This document discuss the following problem: given the 3D locations of a UAV and a ground station, and the gesture of the UAV, how to compute the optimal array responses of the UAV receiver and the ground station transmitter.

Supposing the 3D positions of a UAV and a ground station respectively are $\mathbf{s}_r \in \mathbb{R}^{3\times 1}$ and $\mathbf{s}_t \in \mathbb{R}^{3\times 1}$, and the mmWave communication is line-of-sight (LoS). The relative UAV position is

$$\Delta\mathbf{s} = \mathbf{s}_r - \mathbf{s}_t = [\Delta x, \Delta y, \Delta z]^{\mathrm{T}}. \tag{1}$$

Considering the UAV as the receiver and the ground station as the transmitter, the angle of departure (AoD) and the zenith of departure (ZoD) respectively are

$$\phi_{\text{AoD}} = \arctan(\frac{\Delta y}{\Delta x}), \tag{2}$$

$$\theta_{\text{ZoD}} = \arctan(\frac{\sqrt{\Delta x^2 + \Delta y^2}}{\Delta z}). \tag{3}$$

According to the LoS assumption and supposing the ground station as the global coordination system (GCS), the angle of arrival (AoA) and the zenith of arrival (ZoA) respectively are

$$\phi_{\text{AoA}} = \pi + \phi_{\text{AoD}}, \tag{4}$$

$$\theta_{\text{ZoA}} = \pi - \theta_{\text{ZoD}}. \tag{5}$$

The ground station is fixed, meanwhile the UAV rotates and its gesture can be changing. The rotation angles along the x, y, z-axis respectively are pitch, roll, yaw, denoted as α, β, γ. Then, in the local coordination system (LCS) of the UAV, the angle of arrival (AoA) and the zenith of arrival (ZoA) respectively are ϕ'_{AoA} and θ'_{ZoA}.

$$\mathbf{R}_\alpha = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) \\ 0 & \sin(\alpha) & \cos(\alpha) \end{bmatrix}. \tag{6}$$

$$\mathbf{R}_\beta = \begin{bmatrix} \cos(\beta) & 0 & \sin(\alpha) \\ 0 & 1 & 0 \\ -\sin(\alpha) & 0 & \cos(\alpha) \end{bmatrix}. \tag{7}$$

$$\mathbf{R}_\gamma = \begin{bmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{8}$$

The rotation equation is

$$\mathbf{R}(\alpha, \beta, \gamma) = \mathbf{R}_\gamma \mathbf{R}_\beta \mathbf{R}_\alpha. \tag{9}$$

The AoA and ZoA in the LCS respectively are

$$\phi'_{\text{AoA}} = \arccos\left([0,0,1]^\mathsf{T} \mathbf{R}(\alpha,\beta,\gamma) \boldsymbol{\rho}(\phi_{\text{AoA}}, \theta_{\text{ZoA}})\right), \tag{10}$$

$$\theta'_{\text{ZoA}} = \angle\left([1,\jmath,0]^\mathsf{T} \mathbf{R}(\alpha,\beta,\gamma)\right). \tag{11}$$

where

$$\boldsymbol{\rho}(\phi,\theta) = [\sin(\theta)\cos(\phi), \sin(\theta)\sin(\phi), \cos(\theta)]^\mathsf{T}. \tag{12}$$

The AoA ϕ'_{AoA} and ZoA θ'_{ZoA} are the LoS direction of the UAV receiver, and the AoD ϕ_{AoD} and ZoD θ_{ZoD} are the LoS direction of the ground station transmitter. Given the azimuth and zenith angles ϕ and θ, the optimal planer array response is given as follows

$$\boldsymbol{\psi}(\phi,\theta) = \boldsymbol{a}_{\text{xy}}(\phi,\theta) \otimes \boldsymbol{a}_{\text{z}}(\theta) \tag{13}$$

where

$$\boldsymbol{a}_{\text{xy}}(\phi,\theta) = \frac{1}{\sqrt{N_\phi}}[1, e^{\jmath\pi\sin\phi\sin\theta}, \cdots, e^{\jmath\pi(N_\phi-1)\sin\phi\sin\theta}]^\mathsf{T}, \tag{14}$$

$$\boldsymbol{a}_{\text{z}}(\theta) = \frac{1}{\sqrt{N_\theta}}[1, e^{\jmath\pi\cos\theta}, \cdots, e^{\jmath\pi(N_\theta-1)\cos\theta}]^\mathsf{T} \tag{15}$$

where N_ϕ and N_θ respectively are the azimuth and zenith antenna number, and $N = N_\phi N_\theta$.