Homework # 2: Problems

2.4 Give context-free grammars that generate the following languages. In all parts, the alphabet Σ is $\{0,1\}$.

- [A]**a.** $\{w \mid w \text{ contains at least three 1s}\}$
- **b.** $\{w \mid w \text{ starts and ends with the same symbol}\}$
- **c.** $\{w \mid \text{the length of } w \text{ is odd}\}$
- [A]**d.** $\{w \mid \text{the length of } w \text{ is odd and its middle symbol is a 0}\}$
- **e.** $\{w \mid w = w^{\mathcal{R}}, \text{ that is, } w \text{ is a palindrome}\}$
- **f.** The empty set

2.9 Give a context-free grammar that generates the language

$$A = \{\mathtt{a}^i\mathtt{b}^j\mathtt{c}^k \mid i = j \text{ or } j = k \text{ where } i, j, k \geq 0\}.$$

Is your grammar ambiguous? Why or why not?

2.18 Consider the following CFG G:

$$\begin{aligned} S &\to SS \mid T \\ T &\to \mathtt{a}T\mathtt{b} \mid \mathtt{ab} \end{aligned}$$

Describe $L(G)$ and show that G is ambiguous. Give an unambiguous grammar H where $L(H) = L(G)$ and sketch a proof that H is unambiguous.

2.43 Let B be the language of all palindromes over $\{0,1\}$ containing equal numbers of 0s and 1s. Show that B is not context free.

2.44 Let $\Sigma = \{1, 2, 3, 4\}$ and $C = \{w \in \Sigma^* \mid \text{in } w, \text{ the number of 1s equals the number of 2s, and the number of 3s equals the number of 4s}\}$. Show that C is not context free.

2.56 If A and B are languages, define $A \diamond B = \{xy \mid x \in A \text{ and } y \in B \text{ and } |x| = |y|\}$. Show that if A and B are regular languages, then $A \diamond B$ is a CFL.