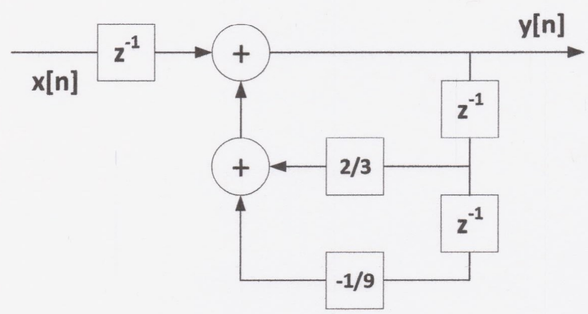
Midterm Exam 3
May 7, 2020

Signals and Systems ELC321
Dept. of Electrical and Computer Engineering
The College of New Jersey
Prof. MacDonald

Instructions:

1. This exam must be completed, scanned, and sent electronically by 12:00 midnight today, May 7.
2. The exam is open book / open notes.
3. Print your name and problem number on each page.
4. You may use the sheets provided or ones of your choice.

Problem 1 (40 points). *A system described by the block diagram below is excited by an impulse at n = 0, and the output is sampled at 1 ms intervals.*



a) Determine the Difference Equation for the system.
b) Determine the system Transfer Function, H(z).
c) Determine the impulse response, h[n].
d) Determine fmax, which is the maximum frequency in Hz at which the system can be analyzed in order to avoid aliasing.
e) Find the Magnitude and Phase of the Frequency Response, H(ω), at (i) DC; (ii) ½fmax ; and (iii) fmax.
f) Compared to DC, what is the filter rejection at fmax, in dB.

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1) a) $y[n] = x[n-1] + \frac{2}{3}y[n-1] + \frac{-1}{9}y[n-2]$

b) $Y(z) = X(z)z^{-1} + \frac{2}{3}Y(z)z^{-1} - \frac{1}{9}Y(z)z^{-2}$

$$Y(z)\left(1 + \frac{1}{9z^2} - \frac{2}{3z}\right) = X(z)z^{-1}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^{-1}}{1 + \frac{1}{9z^2} - \frac{2}{3z}}\left(\frac{z^2}{z^2}\right) = \boxed{\frac{z}{z^2 - \frac{2}{3}z + \frac{1}{9}}}$$

c) $H(z) = \frac{z}{(z-\frac{1}{3})^2}$ Using the form: $na^n = \frac{az}{(z-a)^2}$

$$H(z) = 3 \cdot \frac{\frac{1}{3}z}{(z-\frac{1}{3})^2}$$

$$\boxed{h[n] = 3n\left(\tfrac{1}{3}\right)^n}$$

d) according to Shannon's Sampling Theorem, we must sample at a frequency $f_s \geq 2f_{max}$

given that we're sampling $T = 1ms$ $f_s = \frac{1}{0.001} = 1000Hz$

to maximize f_{max}, $f_s = 2f_{max}$ $f_{max} = \frac{1000}{2}Hz = \boxed{500Hz}$

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e) finding frequency response @ DC $\omega = 0 \rightarrow \Omega = 0$

$$H(1) = \frac{1}{1 - \frac{2}{3} + \frac{1}{9}} = \frac{1}{\frac{4}{9}} = \boxed{\frac{9}{4} \angle 0^\circ} \qquad e^{j\Omega} = 1$$

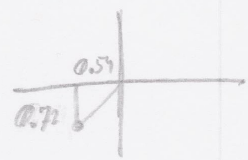
@ $\frac{1}{2} f_{max}$ $\quad \omega = \frac{2\pi}{2} f_{max} = \frac{2\pi f_s}{4} = \frac{\omega_s}{4}$

$$\Omega = \pi/2 \rightarrow e^{j\Omega} = j \qquad H(j) = \frac{j}{(-1) - \frac{2}{3}j + \frac{1}{9}}$$

$$H(j) = \frac{j}{-\frac{8}{9} - \frac{2}{3}j} = \frac{-1}{-\frac{8}{9}j + \frac{2}{3}} = \frac{1}{\frac{8j-6}{9}} = \frac{9}{8j-6}$$

$$H(j) = -0.54 - 0.72j$$

$$\boxed{H(j) = 0.9 \angle 233^\circ}$$



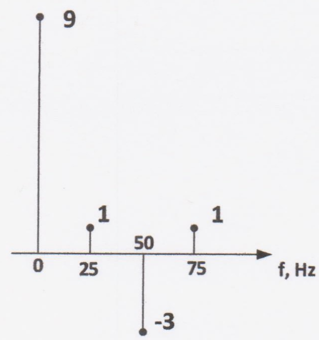
$$180 + \tan^{-1}\left(\frac{0.72}{0.54}\right)$$

@ f_{max} $\quad \omega = \frac{\omega_s}{2} \rightarrow \Omega = \pi \rightarrow e^{j\pi} = -1 \rightarrow H(-1)$

$$H(-1) = \frac{1}{1 + \frac{2}{3} + \frac{1}{9}} = \frac{1}{\frac{16}{9}} = \boxed{\frac{9}{16} \angle 0^\circ}$$

f) $20 \log_{10}\left(\frac{9/16}{9/4}\right) = \boxed{-12 \text{ dB}}$

Problem 2 (20 points). *A 4-point DFT sequence is shown in the figure below. Each sample is separated in frequency by 25 Hz, as indicated in the figure.*



a) Find the 4-sample time sequence from which this DFT was derived.
b) At what time interval were the samples taken?

Problem 3 (20 points). *A system has impulse response, h[n], and input, x[n], given below.*

$$h[n] = a^{n-2}u[n-2] \qquad x[n] = b^{n}u[n]$$

Determine a closed-form solution for the output, y[n], using the convolution sum.

$e^{j\pi/2} = j \qquad e^{j\frac{3\pi}{2}} = -j$

$e^{j\pi} = -1 \qquad e^{2\pi j} = 1$

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2) Inverse DFT: $x[k] = \frac{1}{N}\sum_{n=0}^{N-1} X[n] e^{j2\pi kn/N}$ $\quad k=0\ldots1\ldots N-1$

$X[n] = 9, 1, -3, 1 \qquad f_s = 25\text{ Hz} \qquad \text{Size} = 4$

$x[k] = \frac{1}{4}\sum_{n=0}^{3} X[n] e^{j2\pi kn/4}$

$X[k] = \frac{1}{4}x[0]e^{0} + \frac{1}{4}x[1]e^{j2\pi k/4} + \frac{1}{4}x[2]e^{j4\pi k/4} + \frac{1}{4}x[3]e^{j6\pi k/4}$

$X[k] = \frac{9}{4} + \frac{1}{4}e^{jk\frac{\pi}{2}} + \frac{-3}{4}e^{jk\pi} + \frac{1}{4}e^{jk\frac{3\pi}{2}}$

$x[0] = \frac{9}{4} + \frac{1}{4} - \frac{3}{4} + \frac{1}{4} = 2$

$x[1] = \frac{9}{4} + \frac{1}{4}e^{j\pi/2} - \frac{3}{4}e^{j\pi} + \frac{1}{4}e^{j\frac{3\pi}{2}} = 3$

$x[2] = \frac{9}{4} + \frac{1}{4}\underbrace{e^{j\pi}}_{-1} - \frac{3}{4}\underbrace{e^{j2\pi}}_{1} + \frac{1}{4}\underbrace{e^{j3\pi}}_{-1} = 1$

$x[3] = \frac{9}{4} + \cancel{\frac{1}{4}\underbrace{e^{j\frac{3\pi}{2}}}_{-j}} - \frac{3}{4}\underbrace{e^{j3\pi}}_{-1} \cancel{+\frac{1}{4}\underbrace{e^{j\frac{9\pi}{2}}}_{j}} = \frac{12}{4} = 3$

$\boxed{x[k] = 2, 3, 1, 3}$

b) $T = 1/f_s = 1/25 = \boxed{0.04\text{ sec}}$

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3) $h[n] = a^{n-2} u[n-2] \qquad x[n] = b^{n} u[n]$

$y[n] = h[n] * x[n]$

$y[n] = \sum_{i=-\infty}^{\infty} x[i]\, h[n-i]$

$y[n] = \underset{1}{\cancel{b^{2} a^{0}}} + b^{3} a + b^{4} a^{2} + b^{5} a^{3} + b^{6} a^{4} \ldots$

$y[n] = \sum_{i=0}^{n} b^{i+2} a^{i} \rightarrow \boxed{y[n] = \left(\frac{1-a^{n+1}}{1-a}\right)\left(\frac{1-b^{n+1}}{1-b}\right) b^{2}}$

Problem 4 (20 points). *A single-input / single-output system is defined by the state equations given below.*

$$\bar{x}[n+1]=\begin{bmatrix} 0 & 1 \\ -\frac{2}{3} & \frac{1}{3} \end{bmatrix}\bar{x}[n]+\begin{bmatrix} 0 \\ 1 \end{bmatrix}u[n]$$

$$y[n]=\begin{bmatrix} 1 & 1 \end{bmatrix}\bar{x}[n]+2u[n]$$

$$\bar{x}[0]=\begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Given that the input to the system is a unit step at n = 0, evaluate the following:

a) *Find y[n] for $0 \le n \le 4$*
b) *Determine whether the system is stable, and if so, what is the steady-state output?*

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4)

a) $y[n] = [1 \; 1]\,\bar{x}[n] + 2u[n]$

$y[0] = [1 \; 1]\begin{bmatrix}1\\2\end{bmatrix} + 2u[0]$

$y[0] = 1+2+2 = \boxed{5}$

$y[1] = [1 \; 1]\,\bar{x}[1] + 2$

$\bar{x}[1] = \begin{bmatrix}0 & 1\\ -2/3 & 1/3\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix} + \begin{bmatrix}0\\1\end{bmatrix}u[n]$

$\bar{x}[1] = \begin{bmatrix}2\\0\end{bmatrix} + \begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}2\\1\end{bmatrix}$

$y[1] = [1 \; 1]\begin{bmatrix}2\\1\end{bmatrix} + 2 = 2+1+2 = \boxed{5}$

$y[2] = [1 \; 1]\,x[2] + 2$

$\bar{x}[2] = \begin{bmatrix}0 & 1\\ -2/3 & 1/3\end{bmatrix}\begin{bmatrix}2\\1\end{bmatrix} + \begin{bmatrix}0\\1\end{bmatrix}$

$\bar{x}[2] = \begin{bmatrix}1\\-1\end{bmatrix} + \begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix}$

$y[2] = [1 \; 1]\begin{bmatrix}0\\1\end{bmatrix} + 2 = \boxed{3}$

$y[3] = [1 \; 1]\,\bar{x}[3] + 2$

$\bar{x}[3] = \begin{bmatrix}0 & 1\\ -2/3 & 1/3\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} + \begin{bmatrix}0\\1\end{bmatrix}$

$\bar{x}[3] = \begin{bmatrix}0\\-2/3\end{bmatrix} + \begin{bmatrix}0\\1\end{bmatrix}$

$\bar{x}[3] = \begin{bmatrix}0\\1/3\end{bmatrix}$

$y[3] = [1 \; 1]\begin{bmatrix}0\\1/3\end{bmatrix} + 2$

$y[3] = \boxed{7/3}$

$y[4] = [1 \; 1]\,\bar{x}[4] + 2$

$\bar{x}[4] = \begin{bmatrix}0 & 1\\ -2/3 & 1/3\end{bmatrix}\begin{bmatrix}0\\1/3\end{bmatrix} + \begin{bmatrix}0\\0\end{bmatrix}$

$\bar{x}[4] = \begin{bmatrix}1/3\\10/9\end{bmatrix}$

$y[4] = [1 \; 1]\begin{bmatrix}1/3\\10/9\end{bmatrix} + 2$

$y[4] = \frac{13}{9} + \frac{18}{9} = \boxed{\frac{31}{9}}$

$y[n] = 5, 5, 3, 7/3, \frac{31}{9} \ldots$

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b) The System is stable and the steady State output is 3.5.