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WINTER 2020-21
MA202 Numerical Techniques
LAB#7 Solution of non-linear equations[1]

1. **Method 1: Bisection method** The bisection method can be applied for solving nonlinear equations like $f(x) = 0$, only in the case where we know some interval $[a, b]$ on which $f(x)$ is continuous and the solution uniquely exists and, most importantly, $f(a)$ and $f(b)$ have the opposite signs. The procedure toward the solution of $f(x) = 0$ is described as follows

Step 1 Initialize the iteration number $k = 0$.
Step 2 Let $m = \frac{(a+b)}{2}$. If $f(m) \approx 0$ or $\frac{(b-a)}{2} \approx 0$,
step 3 If $f(a)f(m) > 0$, then let $b \leftarrow m$. Go back to step 2.

Method 2: Fixed point iteration method Suppose a function $g(x)$ is defined and its first derivative $g'(x)$ exists continuously on some interval $I = [x^o - r, x^o + r]$ around the fixed point x^o of $g(x)$ such that $g(x^o) = x^o$

$$g(x^o) = x^o \tag{1}$$

Then, if the absolute value of $g'(x)$ is less than or equal to a positive number α that is strictly less than one, that is,

$$|g'(x)| \leq \alpha < 1 \tag{2}$$

the iteration starting from any point $x_0 \in I$

$$x_{k+1} = g(x_k) \;\; with \;\; x_0 \in I \tag{3}$$

converges to the (unique) fixed point x^o of $g(x)$.

Method 3: NEWTON(–RAPHSON) METHOD

Consider the problem of finding numerically one of the solutions, x^o, for a nonlinear equation

$$f(x) = (x - x^o)^m g(x) = 0 \tag{4}$$

where $f(x)$ has $(x - x^o)^m$(m is an even number) as a factor and so its curve is tangential to the x-axis without crossing it at $x = x^o$. In this case, the signs of $f(x^o)$ and $f(x^o+)$ are the same and we cannot find any interval $[a, b]$ containing only x^o as a solution such that $f(a)f(b) < 0$. Consequently, bracketing methods such as the bisection or false position ones are not applicable to this problem. Neither can the MATLAB built-in routine fzero() be applied to solve as simple an equation as $x^2 = 0$, which you would not believe until you try it for yourself. Then, how do we solve it? The Newton(–Raphson) method can be used for this kind of problem as well as general nonlinear equation problems, only if the first derivative of $f(x)$ exists and is continuous around the solution. The strategy behind the Newton(–Raphson) method is to approximate the curve of $f(x)$ by its tangential line at some estimate x_k

$$y - f(x_k) = f'(x_k)(x - x_k) \tag{5}$$

and set the zero (crossing the x-axis) of the tangent line to the next estimate x_{k+1}.

$$0 - f(x_k) = f'(x_k)(x_{k+1} - x_k) \tag{6}$$

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$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)} \tag{7}$$

Q. 1: Consider the nonlinear equations

a. $f(x) = 2.0 - x + ln(x) = 0$
b. $f(x) = x^2 - 3x + 1 = 0$

Write a MATLAB function to solve the non-linear equations using Bisection method, Fixed point iteration method, Newton-Raphson method. Use fzero() and fsolve() MATLAB functions to verify your answers.