IIIT Vadodara
WINTER 2020-21
MA202 Numerical Techniques
LAB#6 Numerical Integration[1]

1. **Various methods of numerical integration** The integrations over two segments by the trapezoidal rule, and Simpson's rule, which are referred to as Newton–Cotes formulas for being based on the approximate polynomial and are implemented by the following formulas:
1. Trapezoidal Rule

$$\int_a^{a+h} f(x)dx = \frac{h * [f(a) + f(a+h)]}{2}, \tag{1}$$

where h is $b - a$.
2. Simpson's one by third rule

$$\int_a^{a+h} f(x)dx = \frac{h * [f(a) + 4f(a+h) + f(a+2h)]}{3}, \tag{2}$$

where h is $(b - a)/2$.
3. Simpson's three by Eighth rule

$$\int_a^{a+h} f(x)dx = \frac{3h * [f(a) + 3f(a+h) + 3f(a+2h) + f(a+3h)]}{8}, \tag{3}$$

where h is $(b - a)/3$.

Local and global truncation errors for single application of Newton-Cotes formulae

1. For the trapezoidal method local truncation error (LTE) will be in the order of h^3 and global truncation error (GTE) will be in the order of h^2.
2. The simpson's one by third rule gives a LTE in the order of h^5, and GTE is in the order of h^4.
3. The simpson's three by eighth rule gives a LTE in the order of h^5, and GTE is in the order of h^4.

Multiple applications of all the rules mentioned above can be summed over the number of intervals to calculate the approximated numerical value for integration and further the GTE.

[1] submission deadline : 14^{th} March 11 PM

Q. 1: Compute the following integrals by using the trapezoidal rule, the Simpson's one by third rule, and Simpson's three by eighth rule for the mentioned limits.

a. Verify the order of errors (LTE) for all the three methods calculating numerical integral using single application of Newton-cotes formulae. Choose $h = 0.1$

b. Choose $h = 0.01$ and repeat the previous sub-division.

b. Verify the order of errors (GTE) for all the three methods calculating numerical integral using multiple application of Newton-cotes formulae. Choose $n = 10$. Change value to 100 and comment on results.

c. Use MATLAB functions *trapz* and *quad* to do the same and check for the errors. Use section wise codes for calculating errors in the single script.

d. Vary the number of intervals and comment on observations.

1. $2 - x + ln(x)$, where $a = 1$ and $b = 2$
2. $x^3 - 2x$, where $a = 0$ and $b = pi/2$