MM I

Reducibles a variable separable

$$y' = f(ax+by+c) \to \begin{matrix} z = ax+by+c \\ y' = (z'-a)/b \end{matrix} \to z' = bf(z)+a$$

Homogéneas

$$y' = f(kx,ky) = f(x,y)\ \forall k \to \begin{matrix} z = y/x \\ y' = z+xz' \end{matrix} \to \text{Separable}$$

Reducibles a homogénea

$$y' = f\left(\frac{ax+by+c}{a'x+b'y+c'}\right);\ \boxed{\text{Caso A. Si } c = c' = 0 \implies \text{Homogénea}}$$

$$\boxed{\text{B. Si } \begin{vmatrix} a & b \\ a' & b' \end{vmatrix} \neq 0;\ \begin{cases} x = \alpha + h \\ y = \beta + k \end{cases} : \frac{ax+by+c}{a'x+b'y+c'} = \frac{a\alpha + b\beta}{a'\alpha + b'\beta}}$$

$$\boxed{\text{C. Si } \begin{vmatrix} a & b \\ a' & b' \end{vmatrix} = 0;\ y' = f\left(\frac{ax+by+c}{d(ax+by)+c'}\right);\ \begin{matrix} z = ax+by \\ \text{Sep.} \end{matrix}}$$

Exactas

$M(x,y)dx + N(x,y)dy = 0$ es exacta $\iff M = \dfrac{\partial f}{\partial x};\ N = \dfrac{\partial f}{\partial y} \iff$ $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$; Entonces encontramos f y la solución es $f(x,y) = C$

Cuasi-exactas Si $M(x,y)dx + N(x,y)dy = 0$ no es exacta, se puede encontrar un $\mu(x,y) : \mu \cdot (M(x,y)dx + N(x,y)dy) = 0$ exacta.

$$\frac{\partial(\mu M)}{\partial y} = \frac{\partial(\mu N)}{\partial x} \to \frac{1}{\mu}\left(N\frac{\partial \mu}{\partial x} - M\frac{\partial \mu}{\partial y}\right) = \frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}$$

Solo puede resolverse suponiendo $\mu = \mu(x)$ o $\mu = \mu(y)$

$$\mu = \mu(x) : \frac{\mu'}{\mu} = \left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)/N = g(x) \to \mu = e^{\int g(x)dx}$$

$$\mu = \mu(y) : \frac{\mu'}{\mu} = \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)/M = h(y) \to \mu = e^{\int h(y)dy}$$

Lineales

$y' + p(x)y = q(x)$ o $\dfrac{dx}{dy} + p(y)x = g(y)$ $\boxed{\text{Caso A. } q(x) = 0 \implies \text{Sep.}}$

Caso B. Si $q(x) \neq 0$; Resolvemos $y_h' + p(x)y_h = 0 \to y = c(x)y_h(x)$

$$c'y_h = q(x) \implies \text{Sep.}$$

Bernoulli

$$y' + p(x)y = q(x)y^n,\quad z = 1/y^{n-1};\quad \frac{-1}{n-1}z' + zp(x) = q(x)$$

Paramétricas

Caso A. $f(y,y') = 0 : y = g(y') \to y' = p;\ \boxed{dx = g'(p)dp/p}$

Caso B. $f(x,y') = 0 : x = g(y') \to y' = p;\ \boxed{dy = pg'(p)dp}$

Lagrange

$$y = xf(y') + g(y') \to y' = p;\ \boxed{(f(p)-p)\frac{dx}{dp} + xf'(p) = -g'(p)}$$

Clairut

$$y = xy' + g(y')\ (\text{Lagrange}) \to dp[x + g'(p)] = 0$$

$$x + g'(p) = 0\ (\text{SP});\quad dp = 0\ (\text{SG})$$

Ricatti

$y' + a(x)y^2 + b(x)y + c = 0$ & SP $y_1 \to y = y_1 + z;\quad y' = y_1' + z'$

Sustituimos y SP = 0 $\to \boxed{z' + (2ay_1 + b)z + az^2 = 0}$ (Bernoulli)

$y' + a(x)y^2 + b(x)y + c = 0$ & 2 SP $y_1;\ y_2 \to \boxed{\dfrac{y-y_1}{y-y_2} = Ce^{\int a(y_2-y_1)dx}}$

Familias de curvas

$h(x,y,\{a_i\}) = 0 \to \dfrac{d^ih}{dx^i} = 0 \to$ quitamos a^i; $F\left(x,y,\{y^{(i)}\}\right) = 0$

$F\left(x,y,-\dfrac{1}{y'}\right)$ es la trayectoria ortogonal a la familia de curvas

Reducción de orden

$$F(x,y^{(k)},\ldots,y^{(n)}) = 0 \to u = y^{(k)} \to F(x,u,\ldots,u^{(n-k)}) = 0$$

$$F(y,\{y^{(i)}\}) = 0 \to y' = p(y);\ y'' = p'p;\ \cdots \to F(y,p,\{p^{(j)}\}) = 0$$

$$F(z,ty,\{ty^{(i)}\}) = t^kF(z,y,\{y^{(i)}\})\ \begin{matrix} y = e^{\int zdx} \\ y' = ze^{\int zdx} \end{matrix}, \ldots F(x,z,\{z^{(j)}\})$$

Teorema de Picard. Si $f(x,y)$ y $\partial f/\partial y$ son funciones continuas sobre un cerrado **R** entonces por cada punto (x_0,y_0) del interior de **R** pasa una única curva integral de la ecuación $dy/dx = f(x,y)$.

Teorema de exist. y unicidad 2º orden lineal. Sean $P(x)$, $Q(x)$, $R(x)$ continuas en un intervalo cerrado $[a,b]$, entonces si $x_0 \in [a,b]$ e y_0 , y_0' son arbitrarios, la EDO: $y'' + P(x)y' + Q(x)y = R(x)$ tiene una única solución $y(x)$ en $[a,b]$ tal que $y(x_0) = y_0$ y $y'(x_0) = y_0'$.

SG 2º orden lineal homogénea y_1 e y_2 SPH lin. indp.

$y(x) = C_1y_1(x) + C_2y_2(x)$ SG; $W = y_1y_2' - y_2y_1' = Ke^{-\int Pdx} \neq 0$

Conocida y_1 SPH ; $y_2 = u(x)y_1$ también SPH; $u = \displaystyle\int \frac{e^{-\int Pdx}}{y_1^2}dx$

SG 2º orden lineal homogénea Coef. Cte.

$$y'' + py' + qy = 0;\quad y(x) = e^{mx};\quad m^2 + pm + q = 0$$

$$y = C_1e^{m_1x} + C_2e^{m_2x};\ y = e^{ax}(C_1\cos bx + C_2\sin bx)$$

$$y = C_1e^{-px/2} + C_2xe^{-px/2}[m_1 \neq m_2(\mathbb{R}); a \pm bi(\mathbb{C}); m_1 = m_2(\mathbb{R})]$$

Ecuación equidimensional de Euler

$$x^2y'' + pxy' + qy = 0;\ x = e^z(\text{coef. cte.})\ \text{o}\ y = x^m$$

Transformar EDOH a coef. cte

$$\frac{Q' + 2PQ}{Q^{3/2}} = \alpha \implies z = \int \sqrt{Q}dx, y''(z) + \frac{\alpha}{2}y'(z) + y(z) = 0$$

Lineal inhomogénea de 2 orden

$$y(x) = y_h(x) + y_p(x);\quad y_h \text{ SGH y } y_p \text{ SPI}$$

Coeficientes indeterminados

$y'' + py' + qy = R(x);\ R(x) : e^{ax}$ (a); $\sin ax$ o $\cos ax$ (b); $\displaystyle\sum_{i=0}^{n} a_ix^i$ (c)

(a) $y_p = Ae^{ax}; a^2 + pa + q \neq 0 \,|\, y_p = Axe^{ax}; 2a + P \neq 0 \,|\, y_p = Ax^2e^{ax}$

(b) $y_p = A\cos ax + B\sin ax; (b-q)^2 + (bp)^2 \neq 0 \mid \bar{y}_p = xy_p$

(b) $y_p = \displaystyle\sum_{i=0}^{n} A_ix^i; q \neq 0 \mid y_p = x\sum_{i=0}^{n} A_ix^i$

Variación de ctes. (Cualquier EDOI 2º)

$$y_p = u_1(x)y_1(x) + u_2(x)y_2(x) \text{ a partir de la EDOH}$$

$$u_1'y_1 + u_2'y_2 = 0;\ u_1'y_1' + u_2'y_2' = R(x); \text{Resolver sist. lineal}$$

Series de Potencias

$$\sum_{n=0}^{\infty} a_nx^n; R = \lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right|;\ f(x)\cdot g(x) = \sum_{n=0}^{\infty}\sum_{i=0}^{n} p_iq_{n-i}x^n$$

Teorema: Sea x_0 un punto ordinario de la EDO $y'' + y'' + P(x)y' + Q(x)y = 0$ y sean a_0, a_1 constantes arbitrarias $\implies$ existe una única función $y(x)$ analítica en x_0 que es solución de la EDO en un mismo entorno de x_0 que $f(x)$ y $g(x)$ y satisface $y(x0) = a0$; $y'(x_0) = a_1$.