Automata Theory (CSE4058-01)
Homework # 01

1.12 Let $D = \{w|\ w$ contains an even number of a's and an odd number of b's and does not contain the substring ab$\}$. Give a DFA with five states that recognizes D and a regular expression that generates D. (Suggestion: Describe D more simply.)

1.22 In certain programming languages, comments appear between delimiters such as /# and #/. Let C be the language of all valid delimited comment strings. A member of C must begin with /# and end with #/ but have no intervening #/. For simplicity, assume that the alphabet for C is $\Sigma = \{\mathtt{a}, \mathtt{b}, /, \#\}$.

a. Give a DFA that recognizes C.

b. Give a regular expression that generates C.

1.30 Describe the error in the following "proof" that 0^*1^* is not a regular language. (An error must exist because 0^*1^* *is* regular.) The proof is by contradiction. Assume that 0^*1^* is regular. Let p be the pumping length for 0^*1^* given by the pumping lemma. Choose s to be the string 0^p1^p. You know that s is a member of 0^*1^*, but Example 1.73 shows that s cannot be pumped. Thus you have a contradiction. So 0^*1^* is not regular.

1.31 For languages A and B, let the ***perfect shuffle*** of A and B be the language

$$\{w|\ w = a_1b_1 \cdots a_kb_k,\ \text{where } a_1 \cdots a_k \in A \text{ and } b_1 \cdots b_k \in B,\ \text{each } a_i, b_i \in \Sigma\}.$$

Show that the class of regular languages is closed under perfect shuffle.

1.40 Let Σ_2 be the same as in Problem 1.38. Consider the top and bottom rows to be strings of 0s and 1s, and let

$$E = \{w \in \Sigma_2^*|\ \text{the bottom row of } w \text{ is the reverse of the top row of } w\}.$$

Show that E is not regular.

1.52 Let $\Sigma = \{1, \#\}$ and let

$$Y = \{w|\ w = x_1\#x_2\#\cdots\#x_k \text{ for } k \geq 0,\ \text{each } x_i \in 1^*,\ \text{and } x_i \neq x_j \text{ for } i \neq j\}.$$

Prove that Y is not regular.

For reference

1.38 Let

$$\Sigma_2 = \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}.$$

Here, Σ_2 contains all columns of 0s and 1s of height two. A string of symbols in Σ_2 gives two rows of 0s and 1s. Consider each row to be a binary number and let

$$C = \{w \in \Sigma_2^* |\ \text{the bottom row of } w \text{ is three times the top row}\}.$$

For example, $\begin{bmatrix} 0 \\ 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \end{bmatrix} \in C$, but $\begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} \notin C$. Show that C is regular.