人工智能引论 第一次作业

周子锐 2100011032

2023 年 3 月 12 日

1 Problem 1

解答. 记事件 A 为" 选中的射手击中十环", 事件 B_i 为" 这名射手来自第 i 组"($i = 1, 2, 3, 4$), 则根据全概率公式有

$$P(A) = \sum_{i=1}^{4} P(B_i)P(A|B_i) = \frac{4}{20} \times 0.9 + \frac{6}{20} \times 0.8 + \frac{7}{20} \times 0.5 + \frac{3}{20} \times 0.3 = 0.64$$

2 Problem 2

解答. 记事件 A 为" 此人是色盲", 事件 B 为" 此人是男性". 则根据 Bayes 公式有

$$\begin{aligned}
P(B|A) =& \frac{P(B)P(A|B)}{P(A)} \\
=& \frac{P(B)P(A|B)}{P(B)P(A|B) + P(\bar{B})P(A|\bar{B})} \\
=& \frac{0.5 \times 0.04}{0.5 \times 0.04 + 0.5 \times 0.002} \\
=& \frac{20}{21}
\end{aligned}$$

3 Problem 3

解答.

(1) 记事件 A_i 为" 第 i 次作业合格"($i = 1, 2$), 则

$$P(A_1A_2) = 1 - P(\overline{A_1A_2}) = 1 - P(\overline{A_1})P(\overline{A_2}|\overline{A_1}) = 1 - (1-p)(1 - \frac{p}{3}) = \frac{4p - p^2}{3}$$

(2) 根据 Bayes 公式有

$$
\begin{aligned}
P(A_1|A_2) =& \frac{P(A_2|A_1)P(A_1)}{P(A_2)} \\
=& \frac{P(A_2|A_1)P(A_1)}{P(A_2|A_1)P(A_1) + P(A_2|\overline{A_1})P(\overline{A_1})} \\
=& \frac{p \cdot p}{p \cdot p + (1-p) \cdot \frac{p}{3}} \\
=& \frac{3p}{2p+1}
\end{aligned}
$$

4 Problem 4

解答.

(1) (a) 若 $Z = z = 2k+1 (k \in \mathbb{N})$ 则有 $P(Z = z) = (1-0.6)^k \cdot (1-0.7)^k \cdot 0.6 = 0.12^k \cdot 0.6$.

(b) 若 $Z = z = 2k+2 (k \in \mathbb{N})$ 则有 $P(Z = z) = (1-0.6)^{k+1} \cdot (1-0.7)^k \cdot 0.7 = 0.12^k \cdot 0.28$.

(2) 甲投了 x 次篮说明在前 $x-1$ 轮中两人均未投中, 且第 x 轮中必有一人投中.

故 $P(X = x) = (1-0.6)^{x-1} \cdot (1-0.7)^{x-1} \cdot (0.6 + (1-0.6) \cdot 0.7) = 0.12^{x-1} \cdot 0.88$.

(3) 乙投了 $y (y > 0)$ 次篮说明在前 $y-1$ 轮中两人均未投中, 且要么第 y 轮中甲未投中但乙投中, 或者这一轮中两人均未投中但下一轮中甲投中.

$P(Y = y) = (1-0.6)^y \cdot (1-0.7)^{y-1} \cdot 0.7 + (1-0.6)^y \cdot (1-0.7)^y \cdot 0.6 = 0.12^{y-1} \cdot 0.352$.

特别地, $P(Y = 0) = 0.6$.

5 Problem 5

解答.

(1) 由归一化条件得

$$\int_{-\infty}^{\infty} f(x)\mathrm{d}x = 2A \int_0^{\infty} e^{-x}\mathrm{d}x = 2A = 1$$

解得

$$A = \frac{1}{2}$$

(2) 对概率分布函数 $F(x)$ 有

(a) $x \le 0$ 时

$$F(x) = \int_{-\infty}^{x} f(t)\mathrm{d}t = \frac{1}{2}\int_{-\infty}^{x} e^t\mathrm{d}t = \frac{e^x}{2}$$

(b) $x > 0$ 时

$$F(x) = \int_{-\infty}^{x} f(t)\mathrm{d}t = \frac{1}{2} + \frac{1}{2}\int_0^x e^{-t}\mathrm{d}t = 1 - \frac{e^{-x}}{2}$$

(3)

$$P(-1 \le X \le 2) = \int_{-1}^{2} f(x)\mathrm{d}x = F(2) - F(-1) = 1 - \frac{e+1}{2e^2}$$

6 Problem 6

解答. $P(X=k) = \frac{\binom{12}{5-k}\binom{3}{k}}{\binom{15}{5}} = \frac{\binom{12}{5-k}\binom{3}{k}}{3003}$.

X 具有如下的分布列:

k	0	1	2	3	4	5
$P(X=k)$	$\frac{24}{91}$	$\frac{45}{91}$	$\frac{20}{91}$	$\frac{2}{91}$	0	0

故 $E(X) = \sum_{k=0}^{5} kP(X=k) = 1$.

7 Problem 7

解答.

(1)

$$\begin{aligned} E(X) &= \int_{-\infty}^{\infty} xf(x)\mathrm{d}x \\ &= \frac{1}{2}\left(\int_{-\infty}^{0} xe^{x}\mathrm{d}x + \int_0^{\infty} xe^{-x}\mathrm{d}x\right) \\ &= \frac{1}{2}\left(-\int_0^{\infty} xe^{-x}\mathrm{d}x + \int_0^{\infty} xe^{-x}\mathrm{d}x\right) \\ &= 0 \end{aligned}$$

(2)

$$\begin{aligned} E(X^2) &= \int_{-\infty}^{\infty} x^2 f(x)\mathrm{d}x \\ &= \int_0^{\infty} x^2 e^{-x}\mathrm{d}x \\ &= 2 \end{aligned}$$

故

$$D(X) = E(X^2) - (E(X))^2 = 2 - 0 = 2$$