ÉRETTSÉGI VIZSGA • 2024. október 15.

MATEMATIKA ANGOL NYELVEN

KÖZÉPSZINTŰ ÍRÁSBELI VIZSGA

JAVÍTÁSI-ÉRTÉKELÉSI ÚTMUTATÓ

OKTATÁSI HIVATAL

Instructions to examiners

Formal requirements:

1. Mark the paper **legibly, in ink, different in colour** from that used by the candidate.
2. The first of the rectangles next to each problem shows the maximum attainable score on that problem. The **score** given by the examiner is to be entered **into the rectangle** next to this.
3. **If the solution is perfect,** enter maximum score and, with a checkmark, indicate that you have seen each item leading to the solution and consider them correct.
4. If the solution is incomplete or incorrect, please **mark the error** and also indicate the individual **partial scores**. It is also acceptable to indicate the points lost by the candidate if it makes grading the paper easier. After correcting the paper, it must be clear about every part of the solution whether that part is correct, incorrect or unnecessary.
5. Please, **use the following symbols** when correcting the paper:
   - correct calculation: *checkmark*
   - principal error: *double underline*
   - calculation error or other, not principal, error: *single underline*
   - correct calculation with erroneous initial data: *dashed checkmark or crossed checkmark*
   - incomplete reasoning, incomplete list, or other missing part: *missing part symbol*
   - unintelligible part: *question mark* and/or *wave*
6. Do not assess anything written **in pencil**, except for diagrams

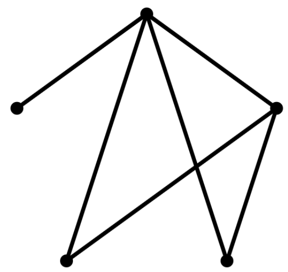
Assessment of content:

1. The answer key may contain more than one solution for some of the problems. If the **solution given by the candidate is different**, allocate the points by identifying parts of the solution equivalent to those given in the answer key.
2. Subtotals may be **further divided, unless stated otherwise in the answer key**. However, scores awarded must always be whole numbers.
3. If there is a **calculation error** or inaccuracy in the solution, take points off only for the part where the error occurs. If the reasoning remains correct and the error is carried forward while the nature of the problem remains unchanged, points for the rest of the solution must be awarded.
4. **In case of a principal error,** no points should be awarded at all for that section of the solution, not even for steps that are formally correct. (These logical sections of the solutions are separated by double lines in the answer key.) However, if the erroneous information obtained through principal error is carried forward to the next section or the next part of the problem, and it is used there correctly, the maximum score is due for that part, provided that the error has not changed the nature of the task to be completed.
5. Where the answer key shows a **remark** in brackets, the solution should be considered complete without that remark as well.

6. Deduct points for **missing units** only if the missing unit is part of the answer or a unit exchange (without parentheses).
7. If there are more than one different approach to a problem, **assess only the one indicated by the candidate**. Please, mark clearly which attempt was assessed.
8. **Do not give extra points** (i.e. more than the score due for the problem or part of problem).
9. The score given for the solution of a problem, or part of a problem, **may never be negative**.
10. **Do not take points off** for steps or calculations that contain errors but are not actually used by the candidate in the solution of the problem.
11. **The use of calculators** in the reasoning behind a particular solution **may be accepted without further mathematical explanation in case of the following operations**: addition, subtraction, multiplication, division, calculating powers and roots, $n!$, $\binom{n}{k}$, replacing the tables found in the 4-digit Data Booklet (sin, cos, tan, log, and their inverse functions), approximate values of the numbers π and e, finding the solutions of the standard quadratic equation. No further explanation is needed when the calculator is used to find the mean and the standard deviation, as long as the text of the question does not explicitly require the candidate to show detailed work. **In any other cases, results obtained through the use of a calculator are considered as unexplained and points for such results will not be awarded.**
12. Using **diagrams** to justify a solution (e.g. reading data off diagrams by way of direct measurement) is unacceptable.
13. **Probabilities** may also be given in percentage form (unless stated otherwise in the text of the problem).
14. If the text of the problem does not instruct the candidate to round their solution in a particular way, any solution **rounded reasonably and correctly** is acceptable even if it is different from the one given in the answer key.
15. **Assess only two out of the three problems in part B of Paper II.** The candidate was requested to indicate in the appropriate square the number of the problem not to be assessed and counted towards their total score. Should there be a solution to that problem, it does not need to be marked. However, if the candidate did not make a selection and neither is their choice clear from the paper, assume automatically that it is the last problem in the examination paper that is not to be assessed.

I.

1.		
$A \cap B = \{1; 2; 4\}$	1 point	
$A \cup B = \{1; 2; 3; 4; 8\}$	1 point	
$A \setminus B = \{3\}$	1 point	
Total:	**3 points**	

2.		
An appropriate graph, for example: 	2 points	*Non-simple graphs may also be accepted.*
Total:	**2 points**	

3.		
(22 + 17 – 30 =) 9 customers bought both types of bread.	2 points	
Total:	**2 points**	

4.		
$a_4 = \left(\frac{6+36}{2} = \right) 21$	2 points	$d = 5$
Total:	**2 points**	

5.		
D	2 points	*Not to be divided.*
Total:	**2 points**	

6.		
$\left(\frac{8 \cdot 5}{2} = \right) 20$	2 points	
Total:	**2 points**	

7.		
The median bisects the 5 cm side.	1 point	*Award this point if the correct reasoning is reflected only by the solution.*
According to the Pythagorean Theorem: $s^2 = 2{,}5^2 + 6^2$	1 point	
The length of the median is s = 6.5 cm.	1 point	
Total:	**3 points**	

8.		
The number will be divisible by 4 if it ends in 24, 32 or 52.	1 point	3524, 5324, 4532, 5432, 3452, 4352
There may be 2 different possibilities in all three cases.	1 point	
There will be a total 2 · 3 = 6 suitable solutions.	1 point	
Total:	**3 points**	

9.		
Béla has $\left(\frac{6500}{5}\cdot 4 =\right)$ points.	2 points	
Total:	**2 points**	

10.		
(As the mean of the grades is 4, the standard deviation is $\sqrt{\frac{4\cdot 1^2}{8}}$ =) $\sqrt{\frac{1}{2}} \approx 0.707$.	2 points	$\frac{\sqrt{2}}{2}$
Total:	**2 points**	

11.		
The common ratio is $10^3 = 1000$.	1 point	
The first term is $\left(\frac{a_8}{q^7} = \frac{10^{20}}{(10^3)^7} = \frac{10^{20}}{10^{21}} =\right) 10^{-1} = 0.1$.	2 points	
Total:	**3 points**	

12.		
There are 36 possible outcomes with two dice. (Total number of cases.)	1 point	
The sum of the numbers thrown may be 4 or 9. The possible cases are 1-3, 2-2, 3-1, 3-6, 4-5, 5-4, 6-3. There are 7 favourable cases.	2 points	
The probability is $\frac{7}{36} \approx 0.194$.	1 point	
Total:	**4 points**	

II. A

13. a)		
Apply a common denominator for all fractions in the equation: $\frac{5x+15}{20}+\frac{4x+4}{20}=-\frac{10x}{20}$.	2 points	*Award these points if the candidate correctly multiplies both sides of the equation by 20.*
Rearranged: $9x + 19 = -10x$.	1 point	
$x=-1$	1 point	
Check by substitution or reference to equivalent steps.	1 point	
Total:	**5 points**	

13. b)		
$b(60)=6\cdot 1.015^{60}\approx 14.7$ thousands	2 points	
Total:	**2 points**	

13. c)		
The equation $600=6\cdot 1.015^{p}$ is to be solved.	1 point	
$100=1.015^{p}$.	1 point	
$p=\log_{1.015}100\approx 309$ minutes	1 point	$p=\frac{\log 100}{\log 1.015}$
According to the model, the number of bacteria will reach 600 thousand in the 6th hour of the experiment.	1 point	
Total:	**4 points**	

14. a)		
$f(2.5) = ((2.5-3)^2 - 4 =) - 3.75$	2 points	
Total:	**2 points**	

14. b)		
The equation $(x-3)^2 - 4 = 0$ is to be solved.	1 point	
$x^2 - 6x + 5 = 0$	1 point	$\lvert x-3 \rvert = 2$
The zeros: $x = 1$ and $x = 5$.	2 points	
Total:	**4 points**	

14. c)		
$d_{PQ} = \sqrt{(6-2)^2 + (5-(-3))^2} = \sqrt{80} \approx 8.94$	2 points	
Total:	**2 points**	

14. d) Solution 1		
The gradient of the line is: $m = \left(\frac{5-(-3)}{6-2} = \right) 2$.	2 points	
Substitute the coordinates of, for example, point P into the equation $y = 2x + b$ we get $-3 = 2 \cdot 2 + b$,	1 point	*Substitute the appropriate values into the equation* $y - y_0 = m(x - x_0)$: $y + 3 = 2(x - 2)$ *or* $y - 5 = 2(x - 6)$.
$b = -7$, therefore $y = 2x - 7$.	1 point	
Total:	**4 points**	

14. d) Solution 2		
Look for the equation of the line in the form $y = mx + b$. The following system is to be solved: $\left.\begin{array}{r} -3 = m \cdot 2 + b \\ 5 = m \cdot 6 + b \end{array}\right\}$.	1 point	
(Subtract the first equation from the second and rearrange:) $m = 2$.	2 points	
Then $b = -7$, so the equation of the line is $y = 2x - 7$.	1 point	
Total:	**4 points**	

14. d) Solution 3		
A direction vector of the line is $\overrightarrow{PQ} = (6-2; 5-(-3)) = (4; 8)$.	2 points	*A normal vector of the line is* (8; –4).
The equation of the line is $8x - 4y = 28$.	2 points	
Total:	**4 points**	

Note: Award maximum score if the candidate gives the proper answer by correctly substituting into the formula about the equation of a line through two given points.

15. a)		
The area of the parallelogram is $A_{ABCD} = 5 \cdot 6 \cdot \sin 70° \approx$	1 point	*The height that belongs to side AB of the parallelogram is* $m = 5 \cdot \sin 70° \approx 4.7$ cm.
≈ 28.2 cm^2.	1 point	$A = 6 \cdot 4.7 = 28.2$ cm^2
Total:	**2 points**	

15. b)		
According to the Cosine Rule: $BD^2 = 5^2 + 6^2 - 2 \cdot 5 \cdot 6 \cdot \cos 70° \approx 40.48$.	1 point	
$BD \approx 6.36$ cm	1 point	
According to the Sine Rule: $\frac{\sin\beta}{\sin 70°} = \frac{5}{6.36}$,	1 point*	$\sin\beta = \frac{m}{BD} = \frac{4.7}{6.36}$
$\sin\beta \approx 0.7388$.	1 point*	
Here $\beta = 47.6°$ (the supplementary angle is 132.4°, which is impossible).	1 point	
The other unknown angle is: $\gamma = 180° - 70° - 47.6° = 62.4°$.	1 point	
Total:	**6 points**	

*The 2 points marked * may also be given for the following reasoning:*

$5^2 = 6^2 + 6.36^2 - 2 \cdot 6 \cdot 6.36 \cdot \cos\beta$	1 point	
$\cos\beta \approx 0.6741$	1 point	

15. c)		
The statement is false.	1 point	
An appropriate counter-example (a cyclic trapezium or a kite that is not a parallelogram).	1 point	
Total:	**2 points**	

15. d)		
The converse of the statement: If a quadrilateral has central symmetry then it also has a line symmetry.	1 point	
The statement is false.	1 point	
An appropriate counter-example (a parallelogram that is not a rhombus or rectangle).	1 point	
Total:	**3 points**	

II. B

16. a) **Solution 1**		
Let k be the price of 1 kg of granulated sugar, in forints, and let b the price of 1 kg of brown sugar. Then: $\left.\begin{array}{r} 4k+b=2600 \\ 3k+2b=3275 \end{array}\right\}$.	1 point	
From the first equation $b = 2600 - 4k$.	1 point	*Multiplying the first equation by 2,*
Substituting this into the second equation $3k + 5200 - 8k = 3275$.	1 point	*and subtracting the second equation from this* $5k = 1925$.
So $k = 385$ (1 kg of granulated sugar costs 385 Ft),	1 point	
and $b = (2600 - 4\cdot 385 =)\ 1060$ (1 kg of brown sugar costs 1060 Ft).	1 point	
Check against the original text.	1 point	
Total:	**6 points**	

16. a) **Solution 2**		
Emese is buying a total of 5 kg of sugar in both cases, the only difference is that she switches 1 kg of granulated sugar to 1 kg of brown sugar in the second case. From there it follows that 1 kg of brown sugar costs $3275 - 2600 = 675$ Ft more than 1 kg of granulated sugar.	2 points	
If 1 kg of granulated sugar costs k forints, then $4k + k + 675 = 2600$.	1 point	
So $k = 385$ (1 kg of granulated sugar costs 385 Ft),	1 point	
and $385 + 675 = 1060$ Ft is the price of 1 kg of brown sugar.	1 point	
Check against the original text.	1 point	
Total:	**6 points**	

16. b)		
1 ounce $= \frac{1}{35.3}$ kg ≈ 0.0283 kg $= 28.3$ g	1 point	1 ounce $= \frac{1000}{35.3}$ g ≈ 28.3 g
5 ounce = 141.5 g	1 point	
With the expected precision Emese must use 140 g of sugar.	1 point	*Do not award this point if the solution is not rounded or rounded incorrectly.*
Total:	**3 points**	

16. c)		
The number of packages must be a divisor of both 72 and 96. The greatest common divisor of the two must be found.	1 point	*Award this point if the correct reasoning is reflected only by the solution.*
$72 = 2^3 \cdot 3^2$	1 point	*Listing all divisors of* 72.
$96 = 2^5 \cdot 3$	1 point	*Listing all divisors of* 96.
The maximum number of Emese's packages is, therefore, (72; 96) = 24.	1 point	
Total:	**4 points**	

Note: Award 2 points if the candidate gives the correct answer without explaining why more than 24 packages are impossible to make.

16. d)		
The total number of cases is $\binom{25}{5} = 53\ 130$.	1 point	
The favourable number of cases is $\binom{10}{2}\binom{15}{3} = 20\ 475$.	2 points	
The probability: $\frac{20\,475}{53130} \approx 0.385$.	1 point	
Total:	**4 points**	

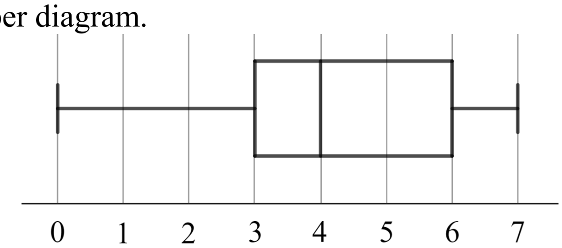
17. a) **Solution 1**		
The radius of the size 3 ball is 9 cm, the radius of the size 5 ball is 10.75 cm.	1 point	
The volume of the size 3 ball is $\frac{4}{3}\cdot 9^3 \cdot \pi \approx 3054$ cm^3,	1 point	
The volume of the size 5 ball is $\frac{4}{3}\cdot 10.75^3 \cdot \pi \approx 5204$ cm^3.	1 point	
The ratio of the volumes: $\frac{5204}{3054} \approx 1.7$,	1 point	
The volume of the size 5 ball is about 70% greater than the volume of the size 3 ball.	1 point	
Total:	**5 points**	

17. a) **Solution 2**		
Any two spheres are similar.	1 point	*Award these 2 points if the correct reasoning is reflected only by the solution.*
The ratio of the volumes of similar objects is equal to the cube of the ratio of similarity.	1 point	
The ratio of similarity in case of the size 3 and size 5 balls is $\frac{21.5}{18} \approx 1.194$,	1 point	
so the ratio of their volumes is $1.194^3 \approx 1.7$.	1 point	
The volume of the size 5 ball is about 70% greater than the volume of the size 3 ball.	1 point	
Total:	**5 points**	

17. b) **Solution 1**		
Each team played three games in the group stage of the tournament.	1 point	*Award this point if the correct reasoning is reflected only by the solution.*
The team that scored 7 points must have (won twice and) got tied once. The team that scored 5 points must have (won once and) got tied twice. The team that scored 4 points must have (won once, lost once and) got tied once. (The team that scored 0 points must have lost all three times.)	2 points	
(As there were 4 ties among the team scores in the group) two games must have ended in a tie.	1 point	
Total:	**4 points**	

17. b) Solution 2		
There were $\left(\frac{4\cdot 3}{2}=\right)6$ games played in each group.	1 point	
If there is no tie in the group then the two participating teams score a total 3 points together, and the total of the scores will be 18. Each tie decreases this score by 1 point (as the two teams together only score $1 + 1 = 2$ points, instead of 3).	1 point	
In the case given the total score is 16,	1 point	
meaning there must have been two ties.	1 point	
Total:	**4 points**	

17. c)		
The mean is $\left(\frac{2\cdot 0+3\cdot 1+4\cdot 3+10\cdot 4+2\cdot 5+8\cdot 6+3\cdot 7}{32}=\right)4.1875.$	2 points	
Total:	**2 points**	

17. d)		
minimum: 0; lower quartile: 3; median: 4; upper quartile: 6; maximum: 7	4 points	*Award 1 point for the correct minimum and maximum, and 1 point each for the other three measures, if correct.*
Proper diagram. 	2 points	
Total:	**6 points**	

minimum	lower quartile	median	upper quartile	maximum
0	3	4	6	7

18. a)		
Observing 9 flashes in 12 seconds would mean 45 flashes in 60 seconds,	1 point	
which corresponds to the level 1 storm signal.	1 point	
Total:	**2 points**	

18. b)		
When there is a level 1 signal in the middle basin there might be any level in the other two.	1 point	
This is $3 \cdot 3 = 9$ possible options.	1 point	
When the level in the middle basin is standby or level 2 then there are two possible levels for each of the other two basins.	1 point	
This means $2 \cdot 2 \cdot 2 = 8$ more possible options.	1 point	
The total number of possible options is then $(9 + 8 =)$ 17 for the whole lake Balaton.	1 point	
Total:	**5 points**	

Note: Award maximum score if the candidate gives the correct answer using a logically ordered list of all possibilities.

western	0					1							2				
middle	0		1			0		1			2		1			2	
eastern	0	1	0	1	2	0	1	0	1	2	1	2	0	1	2	1	2

18. c)		
The radius of the base circle of the truncated cone is $\frac{11000}{2\pi} \approx 1751$ m.	1 point	
The volume of the truncated cone: $\frac{330 \cdot \pi}{3}(1751^2 + 1751 \cdot 600 + 600^2) \approx$	1 point	
$\approx 1\,547\,000\,000\ \text{m}^3 =$	1 point	
$= 1.547\ \text{km}^3$.	1 point	
The statement is, therefore, true.	1 point	
Total:	**5 points**	

18. d)		
The amounts (in hectolitres) of wine produced in each consecutive year form the first ten terms of a geometric sequence with a common ratio of 1.05 and a sum of 1000 for the first ten terms.	1 point	*Award this point if the correct reasoning is reflected only by the solution.*
$a_1 \cdot \frac{1.05^{10} - 1}{1.05 - 1} = 1000$	1 point	
So $a_1 \approx 79.5$ hectolitres for the first year.	1 point	
In the tenth year it will be $a_{10} = 79.5 \cdot 1.05^9 \approx 123{,}3$ hektolitres.	2 points	
Total:	**5 points**	