Azonosító jel:															

ÉRETTSÉGI VIZSGA • 2025. május 6.

MATEMATIKA
ANGOL NYELVEN

EMELT SZINTŰ
ÍRÁSBELI VIZSGA

2025. május 6. 9:00

Időtartam: 300 perc

Pótlapok száma	
Tisztázati	
Piszkozati	

OKTATÁSI HIVATAL

Instructions to candidates

1. The time allowed for this examination paper is 300 minutes. When that time is up, you will have to stop working.

2. You may solve the problems in any order.

3. In part II, you are only required to solve four of the five problems. **When you have finished the examination, enter the number of the problem not selected in the square below**. *If it is not clear* for the examiner which problem you do not want to be assessed, the last problem in this examination paper will not be assessed.

4. On solving the problems, you may use a calculator that cannot store and display textual information. You may also use any edition of the four-digit data tables. The use of any other electronic device or printed or written material is forbidden!

5. **Always write down the reasoning used to obtain the answers. A major part of the score will be awarded for this.**

6. **Make sure that calculations of intermediate results are also possible to follow.**

7. **The use of calculators** in the reasoning behind a particular solution **may be accepted without further mathematical explanation in case of the following operations**: addition, subtraction, multiplication, division, calculating powers and roots, $n!$, $\binom{n}{k}$, replacing the tables found in the 4-digit Data Booklet (sin, cos, tan, log, and their inverse functions), approximate values of the numbers π and e, finding the solutions of the standard quadratic equation. No further explanation is needed when the calculator is used to find the mean and the standard deviation, as long as the text of the question does not explicitly require the candidate to show detailed work. **In any other cases, results obtained through the use of a calculator are considered as unexplained and points for such results will not be awarded.**

8. On solving the problems, theorems studied and given a name in class (e.g. the Pythagorean Theorem or the height theorem) do not need to be stated precisely. It is enough to refer to them by name, but their applicability needs to be briefly explained. Reference to other theorems will be fully accepted only if the theorem and all its conditions are stated correctly (proof is not required) and the applicability of the theorem to the given problem is explained.

9. Always state the final result (the answer to the question of the problem) in words, too!

10. Write in pen. Diagrams may be drawn in pencil. The examiner is instructed not to mark anything written in pencil, other than diagrams. If you cancel any solution or part of a solution by crossing it over, it will not be assessed.

11. Only one solution to each problem will be assessed. In case of more than one attempt to solve a problem, **indicate clearly** which attempt you wish to be marked.

12. Please, **do not write in the grey rectangles**.

I.

1. **a)** Solve the following equation over the set of real numbers.

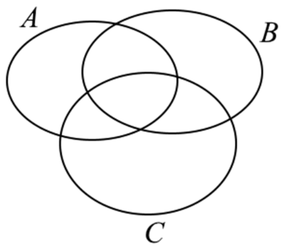
$$2^{x+3} - 2^{x} + 2^{x-1} = 60$$

b) Solve the following equation system over the set of pairs of real numbers.

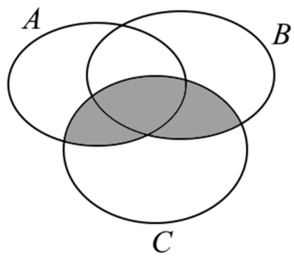
$$\left.\begin{aligned} x + y &= 3(x-1) \\ |x + 2y + 1| &= 5 \end{aligned}\right\}$$

a)	6 points	
b)	7 points	
T.:	13 points	

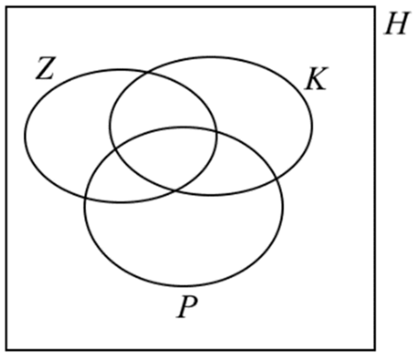
2. **a)** In the following diagram, shade the set $B \setminus (A \cap C)$.



b) Use set operations to describe the subset shaded grey in the following diagram.



Let *H* be the fundamental set of all functions and let *Z*, *K* and *P* be subsets of *H*, such that:
Z = {functions that have zero(s)}
K = {one-to-one functions}
P = {odd functions}



c) Place the letter corresponding to each function below into the appropriate region of the diagram.

f: $\mathbf{R} \to \mathbf{R}$, $x \mapsto |x|$

g: $\mathbf{R} \to \mathbf{R}$, $x \mapsto x^2 + 2$

h: $\mathbf{R}^+ \to \mathbf{R}$, $x \mapsto \log x$

i: $\mathbf{R} \to \mathbf{R}$, $x \mapsto \sin x$

A graph has four vertices representing the above functions *f*, *g*, *h* and *i*. Two vertices are connected with an edge if and only if the functions represented by the two vertices have any common element(s) in their ranges.

d) Draw the graph described above. You do not have to justify your answer here.

a)	2 points	
b)	2 points	
c)	4 points	
d)	3 points	
T.:	11 points	

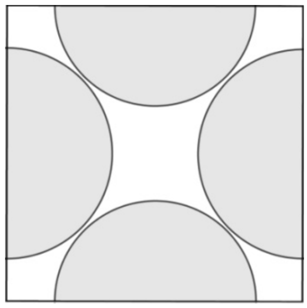
3. A pizzeria sells 2 ham pizzas and 3 peach juices for a total of 7600 Ft, while 3 ham pizzas and 5 peach juices would cost 11 700 Ft there.

a) How much does a ham pizza cost and what is the price of a peach juice?

While preparing pizzas, the ratio of the cost of ingredients vs. other costs (utilities, labour, etc.) is 7 : 3. The cost of ingredients has increased by 15% over the past year, while other costs have increased by 25%.

b) By what percentage does preparing a pizza cost more this year, than it did last year?

Trying to save on utilities, Peter uses a 46 cm × 46 cm oven tray to bake two pizzas of diameter 32 cm at the same time. As they do not fit, Peter cuts each pizza in half and tries to arrange them as shown in the diagram. (The centre of each half-pizza coincides with the midpoint of a side of the oven tray and the pizza-halves do not overlap.)



c) Will the two pizzas fit into the tray this way?

a)	5 points	
b)	4 points	
c)	4 points	
T.:	13 points	

4. While memorising the year of various historic events, Bence noticed that three of these years formed three consecutive terms of a geometric sequence. The smallest of the numbers was 732, the largest was 1647.

a) What year is the middle term?

Both the mean and median of Bence's 10 Mathematics grades is 4, while their single mode is 5. (All grades are integers, 1 through 5.)

b) Determine Bence's Mathematics grades.

Bence buys three chimney cakes for his family. The shop sells cakes flavoured with either walnuts, cinnamon, cocoa or vanilla.

c) In how many different ways can Bence select the three cakes? (He may buy more than one cake of the same flavour. Two selections are considered different if there is at least one flavour that appears at a different number of times.)

a)	3 points	
b)	7 points	
c)	4 points	
T.:	14 points	

II.

You are required to solve any four out of the problems 5 to 9.
Write the number of the problem NOT selected in the blank square on page 2.

5. **a)** The third term of an arithmetic sequence is 5, the thirteenth term is 22. Determine the sum of the first 100 terms of this sequence.

b) The first term of an arithmetic sequence is 91, the common difference is 2. Determine the value of the positive integer n, such that the sum of the first n terms of the sequence is equal to n^3.

c) The first term of a geometric sequence is 1.6, the common ratio is 2. Starting with the first term, at least how many terms must be added, so that the sum is greater than one billion?

a)	4 points	
b)	6 points	
c)	6 points	
T.:	16 points	

You are required to solve any four out of the problems 5 to 9.
Write the number of the problem NOT selected in the blank square on page 2.

6. The quadrilateral *ABCD* is cyclic (all sides are chords of the same circle).
The length of diagonal *BD* is 20 cm, $ABD\sphericalangle = BCA\sphericalangle = 35°$, $DAC\sphericalangle = 48°$.

a) Prove that $BDA\sphericalangle = 35°$ and $ACD\sphericalangle = 35°$.

b) Calculate the perimeter of the cyclic quadrilateral *ABCD*.

The following are known about triangle *EFG*: *EF* = 110 cm, *FG* = 50 cm, *EG* = 104 cm.

c) Calculate the length of both parts of side *EG* divided by the interior angle bisector drawn from vertex *F*.

a)	3 points	
b)	9 points	
c)	4 points	
T.:	16 points	

You are required to solve any four out of the problems 5 to 9.
Write the number of the problem NOT selected in the blank square on page 2.

7. Anna and Balázs take turns rolling a fair gambling die. Anna is first to roll. The winner is the player that first rolls a six. (The game ends when either of the players rolls the first six.)

a) Prove that the probability that Balázs wins the game on his first roll is $\frac{5}{36}$.

b) Calculate the probability that Anna wins the game on one of her first three rolls.

c) Calculate the probability that Anna wins the game.

Two fair dice are rolled in an experiment. The outcome of the experiment is the number, out of the two numbers shown, that is not greater than the other.

d) Calculate the expected value of the outcome of this experiment.

a)	3 points	
b)	4 points	
c)	5 points	
d)	4 points	
T.:	16 points	

You are required to solve any four out of the problems 5 to 9.
Write the number of the problem NOT selected in the blank square on page 2.

8. **a)** Give the letter corresponding to every statement below that is the negation of the statement *“All flowers are fragrant”*.

A) There exists a flower that is fragrant.
B) There exists a flower that is not fragrant.
C) None of the flowers are fragrant.
D) Not all flowers are fragrant.

Six different flowers are placed into a red, a white and a green vase.

b) How many different arrangements are possible to place the flowers into the vases if each vase must contain at least one flower? (Two arrangements are considered different if there is at least one vase that does not contain exactly the same flowers.)

The shape of a vase is a cylinder, open at the top, and its volume is 2 litres.

c) How many centimetres should the height of the vase be to minimise the interior surface area?

a)	2 points	
b)	6 points	
c)	8 points	
T.:	16 points	

You are required to solve any four out of the problems 5 to 9.
Write the number of the problem NOT selected in the blank square on page 2.

9. One endpoint of the base of the isosceles triangle *ABC* is the point *B*(0; 4), the point of intersection of the two equal sides (legs) is *A*(3; 0). The other endpoint of the base of the triangle is on the line *e*: $x + 2y = 8$.

a) Determine the coordinates of vertex *C* of the triangle.

Given is a parabola: $y = -\frac{1}{2}x^2 + \frac{9}{2}x - 4$ and the line *f*: $y = -\frac{1}{2}x + \frac{13}{2}$.

b) Determine the area of the bounded planar region between the graphs of the parabola and line *f*.

a)	8 points	
b)	8 points	
T.:	16 points	

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<tr><td></td><td rowspan="2">Number of problem</td><td colspan="4">score</td></tr>
<tr><td></td><td>maximum</td><td>awarded</td><td>maximum</td><td>awarded</td></tr>
<tr><td rowspan="4">Part I</td><td>1.</td><td>13</td><td></td><td rowspan="4">51</td><td rowspan="4"></td></tr>
<tr><td>2.</td><td>11</td><td></td></tr>
<tr><td>3.</td><td>13</td><td></td></tr>
<tr><td>4.</td><td>14</td><td></td></tr>
<tr><td rowspan="5">Part II</td><td></td><td>16</td><td></td><td rowspan="4">64</td><td rowspan="4"></td></tr>
<tr><td></td><td>16</td><td></td></tr>
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<tr><td colspan="4">Total score on written examination</td><td>115</td><td></td></tr>
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date

examiner

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<tr><td></td><td>elért</td><td>programba beírt</td></tr>
<tr><td>I. rész</td><td></td><td></td></tr>
<tr><td>II. rész</td><td></td><td></td></tr>
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dátum

dátum

javító tanár

jegyző