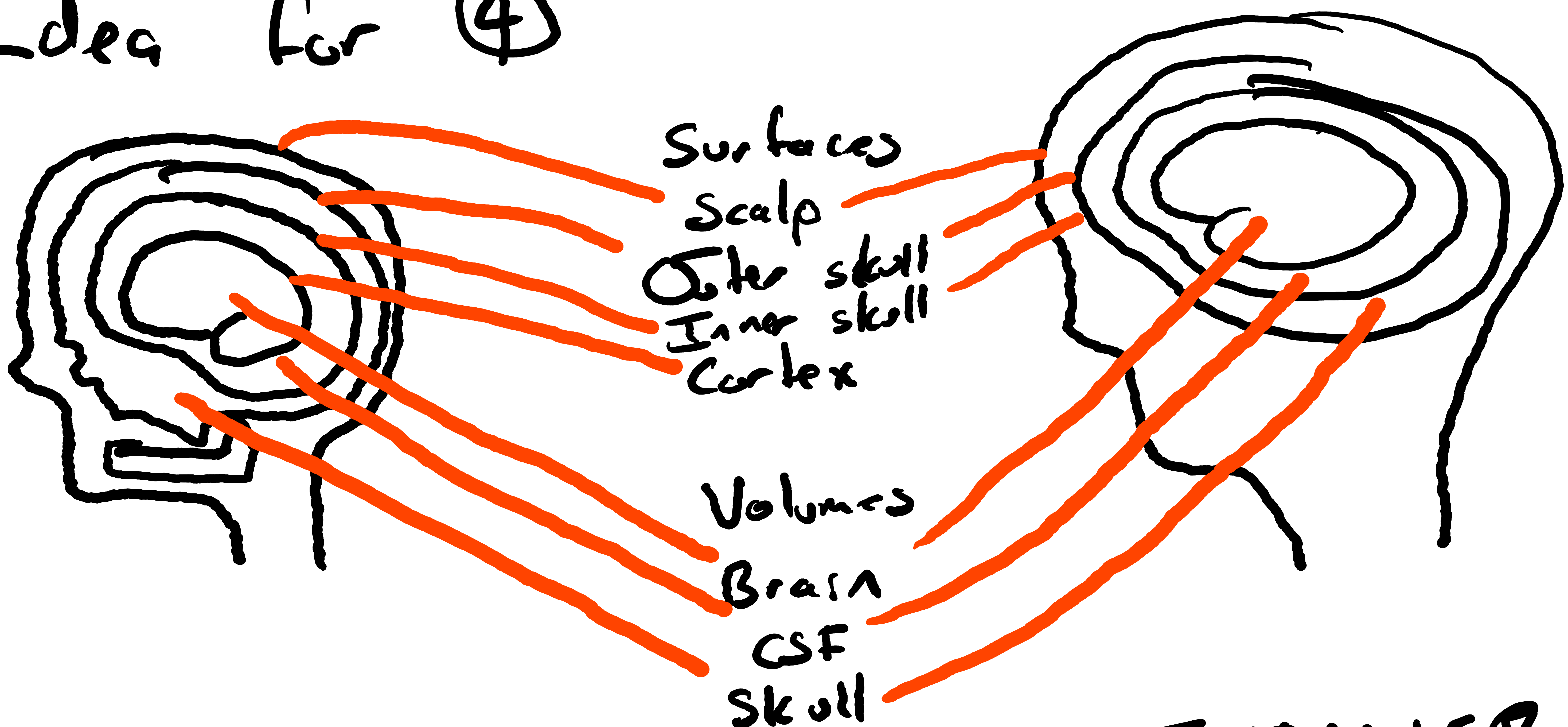
3

2


Volumes:
Brain
CSF
Skull
Multiple tissues
Cortex
Inner skull
Outer skull
Scalp
Surfaces:
Brain
CSF
Skull
Air
3-sphere model

4

Idea for 4



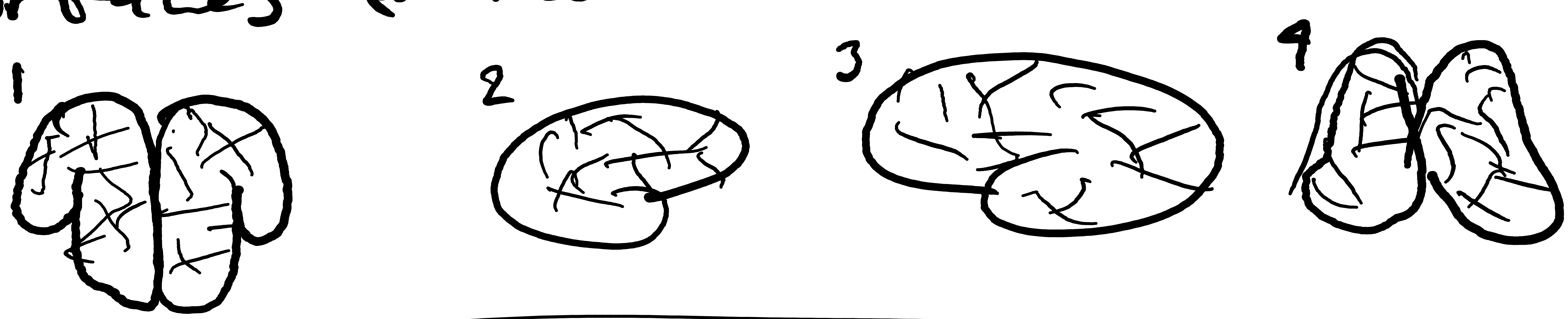
Cartoon

ICBM 152
(MRI)?

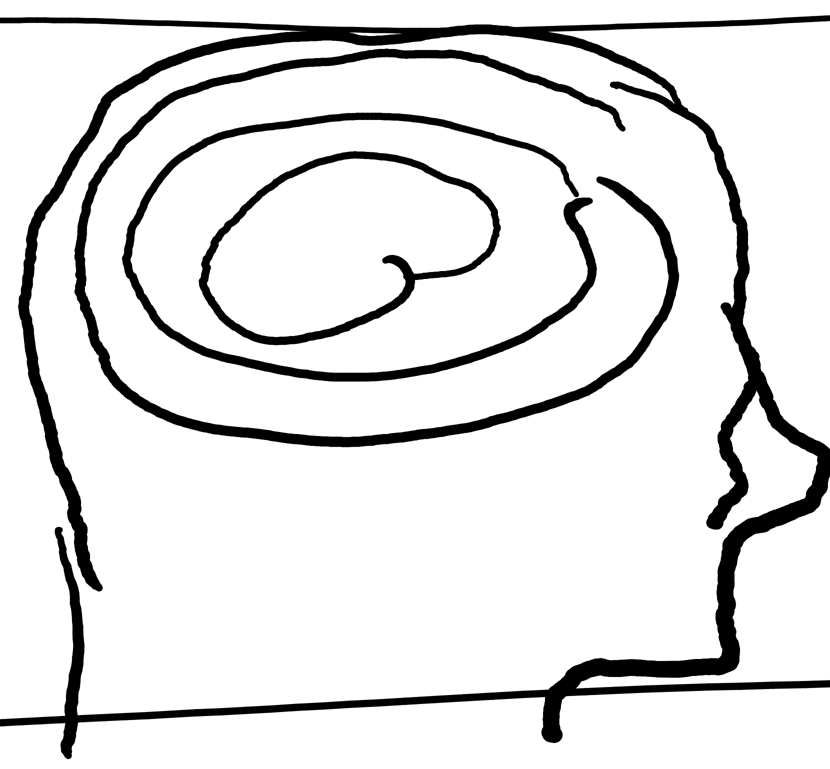
Synthetic data setup 5-20 system

Surfaces @ model

A



B



(same views)

All surfaces

B same with FEM ?

True source, squae