Formlenes forutsetninger og gyldighetsområde antas å være kjent. Det antas også at symbolenes betydning framgår av konteksten og ligningene de framkommer i.

Kraft og bevegelse

Bevegelsesligninger (konstant akselerasjon):

$$\vec{v} = \vec{v}_0 + \vec{a}\,t \qquad \vec{s} = \frac{1}{2}(\vec{v}_0 + \vec{v})\,t$$

$$\vec{s} = \vec{v}_0\,t + \frac{1}{2}\vec{a}\,t^2 \qquad 2\,\vec{a}\,\vec{s} = \vec{v}^2 - \vec{v}_0^2$$

Newtons lover (i et inertialsystem):

N. 1. lov: $\sum \vec{F} = 0 \Leftrightarrow \vec{v} = 0$ eller $\vec{v} =$ konst.,

N. 2. lov: $\sum \vec{F} = m\vec{a}$

N. 3. lov: $\vec{F}_{\mathrm{AB}} = -\vec{F}_{\mathrm{BA}}$

Polarkoordinater og bevegelse:

$$x = r\cos\theta\,, \quad y = r\sin\theta$$

$$\vec{e}_{\mathrm{r}} = \cos\theta\,\vec{i} + \sin\theta\,\vec{j}\,, \quad \vec{e}_\theta = -\sin\theta\,\vec{i} + \cos\theta\,\vec{j}$$

Vinkelfart: $\omega = \dfrac{d\theta}{dt}$

Vinkelakselerasjon: $\alpha = \dfrac{d\omega}{dt}$

$$\vec{v} = \frac{dr}{dt}\,\vec{e}_{\mathrm{r}} + \omega r \vec{e}_\theta$$

$$\vec{a} = \left(\frac{d^2r}{dt^2} - \omega^2 r\right)\vec{e}_{\mathrm{r}} + \left(\alpha r + 2\omega\frac{dr}{dt}\right)\vec{e}_\theta$$

N. 2. lov: $\sum F_r = m\left(\dfrac{d^2r}{dt^2} - \omega^2 r\right)$

$$\sum F_\theta = m\left(\alpha r + 2\omega\frac{dr}{dt}\right)$$

Noen kraftmodeller:

Friksjon (kinetisk): $R = \mu N$

Tyngde: $\vec{G} = m\vec{g}\,, \quad g = 9.81\mathrm{m/s^2}$

Fjær: $F = -kx$

Viskøs dempning: $\vec{F} = -b\vec{v}$

Coulomb–kraften: $\vec{F}_{\mathrm{C}} = -k_{\mathrm{e}}\dfrac{qQ}{r^2}\,\vec{e}_{\mathrm{r}}$

$$k_{\mathrm{e}} = 8.99 \cdot 10^9\ \mathrm{Nm^2/C^2}$$

Mekanisk arbeid og energi

Arbeid:

$$W = \int_{\vec{s}} \vec{F} \cdot d\vec{s}$$

Ved rettlinjet bevegelse og konstant kraft:

$$W = \vec{F} \cdot \vec{s} = Fs\cos\theta$$

Kinetisk energi:

$$E_{\mathrm{k}} = \frac{1}{2}mv^2$$

Arbeid-energi teoremet:

$$W_{\mathrm{tot}} = \Delta E_{\mathrm{k}} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

Potensiell energi:

Når kraften er konservativ:

$$E_p(\mathbf{b}) - E_p(\mathbf{a}) = -\int_{\mathbf{a}}^{\mathbf{b}} \vec{F} \cdot d\vec{s}$$

Tyngdefeltet: $E_{\mathrm{p}} = mgh$

Elastisk fjær: $E_{\mathrm{p}} = \dfrac{1}{2}kx^2$

Bevaring av mekanisk energi:

Når kun konservative krefter utfører arbeid:
$E_{\mathrm{tot}} = E_{\mathrm{k}} + E_{\mathrm{p}} =$ konstant
eller
$E_{\mathrm{k1}} + E_{\mathrm{p1}} = E_{\mathrm{k2}} + E_{\mathrm{p2}}$

Elektrisitet og magnetisme

Strøm:

$$I = \frac{dQ}{dt}$$

I defineres som positiv når positive ladninger går i (vilkårlig) valgt positiv retning.

Elektrisk felt:

$$\vec{E} = \frac{\vec{F}}{Q}$$

Når $\vec{E}$ er konservativt:

$$\oint \vec{E} \cdot d\vec{s} = 0$$

Elektrisk potensial: $U = \dfrac{E_p}{Q}$

Kapasitans: $C = \dfrac{Q}{U_b - U_a}$

Faradays induksjonslov:

$$\mathcal{E} = -\frac{d\Phi}{dt}$$

Selvfluks og selvinduktans:
$\Phi = L \cdot I$

Svingninger

Enkel harmonisk oscillator:

Bev. lign.: $\frac{d^2x}{dt^2} + \omega^2 x = 0$

Løsning: $x(t) = x_m \cos(\omega t + \phi)$,

$$x_m = \sqrt{x_0^2 + (v_0/\omega)^2}$$

$$\phi = \tan^{-1}(-v_0/(\omega x_0)) - \omega t_0$$

Periode og frekvens: $T = 2\pi/\omega$

$f = 1/T$

Kloss-fjærsystem: $\omega = \sqrt{k/m}$

Elektrisk svingekrets: $\omega = 1/\sqrt{LC}$

Bølger

Bølgeligningen:

$$\frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2}$$

Harmonisk bølge:

$$y(x,t) = y_m \sin(kx \pm \omega t - \phi)$$

$$\lambda = 2\pi/k\,,\ T = 2\pi/\omega\,,\ f = 1/T\,,\ v = \omega/k = f \cdot \lambda$$

Eksempel på interferens av to harmoniske bølger (med samme y_m, λ og v):

$$y(x,t) = 2y_m \cos\frac{\phi_2 - \phi_1}{2} \sin\left(kx - \omega t - \frac{\phi_1 + \phi_2}{2}\right)$$

Eksempel på stående bølge:

$$y(x,t) = A \sin kx \cos \omega t$$

$$k = n\pi/L\,, \quad n = 1, 2, 3, \ldots$$

Varmeledning og temperatur

Fouriers lov:

Vektorform: $\vec{j} = -K\nabla T$

Én dimensjon: $j = -K\frac{dT}{dx}$

Varmeledningsligningen i én dimensjon:

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial x^2}$$

Eksempel på løsning (tynn plate):

$$T(x,t) = T_0 \sin\left(\frac{\pi x}{L}\right) e^{-\frac{\kappa \pi^2 t}{L^2}}$$

Differensialligninger og numeriske algoritmer

Ordinære differensialligninger

Eulers metode:

$x_{n+1} = x_n + f\left(t_n, x_n\right) \Delta t\,,$
når $f(t,x)$ er kjent funksjon.

Eulers midtpunktsmetode:

$$x_{n+1} = x_n + f\left[t_n + \frac{\Delta t}{2}, x_n + f\left(t_n, x_n\right) \frac{\Delta t}{2}\right] \Delta t\,,$$

når $f(t,x)$ er kjent funksjon.

Midtpunktsmetoden for 2. ordens diff. lign.:

$$\frac{d^2x}{dt^2} + p(t)\frac{dx}{dt} + q(t)x(t) = r(t)$$

$$\Updownarrow$$

$$\begin{cases} \frac{dx}{dt} = f(t) \\ \frac{df}{dt} = -p(t)f(t) - q(t)x(t) + r(t) \end{cases}$$

$$\Downarrow$$

$$\begin{cases} x_{n+1} = x_n + f_{n+\frac{1}{2}} \cdot \Delta t \\ f_{n+\frac{1}{2}} = f_n + \left(-p_n f_n - q_n x_n + r_n\right) \cdot \frac{\Delta t}{2} \end{cases}$$

Partielle differensialligninger

Varmeledningsligningen (eksplisitt metode):

$$T_i^{n+1} = \alpha\left(T_{i+1}^n + T_{i-1}^n\right) + (1 - 2\alpha)\,T_i^n$$

Stabilitetskrav : $\alpha = \dfrac{\kappa \Delta t}{\Delta x^2} \leq \dfrac{1}{2}$

USEFUL MATHEMATICAL RELATIONS

Algebra

$a^{-x} = \dfrac{1}{a^x}$ $\quad a^{(x+y)} = a^x a^y$ $\quad a^{(x-y)} = \dfrac{a^x}{a^y}$

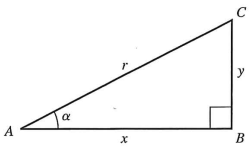
Logarithms: If $\log a = x$, then $a = 10^x$. $\quad \log a + \log b = \log(ab)$ $\quad \log a - \log b = \log(a/b)$ $\quad \log(a^n) = n \log a$

If $\ln a = x$, then $a = e^x$. $\quad \ln a + \ln b = \ln(ab)$ $\quad \ln a - \ln b = \ln(a/b)$ $\quad \ln(a^n) = n \ln a$

Quadratic formula: If $ax^2 + bx + c = 0$, $\quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Trigonometry

In the right triangle ABC, $x^2 + y^2 = r^2$.



Definitions of the trigonometric functions:

$\sin\alpha = y/r$ $\quad \cos\alpha = x/r$ $\quad \tan\alpha = y/x$

Identities:

$\sin^2\alpha + \cos^2\alpha = 1$ $\qquad \tan\alpha = \dfrac{\sin\alpha}{\cos\alpha}$

$\sin 2\alpha = 2\sin\alpha\cos\alpha$ $\qquad \cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha$

$\sin\frac{1}{2}\alpha = \sqrt{\dfrac{1-\cos\alpha}{2}}$ $\qquad \cos\frac{1}{2}\alpha = \sqrt{\dfrac{1+\cos\alpha}{2}}$

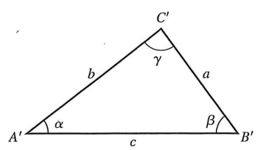
$\sin(-\alpha) = -\sin\alpha$ $\qquad \sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\cos(-\alpha) = \cos\alpha$ $\qquad \cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\sin(\alpha \pm \pi/2) = \pm\cos\alpha$ $\qquad \sin\alpha + \sin\beta = 2\sin\frac{1}{2}(\alpha+\beta)\cos\frac{1}{2}(\alpha-\beta)$

$\cos(\alpha \pm \pi/2) = \mp\sin\alpha$ $\qquad \cos\alpha + \cos\beta = 2\cos\frac{1}{2}(\alpha+\beta)\cos\frac{1}{2}(\alpha-\beta)$

For *any* triangle $A'B'C'$ (not necessarily a right triangle) with sides a, b, and c and angles α, β, and γ:



Law of sines: $\dfrac{\sin\alpha}{a} = \dfrac{\sin\beta}{b} = \dfrac{\sin\gamma}{c}$

Law of cosines: $c^2 = a^2 + b^2 - 2ab\cos\gamma$

Geometry

Circumference of circle of radius r:	$C = 2\pi r$	Surface area of sphere of radius r:	$A = 4\pi r^2$
Area of circle of radius r:	$A = \pi r^2$	Volume of cylinder of radius r and height h:	$V = \pi r^2 h$
Volume of sphere of radius r:	$V = 4\pi r^3/3$		

Calculus

Derivatives:

$\dfrac{d}{dx}x^n = nx^{n-1}$ $\qquad \dfrac{d}{dx}\ln ax = \dfrac{1}{x}$ $\qquad \dfrac{d}{dx}e^{ax} = ae^{ax}$

$\dfrac{d}{dx}\sin ax = a\cos ax$ $\qquad \dfrac{d}{dx}\cos ax = -a\sin ax$

Integrals:

$\displaystyle\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad (n \neq -1)$ $\qquad \displaystyle\int \frac{dx}{x} = \ln x$ $\qquad \displaystyle\int e^{ax}\,dx = \frac{1}{a}e^{ax}$

$\displaystyle\int \sin ax\,dx = -\frac{1}{a}\cos ax$ $\qquad \displaystyle\int \cos ax\,dx = \frac{1}{a}\sin ax$ $\qquad \displaystyle\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a}$

$\displaystyle\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln\left(x + \sqrt{x^2 + a^2}\right)$ $\qquad \displaystyle\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\arctan\frac{x}{a}$ $\qquad \displaystyle\int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{1}{a^2}\frac{x}{\sqrt{x^2 + a^2}}$

$\displaystyle\int \frac{x\,dx}{(x^2 + a^2)^{3/2}} = -\frac{1}{\sqrt{x^2 + a^2}}$

Length
1 m = 100 cm = 1000 mm = 10^6 μm = 10^9 nm
1 km = 1000 m = 0.6214 mi
1 m = 3.281 ft = 39.37 in.
1 cm = 0.3937 in.
1 in. = 2.540 cm
1 ft = 30.48 cm
1 yd = 91.44 cm
1 mi = 5280 ft = 1.609 km
1 Å = 10^{-10} m = 10^{-8} cm = 10^{-1} nm
1 nautical mile = 6080 ft
1 light-year = 9.461 × 10^{15} m

Area
1 cm^2 = 0.155 $in.^2$
1 m^2 = 10^4 cm^2 = 10.76 ft^2
1 $in.^2$ = 6.452 cm^2
1 ft^2 = 144 $in.^2$ = 0.0929 m^2

Volume
1 liter = 1000 cm^3 = 10^{-3} m^3 = 0.03531 ft^3 = 61.02 $in.^3$
1 ft^3 = 0.02832 m^3 = 28.32 liters = 7.477 gallons
1 gallon = 3.788 liters

Time
1 min = 60 s
1 h = 3600 s
1 d = 86,400 s
1 y = 365.24 d = 3.156 × 10^7 s

Angle
1 rad = 57.30° = 180°/π
1° = 0.01745 rad = π/180 rad
1 revolution = 360° = 2π rad
1 rev/min (rpm) = 0.1047 rad/s

Speed
1 m/s = 3.281 ft/s
1 ft/s = 0.3048 m/s
1 mi/min = 60 mi/h = 88 ft/s
1 km/h = 0.2778 m/s = 0.6214 mi/h
1 mi/h = 1.466 ft/s = 0.4470 m/s = 1.609 km/h
1 furlong/fortnight = 1.662 × 10^{-4} m/s

Acceleration
1 m/s^2 = 100 cm/s^2 = 3.281 ft/s^2
1 cm/s^2 = 0.01 m/s^2 = 0.03281 ft/s^2
1 ft/s^2 = 0.3048 m/s^2 = 30.48 cm/s^2
1 mi/h · s = 1.467 ft/s^2

Mass
1 kg = 10^3 g = 0.0685 slug
1 g = 6.85 × 10^{-5} slug
1 slug = 14.59 kg
1 u = 1.661 × 10^{-27} kg
1 kg has a weight of 2.205 lb when g = 9.80 m/s^2

Force
1 N = 10^5 dyn = 0.2248 lb
1 lb = 4.448 N = 4.448 × 10^5 dyn

Pressure
1 Pa = 1 N/m^2 = 1.450 × 10^{-4} $lb/in.^2$ = 0.0209 lb/ft^2
1 bar = 10^5 Pa
1 $lb/in.^2$ = 6895 Pa
1 lb/ft^2 = 47.88 Pa
1 atm = 1.013 × 10^5 Pa = 1.013 bar
= 14.7 $lb/in.^2$ = 2117 lb/ft^2
1 mm Hg = 1 torr = 133.3 Pa

Energy
1 J = 10^7 ergs = 0.239 cal
1 cal = 4.186 J (based on 15° calorie)
1 ft · lb = 1.356 J
1 Btu = 1055 J = 252 cal = 778 ft · lb
1 eV = 1.602 × 10^{-19} J
1 kWh = 3.600 × 10^6 J

Mass-Energy Equivalence
1 kg ↔ 8.988 × 10^{16} J
1 u ↔ 931.5 MeV
1 eV ↔ 1.074 × 10^{-9} u

Power
1 W = 1 J/s
1 hp = 746 W = 550 ft · lb/s
1 Btu/h = 0.293 W

Quantity	Name of unit	Symbol	Equivalent units
	SI base units		
length	meter	m	
mass	kilogram	kg	
time	second	s	
electric current	ampere	A	
thermodynamic temperature	kelvin	K	
amount of substance	mole	mol	
luminous intensity	candela	cd	
	SI derived units		**Equivalent units**
area	square meter	m^2	
volume	cubic meter	m^3	
frequency	hertz	Hz	s^{-1}
mass density (density)	kilogram per cubic meter	kg/m^3	
speed, velocity	meter per second	m/s	
angular velocity	radian per second	rad/s	
acceleration	meter per second squared	m/s^2	
angular acceleration	radian per second squared	rad/s^2	
force	newton	N	$kg \cdot m/s^2$
pressure (mechanical stress)	pascal	Pa	N/m^2
kinematic viscosity	square meter per second	m^2/s	
dynamic viscosity	newton-second per square meter	$N \cdot s/m^2$	
work, energy, quantity of heat	joule	J	N · m
power	watt	W	J/s
quantity of electricity	coulomb	C	A · s
potential difference, electromotive force	volt	V	J/C, W/A
electric field strength	volt per meter	V/m	N/C
electrical resistance	ohm	Ω	V/A
capacitance	farad	F	A · s/V
magnetic flux	weber	Wb	V · s
inductance	henry	H	V · s/A
magnetic flux density	tesla	T	Wb/m^2
magnetic field strength	ampere per meter	A/m	
magnetomotive force	ampere	A	
luminous flux	lumen	lm	cd · sr
luminance	candela per square meter	cd/m^2	
illuminance	lux	lx	lm/m^2
wave number	1 per meter	m^{-1}	
entropy	joule per kelvin	J/K	
specific heat capacity	joule per kilogram-kelvin	J/kg · K	
thermal conductivity	watt per meter-kelvin	W/m · K	