1 Exercise 1004

Solve the following equation:

$$\left(x^2+6x+1\right)\left(x^2+6x-3\right)=5$$

$$\begin{aligned}
\left(x^2+6x+1\right)\left(x^2+6x-3\right) &= 5 \\
\text{let } u &= x^2+6x \\
(u+1)(u-3) &= 5 // \text{ substitute u} \\
u^2-2u-3 &= 5 // \text{ factor} \\
u^2-2u-8 &= 0 \\
(u-4)(u+2) &= 0 \\
(u-4)=0 \text{ and } (u+2) &= 0 // \text{ the two solutions} \\
u &= 4,\ u=-2
\end{aligned}$$

solution 1 for substitution: $u=4$

$$\begin{aligned}
x^2+6x+9 &= 4+9 \\
(x+3)^2 &= \sqrt{13} \\
x &= -3\pm\sqrt{13} \text{ // } \textbf{real solution 1}
\end{aligned}$$

solution 2 for substitution: $u=-2$

$$\begin{aligned}
x^2+6x+9 &= -2+9 \\
(x+3)^2 &= \sqrt{7} \\
x &= -3\pm\sqrt{7} \text{ // } \textbf{real solution 2}
\end{aligned}$$

Notes

- we haven't multiplied the two together because it would result in a complex mess where easy to make mistakes
- we used substitution
- to find the correct numbers to transform the

  $$u^2-2u-8=0$$

  to

  $$(u-4)(u+2)=0$$

  we used the trick described below.

- the magic to find the integer added to the

$$x^2 + 6x$$

  to get a perfect square is described below too.

- we haven't used the quadratic formula due to ...