ÉRETTSÉGI VIZSGA • 2025. május 6.

MATEMATIKA ANGOL NYELVEN

EMELT SZINTŰ ÍRÁSBELI VIZSGA

JAVÍTÁSI-ÉRTÉKELÉSI ÚTMUTATÓ

OKTATÁSI HIVATAL

Instructions to examiners

Formal requirements:

1. Mark the paper **legibly, in ink, different in colour** from that used by the candidate.
2. The first of the rectangles next to each problem shows the maximum attainable score on that problem. The **score** given by the examiner is to be entered **into the rectangle** next to this.
3. **If the solution is perfect**, enter maximum score and, with a checkmark, indicate that you have seen each item leading to the solution and consider them correct.
4. If the solution is incomplete or incorrect, please **mark the error** and also indicate the individual **partial scores**. It is also acceptable to indicate the points lost by the candidate if it makes grading the paper easier. After correcting the paper, it must be clear about every part of the solution whether that part is correct, incorrect or unnecessary.
5. Please, **use the following symbols** when correcting the paper:
   - correct calculation: *checkmark*
   - principal error: *double underline*
   - calculation error or other, not principal, error: *single underline*
   - correct calculation with erroneous initial data: *dashed checkmark or crossed checkmark*
   - incomplete reasoning, incomplete list, or other missing part: *missing part symbol*
   - unintelligible part: *question mark* and/or *wave*
6. Do not assess anything written **in pencil**, except for diagrams

Assessment of content:

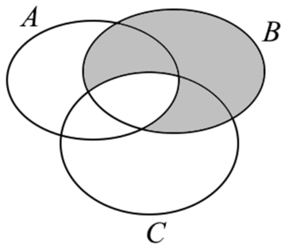
1. The answer key may contain more than one solution for some of the problems. If the **solution given by the candidate is different**, allocate the points by identifying parts of the solution equivalent to those given in the answer key.
2. Subtotals may be **further divided, unless stated otherwise in the answer key**. However, scores awarded must always be whole numbers.
3. If there is a **calculation error** or inaccuracy in the solution, take points off only for the part where the error occurs. If the reasoning remains correct and the error is carried forward while the nature of the problem remains unchanged, points for the rest of the solution must be awarded.
4. **In case of a principal error**, no points should be awarded at all for that section of the solution, not even for steps that are formally correct. (These logical sections of the solutions are separated by double lines in the answer key.) However, if the erroneous information obtained through principal error is carried forward to the next section or the next part of the problem, and it is used there correctly, the maximum score is due for that part, provided that the error has not changed the nature of the task to be completed.
5. Where the answer key shows a **remark** in brackets, the solution should be considered complete without that remark as well.

6. Deduct points for **missing units** only if the missing unit is part of the answer or a unit exchange (without parentheses).
7. If there are more than one different approach to a problem, **assess only the one indicated by the candidate**. Please, mark clearly which attempt was assessed.
8. **Do not give extra points** (i.e. more than the score due for the problem or part of problem).
9. The score given for the solution of a problem, or part of a problem, **may never be negative**.
10. **Do not take points off** for steps or calculations that contain errors but are not actually used by the candidate in the solution of the problem.
11. **The use of calculators** in the reasoning behind a particular solution **may be accepted without further mathematical explanation in case of the following operations**: addition, subtraction, multiplication, division, calculating powers and roots, $n!$, $\binom{n}{k}$, replacing the tables found in the 4-digit Data Booklet (sin, cos, tan, log, and their inverse functions), approximate values of the numbers π and e, finding the solutions of the standard quadratic equation. No further explanation is needed when the calculator is used to find the mean and the standard deviation, as long as the text of the question does not explicitly require the candidate to show detailed work. **In any other cases, results obtained through the use of a calculator are considered as unexplained and points for such results will not be awarded.**
12. Using **diagrams** to justify a solution (e.g. reading data off diagrams by way of direct measurement) is unacceptable.
13. **Probabilities** may also be given in percentage form (unless stated otherwise in the text of the problem).
14. If the text of the problem does not instruct the candidate to round their solution in a particular way, any solution **rounded reasonably and correctly** is acceptable even if it is different from the one given in the answer key.
15. **Assess only four out of the five problems in part II of this paper**. The candidate was requested to indicate in the appropriate square the number of the problem not to be assessed and counted towards their total score. Should there be a solution to that problem, it does not need to be marked. However, if the candidate did not make a selection and neither is their choice clear from the paper, assume automatically that it is the last problem in the examination paper that is not to be assessed.

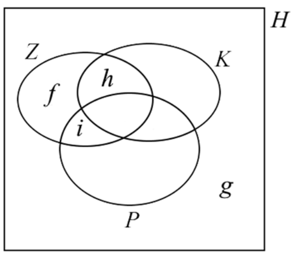
I.

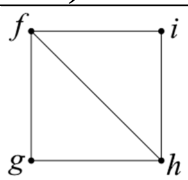
1. a)		
$8\cdot 2^x - 2^x + \frac{2^x}{2} = 60$	1 point	
Multiply both sides by 2: $16\cdot 2^x - 2\cdot 2^x + 2^x = 120$.	1 point	$7.5\cdot 2^x = 60$
Rearranged: $15\cdot 2^x = 120$.	1 point	
$2^x = 8 (= 2^3)$	1 point	
(As the exponential function is one-to-one:) $x = 3$.	1 point	
Check by substitution or reference to equivalent steps.	1 point	
Total:	**6 points**	

1. b)		
From the first equation: $y = 2x - 3$.	1 point	$x = \frac{y+3}{2}$
Substituting into the second equation: $\lvert 5x-5 \rvert = 5$.	1 point	$\lvert y+1 \rvert = 2$
If $5x - 5 = 5$ then $x = 2$,	1 point	*If $y + 1 = 2$ then $y = 1$,*
in which case $y = 1$. (One of the solutions is (2; 1).)	1 point	*in which case $x = 2$.*
If $5x - 5 = -5$, then $x = 0$,	1 point	*If $y + 1 = -2$, then $y = -3$,*
in which case $y = -3$, (The other solution is (0; –3).)	1 point	*in which case $x = 0$.*
Check.	1 point	
Total:	**7 points**	

2. a)		
	2 points	
Total:	**2 points**	

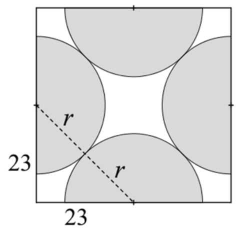
2. b)		
For example $(A \cup B) \cap C$.	2 points	$(A \cap C) \cup (B \cap C)$
Total:	**2 points**	

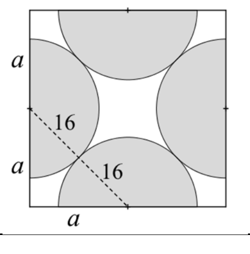
2. c)		
	1-1 point	
Total:	**4 points**	

2. d)		
	3 points	*Deduce one point for every edge that is either incorrect or missing.*
Total:	**3 points**	

3. a)		
Let p be the price of the ham pizza and b be the price of a peach juice. In this case: $\left.\begin{aligned} 2p+3b&=7600 \\ 3p+5b&=11\,700 \end{aligned}\right\}$.	1 point	
Express p from the first equation: $p = 3800 - 1.5b$.	1 point	*Subtract the triple of the first equation from the double of the second:* $b = 600$.
Substitute it into the second equation: $3(3800 - 1.5b) + 5b = 11\ 700$. This gives $b = 600$ (a peach juice costs 600 Ft).	1 point	
Then $p = 2900$ (the cost of a pizza is 2900 Ft).	1 point	
Check against the text of the question.	1 point	
Total:	**5 points**	

3. b)		
Let $7x$ be the cost of ingredients last year and let $3x$ represent the other costs (total cost is $10x$).	1 point	
After inflation this will be $1.15 \cdot 7x + 1.25 \cdot 3x =$	1 point	
$= 11.8x$ Ft (which is 1.18 times the original costs),	1 point	
so a pizza costs 18% more this year.	1 point	
Total:	**4 points**	

3. c) Solution 1		
Let us calculate the diameter of the largest pizzas that are still possible to be placed in the oven this way. In this case, the semicircles are tangent to one another. The distance between the centres of two tangential semicircles is the sum of the radii of the semicircles.	1 point	*Award this point if the correct reasoning is reflected only by the solution.*
(Let r be the radius of the semicircles:) Use the Pythagorean Theorem: $(2r)^2 = 23^2 + 23^2$,	1 point	
$2r\ (= 23\sqrt{2}) \approx 32.5$ cm.	1 point	
This is more than 32 cm, so Peter's pizzas will fit into the oven tray.	1 point	
Total:	**4 points**	

3. c) Solution 2		
Let us determine the size of the tray that accommodates two 32-cm pizzas the way Peter wants them. In this case, pizzas will touch one another. The distance between the centres of two tangential semicircles is the sum of the radii of the semicircles.	1 point	*Award this point if the correct reasoning is reflected only by the solution.*
(Let $2a$ be the side of the tray:) Use the Pythagorean Theorem: $32^2 = a^2 + a^2$,	1 point	
$a \approx 22.63$ cm.	1 point	
$2a < 46$, so the pizza-halves will fit into the tray.	1 point	
Total:	**4 points**	

4. a)		
Let q be the common ratio of the geometric sequence: $732 \cdot q^2 = 1647$,	1 point	*As per the properties of geometric sequences:* $x^2 = 732 \cdot 1647$.
(as $q > 0$) $q = 1.5$,	1 point	
the middle term is, therefore, $(732 \cdot 1.5 =)$ 1098.	1 point	(*as* $x > 0$) $x = 1098$.
Total:	**3 points**	

4. b)		
When the grades are arranged in order, there are two possible ways for the median to be 4: case 1: the fifth grade is 3 and the sixth is 5; case 2: both the fifth and the sixth grade are 4-s.	1 point*	*Case 1:* _ _ _ _ 3 5 _ _ _ _ *Case 2:* _ _ _ _ 4 4 _ _ _ _
In case 1 there would be five 5-s which would also require five 3-s to make the mean 4 but this is prohibited by the single mode.	1 point*	
In case 2 there must be four 5-s as, should there be less than that, there should be at least the same number of 4-s and so 5 could not be the single mode.	1 point*	_ _ _ _ 4 4 5 5 5 5 *is possible.*
As the mean is 4, the sum of the ten grades is 40, so the sum of the four smallest grades would be $40 - 4 \cdot 5 - 2 \cdot 4 = 12$.	1 point	
If there are no 4-s among these, the only possibility to get a sum of 12 is four 3-s, but in that case 5 could not be the single mode.	1 point	
If there is one more 4 (not more than that, because of the mode) than the other three grades must be 2, 3, 3,	1 point	
and so the ten grades must be 2, 3, 3, 4, 4, 4, 5, 5, 5, 5. (Which meets all conditions.)	1 point	
Total:	**7 points**	

*The 3 points marked * may also be given for the following reasoning:*

The number of 5-s is minimum three, because of the mode, and maximum five, because of the median.	1 point	
Three 5-s would not be enough, as the sum of the grades then could not be more than $(3 \cdot 5 + 2 \cdot 4 + 2 \cdot 3 + 2 \cdot 2 + 1 \cdot 1 =)$ 34 (and then the mean could not be 4).	1 point	
Also, there must not be five 5-s, as there should be five 3-s as well to make the mean 4. This is also forbidden (by the mode). Therefore, there must only be four 5-s.	1 point	

4. c) **Solution 1**		
There are 4 options if all three flavours are the same.	1 point	
There are also 4 options if all three flavours are different.	1 point	
There are $4 \cdot 3 = 12$ options to select two cakes of a certain flavour and one more of a different kind.	1 point	$\binom{4}{2} \cdot 2$
In total, there are $4 + 4 + 12 = 20$ different possibilities for Bence to select the cakes.	1 point	
Total:	**4 points**	

4. c) **Solution 2**		
Three cakes must be selected out of 4 different flavours, such that the order is not considered and the flavours may repeat.	1 point	*Award this point if the correct reasoning is reflected only by the solution.*
This is a case of combination with repeat, three items out of 4. The number of possibilities is: $\binom{4+3-1}{3} =$	2 points	
$= \binom{6}{3} = 20.$	1 point	
Total:	**4 points**	

Note: Award full marks if the candidate gives the correct answer by listing all possible cases (in an organised, logical manner).

II.

5. a)		
The common difference of the arithmetic sequence: $d=\frac{22-5}{10}=1.7.$	1 point	
The first term of the sequence: $a_1=5-2\cdot1.7=1.6$.	1 point	
The sum of the first 100 terms: $S_{100}=\frac{2\cdot1.6+99\cdot1.7}{2}\cdot100=8575.$	2 points	$a_{100}=1.6+99\cdot1.7=169.9$ $S_{100}=\frac{1.6+169.9}{2}\cdot100=$ $=8575$
Total:	**4 points**	

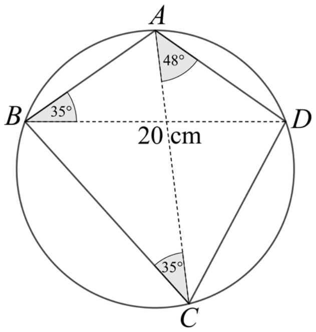
5. b)		
The sum of the first n terms of the arithmetic sequence: $S_n=\frac{2\cdot91+(n-1)\cdot2}{2}\cdot n=n^3$.	2 points	$a_n=91+(n-1)\cdot2=2n+89$ $S_n=\frac{91+(2n+89)}{2}\cdot n=n^3$
$(90+n)\cdot n=n^3$ (Divide by $n\neq0$ and rearrange to zero:) $n^2-n-90=0$.	2 points	
$n=-9$ is not correct (as $n>0$).	1 point	
$n=10$ is correct ($S_{10}=1000=10^3$).	1 point	
Total:	**6 points**	

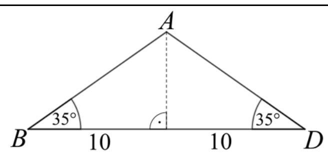
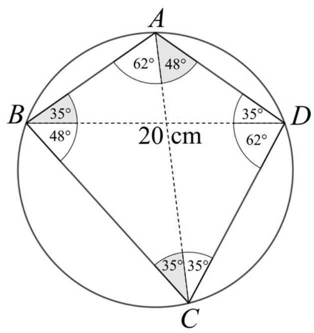
5. c)		
The sum of the first n terms of the geometric sequence: $S_n=1.6\cdot\frac{2^n-1}{2-1}>1\,000\,000\,000.$	2 points	
$2^n>625\,000\,001$	1 point	
As the base 2 logarithm function is strictly monotone increasing,	1 point	*The common logarithm function is strictly monotone increasing, so*
$n>\log_2 625\,000\,001\approx29.2.$	1 point	$\log2^n>\log625\,000\,001$ $n\log2>\log625\,000\,001$ ($\log2>0$, *so*) $n>\frac{\log625\,000\,001}{\log2}\approx29.2$
At least 30 terms must be added.	1 point	
Total:	**6 points**	

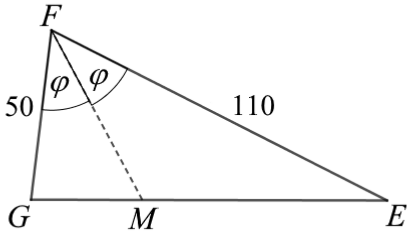
Notes:

1. Award 4 points for the correct solution of the equation if the candidate uses an equation instead of an inequality. Award full marks, if they then give the correct answer by also referring to the strictly monotone increase.

2. Award 3 points if the candidate proves that $n = 29$ *is incorrect,* ***and*** $n = 30$ *is correct. Award full marks, if they then give the correct answer by also referring to the strictly monotone increase.*

6. a)		
 According to the theorem on inscribed angles	1 point	
$ADB\sphericalangle = ACB\sphericalangle = 35°$ (as they are both subtended by the same arc *AB*),	1 point	
also, $ABD\sphericalangle = ACD\sphericalangle = 35°$ (as they are both subtended by the same arc *AD*),	1 point	
Total:	**3 points**	

6. b)		
(The base *BD* of the isosceles triangle *ABD* is 20 cm, therefore) $AB = AD = \frac{10}{\cos 35°} \approx 12.2$ cm.	2 points	
(Because of inscribed angles or the sum of the interior angles of a triangle:) $CAB\sphericalangle = 62°$, $DBC\sphericalangle = 48°$, $BDC\sphericalangle = 62°$.	2 points	
(as $BCD\sphericalangle = 70°$) apply the Sine Rule in triangle *BCD*: $\frac{BC}{20} = \frac{\sin 62°}{\sin 70°}$,	1 point	
$BC \approx 18.8$ cm.	1 point	
Apply the Cosine Rule: $CD = \sqrt{20^2 + 18.8^2 - 2\cdot 20\cdot 18.8\cdot \cos 48°} \approx$	1 point	*Apply the Sine Rule:* $\frac{CD}{20} = \frac{\sin 48°}{\sin 70°}$
≈ 15.8 cm.	1 point	$CD \approx 15.8$ cm.
The perimeter of the cyclic quadrilateral: (12.2 + 12.2 + 18.8 + 15.8 =) 59 cm.	1 point	
Total:	**9 points**	

6. c)		
 (Let M be the point of intersection of the bisector of the interior angle at F and the side EG.) Apply the theorem on angle bisectors: $\frac{EM}{MG}=\frac{FE}{FG}$,	1 point	*As per the theorem about angle bisectors, side EG must be divided at a ratio of* 110:50.
from where $EM=\frac{104}{110+50}\cdot 110=71.5$ cm.	2 points	
Then MG = (104 – 71.5 =) 32.5 cm. (The angle bisector divides side EG into a 71.5 and a 32.5 cm long segment.)	1 point	
Total:	**4 points**	

Note: Deduce a maximum of 1 point for the entire question if the candidate omits units of length.

7. a)		
The probability that Anna does not win on her first roll (giving Balázs a chance) is $\frac{5}{6}$.	1 point	
The probability that Balázs then wins (regardless of Anna's first roll) is $\frac{1}{6}$.	1 point	
The final probability is $\frac{5}{6}\cdot\frac{1}{6}=\frac{5}{36}$, indeed.	1 point	
Total:	**3 points**	

7. b)		
The probability that Anna wins on her first roll is $\frac{1}{6}$.	1 point	
Anna wins on her second roll if neither of them roll six on their first roll but Anna's second roll is a six. The probability of this is $\left(\frac{5}{6}\right)^2 \cdot \frac{1}{6}$ (≈ 0.116).	1 point	
Anna wins on her third roll if neither of them roll six on their first two rolls but Anna's third roll is a six. The probability of this is $\left(\frac{5}{6}\right)^4 \cdot \frac{1}{6}$ (≈ 0.080).	1 point	
The final probability is the sum of the above: $\frac{1}{6}+\left(\frac{5}{6}\right)^2 \cdot \frac{1}{6}+\left(\frac{5}{6}\right)^4 \cdot \frac{1}{6}=\frac{1}{6}\cdot\left(1+\frac{25}{36}+\frac{625}{1296}\right)\approx 0.363$.	1 point	$\frac{2821}{7776}$
Total:	**4 points**	

7. c) **Solution 1**		
(The probability that Anna wins on her first roll is $\frac{1}{6}$. The probability that she wins on her second roll is $\left(\frac{5}{6}\right)^2 \cdot \frac{1}{6}$. The probability that she wins on her third roll is $\left(\frac{5}{6}\right)^4 \cdot \frac{1}{6}$.) And so on, the probability that Anna wins on her n^{th} roll is $\left(\frac{5}{6}\right)^{2n-2} \cdot \frac{1}{6}$ (where n is a positive integer).	1 point	*Award this point if the correct reasoning is reflected only by the solution.*
The above probabilities form consecutive terms of a geometric sequence whose first term is $a_1=\frac{1}{6}$ and the common ratio is $q=\left(\frac{5}{6}\right)^2=\frac{25}{36}$.	2 points	
The rolls may continue indefinitely, so the sum of an infinite geometric sequence must be determined (which exists, as $0 < q < 1$).	1 point	*Award this point if the correct reasoning is reflected only by the solution.*
The probability that Anna wins is $S=\frac{a_1}{1-q}=\frac{\frac{1}{6}}{1-\frac{25}{36}}=\frac{\frac{1}{6}}{\frac{11}{36}}=\frac{6}{11}$ (≈ 0.545).	1 point	
Total:	**5 points**	

7. c) Solution 2		
The probability that Anna wins on her first roll is $\frac{1}{6}$. The probability that neither player rolls six on their first trial is $\frac{5}{6}\cdot\frac{5}{6}=\frac{25}{36}$.	1 point	
The case when neither player rolls six may as well be considered as if the game restarted. At this moment the probability p that Anna wins is equal to the probability of her winning at the very beginning of the game.	1 point	
That is: $p=\frac{1}{6}+\frac{25}{36}\cdot p$,	2 points	
$p=\frac{6}{11}$.	1 point	
Total:	**5 points**	

7. d)		
When two dice are rolled, there are a total $6\cdot 6 = 36$ (equally likely) outcomes. Of these, there are 11, 9, 7, 5, 3 and 1 cases when the number not greater than the other is 1, 2, 3, 4, 5 and 6, respectively.	2 points	see tables below
The expected value: $\frac{11}{36}\cdot 1+\frac{9}{36}\cdot 2+\frac{7}{36}\cdot 3+\frac{5}{36}\cdot 4+\frac{3}{36}\cdot 5+\frac{1}{36}\cdot 6=$	1 point	
$=\frac{91}{36}$ (≈ 2.528).	1 point	
Total:	**4 points**	

the number that is not greater than the other	1	2	3	4	5	6
the probability of this	$\frac{11}{36}$	$\frac{9}{36}$	$\frac{7}{36}$	$\frac{5}{36}$	$\frac{3}{36}$	$\frac{1}{36}$

	1	**2**	**3**	**4**	**5**	**6**
1	1	1	1	1	1	1
2	1	2	2	2	2	2
3	1	2	3	3	3	3
4	1	2	3	4	4	4
5	1	2	3	4	5	5
6	1	2	3	4	5	6

8. a)		
B, D	2 points	*Award 1 point for one correct answer or for two correct and one incorrect answers. Award 0 points in any other case.*
Total:	**2 points**	

8. b) Solution 1		
(There are three possible arrangements according to the number of flowers.) Case 1: There are 4 flowers in one vase, 1-1 in the other two. There are $\binom{6}{4}\cdot\binom{2}{1}\cdot\binom{1}{1}=30$ different possibilities to place 4 flowers into, for example, the red vase and 1-1 into the white and the green vase.	1 point	
The vase that contains 4 flowers can be either of the three, so there are a total $30\cdot 3 = 90$ possibilities in this case.	1 point	
Case 2: There are 3 flowers in one vase, 2 in another and 1 in the third. There are $\binom{6}{3}\cdot\binom{3}{2}\cdot\binom{1}{1}=60$ different possibilities to place 3 flowers into, for example, the red vase, 2 flowers into the white and 1 into the green vase.	1 point	
There are $3! = 6$ different orders for the colours of the vases, so there are a total $60\cdot 6 = 360$ possibilities in this case.	1 point	
Case 3: There are 2 flowers in each vase. There are $\binom{6}{2}\cdot\binom{4}{2}\cdot\binom{2}{2}=90$ possibilities in this case.	1 point	
The total number of the different arrangements of the flowers is, therefore, $90 + 360 + 90 = 540$.	1 point	
Total:	**6 points**	

8. b) Solution 2		
Permitting empty vases, too, there could be a total $3^6 = 729$ different arrangements.	1 point	
Of these, there are 2^6 cases when the red vase is empty, two of which also involves another empty vase, too, so there are $2^6 - 2 = 62$ cases when the red vase is the only empty one.	2 points	
Either of the vases may be the only empty one, so there are $3\cdot 62 = 186$ cases when there is exactly one empty vase.	1 point	

There are also another three cases when there are exactly two empty vases (i.e. all 6 flowers are in the same vase).	1 point	
The total number of proper arrangements is, therefore, 729 – 186 – 3 = 540.	1 point	
Total:	**6 points**	

8. c) Solution 1		
(Let r be the radius of the base circle and let m be the height of the cylinder, in cm-s.) The volume of the cylinder is $V = r^2\pi m = 2000\ \text{cm}^3$,	1 point	
from which the height of the vase is: $m = \frac{2000}{r^2\pi}$.	1 point	
The surface area of the open-top cylinder is $A = r^2\pi + 2r\pi m = r^2\pi + 2r\pi\cdot\frac{2000}{r^2\pi} = r^2\pi + \frac{4000}{r}$.	1 point	
The derivative of the function $f(r) = r^2\pi + \frac{4000}{r}$ ($r > 0$) is $f'(r) = 2r\pi - \frac{4000}{r^2}$.	1 point*	
(Function f has its minimum where $f'(r) = 0$) $2r\pi - \frac{4000}{r^2} = 0$.	1 point*	
$r\left(=\sqrt[3]{\frac{2000}{\pi}}\right) \approx 8.6$ cm.	1 point*	
As $f''(r) = 2\pi + \frac{8000}{r^3} > 0$, this is a minimum, indeed.	1 point*	*The first derivative is negative where* $0 < r < 8.6$. *It is positive where* $r > 8.6$, *therefore* $r = 8.6$ *is, indeed, a minimum.*
The height of the vase should be $m = \frac{2000}{8{,}6^2\pi} \approx 8.6$ cm.	1 point	
Total:	**8 points**	

*The 4 points marked * may also be given for the following reasoning:*

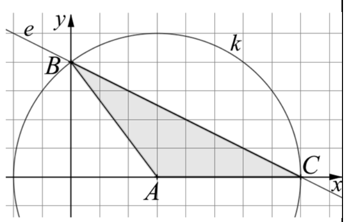
Apply the relation between the arithmetic and geometric means for the terms $r^2\pi$, $\frac{2000}{r}$, $\frac{2000}{r}$:	1 point	
$f(r) = r^2\pi + \frac{2000}{r} + \frac{2000}{r} \geq 3\cdot\sqrt[3]{2000^2\pi} \approx 697.5$.	1 point	
Equality will occur if and only if $r^2\pi = \frac{2000}{r}$,	1 point	

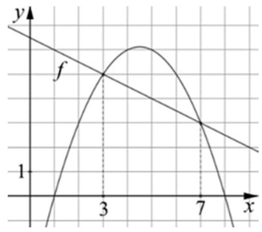
i.e. $r\left(=\sqrt[3]{\frac{2000}{\pi}}\right)\approx 8.6$ cm.	1 point	

8. c) Solution 2		
(Let r be the radius of the base circle and let m be the height of the cylinder, in dm-s.) The volume of the cylinder is $V=r^2\pi m=2$ dm^3,	1 point	
from which the base radius is: $r=\sqrt{\frac{2}{\pi m}}$.	1 point	
The surface area of the open-top cylinder is $A=r^2\pi+2r\pi m=\frac{2}{\pi m}\cdot\pi+2\cdot\sqrt{\frac{2}{\pi m}}\cdot\pi m=\frac{2}{m}+2\sqrt{2\pi m}$.	1 point	
The derivative of the function $f(m)=\frac{2}{m}+2\sqrt{2\pi m}$ ($m>0$) is $f'(m)=-\frac{2}{m^2}+\frac{\sqrt{2\pi}}{\sqrt{m}}$.	1 point*	
(Function f has its minimum where $f'(m)=0$) $\frac{2}{m^2}=\frac{\sqrt{2\pi}}{\sqrt{m}}$.	1 point*	
Square this: $\frac{4}{m^4}=\frac{2\pi}{m}$, and then $m=\sqrt[3]{\frac{2}{\pi}}$.	1 point*	
As $f''(m)=\frac{4}{m^3}-\sqrt{\frac{\pi}{2}}\cdot\frac{1}{\sqrt{m^3}}$, $f''\left(\sqrt[3]{\frac{2}{\pi}}\right)=2\pi-\frac{\pi}{2}>0$, this is a minimum, indeed.	1 point*	
The height of the vase should be $m=\sqrt[3]{\frac{2}{\pi}}\approx 0.86$ dm = 8.6 cm.	1 point	
Total:	**8 points**	

*The 4 points marked * may also be given for the following reasoning:*

Apply the relation between the arithmetic and geometric means for the terms $\frac{2}{m}$, $\sqrt{2\pi m}$, $\sqrt{2\pi m}$:	1 point	
$f(m)=\frac{2}{m}+\sqrt{2\pi m}+\sqrt{2\pi m}\geq 3\cdot\sqrt[3]{4\pi}\approx 6.97$.	1 point	
Equality will occur if and only if $\frac{2}{m}=\sqrt{2\pi m}$,	1 point	
i.e. $m=\sqrt[3]{\frac{2}{\pi}}$.	1 point	

<table>
<tr><td colspan="3">9. a)</td></tr>
<tr><td>Vertex C falls onto the arc k with centre A and radius AB.</td><td>1 point</td><td>Award this point if the correct reasoning is reflected only by the solution.</td></tr>
<tr><td>The length of radius AB is $\sqrt{9+16}=5$,</td><td>1 point</td><td></td></tr>
<tr><td>the equation of circle k is $(x-3)^2+y^2=25$.</td><td>1 point</td><td></td></tr>
<tr><td>(Possible locations for vertex C are the points of intersection of circle k and line e.)
From the equation of line e: $x = 8 - 2y$.
Substituted into the equation of circle k:
$(5-2y)^2+y^2=25$.
From this $y^2-4y=0$.</td><td>2 points</td><td>$y = 4-\frac{1}{2}x$
$(x-3)^2+\left(4-\frac{1}{2}x\right)^2=25$
$x^2-8x=0$</td></tr>
<tr><td>$y = 4$ or $y = 0$.</td><td>1 point</td><td>$x = 0$ or $x = 8$.</td></tr>
<tr><td>When $y = 4$, $x = 0$, so $C(0; 4)$ which is the same as point B, the three points do not form a triangle, and so this is not a correct solution.</td><td>1 point</td><td rowspan="2">
</td></tr>
<tr><td>When $y = 0$, $x = 8$, so $C(8; 0)$ which is correct. (Points A, B and C are three different, non-collinear points.)</td><td>1 point</td></tr>
<tr><td align="right">Total:</td><td>8 points</td><td></td></tr>
</table>

9. b)		
(About the first coordinate of the points of intersection of the parabola and the line:) $-\frac{1}{2}x^2+\frac{9}{2}x-4=-\frac{1}{2}x+\frac{13}{2}$.	1 point	
$x^2-10x+21=0$	1 point	
$x_1=3$ or $x_2=7$.	1 point	
On [3; 7] the graph of the parabola is above the graph of the line.	1 point	
The area is: $A=\int_3^7\left(\left(-\frac{1}{2}x^2+\frac{9}{2}x-4\right)-\left(-\frac{1}{2}x+\frac{13}{2}\right)\right)dx=$ $=\int_3^7\left(-\frac{1}{2}x^2+5x-\frac{21}{2}\right)dx=$	2 points*	
$=\left[-\frac{1}{6}x^3+\frac{5}{2}x^2-\frac{21}{2}x\right]_3^7=-\frac{49}{6}-\left(-\frac{81}{6}\right)=\frac{16}{3}$.	2 points*	
Total:	**8 points**	

*Note: The 4 points marked * may also be given for the following reasoning:*

The area below the graph of the parabolic arc: $\int_3^7\left(-\frac{1}{2}x^2+\frac{9}{2}x-4\right)dx=\left[-\frac{1}{6}x^3+\frac{9}{4}x^2-4x\right]_3^7=$ $=\frac{301}{12}-\frac{45}{12}=\frac{64}{3}$.	2 points	
The area of the right trapezium: $\frac{5+3}{2}\cdot 4=16$.	1 point	
The area of the region bounded by the parabola and the line is $\frac{64}{3}-16=\frac{16}{3}$.	1 point	