TCNJ THE COLLEGE OF NEW JERSEY

School of Engineering

ELC321-02 **Spring 2020**

Assignment 4

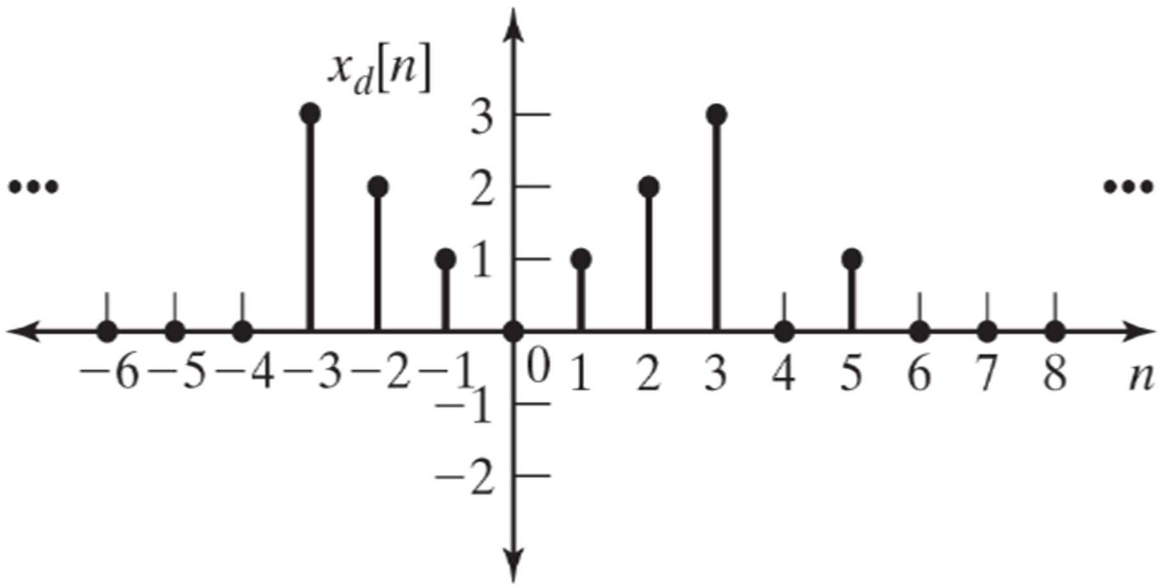
Date: April 14^{th}, 2020

Submitted by: Jeffrey Blanda

Instructor: John MacDonald

Problem 1. For the signal $x_d[n]$ of figure 9.3(d) plot

a) $x_d[2n]$
b) $x_d\left[-\frac{n}{2}\right]$
c) $x_d[-n]$
d) $x_d[2-n]$
e) $x_d[n-2]$
f) $x_d[-2-n]$



Prior to manipulating the discrete signal, reproducing it in MatLab yields the graph:

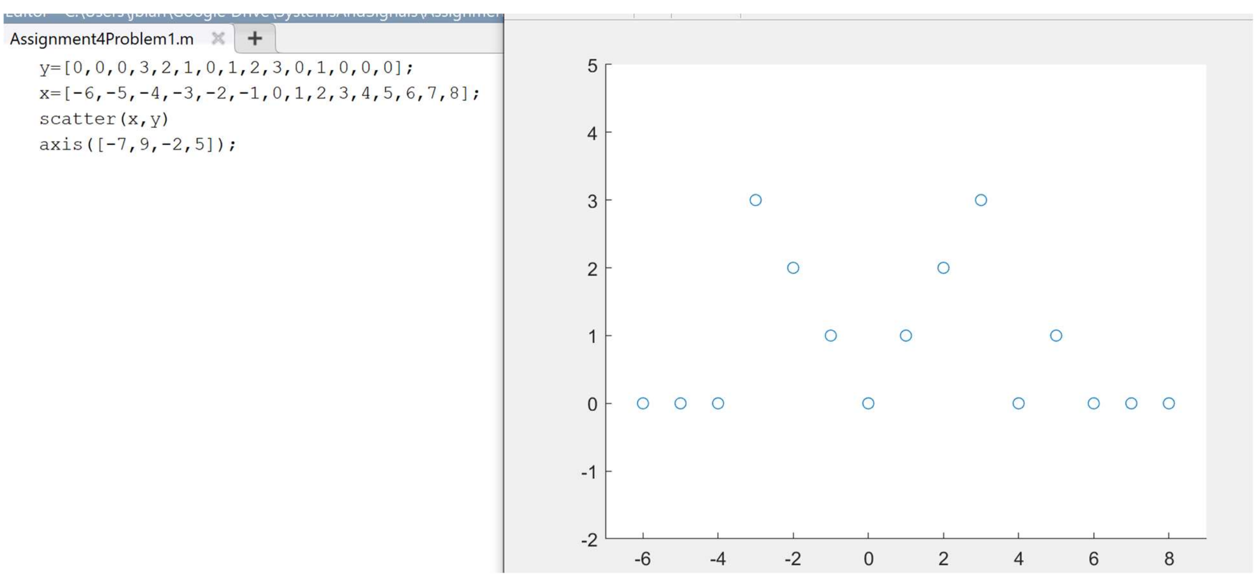


Figure 1: MatLab code and plot for the given discrete signal.

a) $x_d[2n]$ can be expressed by doubling the x axis of the signal, however since data is unavailable for every other point these can be expressed as blank, averages, or zeros. Here they will be left blank.

```
y=[0,0,0,3,2,1,0,1,2,3,0,1,0,0,0];
x=[-12,-10,-8,-6,-4,-2,0,2,4,6,8,10,12,14,16];
scatter(x,y)
axis([-14,18,-2,5]);
```

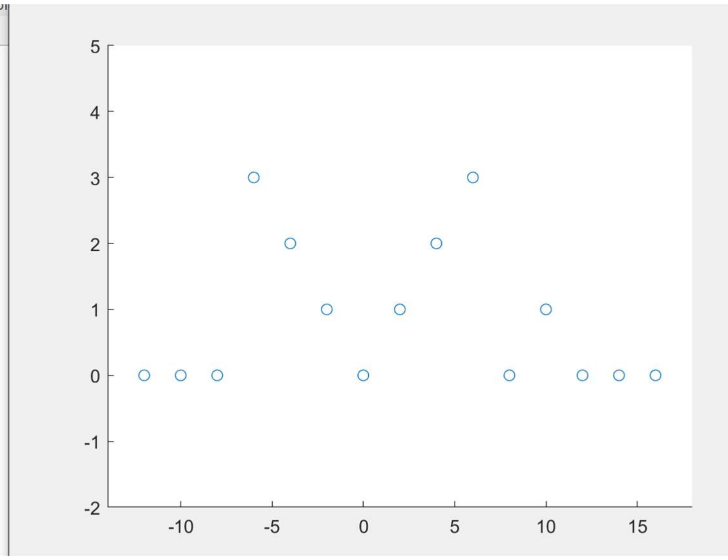


Figure 2. MatLab code and plot for $x_d[2n]$

b) $x_d\left[-\frac{n}{2}\right]$ can be represented as the inversion of the original signal with the x axis shrunk by ½, losing every other data point.

```
%y=[0,0,0,3,2,1,0,1,2,3,0,1,0,0,0];
y=[0,0,2,0,2,0,0,0];
x=[-3,-2,-1,0,1,2,3,4];
scatter(x,y)
axis([-7,9,-2,5]);
```

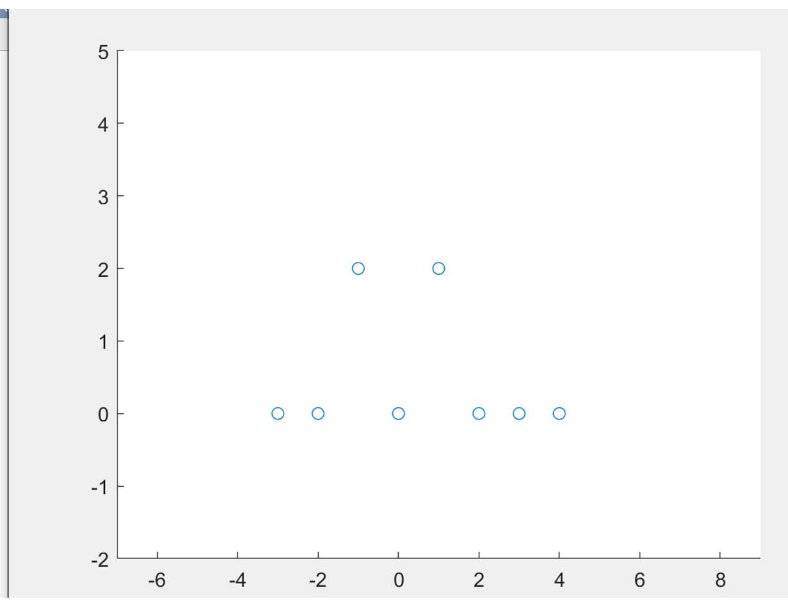


Figure 3. MatLab code and plot for $x_d\left[-\frac{n}{2}\right]$

c) $x_d[-n]$ is simply the original signal, inverted.

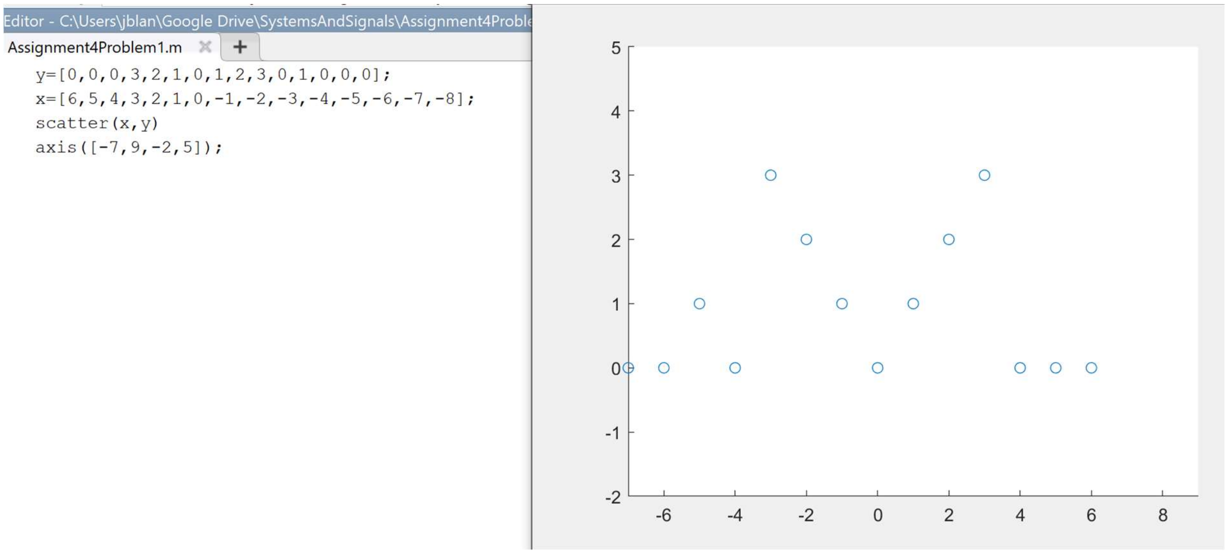


Figure 4. MatLab code and plot for $x_d[-n]$

d) $x_d[2-n]$ can be rewritten as $x_d[-(n-2)]$, or the original signal offset by two, then inverted.

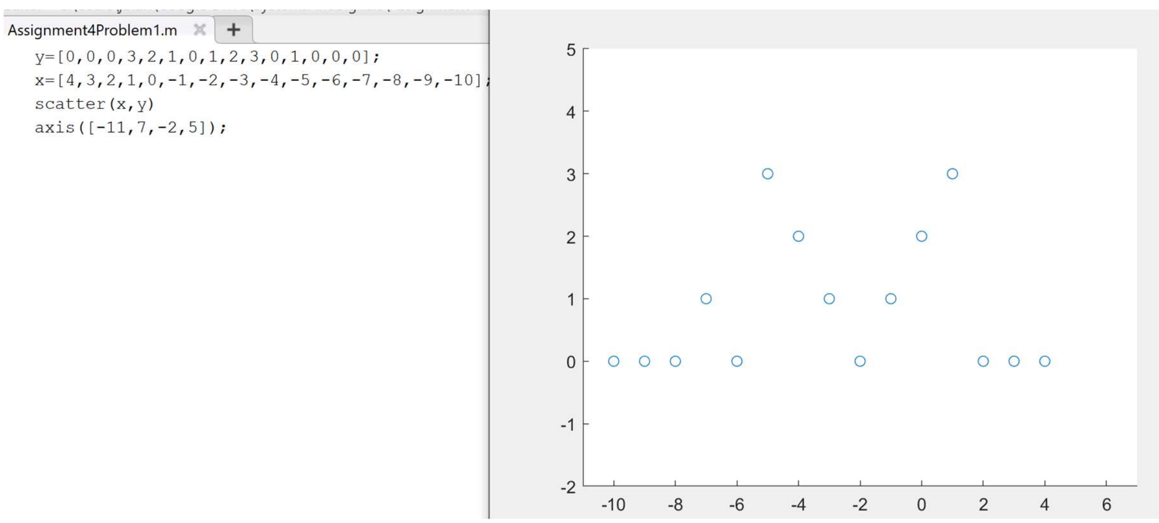


Figure 5. MatLab code and plot for $x_d[2-n]$

e) $x_d[n-2]$ is the original signal, offset by 2.

Assignment4Problem1.m

```
y=[0,0,0,3,2,1,0,1,2,3,0,1,0,0,0];
x=[-4,-3,-2,-1,0,1,2,3,4,5,6,7,8,9,10];
scatter(x,y)
axis([-7,11,-2,5]);
```

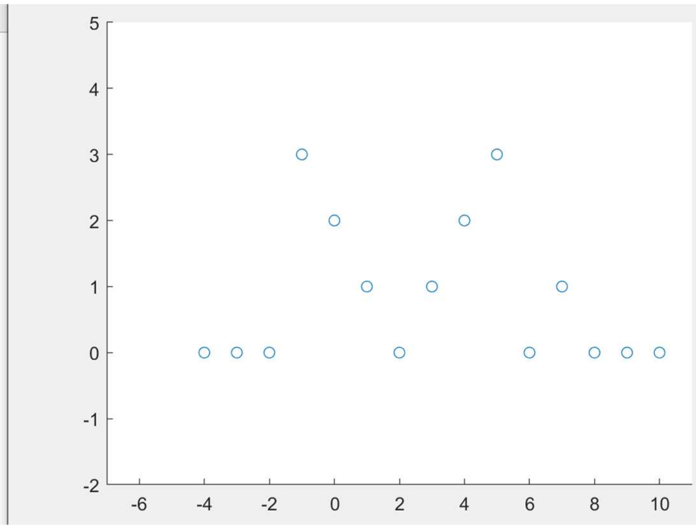


Figure 6. MatLab code and plot of $x_d[n-2]$

f) $x_d[-2-n]$ can also be written as $x_d[-(n+2)]$, or the signal shifted two to the left then inverted.

Assignment4Problem1.m

```
y=[0,0,0,3,2,1,0,1,2,3,0,1,0,0,0];
x=[6,5,4,3,2,1,0,-1,-2,-3,-4,-5,-6,-7,-8];
scatter(x,y)
axis([-9,7,-2,5]);
```

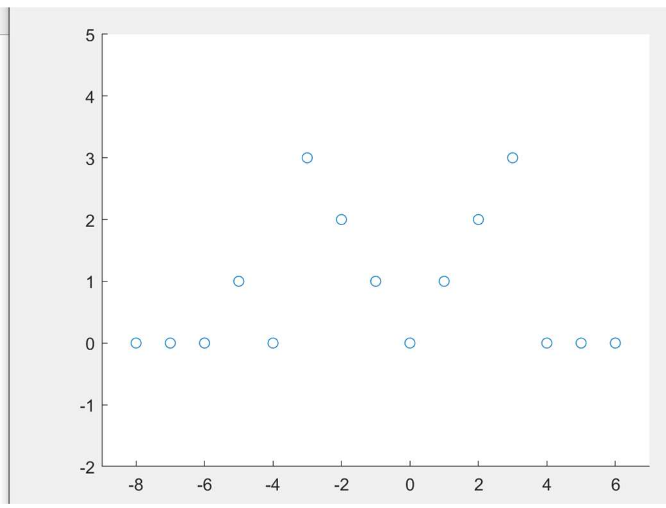


Figure 7. MatLab code and plot of $x_d[-2-n]$

Problem 2.

The signals in P9.4a are zero except as shown. Plot the following for $x_a[n]$

a) $x_a[-n]u[n]$
b) $x_a[n]u[-n]$
c) $x_a[n]u[n-2]$
d) $x_a[n]u[2-n]$
e) $x_a[n]\delta[n-1]$
f) $x_a[n](\delta[n]+\delta[n-2])$

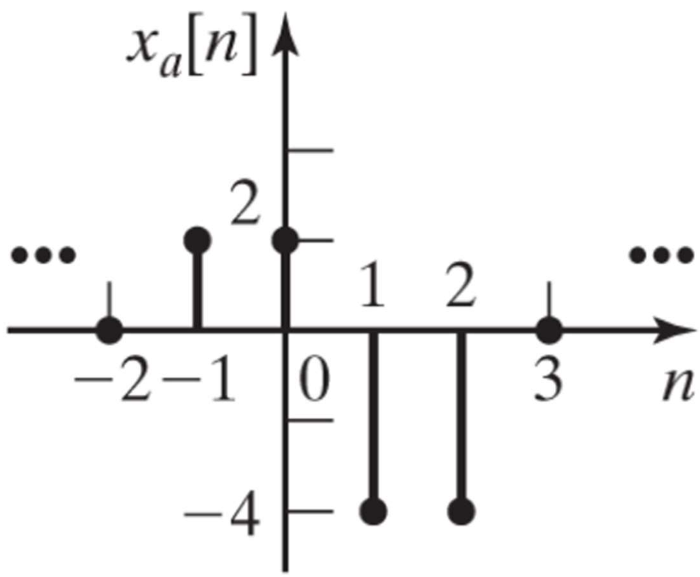


Figure P9.4a

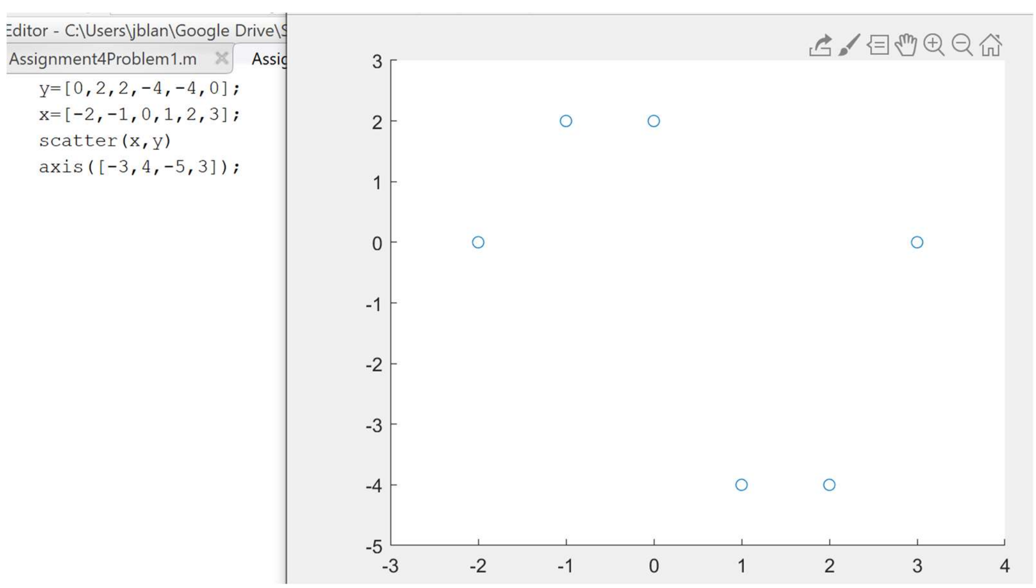


Figure 8. P9.4 recreated in MatLab

a) $x_a[-n]u[n]$

Since the unit step function is 0 except for $n \geq 0$ where the function is 1, the discrete signal can be plotted as the inverted signal at and past 0.

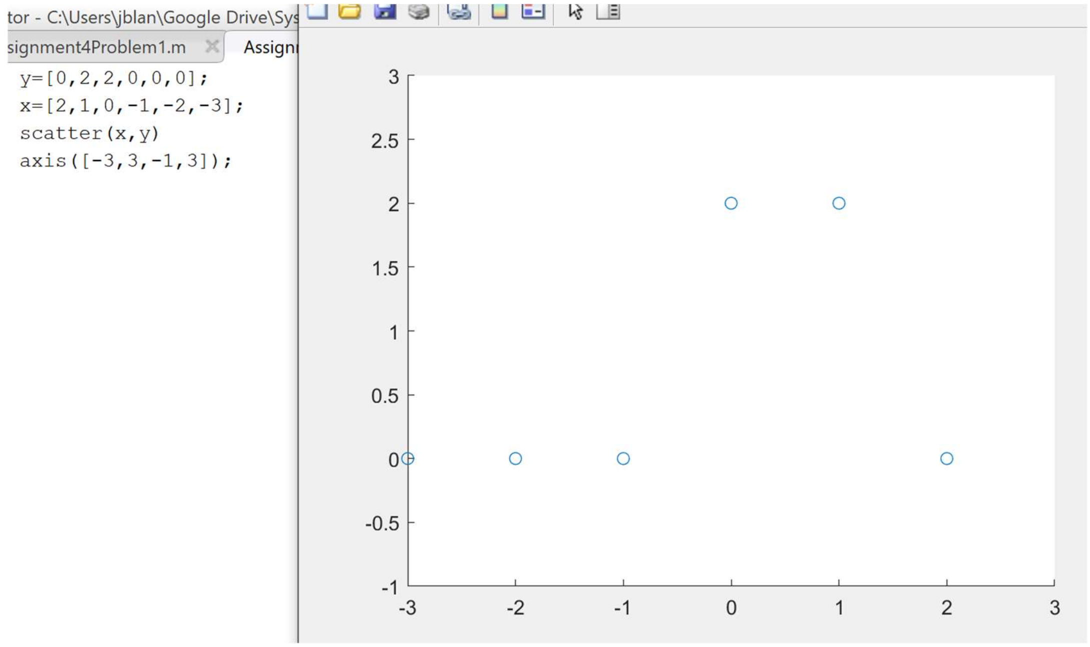


Figure 9. $x_a[-n]u[n]$ plotted in MatLab

b) $x_a[n]u[-n]$

The inverted unit step function evaluates to 1 at any $n \leq 0$ and 0 at $n > 0$.

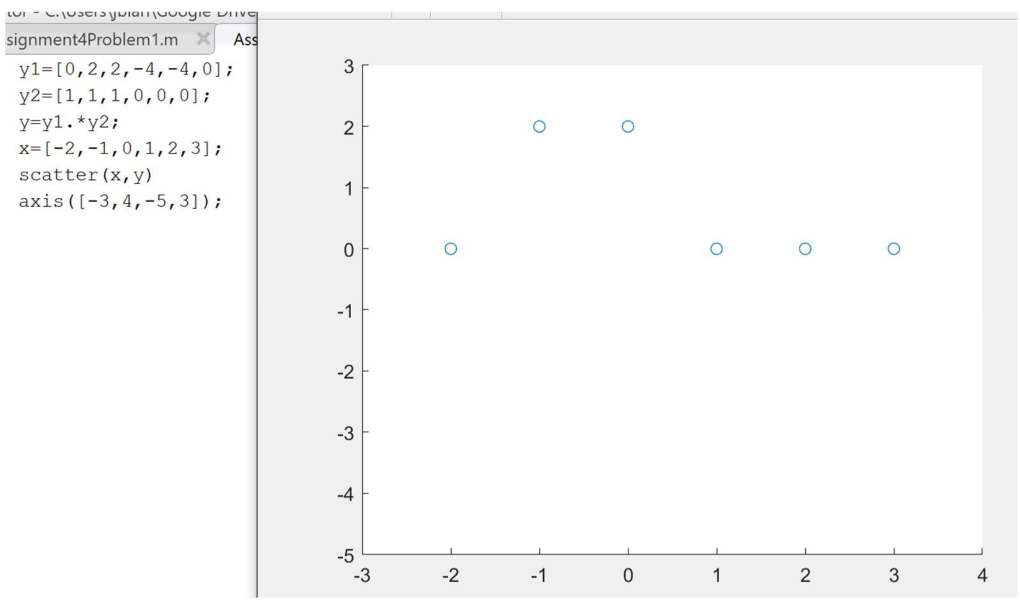


Figure 10. MatLab code and plot of $x_a[n]u[-n]$

c) $x_a[n]u[n-2]$

The offset unit function will evaluate to 1 when $n \geq 2$ or 0 when $n < 2$.

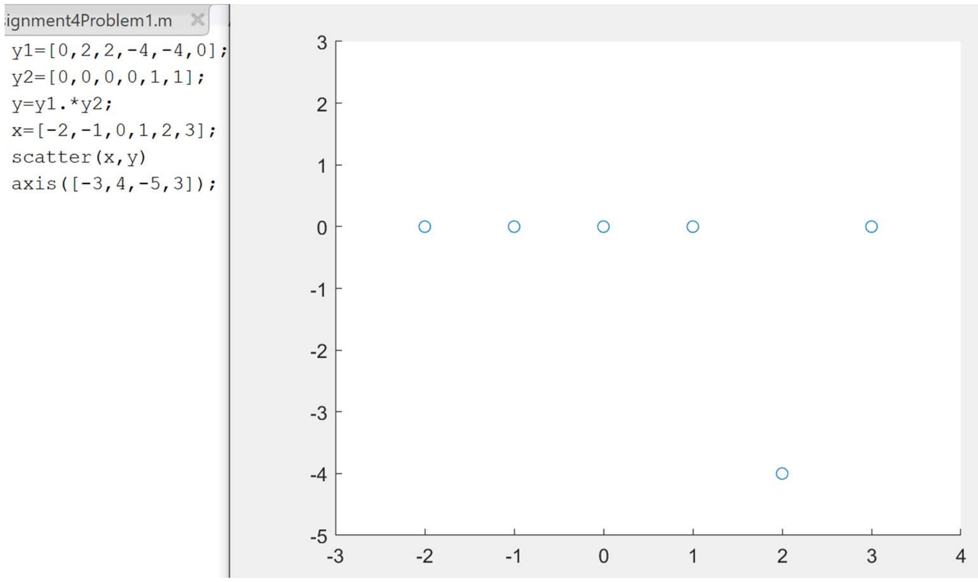


Figure 11. MatLab code and plot of $x_a[n]u[n-2]$

d) $x_a[n]u[2-n]$

This can also be written as $x_a[n]u[-(n-2)]$, or the inverted offset unit step function. This when $n \leq -2$ the function will evaluate to 1, otherwise it will be 0.

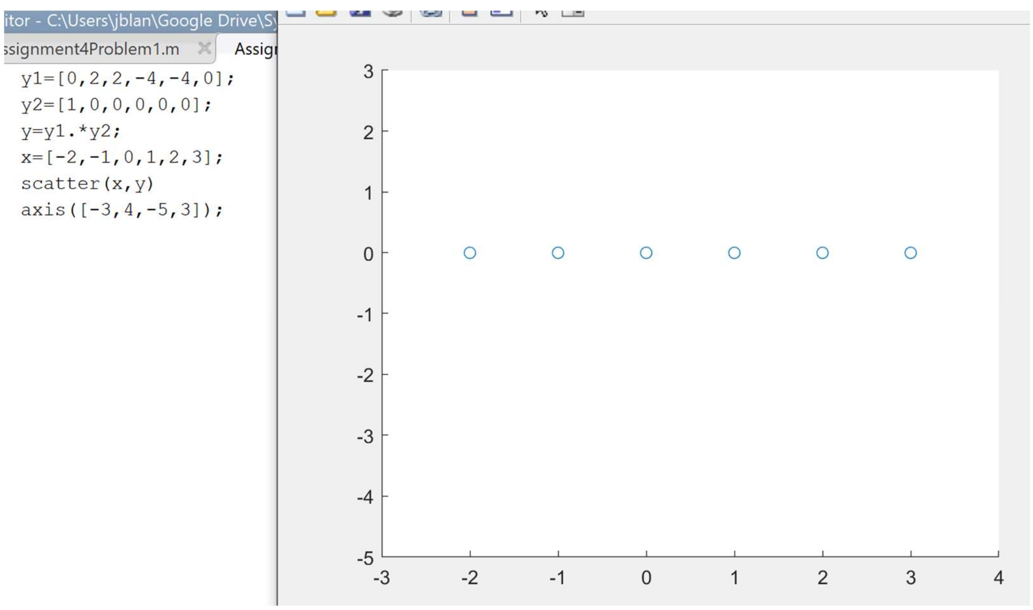


Figure 12. MatLab code and plot for $x_a[n]u[2-n]$

e) $x_a[n]\delta[n-1]$

The delta function only evaluates to 1 at point n=1.

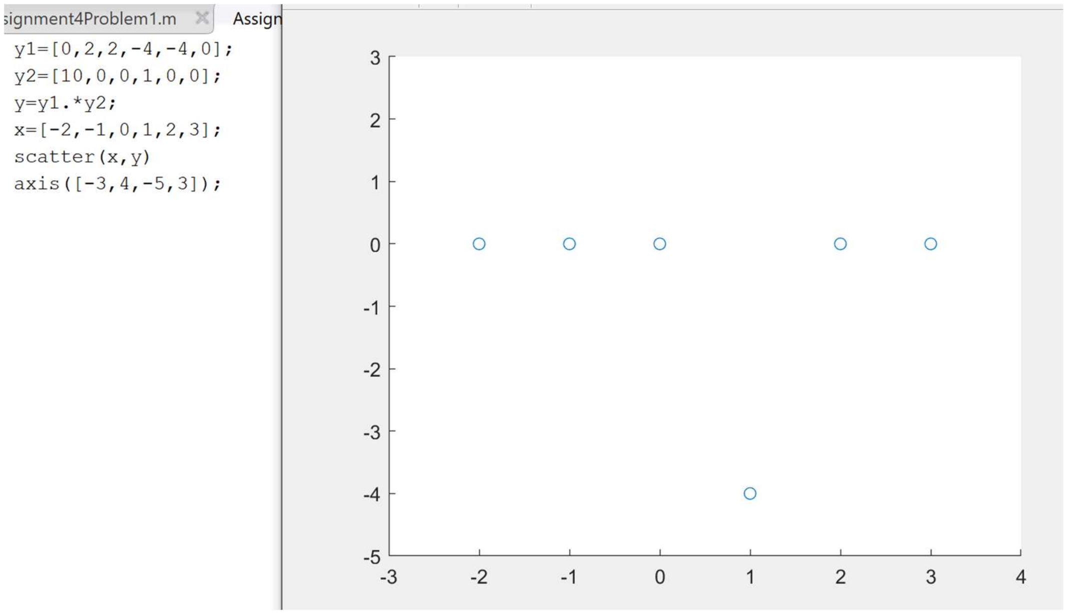


Figure 13. MatLab code and plot for $x_a[n]\delta[n-1]$

f) $x_a[n](\delta[n]+\delta[n-2])$

Inside the parentheses, the function will evaluate to 0 everywhere except at points n=0 and n=2 where it'll evaluate to 1.

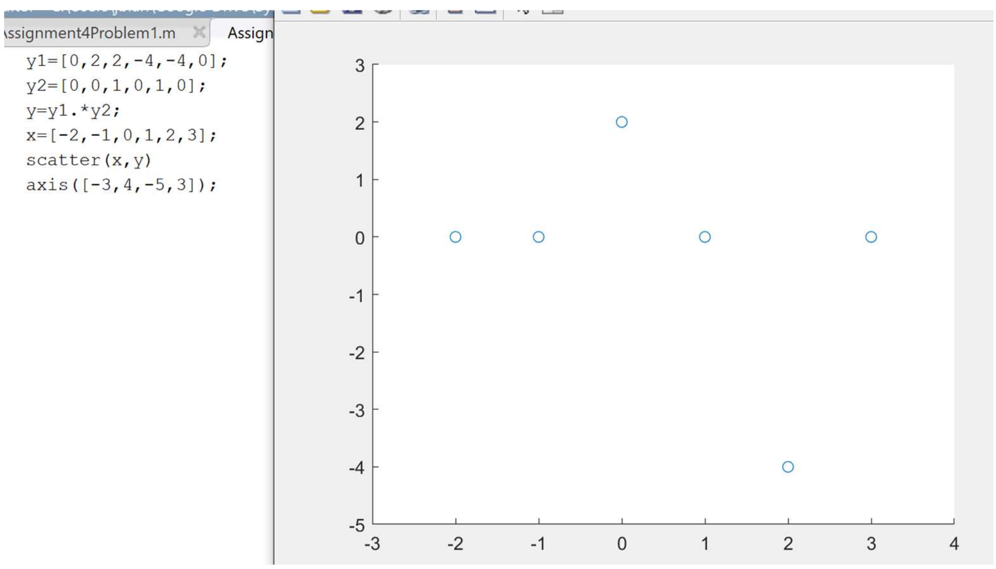


Figure 14. MatLab code and figure for $x_a[n](\delta[n]+\delta[n-2])$

Problem 3.

Given the LTI system of Figure P10.6 with the impulse response $h[n] = \propto^n u[n]$ where α is a constant. This system is excited with the input $x[n] = \beta^n u[n]$ with β != α and β is a constant.

a) Find the system response y[n]. Express y[n] in closed form, using the formulas for geometric series in Appendix C.
b) Evaluate y[4], using the results of Part a.
c) Verify the results of Part b by expanding the convolution sum for y[4], as in (10.15).

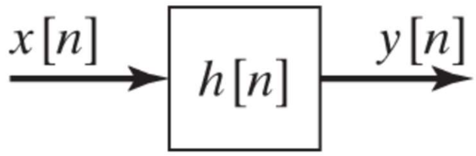


Figure P10.6

a) The convolution sum from the text is

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = x[n] * h[n]$$

$$y[n] = \sum_{k=n}^{\infty} \beta^k \propto^{n-k}$$

Using the formulas for geometric series found in the text,

$$y[n] = \frac{\beta^n}{1-\beta} \sum_{k=0}^{\infty} -\propto^k$$

$$y[n] = -\frac{\beta^n}{1-\beta} \times \frac{1}{1-\alpha}$$

b) $y[n] = -\frac{\beta^n}{1-\beta} \times \frac{1}{1-\alpha}$

$$y[4] = -\frac{\beta^4}{1-\beta} \times \frac{1}{1-\alpha}$$

c) As stated in 10.15, the convolution sum can also be expressed as

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k] = x[n] * h[n]$$

Thus

$$y[n] = \sum_{k=n}^{\infty} \beta^{n-k} \propto^k = -\frac{1}{1-\beta} \times \frac{\propto^n}{1-\alpha}$$

$$y[4] = -\frac{1}{1-\beta} \times \frac{\propto^4}{1-\alpha}$$

Problem 4. Consider the LTI system of Figure P10.6, with the input x[n] and the impulse response h[n], where

$$x[n]\begin{cases}2, & 1 \le n \le 10 \; and \; 20 \le n \le 29\\ 0, & otherwise\end{cases}$$

$$h[n] = \begin{cases}1, & 2 \le n \le 20\\ 0, & otherwise\end{cases}$$

Find and plot y[n].

$$x[n] = 2(u[n-1] - u[n-11]) + 2(u[n-20] - u[n-30])$$

$$h[n] = u[n-2] - u[n-21]$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = x[n] * h[n]$$

$$y[n] = \sum_{k=-\infty}^{\infty} \begin{array}{c}\{2(u[n-1] - u[n-11]) + 2(u[n-20] - u[n-30])\}\\ \{u[(n-k)-2] - u[(n-k)-21]\}\end{array}$$

$$y[n] = \sum_{k=-\infty}^{\infty} \begin{array}{c}\{2(u[n-1] - u[n-11]) + 2(u[n-20] - u[n-30])\}\\ \{u[n-(2+k)] - u[(n-(21+k)]\}\end{array}$$

Doing convolution by hand,

At $n-k < 3, 0$

At $3 \le n-k \le 10, \sum_{3}^{n-k} 2$

At $11 \le n-k \le 18, 20$

At $n-k = 19, 18$

At $20 \le n-k \le 29, 18$

At $30 \le n-k \le 48, 20$

At $48 \le n-k \le 58, 20 - \sum_{48}^{n-k} 2$

At $n-k > 58, 0$

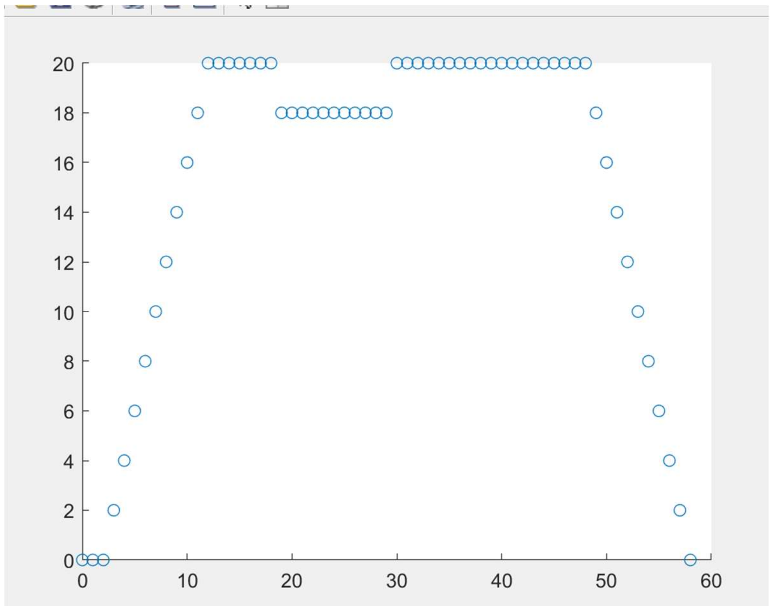


Figure 15. Convolution plot for y[n]

Problem 5.

Consider the discrete-time LTI system of Figure P10.6. This system has the impulse response

$$h[n] = u[n] - u[n-2]$$
$$\textit{and the input signal is given by}$$
$$x[n] = \delta[n+2] + 3(0.7)^n(u[n] - u[n-5])$$

Find and plot y[n].

Using h[n-k] convolving the functions results in

0 at $n-k < -2$

2 at $-2 \leq n-k \leq -1$

6 at $n-k = 0$

$6(0.7)^n + 6(0.7)^{n-1}\ at\ n-k > 0$

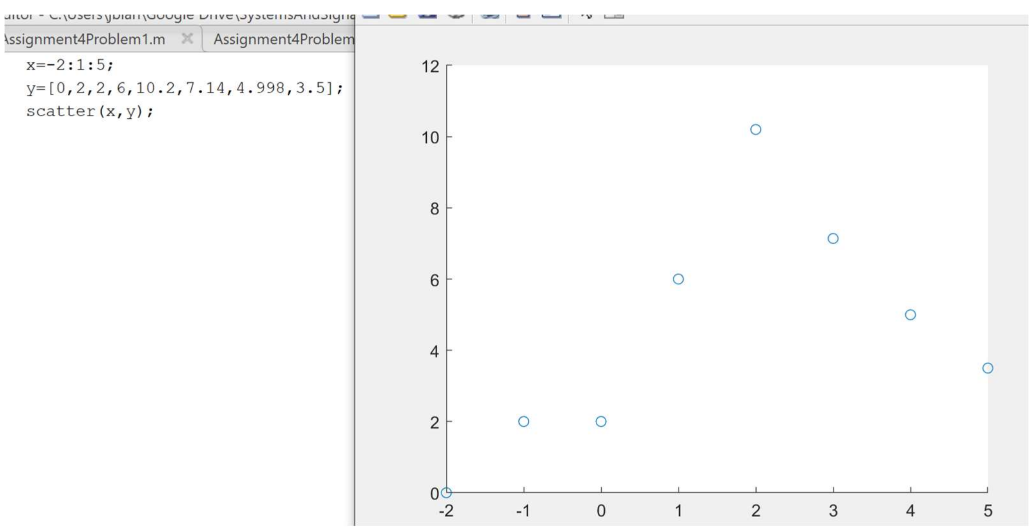


Figure 16. MatLab code and plot for y[n]

Problem 6.

Suppose that the discrete-time LTI system of Figure P10.6 has the impulse response h[n] given in Figure P10.10a. The system input is given in P10.10b. Find the output y[n].

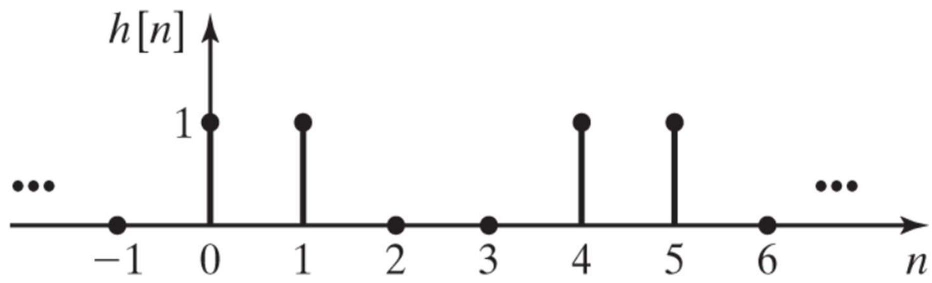


Figure P10.10a

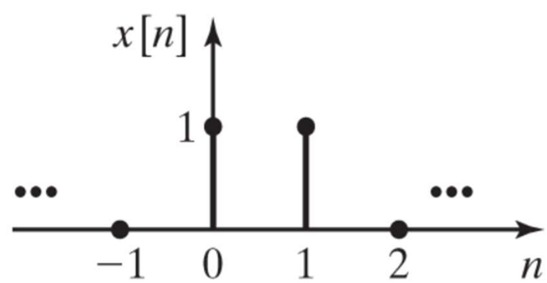


Figure P10.10b

Convolving the two discrete signals yields

0 at $n-k<0$

1 at $n-k=0$

2 at $n-k=1$

1 at $n-k=2$

0 at $n-k=3$

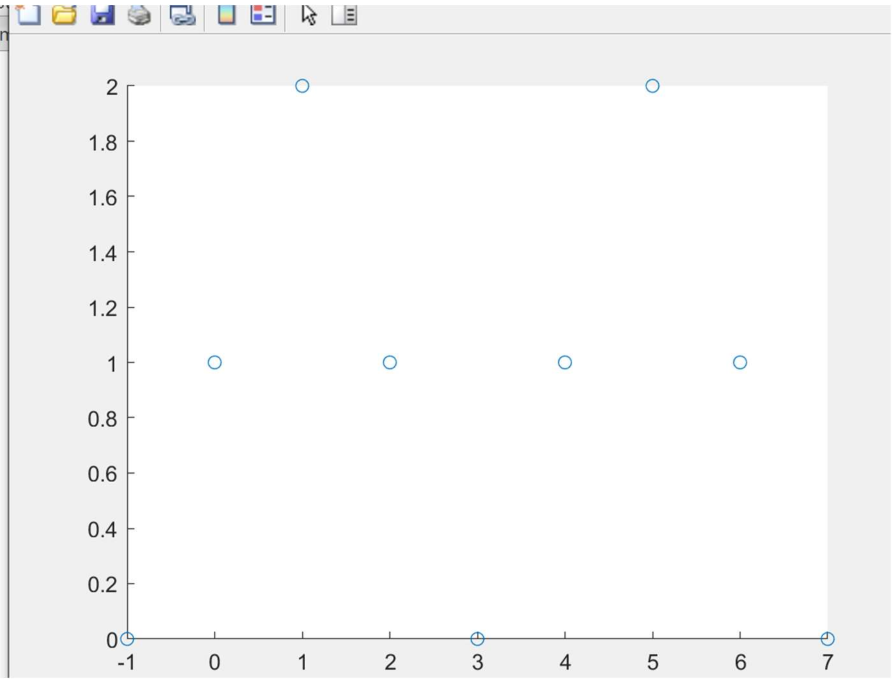
1 at $n-k=4$

2 at $n-k=5$

1 at $n-k=6$

0 everywhere else

```
x=-1:1:7;
y=[0,1,2,1,0,1,2,1,0];
scatter(x,y);
```



MatLab code and plot for y[n]

References

Phillips, C. L., Parr, J. M., and Riskin, E. A, Signals, Systems, and Transforms (4th ed), ISBN: 978-0-13-198923-8, Pearson, 2008.