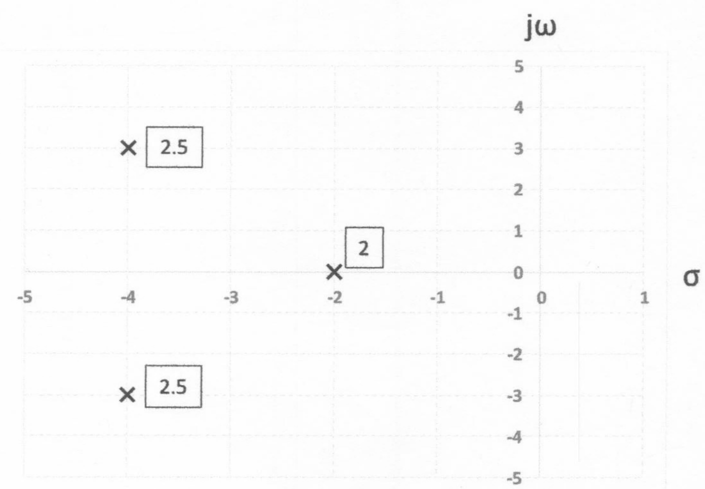
Instructions:

1. This exam must be completed, scanned, and sent electronically by 12:00 midnight today, April 7.
2. The exam is open book / open notes.
3. Print your name on each page
4. Attach additional sheets if necessary.

Problem 1 (30 points). *A system has an input voltage vi(t) and output voltage vo(t). The transfer function, H(s), is described by the pole-zero diagram in the s-plane as shown in the figure below. The boxed number at each pole location indicates its residue.*

a) ***Determine*** *the Transfer Function, H(s)*
b) ***Determine*** *the system impulse response, h(t).*
c) ***Write*** *the differential equation model for the system in terms of vi(t) and vo(t).*
d) ***Find*** *h(0+) and h(∞) using the Initial Value and Final Value Theorems. Assume that h(0-) = 0.*



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$$\frac{2.5}{(s-3j+4)(s+3j+4)} = \frac{2.5}{s^2+8s+25}$$

1)

a) $H(s) = \frac{2}{s+2} + \frac{2.5}{s-(3j-4)} + \frac{2.5}{s-(-3j-4)} = \frac{2}{s+2} + \frac{6.25}{s^2+8s+25}$

b) $h(t) = 2e^{-2t} + 5e^{-4t}\cos(3t)$ From Prof. Mac's Super duper pole-pair table

c) $\frac{6.25(s+2)+2(s^2+8s+25)}{s^3+10s^2+30s+50} = \frac{2s^2+14.25s+37.5}{s^3+10s^2+30s+50}$

$$\boxed{2\frac{d^2V_o}{dt^2} + 14.25\frac{dV_o}{dt} + 37.5V_o = \frac{d^3V_i}{dt} + 10\frac{d^2V_i}{dt} + 30\frac{dV_i}{dt} + 50V_i}$$

d) IVT: $\lim_{s\to\infty} sF(s) - f(0^+) = 0$

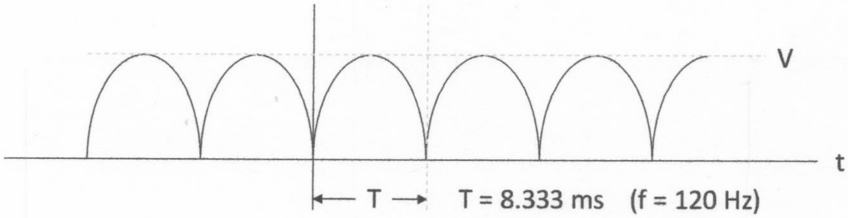
$$\lim_{s\to\infty} \frac{2s}{s+2} + \frac{2.5s}{s+4-3j} + \frac{2.5s}{s+4+3j}$$

$$\lim_{s\to\infty} \frac{2}{1+2/s} + \frac{2.5}{1+\frac{4-3j}{s}} + \frac{2.5}{1+\frac{4+3j}{s}} = 2+5 = 7 \quad \boxed{f(0^+)=7}$$

FVT: $\lim_{s\to 0} sF(s) = f(\infty)$

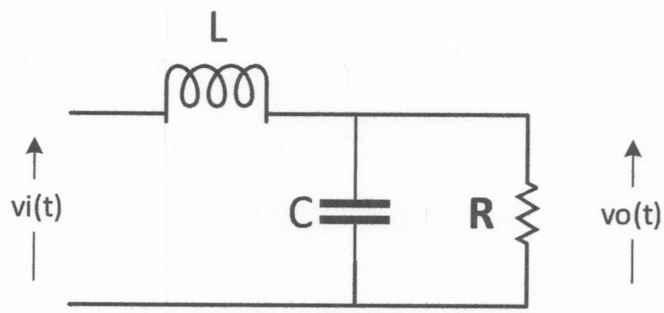
$$\lim_{s\to 0} \frac{2s}{s+2} + \frac{2.5s}{s+4-3j} + \frac{2.5s}{s+4+3j} = 0 \quad \boxed{f(\infty)=0}$$

Problem 2 (40 points). *A full-wave rectified signal, vi(t), is shown in the figure below:*



$$vi(t) = V\left|\sin \omega_0 t\right| = \frac{2V}{\pi} - \frac{4V}{\pi}\sum_{k=1}^{\infty}\frac{\cos k\omega_0 t}{4k^2-1}$$

The signal vi(t) is in turn applied to the low-pass filter shown below in order to reduce the ripple in the load resistor, R. The load resistor has a value of 100 Ω.



a) ***Find*** *the Transfer Function, H(ω) = Vo(ω)/Vi(ω) in terms of R, L, and C.*
b) ***Determine*** *the values of L and C so that the filter meets the Butterworth criteria with a cutoff frequency,* ω_c *of 150 r/s.*
c) ***Sketch*** *the filter magnitude response, including the value at the cutoff frequency.*
d) *V is the amplitude of the full-wave input signal, as shown above.*
   i. *If V = 100 V,* ***find*** *the amplitudes of vi(t) and vo(t) at both the fundamental (120 Hz) and the second harmonic (240 Hz).*
   ii. *The filter rejection is the ratio of output to input amplitude. Determine the filter rejection in dB at the fundamental and second harmonic.*

The Second-order Butterworth Condition: $$|H(\omega)| = \left[1 + \left(\omega/\omega_c\right)^4\right]^{-1/2}$$

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2) a) using impedence which is already in the Fourier domain with the Voltage divider equation,

$$V_o(\omega) = \frac{\left[\frac{1}{R} + j\omega C\right]^{-1}}{\left[\frac{1}{R} + j\omega C\right]^{-1} + j\omega L} V_i(\omega) = \frac{\frac{R}{1+Rj\omega C}}{\frac{R}{1+Rj\omega C} + j\omega L} V_i(\omega)$$

$$H(\omega) = \frac{V_o(\omega)}{V_i(\omega)} = \frac{R}{R + j\omega L(1+j\omega RC)}$$

b) Butterworth Condition: $\frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_c}\right)^4}} = |H(\omega)|$ $R = 100\Omega$

$\omega_c = 150$

$$H(\omega)^2 = \frac{1}{1+\left(\frac{\omega}{\omega_c}\right)^4} = \frac{R^2}{R^2 - \omega^2 L^2 (1 + 2j\omega RC - \omega^2 R^2 C^2)}$$

$$\frac{1}{1+\left(\frac{\omega^4}{22500^2}\right)} = \frac{10000}{10000 - \omega^2 L^2 (1 + 2j\omega 100C - \omega^2 C^2 \cdot 10000)}$$

@ $\omega = \omega_c$

$$\frac{1}{2} = \frac{10000}{10000 - (22500) L^2 (1 + 2j(150)100C - (22500)C^2 \cdot 10000)}$$

$$20000 = 10000 - \left[22500 L^2 \left(\frac{1}{10000} + 3\cancel{0000}jC - 22500\cancel{0000}C^2\right)\right.$$

$$10000 = -22500L^2\left(\frac{1}{10000} + 3jC - 22500C^2\right)$$

$$1 = 22500^2 L^2 C^2 - 2.25L^2 - 67500CL^2 j$$

ex: for $C = 10^{-6}$
$L = 0.65695$

L and C must satisfy this equation

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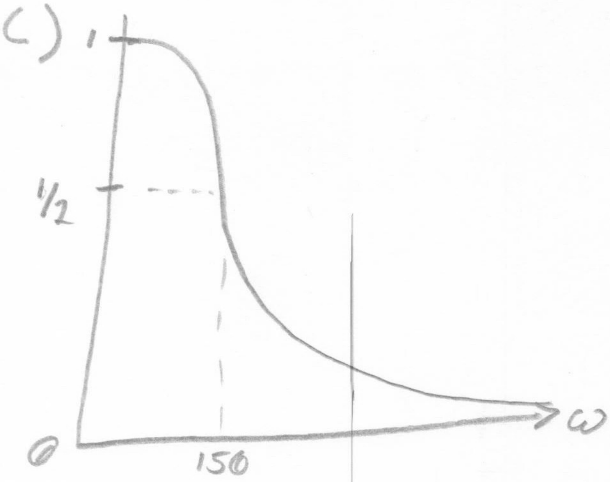
2(c)



d) i) $100 \times H(120) = 61.81\,V = V_o$

$V_i = 100\,v$

$100 \times H(240) = 28.20\,V = V_o$

$V_i = 100\,v$

ii)

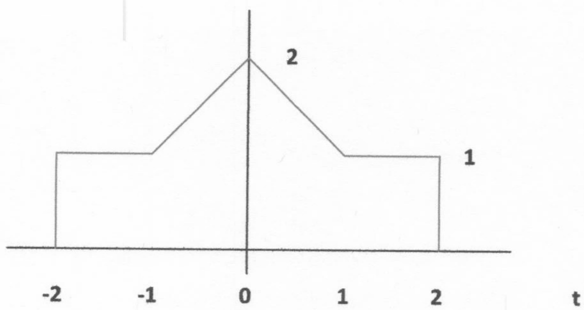
$\frac{61.81}{100} = 0.6181 \qquad 20\log(0.6181) = -4.18\,dB$

$\frac{28.20}{100} = 0.2820 \qquad 20\log(0.282) = -10.995\,dB$

Problem 3 (20 points). *An active filter has a frequency transfer function given by* $H(\omega) = \dfrac{5}{2 + j\omega}$

a) ***Determine** and **sketch** the impulse response, h(t).*
b) ***Determine** and **sketch** the step response, s(t).*
c) ***Determine** the steady-state output, y(t), when the input = 2cos(3t). Express y(t) in standard form; i.e., Acos(ωt+θ).*

Problem 4 (10 points). A time-domain signal, f(t), is shown below. Find the Fourier Transform, F(ω).



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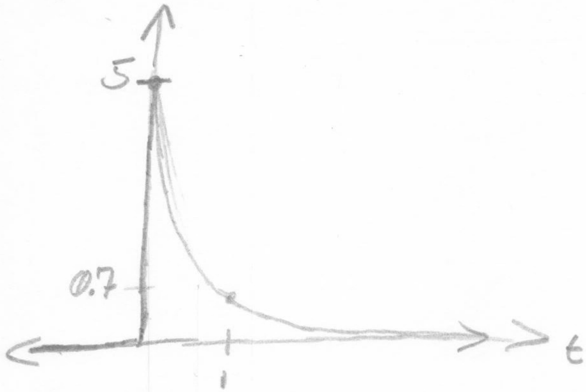
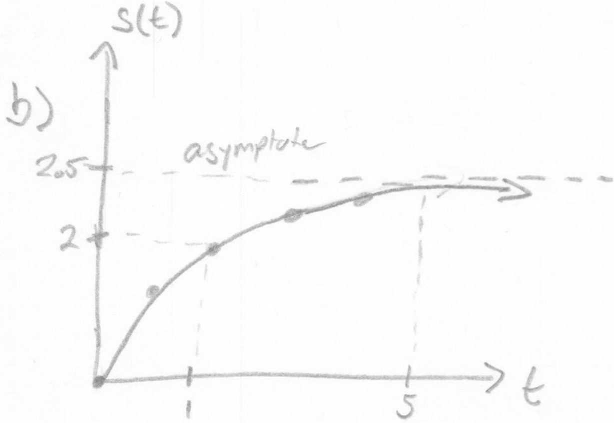
3) $H(\omega) = \frac{5}{2+j\omega}$ $\qquad h(t) = 5e^{-2t}u(t)$

a)



b)



b) System is causal

$$s(t) = \int_0^t h(\tau)\,d\tau$$

$$s(t) = 5\int_0^t e^{-2\tau}\,d\tau$$

$$s(t) = \frac{5}{2}\left[e^{0} - e^{-2t}\right]$$

$$s(t) = \frac{5}{2} - \frac{5}{2}e^{-2t}$$

c) $y = x(t) * h(t)$

$$y(t) = 2\int_{-\infty}^{t} \cos(3\tau)\,d\tau \cdot 5e^{-2t}u(t)$$

$$y(t) = \frac{2}{3}\sin(3\tau)\Big|_{-\infty}^{t} \cdot 5e^{-2t}u(t)$$

$$\boxed{y(t) = \frac{10}{3}e^{-2t}\sin(3t)\,u(t)}$$

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4) ft of $rect(t/4) + tri(t/2)$ from table 5.2

$4\,sinc(2\omega) + 2\,sinc^2(\omega)$