# DBS Extra Credit — Question 4 Part B

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# 1 Question 4 part (b)

You are given the following set F of functional dependencies for a relation R(A,B,C,D,E).

$$F = \{ \tag{1}$$
$$AB \longrightarrow CE, \tag{2}$$
$$A \longrightarrow D, \tag{3}$$
$$BE \longrightarrow D, \tag{4}$$
$$CDE \longrightarrow AB, \tag{5}$$
$$BC \longrightarrow DE\} \tag{6}$$

## 1.1 b (Problem)

If we remove $AB \longrightarrow E$ from $\mathcal{F}$ does it change the closure of $\mathcal{F}$ (i.e. $\mathcal{F}^+$?

To show this check wether $\mathcal{F} - \{AB \longrightarrow E\}$ implies $AB \longrightarrow E$. You can either use inference rules or demonstrate $\{AB\}^+$ in $\mathcal{F} - \{AB \longrightarrow E\{$.

## 1.2 b (Solution)

F becomes

$$F' = \{ \tag{7}$$
$$AB \longrightarrow C, \tag{8}$$
$$A \longrightarrow D, \tag{9}$$
$$BE \longrightarrow D, \tag{10}$$
$$CDE \longrightarrow AB, \tag{11}$$
$$BC \longrightarrow DE\} \tag{12}$$

The most simple way to demonstrate if the removal changes $\mathcal{F}^+$ is to demonstrate $\{AB\}^+$.

| **Output** | **Rule** |
|---|---|
| $AB$ | beginning |
| $ABC$ | $AB \longrightarrow C$ |
| $ABCD$ | $A \longrightarrow D$ |
| $ABCDE$ | $BC \longrightarrow DE.$ |

As the closure is $ABCDE$ (it encloses E) the removal does **not** change $\mathcal{F}^+$. The same input AB can achieve the same output CE. Even if the rule $AB \longrightarrow E$ is removed.

Additionally this property can be demonstrated via inference rules demonstrated below.

| **Output** | **Rule** |
|---|---|
| $AB \longrightarrow C$ (1) | Given |
| $BC \longrightarrow DE$ (2) | Given |
| $AB \longrightarrow BC$ (3) | Reflexivity of B in (1) |
| $AB \longrightarrow DE$ (4) | Transitivity of (3) and (2). |
| $AB \longrightarrow E$ (5) | Decomposition of (4). |

We can see in (5) that using inference rules $AB \longrightarrow E$ can be recovered from $\mathcal{F} - \{AB \longrightarrow E\{$. This means that removing $AB \longrightarrow E$ does **not** change $\mathcal{F}^+$.