Indian Institute of Information Technology Vadodara

MA202: Numerical Techniques Lab
Semester: IV
Lab 5

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Section : **2A**

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Note: I have made PDF from next page using matlab only. They are in parts. I have merged them all.

Question-1

Table of Contents

Question-1(a)

Matlab script to calculate numerical differentiation of arctan(x)

```
a = 1;
truevalue = 1/(1 + a.^2);
h = 1e-4;
disp(['Numerical derivative of tan inverse(x) at x = 1 is ',
 num2str(truevalue)]);
```

```
Numerical derivative of tan inverse(x) at x = 1 is 0.5
```

Question-1(b)

Caluclation of error using forward difference, backward difference, and central difference methods.

```
% Formula for Forward difference
fdiff = (f(a + h) - f(a))/h;
ferr = abs(truevalue - fdiff);
disp(['Error in forward difference:- ', num2str(ferr)]);
% Formula for Backward difference
bdiff = (f(a) - f(a - h))/h;
berr = abs(truevalue - bdiff);
disp(['Error in backward difference:- ', num2str(berr)]);
% Formula for Central difference
cdiff = (f(a + h) - f(a - h))/(2*h);
cerr = abs(truevalue - cdiff);
disp(['Error in central difference:- ', num2str(cerr)]);
% Formula for Improved Forward difference
ifdiff = (-f(a + 2*h) + 4*f(a + h) -3*f(a))/(2*h);
iferr = abs(truevalue - ifdiff);
disp(['Error in improved forward difference:- ', num2str(iferr)]);
% Formula for Improved Backward difference
ibdiff = (3*f(a) - 4*f(a - h) + f(a - 2*h))/(2*h);
iberr = abs(truevalue - ibdiff);
disp(['Error in improved backward difference:- ', num2str(iberr)]);
% Formula for Improved Central difference
icdiff = (8*f(a + h) - 8*f(a - h) - f(a + 2*h) + f(a - 2*h))/(12*h);
icerr = abs(truevalue - icdiff);
disp(['Error in improved central difference:- ', num2str(icerr)]);
```

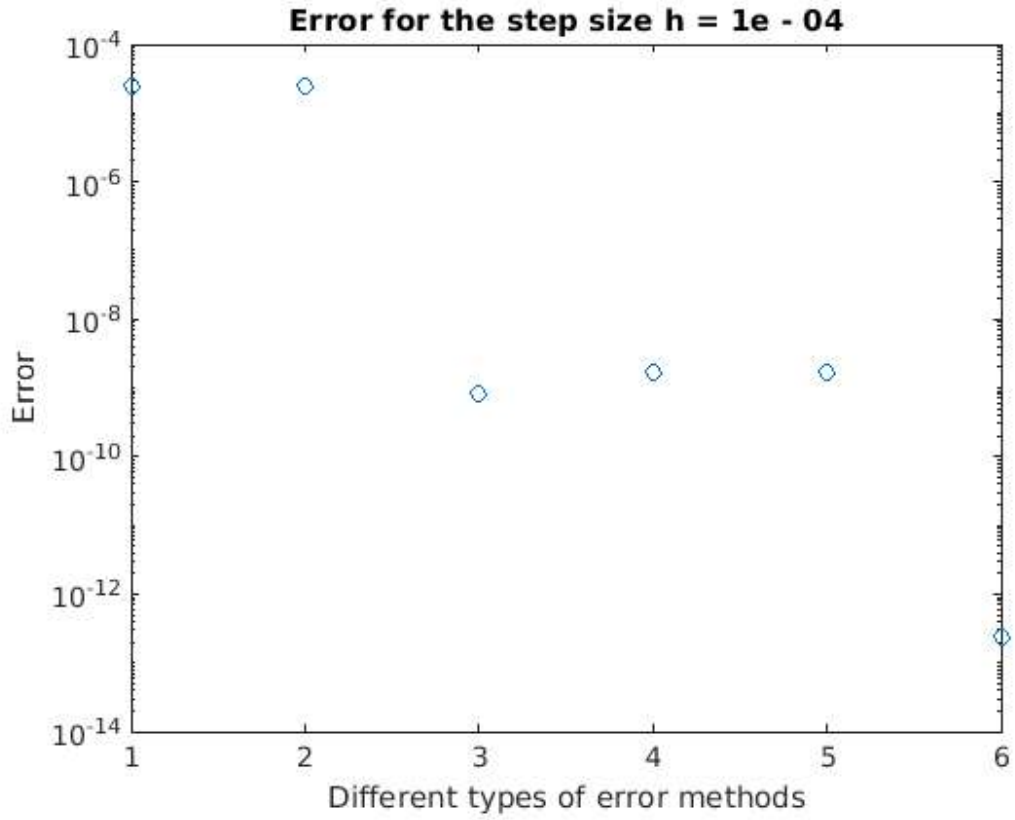
```
Error in forward difference:- 2.4999e-05
```

```
Error in backward difference:- 2.5001e-05
Error in central difference:- 8.3317e-10
Error in improved forward difference:- 1.6665e-09
Error in improved backward difference:- 1.6682e-09
Error in improved central difference:- 2.4014e-13
```

Question-1(c)

Ploting graph for diffrent method

```
semilogy([ferr, berr, cerr, iferr, iberr, icerr], 'o')
title('Error for the step size h = 1e - 04')
ylabel('Error')
xlabel('Different types of error methods')
% Defining function for tan inverse(x)
function fx = f(x)
  fx = atan(x);
end
```



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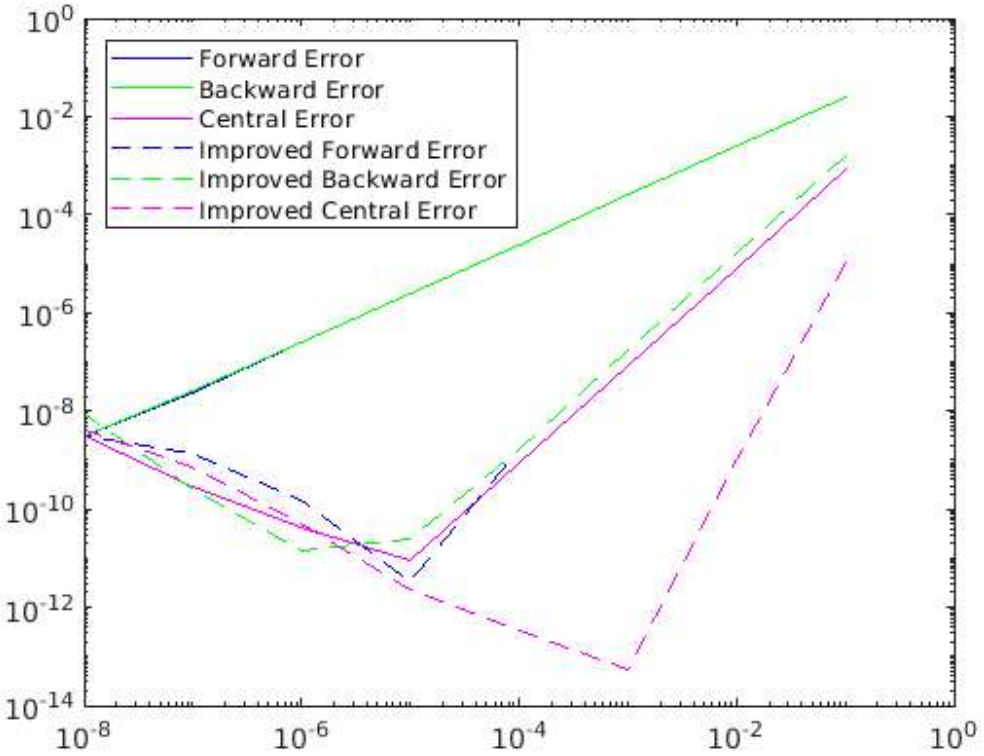
Question-1

Question-1(d)

Matlab script to calculate numerical differentiation of arctan(x)

```
a = 1;
truevalue = 1/(1 + a.^2);
h = 10.^[-1:-1:-8];
% Formula of Forward difference
fdiff = (f(a + h) - f(a))./h;
ferr = abs(truevalue - fdiff);
disp('Error in forward difference:- ');
disp(num2str(ferr));
% Formula of Backward difference
bdiff = (f(a) - f(a - h))./h;
berr = abs(truevalue - bdiff);
disp('Error in backward difference:- ');
disp(num2str(berr));
% Formula of Central difference
cdiff = (f(a + h) - f(a - h))./(2.*h);
cerr = abs(truevalue - cdiff);
disp('Error in central difference:- ');
disp(num2str(cerr));
% Formula of Improved Forward difference
ifdiff = (-f(a + 2*h) + 4*f(a + h) - 3*f(a))./(2.*h);
iferr = abs(truevalue - ifdiff);
disp('Error in improved forward difference:- ');
disp(num2str(iferr));
% Formula of Improved Backward difference
ibdiff = (3*f(a) - 4*f(a - h) + f(a - 2*h))./(2.*h);
ibderr = abs(truevalue - ibdiff);
disp('Error in improved backward difference:- ');
disp(num2str(ibderr));
% Formula of Improved Central difference
icdiff = (-f(a + 2*h) + 8*f(a + h) - 8*f(a - h) + f(a - 2*h))./
(12.*h);
icerr = abs(truevalue - icdiff);
disp('Error in improved central difference:- ');
disp(num2str(icerr));
% Graph plotings
loglog(h, ferr, '-b', h, berr, '-g', h, cerr, '-m');
hold on;
loglog(h, iferr, '--b', h, ibderr, '--g', h, icerr, '--m');
legend('Forward Error', 'Backward Error', 'Central Error', 'Improved
 Forward Error', 'Improved Backward Error', 'Improved Central
 Error', 'Location', 'northwest');
% Function
function fx = f(x)
  fx = atan(x);
end
```

```
Error in forward difference:-
0.024169   0.0024917  0.00024992  2.4999e-05     2.5e-06  2.5006e-07
 2.4133e-08  3.0387e-09
Error in backward difference:-
0.025831   0.0025083  0.00025008  2.5001e-05     2.5e-06  2.4998e-07
 2.4717e-08  3.0387e-09
Error in central difference:-
0.00083082  8.3331e-06  8.3333e-08  8.3317e-10  8.8267e-12  4.1133e-11
  2.9193e-10  3.0387e-09
Error in improved forward difference:-
0.0016374  1.6663e-05  1.6667e-07  1.6665e-09  3.2756e-12  1.5216e-10
 1.4022e-09  3.0387e-09
Error in improved backward difference:-
0.0016249  1.6663e-05  1.6667e-07  1.6682e-09   2.448e-11  1.4378e-11
 2.6318e-10  8.5899e-09
Error in improved central difference:-
1.0176e-05  1.0002e-09  5.5955e-14  3.3268e-13  2.3505e-12  5.0385e-11
  7.5453e-10  3.9639e-09
```



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Question 1(e):

Truncation Error is caused by a truncated Taylor series expansion replacing the spatial derivative and time derivative. The order of truncation error is proportional to #x (grid size) and #t (step size). This implies that as #x and #t decreases, the truncation error decreases. Numerical Differentiation The simplest way to compute a function's derivatives numerically is to use finite difference approximations. Decreasing step size can reduce truncation error, but can also increase the subtractive cancellation or the number of computations, and also, decreased grid size and timestep size result in an increased number of computational operations, which introduces additional error called computational round-off error. As we noticed in the graph, there is a deep fall in between as it is the minima where order of accuracy is highest. Therefore, the trade-off between truncation error and round-off error should be examined carefully.

Question-2

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Question-2(a)

Matlab script to calculate first order numerical differentiation of 2 - x + ln(x) at x = 1

```
a = 1;
truevalue = (1 - a)/a;
disp('First order numerical derivative of 2 - x + ln(x) at x = 1 is
 ');
disp(num2str(truevalue));
h = 1e-4;
```

```
First order numerical derivative of 2 - x + ln(x) at x = 1 is
0
```

(b) ,(c) steps of Q.1 for first order derivative

Question-2(b)

Caluclation of error using forward difference, backward difference, and central difference methods.

```
% Formula of Forward difference
fdiff = (f(a + h) - f(a))/h;
ferr = abs(truevalue - fdiff);
disp('Error in forward difference:- ');
disp(num2str(ferr));
% Formula of Backward difference
bdiff = (f(a) - f(a - h))/h;
berr = abs(truevalue - bdiff);
disp('Error in backward difference:- ');
disp(num2str(berr));
% Formula of Central difference
cdiff = (f(a + h) - f(a - h))/(2*h);
cerr = abs(truevalue - cdiff);
disp('Error in central difference:- ');
disp(num2str(cerr));
% Formula of Improved Forward difference
ifdiff = (-f(a + 2*h) + 4*f(a + h) -3*f(a))/(2*h);
```

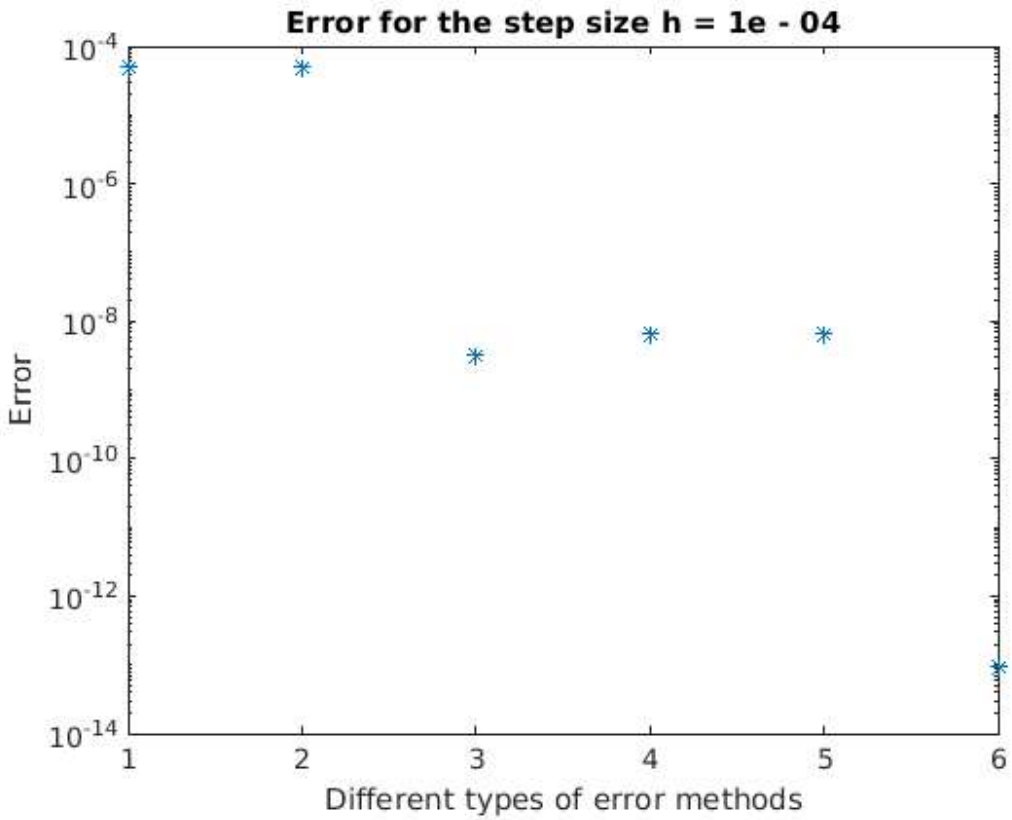
```
iferr = abs(truevalue - ifdiff);
disp('Error in improved forward difference:- ');
disp(num2str(iferr));
% Formula of Improved Backward difference
ibdiff = (3*f(a) - 4*f(a - h) + f(a - 2*h))/(2*h);
iberr = abs(truevalue - ibdiff);
disp('Error in improved backward difference:- ');
disp(num2str(iberr));
% Formula of Improved Central difference
icdiff = (8*f(a + h) - 8*f(a - h) - f(a + 2*h) + f(a - 2*h))/(12*h);
icerr = abs(truevalue - icdiff);
disp('Error in improved central difference:- ');
disp(num2str(icerr));
```

```
Error in forward difference:-
4.9997e-05
Error in backward difference:-
5.0003e-05
Error in central difference:-
3.3334e-09
Error in improved forward difference:-
6.6658e-09
Error in improved backward difference:-
6.668e-09
Error in improved central difference:-
9.2519e-14
```

Question-2(c)

Graph plotings

```
semilogy([ferr, berr, cerr, iferr, iberr, icerr], '*')
title('Error for the step size h = 1e - 04')
ylabel('Error')
xlabel('Different types of error methods')
% Defining Function
function fx = f(x)
 fx = 2 - x + log(x);
end
```



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Question-2

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(d) step of Q.1 for first order derivative

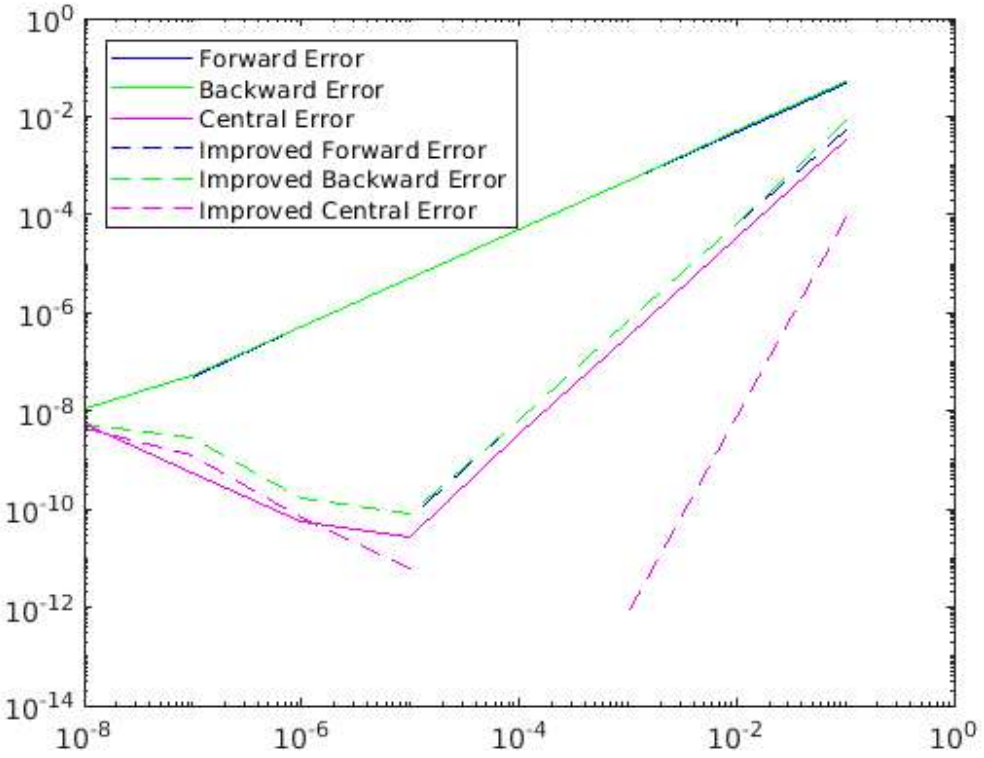
Question-2(d)

Matlab script to calculate first order numerical differentiation of 2 - x + ln(x) at x = 1

```
a = 1;
truevalue = (1 - a)/a;
h = 10.^[-1:-1:-8];
% Formula of Forward difference
fdiff = (f(a + h) - f(a))./h;
ferr = abs(truevalue - fdiff);
disp('Error in forward difference:- ');
disp(num2str(ferr));
% Formula of Backward difference
bdiff = (f(a) - f(a - h))./h;
berr = abs(truevalue - bdiff);
disp('Error in backward difference:- ');
disp(num2str(berr));
% Formula of Central difference
cdiff = (f(a + h) - f(a - h))./(2.*h);
cerr = abs(truevalue - cdiff);
disp('Error in central difference:- ');
disp(num2str(cerr));
% Formula of Improved Forward difference
ifdiff = (-f(a + 2*h) + 4*f(a + h) - 3*f(a))./(2.*h);
iferr = abs(truevalue - ifdiff);
disp('Error in improved forward difference:- ');
disp(num2str(iferr));
% Formula of Improved Backward difference
ibdiff = (3*f(a) - 4*f(a - h) + f(a - 2*h))./(2.*h);
iberr = abs(truevalue - ibdiff);
disp('Error in improved backward difference:- ');
disp(num2str(iberr));
% Formula of Improved Central difference
icdiff = (-f(a + 2*h) + 8*f(a + h) - 8*f(a - h) + f(a - 2*h))./
(12.*h);
icerr = abs(truevalue - icdiff);
disp('Error in improved central difference:- ');
disp(num2str(icerr));
```

```
% Graph plotings
loglog(h, ferr, '-b', h, berr, '-g', h, cerr, '-m');
hold on;
loglog(h, iferr, '--b', h, iberr, '--g', h, icerr, '--m');
legend('Forward Error', 'Backward Error', 'Central Error', 'Improved
 Forward Error', 'Improved Backward Error', 'Improved Central
 Error', 'Location', 'northwest');
hold on;
% Function for 2 - x + ln(x)
function fx = f(x)
  fx = 2 - x + log(x);
end
```

```
Error in forward difference:-
0.046898   0.0049669  0.00049967  4.9997e-05       5e-06  5.0004e-07
 4.996e-08           0
Error in backward difference:-
0.053605   0.0050336  0.00050033  5.0003e-05       5e-06  4.9993e-07
 5.107e-08  1.1102e-08
Error in central difference:-
0.0033535  3.3335e-05  3.3333e-07  3.3334e-09  2.7756e-11  5.5511e-11
 5.5511e-10  5.5511e-09
Error in improved forward difference:-
0.0054042  6.5194e-05  6.6517e-07  6.6658e-09  6.6613e-11           0
          0           0
Error in improved backward difference:-
0.0085074  6.8195e-05  6.6817e-07   6.668e-09  8.3267e-11  1.6653e-10
 2.7756e-09  5.5511e-09
Error in improved central difference:-
8.2954e-05  8.0029e-09  7.5865e-13           0  6.4763e-12  6.4763e-11
  1.2027e-09  4.6259e-09
```



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Question-2

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Question-2(a)

Matlab script to calculate second order numerical differentiation of 2 - x + ln(x) at x = 1

```
a = 1;
truevalue = -1/a.^2;
disp('Second order numerical derivative of 2 - x + ln(x) at x = 1 is
 ');
disp(num2str(truevalue));
h = 1e-4;
```

```
Second order numerical derivative of 2 - x + ln(x) at x = 1 is
-1
```

(b) ,(c) steps of Q.1 for second order derivative

Question-2(b)

Caluclation of error using forward difference, backward difference, and central difference methods. Formula of Forward difference

```
% Formula for Forward difference second order derivative
fdiff = (f(a + 2*h) - 2*f(a + h) + f(a))./(h.*h);
ferr = abs(truevalue - fdiff);
disp(['Error in forward difference for second order derivative:- ',
 num2str(ferr)]);
% Formula for Backward difference for second order derivative
bdiff = (f(a) - 2*f(a - h) + f(a - 2*h))./(h.*h);
berr = abs(truevalue - bdiff);
disp(['Error in backward difference for second order derivative:- ',
 num2str(berr)]);
% Formula for Central difference second order derivative
cdiff = (f(a + h) - 2*f(a) + f(a - h))./(h.*h);
cerr = abs(truevalue - cdiff);
disp(['Error in central difference for second order derivative:- ',
 num2str(cerr)]);
```

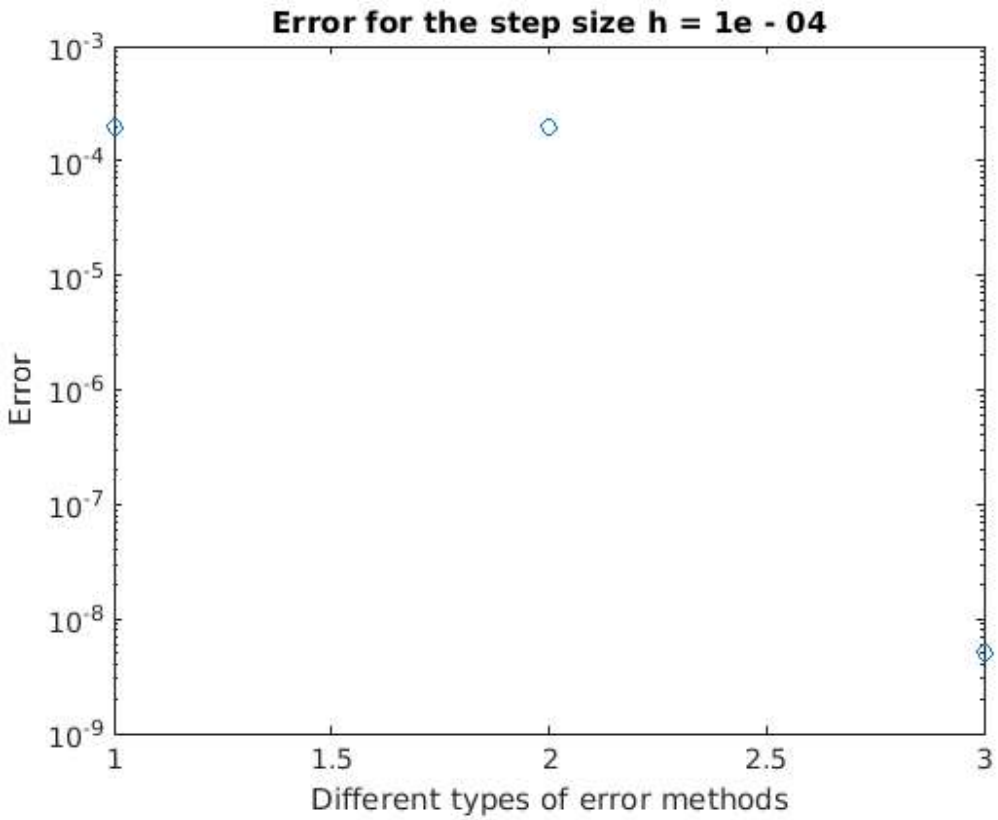
```
Error in forward difference for second order derivative:- 0.00019998
Error in backward difference for second order derivative:- 0.00020003
```

Error in central difference for second order derivative:- 5.0248e-09

Question-2(c)

Graph plotings

```
semilogy([ferr, berr, cerr], 'o')
title('Error for the step size h = 1e - 04')
ylabel('Error')
xlabel('Different types of error methods')
% Defing Function
function fx = f(x)
  fx = 2 - x + log(x);
end
```



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Question-2

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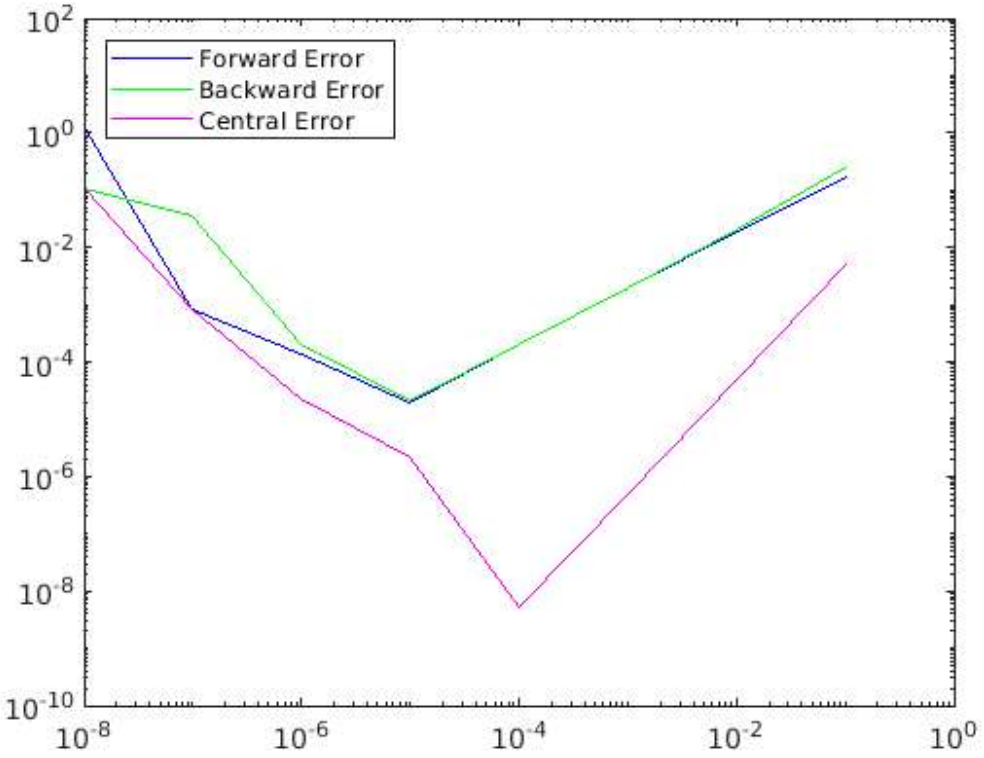
(d) steps of Q.1 for second order derivative

Question-2(c)

Matlab script to calculate second order numerical differentiation of 2 # x + ln(x) at x = 1

```
a = 1;
trueValue = -1/a.^2;
h = 10.^[-1:-1:-8];
% Formula of Forward difference for second order derivative
fdiff = (f(a + 2*h) - 2*f(a + h) + f(a))./(h.*h);
ferr = abs(trueValue - fdiff);
disp('Error in forward difference for second order derivative: ');
disp(num2str(ferr));
% Formula of Backward difference for second order derivative
bdiff = (f(a) - 2*f(a - h) + f(a - 2*h))./(h.*h);
berr = abs(trueValue - bdiff);
disp('Error in backward difference for second order derivative: ');
disp(num2str(berr));
% Formula of Central difference for second order derivative
cdiff = (f(a + h) - 2*f(a) + f(a - h))./(h.*h);
cerr = abs(trueValue - cdiff);
disp('Error in central difference for second order derivative: ');
disp(num2str(cerr));
% Graph Plotings
loglog(h, ferr, '-b', h, berr, '-g', h, cerr, '-m');
legend('Forward Error', 'Backward Error', 'Central
 Error', 'Location', 'northwest');
% Function
function fx = f(x)
  fx = 2 - x + log(x);
end
```

```
Error in forward difference for second order derivative:
0.17012    0.019656   0.0019965  0.00019998  1.9901e-05  0.00013314
 0.00079928      1.2204
Error in backward difference for second order derivative:
0.24225    0.020356   0.0020035  0.00020003  2.1177e-05  0.00019992
 0.034106     0.11022
Error in central difference for second order derivative:
0.0050336  5.0003e-05  5.0018e-07  5.0248e-09  2.1377e-06  2.2122e-05
 0.00079928     0.11022
```



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Question-3

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Question-3(a)

Matlab script to calculate partial derivative of f(x) = sin(x1)exp(-x2)

```
a = [0.5; 1];
h = [10e-6; 10e-6];
truevalue = [cos(a(1))*exp(-a(2)); -sin(a(1))*exp(-a(2))];
disp('Partial derivative of given f(x) is ');
disp(num2str(truevalue));
```

```
Partial derivative of given f(x) is
 0.32284
-0.17637
```

Question-3(b)

Central difference

```
% Partial differentiation w.r.t x
centraldiff(1) = (f(a(1) + h(1), a(2)) - f(a(1) - h(1), a(2)))./
(2.*h(1));
% Partial differentiation w.r.t y
centraldiff(2) = (f(a(1), a(2) + h(2)) - f(a(1), a(2) - h(2)))./
(2.*h(2));
centralerror = abs(truevalue - centraldiff.');
disp('Error in Central Difference is :-');
disp(centralerror)
% The centralerr has two values: The first one is the differentiation
 w.r.t x and second is differentiation w.r.t y.
```

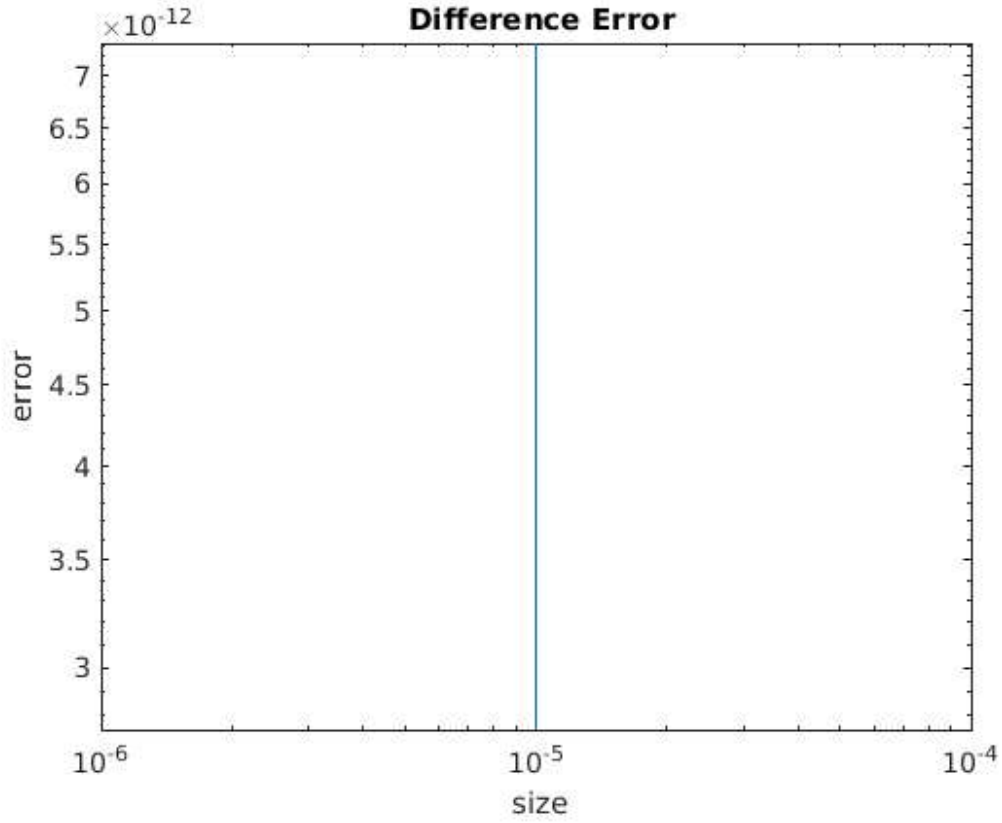
```
Error in Central Difference is :-
   1.0e-11 *

    0.7339
    0.2744
```

Question-3(c)

Graph plotings

```
loglog(h,centralerror);
xlabel(' size');
ylabel('error');
title(' Difference Error');
% Function
function fx = f(x1, x2)
  fx = sin(x1)*exp(-x2);
end
```



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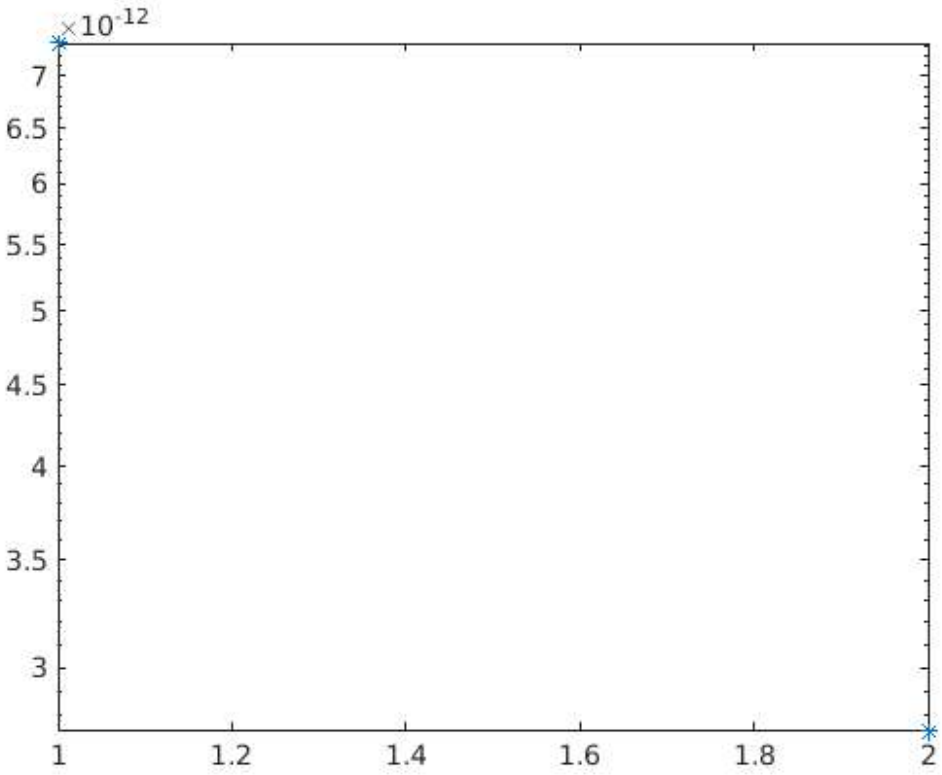
Question-3

Question-3(c)

Graph plotings

```
semilogy(centralerror, '*')
% Function
function fx = f(x1, x2)
  fx = sin(x1)*exp(-x2);
end
```



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