Indian Institute of Information Technology Vadodara

MA202: Numerical Techniques Lab
Semester: IV
Lab 3

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Section : 2A

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Note: I have made PDF from next page using matlab only. They are in parts.

I have merged them all.

Que-1A

```
a = 0.1;
true_val = exp(a);
exp_val = 1;
current_term = 1;
n = 5;
for i = 1:n %loop to calc the expected val of exp(a)
    current_term = current_term*(a/i);
    exp_val(i+1)=exp_val(i)+current_term;
end
Error= abs(true_val-exp_val(n+1));
fprintf('Error!! while calculating e^0.1 for n= %d is
 %1.20f',n,Error);
```

```
Error!! while calculating e^0.1 for n= 5 is 0.00000000140898115397
```

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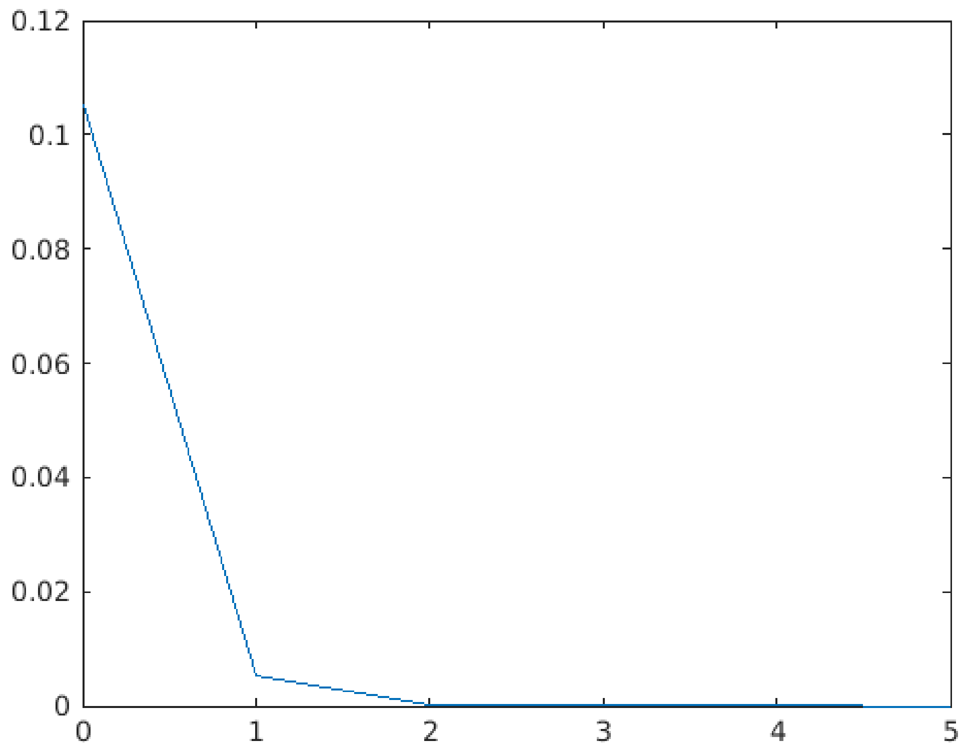
Que-1b

```
a = 0.1;
true_val = exp(0.1);
exp_val = 1;
current_term = 1;
n = 5;
for i = 1:n %loop to calc the expected val of exp(a)
    current_term = current_term*(a/i);
    exp_val(i+1)=exp_val(i)+current_term;
end
Error= abs(exp_val-true_val);%error in calculating
n=[0:1:5];
plot(n,Error)
```



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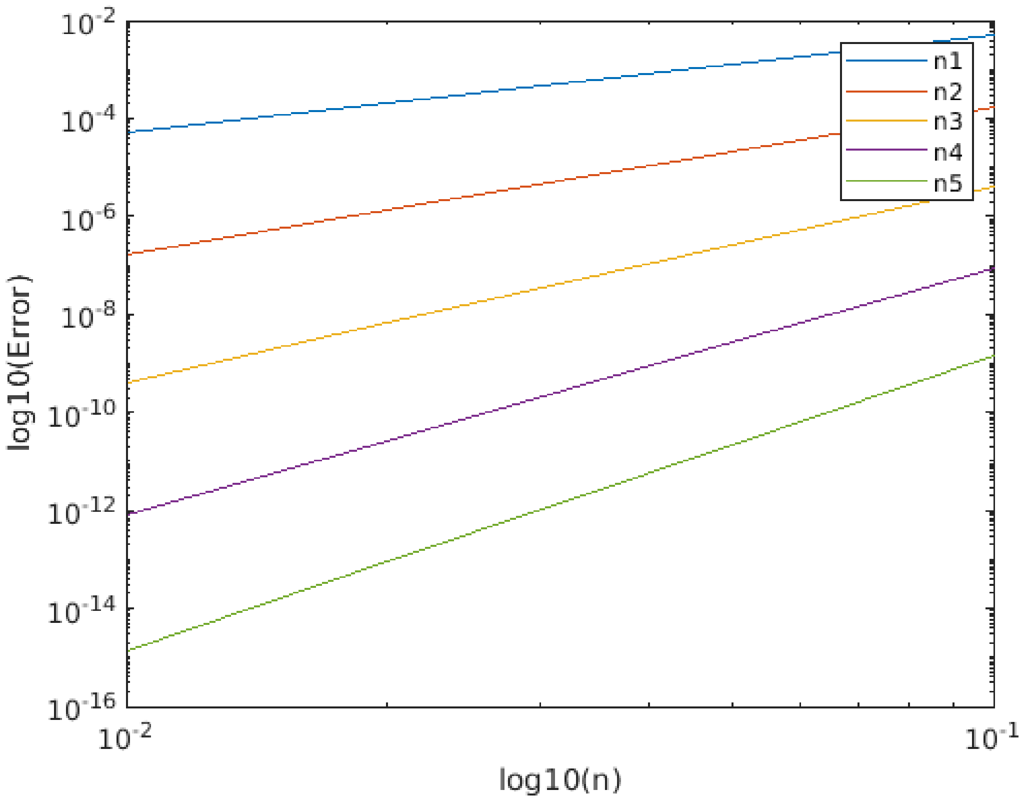
Que-1c

```
stepsizes=[0.1 0.05 0.02 0.01];%array of all the step sizes
value=[1:5];%array of all the values of n
Error=[];
for i=1:length(stepsizes)%loop for calculating all the stepsizes
    a=stepsizes(i);
    term=a.^value./cumprod(value);%makes as array of the value of nth
 term
    exp_val=1+cumsum(term);
    true_val=exp(a);
    Error=[Error;abs(true_val-exp_val)];%makes an matrix of size 4*5
 where esch colum tells the error assocuiated with that stepsize
end
loglog(stepsizes,Error);
xlabel("log10(n)");
ylabel("log10(Error)");
legend('n1','n2','n3','n4','n5');
```



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Question-1(D)

• In the first graph the value of n increases and the value of error decreases simultaneously, so we can say that the computed value is approaching true value hence we can observe increase in accuracy.

• In second graph for smaller step-size, error is small so by this we can say that smaller stepsizes yeild more precise result than the larger step-sizes as well as the value of n increase the graph becomes more and more precise.

Question-1(E)

As we known that for n=1

(x1, y1)= (0.1, 0.005171)

(x2, y2)= (0.01, 5.017×10^{-5})

and also slope is

$$\textbf{Slope} = \frac{\log\left(\frac{y2}{y1}\right)}{\log\left(\frac{x2}{x1}\right)}$$

Hence for,

n=1, Slope = 2.01312178

n=2, Slope = 3.04

n=3, Slope = 4.008

n=4, Slope = 5.008

As we already know that for as n increases accuracy also increases and from the above calculations we saw that as the n increases the slope also increases thus we can say that as slope increase the accuracy also increases.

Question-2

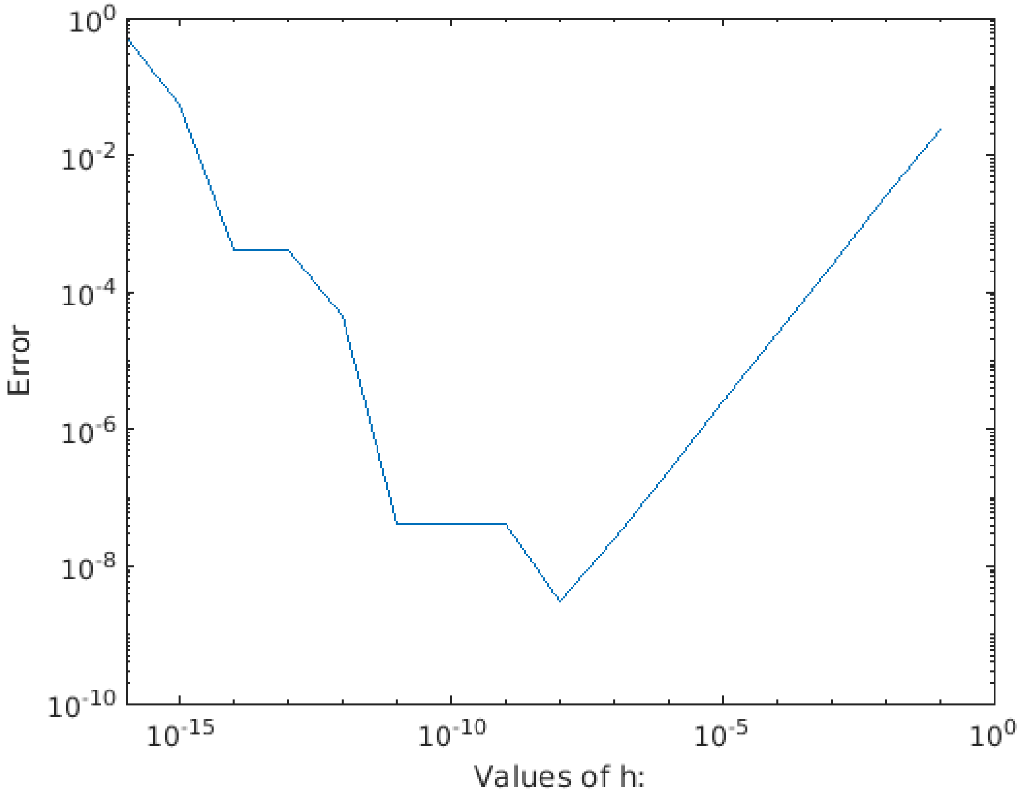
a)

```
x=5;
fprintf("The derivative is: ")
result=1/(1+x*x) %derivative of atan(x)

% b)
a=1;
true_val=1/(1+a*a);%derivative of atan(a)
Error=[];%creation of Error array
for i=1:16 %loop for executive error of different stepsizes
    h=10^-i;
    exp_val=(atan(a+h)-atan(a))/h;
    Error(i)=abs(true_val-exp_val); %creates an array of 1X16 values
 of array at different valyes of stepsizes
end
n=[-1:-1:-16];
h=10.^n; %array of 16 different values of h
loglog(h, Error) %graph betweeen values of h and Error
xlabel('Values of h:');
ylabel('Error');
```

```
The derivative is:
result =

    0.0385
```



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Question-2(C)

- Truncation Error :- Truncation Error is the error made by truncating an infinite sum and approximating and also these error result from using an approx, in place of exact mathematical by a finite sum.

- Machine precision :- The machine precision is single precision floating-point number and the machine precision should be the distance from one floating-point number to another. eps = 2.220446049250313e-16

- Round off Error :- Round off error can be minimized by increasing the number of significant digits.These errors arise because digital computers cannot represent some quantities exactly.

Question-3

```
%a)
A=[1.01 0.99;0.99 1.01];
b_a=[2;2];
fprintf ("The solution for given linear system in part-A is:")
X_a=A\b_a

%b)
A=[1.01 0.99;0.99 1.01];
%b_a=[2;2];
%x=A\b_a
b_b=[2.02;1.98];
fprintf ("The solution for given linear system in part-B is:")
x_b=A\b_b;

%c)
A=[1.01 0.99;0.99 1.01];
b_a=[2;2];
x_a=A\b_a;
b_b=[2.02;1.98];
x_b=A\b_b;
fprintf('The Condition number of the given Matrix is:');
k=cond (A)
fprintf('The upper bound on the possible change in x indicates changes
 in all of the significant digits.')
k*norm(b_a-b_b)/norm(b_a)
fprintf('The actual changes inn x is:');
norm(x_a-x_b)/norm(x_a)
fprintf('The Relative Error observed:')
norm(b_a-b_b)/norm(b_a)
```

```
The solution for given linear system in part-A is:
X_a =

    1.0000
    1.0000

The solution for given linear system in part-B is:The Condition number
 of the given Matrix is:
k =

  100.0000

The upper bound on the possible change in x indicates changes in all
 of the significant digits.
ans =

    1.0000

The actual changes inn x is:
ans =
    1.0000

The Relative Error observed:
ans =

    0.0100
```

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Question-3(C)

a. The maximum by which the relative error in observation can affect the solution is the upper bound of relative error of solution (kappa*norm(b1-b2)/norm(b1)).Since the upper bound of possible change in x i.e.-solution is 1.0000 so the maximum possible error due to this perturbation is 1.0000 and the change which we have got by calculating(norm(x-x2)/norm(x)) is the largest possible change in the solution(1.0000) for this perturbation.

b. Larger the condition number closer the matrix will be towards singularity but here condition number is 100 which is not so large for the machine thus we can say that matrix is well behind being a singular matrix.

Question-4

```
%a)
A=[1.01 0.99;0.99 1.01];
b_a=[2;2]; % observation vector b1
x=A\b_a;
b_b=[2.02;1.98]; % observation vector b2
x2=A\b_b;
fprintf('The Backward error resulted is ');
norm(b_a-b_b)%backward error
```

```
The Backward error resulted is
ans =

    0.0283
```

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Question-4

a. Comment on backward error In the above x is not available and it also impossible to become calculate norm (x-x2) form so,in order to check the accuracy we measure backward error in terms of difference norm(b_a-b_b). We had changed name of variables during solving to remove ambiguity.

b.

$A1X1=b1$ …(1)

$A2X2=b2$ …(2)

Upon subtracting them we get

$A1(X1-X2) = b1 - b2$

Taking magnitude on both sides

$|| A1(X1-X2) || = ||b1-b2||$

Since,$|| (b1-b2)|| = 0$

Hence, $|| A1(X1-X2)|| = 0$

$(X1-X2) = 0$

$X1= X2$