실습1장 수학과 물리

Angles

An angle is defined as the measure of divergence of two rays that share the same origin. See Figure 1.7.

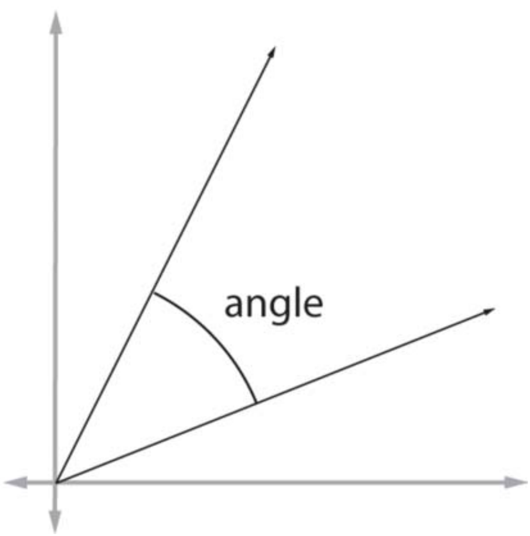

Figure 1.7. An angle

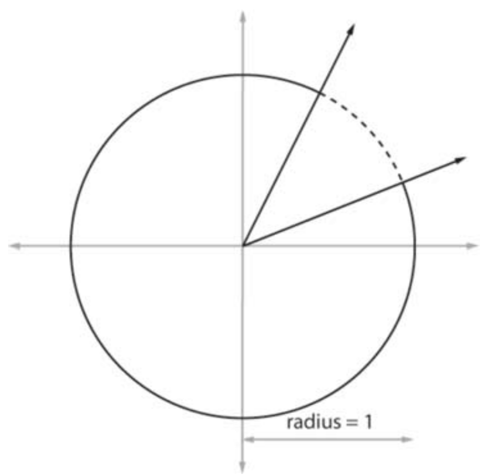

Figure 1.8. The length of the dotted line is the angle in radians between the two rays.

$$perimeter = 2\pi r = 2\pi(1) = 2\pi = num\ radians \qquad (1.40)$$

Therefore, there are 2 π radians in every circle.

360° = 2 π rads

Dividing both sides by 360 we get:

1° = 2 π /360 rads

삼각형

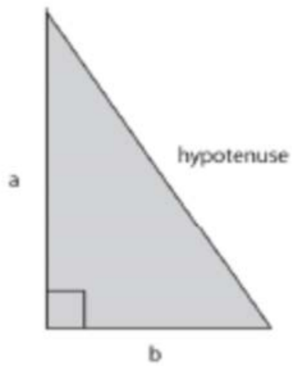

Figure 1.10

If the hypotenuse is denoted as h, the Pythagorean theorem can be written as:

$$h^2 = a^2 + b^2 \qquad (1.41)$$

Taking the square root of both sides gives:

$$h = \sqrt{a^2 + b^2} \qquad (1.42)$$

삼각함수

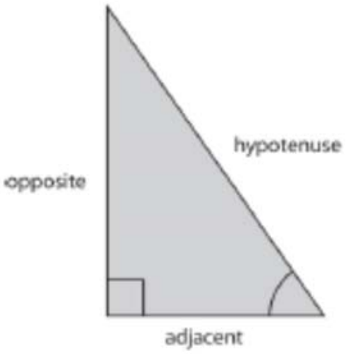

Figure 1.12. Names of the sides of a triangle

$$\sin(\theta) = \frac{opposite}{hypotenuse}$$

$$\cos(\theta) = \frac{adjacent}{hypotenuse}$$

$$\tan(\theta) = \frac{opposite}{adjacent}$$

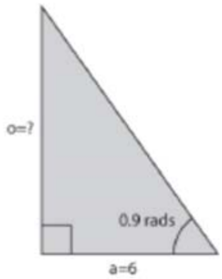

Figure 1.13

$$o = aTan(\theta)$$

$$o = 6Tan(0.9)$$
$$= 7.56$$

Vector

You have learned that a point on the Cartesian plane can be expressed as two numbers, just like this:

$$P = (x, y) \tag{1.55}$$

A 2D vector looks almost the same when written down:

$$\mathbf{v} = (x, y) \tag{1.56}$$

However, although similar, a vector represents two qualities: direction *and* magnitude. The right-hand side of Figure 1.16 shows the vector (9, 6) situated at the origin.

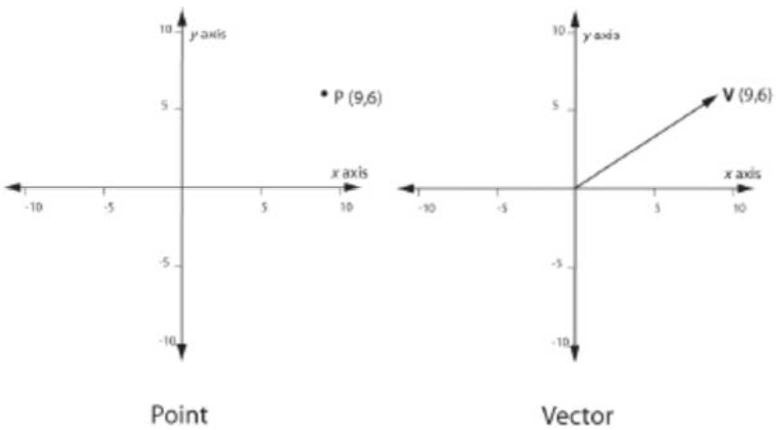

Figure 1.16. A point, P, and a vector, V

Normalizing Vectors

When a vector is normalized, it retains its direction but its magnitude is recalculated so that it is of unit length (a length of 1). To do this you divide each component of the vector by the magnitude of the vector. Mathematicians write the formula like this:

$$\mathbf{N} = \frac{\mathbf{v}}{|\mathbf{v}|} \tag{1.61}$$

Therefore, to normalize the vector (4, 5) you would do this:

$$\begin{aligned} new\ x &= 4/6.403 = 0.62 \\ new\ y &= 5/6.403 = 0.78 \end{aligned} \tag{1.62}$$

벡터 분해(Resolving Vector)

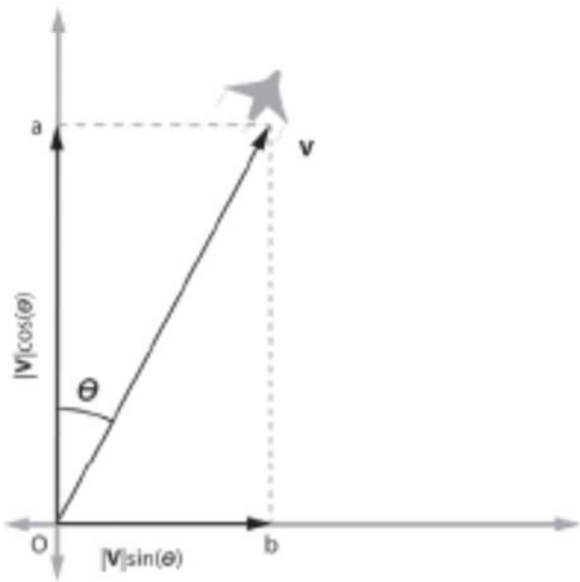

Figure 1.20

$$\cos(\theta) = \frac{adjacent}{hypotenuse} = \frac{Oa}{|\mathbf{v}|}$$

Rearranged, this gives:

$$Oa = |\mathbf{v}|Cos(\theta) = y\ component$$

$$\sin(\theta) = \frac{opposite}{hypotenuse} = \frac{Ob}{|\mathbf{v}|}$$

Giving:

$$Ob = |\mathbf{v}|\sin(\theta) = x\ component$$

내적 (Dot Product)

$$\mathbf{u} \bullet \mathbf{v} = \mathbf{u}_x \mathbf{v}_x + \mathbf{u}_y \mathbf{v}_y$$

$$\mathbf{u} \bullet \mathbf{v} = |\mathbf{u}||\mathbf{v}|\cos(\theta)$$

Rearranging we get:

$$\cos(\theta) = \frac{\mathbf{u} \bullet \mathbf{v}}{|\mathbf{u}||\mathbf{v}|}$$

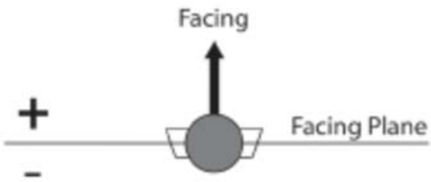

Figure 1.21

Using the dot product it's easy to determine if an object is situated in front or behind the agent. The dot product of the agent's facing vector and the vector from the agent to the object will be positive if the object is forward of the facing plane of the agent and negative if it is behind.

$$\cos(\theta) = \frac{\mathbf{u} \bullet \mathbf{v}}{1 \times 1} = \mathbf{u} \bullet \mathbf{v} \qquad (1.70)$$

← 각각이 단위 벡터일때

Substituting in the equation from (1.67) for the right-hand side gives:

$$\cos(\theta) = \mathbf{u} \bullet \mathbf{v} = \mathbf{u}_x \mathbf{v}_x + \mathbf{u}_y \mathbf{v}_y \qquad (1.71)$$

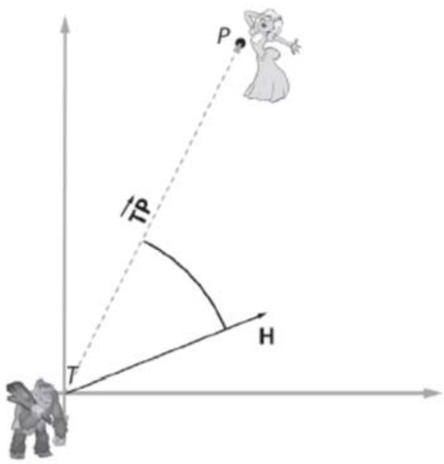

Figure 1.22

트롤이 곤봉을 던져서 공주를 잡으려면 현재 H 방향에서 얼마나 방향 전환을 해야 하는가?

$$\mathbf{N_p} = \frac{\mathbf{P}}{|\mathbf{P}|}$$

The dot product can now be used to determine the angle.

$$\cos(\theta) = \mathbf{N_p} \bullet \mathbf{H}$$

So

$$\theta = \cos^{-1}(\mathbf{N_p} \bullet \mathbf{H})$$

$$\theta = \cos^{-1}(\mathbf{N}_{TP} \bullet \mathbf{H})$$

$$\theta = \cos^{-1}((0.62 \times 1) + (0.78 \times 0))$$

$$\theta = \cos^{-1}(0.62)$$

$$\theta = 0.902 \text{ radians}$$

트롤이 원점 T(0, 0)에 위치해 있고 H(1, 0)를 향하고 있고, 공주는 P(4, 5)
N_p(0.62, 0.78)

Local Space and World Space

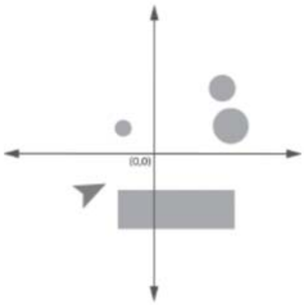

Figure 1.23. Some obstacles and a vehicle shown in world space

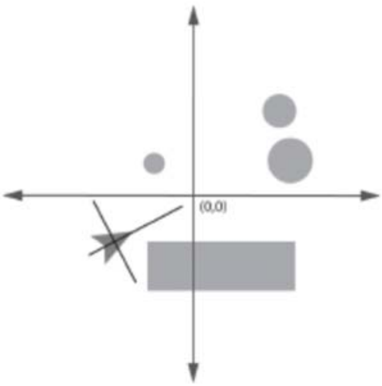

Figure 1.24. The vehicle's local coordinate system

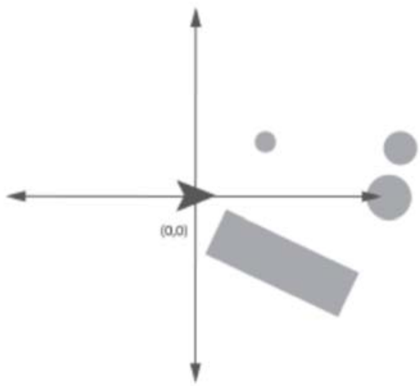

Figure 1.25. Objects transformed into the vehicle's local space

Physics

Velocity

Velocity is a vector quantity (a quantity that has magnitude *and* direction) that expresses *the rate of change of distance over time*. The standard unit of measurement of velocity is meters per second, abbreviated to m/s. This can be expressed mathematically as:

$$v = \frac{\Delta x}{\Delta t} \qquad (1.75)$$

Acceleration

Acceleration is a vector quantity that expresses *the rate of change of velocity over time* and is measured in meters per second per second, written as m/s^2. Acceleration can be expressed mathematically as:

$$a = \frac{\Delta v}{\Delta t} \qquad (1.80)$$

가속도

For example, if a car starts from rest and accelerates at 2 m/s^2, then every second, 2 m/s is added to its velocity. See Table 1.1.

Table 1.1

Time(s)	Velocity(m/s)
0	0
1	2
2	4
3	6
4	8
5	10

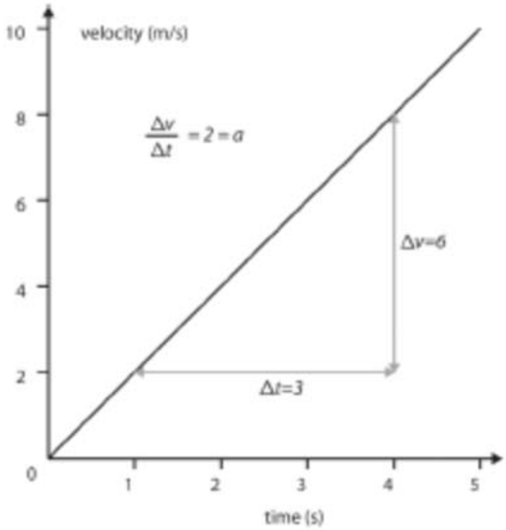

Figure 1.26. The velocity of the car plotted against time

$$v = at + u$$

가속도

$$\Delta x = u\left(\frac{v-u}{a}\right) + \frac{1}{2}a\left(\frac{v-u}{a}\right)^2$$

$$v^2 = u^2 + 2a\Delta x \tag{1.91}$$

This equation is extremely useful. For example, we can use it to determine how fast a ball dropped from the top of the Empire State Building will be traveling when it hits the ground (assuming no air resistance due to wind or velocity). The acceleration of a falling object is due to the force exerted upon it by the Earth's gravitational field and is equivalent to approximately 9.8 m/s^2. The starting velocity of the ball is 0 and the height of the Empire State Building is 381 m. Putting these values into the equation gives:

$$\begin{aligned} v^2 &= 0^2 + 2 \times 9.8 \times 381 \\ v &= \sqrt{7467.6} \\ v &= 86.41\ \text{m/s} \end{aligned} \tag{1.92}$$

Force

$$F = ma$$

$$a = \frac{F}{m}$$

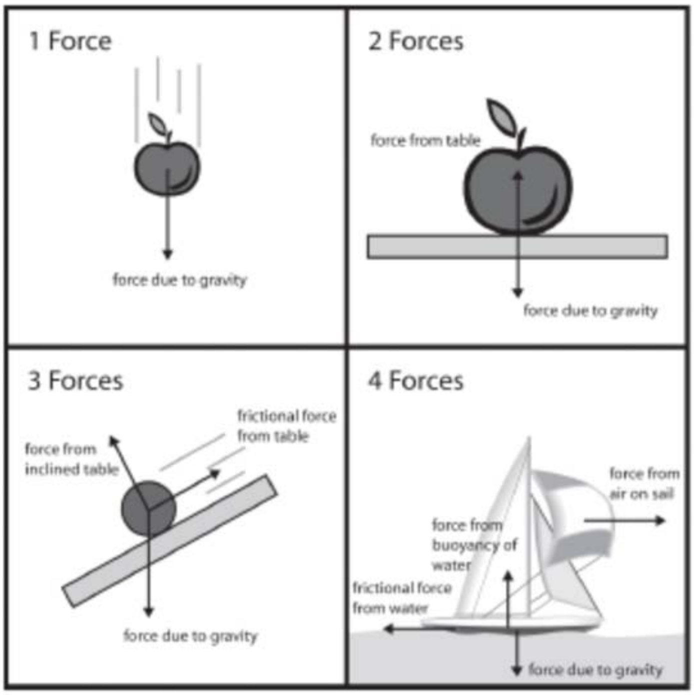

Figure 1.33. From left to right and top to bottom: a falling apple, an apple resting on a table, a ball rolling down an inclined table, and a yacht sailing on water