Automata Theory (CSE4058-01)
Homework # 01 Solution

1.12 Observe that $D \subseteq \mathtt{b}^*\mathtt{a}^*$ because D doesn't contain strings that have ab as a substring. Hence D is generated by the regular expression $(\mathtt{aa})^*\mathtt{b}(\mathtt{bb})^*$. From this description, finding the DFA for D is more easily done.

1.22 **b.** $/\#(\#^*(\mathtt{a} \cup \mathtt{b}) \cup /)^*\#^+/$

1.30 The error is that $s = 0^p1^p$ *can* be pumped. Let $s = xyz$, where $x = 0$, $y = 0$ and $z = 0^{p-2}1^p$. The conditions are satisfied because

i) for any $i \geq 0$, $xy^iz = 00^i0^{p-2}1^p$ is in 0^*1^*.
ii) $|y| = 1 > 0$, and
iii) $|xy| = 2 \leq p$.

1.31 We construct a DFA which alternately simulates the DFAs for A and B, one step at a time. The new DFA keeps track of which DFA is being simulated. Let $M_1 = (Q_1, \Sigma, \delta_1, s_1, F_1)$ and $M_2 = (Q_2, \Sigma, \delta_2, s_2, F_2)$ be DFAs for A and B. We construct the following DFA $M = (Q, \Sigma, \delta, s_0, F)$ for the perfect shuffle of A and B.

i) $Q = Q_1 \times Q_2 \times \{1, 2\}$.
ii) For $q_1 \in Q_1$, $q_2 \in Q_2$, $b \in \{1, 2\}$, and $a \in \Sigma$:

$$\delta((q_1, q_2, b), a) = \begin{cases} (\delta_1(q_1, a), q_2, 2) & b = 1 \\ (q_1, \delta_1(q_2, a), 1) & b = 2. \end{cases}$$

iii) $s_0 = (s_1, s_2, 1)$.
iv) $F = \{(q_1, q_2, 1) | \; q_1 \in F_1 \text{ and } q_2 \in F_2\}$.

1.40 Assume language E is regular. Use the pumping lemma to a get a pumping length p satisfying the conditions of the pumping lemma. Set $s = \left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]^p \left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]^p$. Obviously, $s \in E$ and $|s| \geq p$. Thus, the pumping lemma implies that the string s can be written as xyz with $x = \left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]^a$, $y = \left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]^b$, $z = \left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]^c \left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]^p$, where $b \geq 1$ and $a + b + c = p$. However, the string $s' = xy^0z = \left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]^{a+c} \left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]^p \notin E$, since $a + c < p$. That contradicts the pumping lemma. Thus E is not regular.

1.52 One short solution is to observe that $\overline{Y} \cap 1^*\#1^* = \{1^n\#1^n | \; n \geq 0\}$. This language is clearly not regular, as may be shown using a straightforward application of the pumping lemma. However, if Y were regular, this language would be regular, too, because the class of regular languages is closed under intersection and complementation. Hence Y isn't regular.

Alternatively, we can show Y isn't regular directly using the pumping lemma. Assume to the contrary that Y is regular and obtain its pumping length p. Let $s = 1^{p!}\#1^{2p!}$. The pumping lemma says that $s = xyz$ satisfying the three conditions. By condition 3, y appears among the left-hand 1s. Let $l = |y|$ and let $k = (p!/l)$. Observe that k is an integer, because l must be a divisor of $p!$. Therefore, adding k copies of y to s will add $p!$ additional 1s to the left-hand 1s. Hence, $xy^{1+k}z = 1^{2p!}\#1^{2p!}$ which isn't a member of Y. But condition 1 of the pumping lemma states that this string is a member of Y, a contradiction.