MM I - Abel Rosado

Variables separable

$$y' = \frac{dy}{dx} = f(x)\cdot g(y) \to \int \frac{dy}{g(y)} = \int f(x)\,dx + C$$

Reducibles a variables separable

$$y' = f(ax+by+c) \to \begin{matrix} z = ax+by+c \\ y' = (z'-a)/b \end{matrix} \to z' = bf(z)+a \implies \text{Separable}$$

Homogéneas

$$y' = f(kx,ky) = f(x,y)\,\forall k \to \begin{matrix} z = y/x \\ y' = z+xz' \end{matrix} \to y' = f(x,y) = f(1,z)$$

$$z + xz' = f(1-z) \to z' = [f(1,z)-z]\frac{1}{x} \to \text{Separable}$$

Reducibles a homogénea

$y' = f\left(\frac{ax+by+c}{a'x+b'y+c'}\right)$; $\boxed{\text{Caso A. Si } c = c' = 0 \implies \text{Homogénea}}$

Caso B. Si $\begin{vmatrix} a & b \\ a' & b' \end{vmatrix} \neq 0$; $\begin{cases} x = \alpha + h \\ y = \beta + k \end{cases} : \frac{ax+by+c}{a'x+b'y+c'} = \frac{a\alpha+b\beta}{a'\alpha+b'\beta}$

$\begin{pmatrix} a & b \\ a' & b' \end{pmatrix}\begin{pmatrix} h \\ k \end{pmatrix} = -\begin{pmatrix} c \\ c' \end{pmatrix} \to y' = \beta' = f\left(\frac{a\alpha+b\beta}{a'\alpha+b'\beta}\right) \implies$ Caso A

Caso C. Si $\begin{vmatrix} a & b \\ a' & b' \end{vmatrix} = 0 \implies \begin{matrix} a' = d\cdot a \\ b' = d\cdot b \end{matrix} \to y' = f\left(\frac{ax+by+c}{d(ax+by)+c'}\right)$

$\begin{matrix} z = ax+by \\ y' = (z'-a)/b \end{matrix} \to z' = a + bf\left(\frac{z+c}{dz+c'}\right) \implies$ Sep.

Exactas

$M(x,y)dx + N(x,y)dy = 0$ es exacta $\iff M = \frac{\partial f}{\partial x}$; $N = \frac{\partial f}{\partial y} \iff$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$; Entonces encontramos f y la solución es $f(x,y) = C$

Cuasi-exactas

Si $M(x,y)dx + N(x,y)dy = 0$ no es exacta, se puede encontrar un $\mu(x,y)$ t.q. $\mu\cdot(M(x,y)dx + N(x,y)dy) = 0$ es exacta.

$$\frac{\partial(\mu M)}{\partial y} = \frac{\partial(\mu N)}{\partial x} \to \frac{1}{\mu}\left(N\frac{\partial\mu}{\partial x} - M\frac{\partial\mu}{\partial y}\right) = \frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}$$

Solo puede resolverse suponiendo $\mu = \mu(x)$ o $\mu = \mu(y)$

$\mu = \mu(x) : \frac{\mu'}{\mu} = \left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)/N = g(x)$ función solo de x.

$\mu = \mu(y) : \frac{\mu'}{\mu} = \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)/M = h(y)$ función solo de y.

Si g depende también de la otra variable, no podemos resolverla.

$\frac{1}{\mu}\frac{d\mu}{dx} = g(x) \to$ Sep. $\to \mu\cdot(M(x,y)dx + N(x,y)dy) = 0$ exacta

Lineales

$y' + p(x)y = q(x)$ $\boxed{\text{Caso A. Si } q(x) = 0 \implies \text{Sep.}}$

Caso B. Si $q(x) \neq 0$; Resolvemos $y_h' + p(x)y_h = 0 \to y = c(x)y_h(x)$

$y' = c'y_h + cy_h' \to c'y_h + cy_h' + p(x)cy_h = q(x)$

$c'y_h + c(y_h' + p(x)y_h) = q(x) \to c'y_h = q(x) \implies$ Sep.

Bernoulli

$y' + p(x)y = q(x)y^n,\ n \neq 0 \neq 1 \to \begin{matrix} z = 1/y^{n-1};\ y = z^{-1/(n-1)} \\ y' = [-1/(n-1)]z^{-n/(n-1)}z' \end{matrix}$

$\frac{-1}{n-1}z^{\frac{-n}{n-1}}z' + p(x)z^{\frac{-1}{n-1}} = q(x)z^{\frac{-n}{n-1}} \to \frac{-1}{n-1}z' + zp(x) = q(x)$ (Lineal)

Paramétricas

Caso A. $f(y,y') = 0 : y = g(y') \to y' = \frac{dy}{dx} = p;\ dy = pdx$

$\boxed{y = g(p)} \to dy = g'(p)dp = pdx \to \boxed{dx = \frac{g'(p)}{p}dp}$ (Sep.)

Caso B. $f(x,y') = 0 : x = g(y') \to y' = \frac{dy}{dx} = p;\ dx = dy/p$

$\boxed{x = g(p)} \to dx = g'(p)dp = dy/p \to \boxed{dy = pg'(p)dp}$ (Sep.)

Lagrange

$y = xf(y') + g(y') \to y' = p;\ dy = f(p)dx + xf'(p)dp + g'(p)dp = pdx$

$p\frac{dx}{dp} = f(p)\frac{dx}{dp} + xf'(p) + g'(p) \to \boxed{(f(p)-p)\frac{dx}{dp} + xf'(p) = -g'(p)}$

$\implies$ (Lineal) $x =$ [sol. lineal]; $y(p) =$ [sol. lineal]$f(p) + g(p)$

Clairut

$y = xy' + g(y')$ (Lagrn.) $(p)dx + xdp + g'(p)dp = pdx \to dp[x + g'(p)] = 0$

$x + g'(p) = 0 \to x = -g'(p)$; $y = -g'(p)p + g(p)$ (Sol. particular)

$dp = 0 \implies p = c \implies y = cx + g(c)$ (Sol. general)

Ricatti

$y' + a(x)y^2 + b(x)y + c = 0$ & SP $y_1 \to y = y_1 + z$; $y' = y_1' + z'$

Sustimos y SP = 0 $\to \boxed{z' + (2ay_1 + b)z + az^2 = 0}$ (Bernoulli)

$y' + a(x)y^2 + b(x)y + c = 0$ & 2 SP $y_1;\ y_2 \to \boxed{\frac{y-y_1}{y-y_2} = Ce^{\int a(y_2-y_1)dx}}$

Familias de curvas

$y = g(x,a) \to y' = \frac{\partial g}{\partial x} \to$ eliminamos a y obtenemos $F(x,y,y') = 0$

$h(x,y,a) = 0 \to \frac{dh}{dx} = \frac{\partial h}{\partial x} + \frac{\partial h}{\partial y}y' = 0 \to$ eliminamos a..., $F(x,y,y') = 0$

$F(x,y,-\frac{1}{y})$ es la trayectoria ortogonal a la familia de curvas

$h(x,y,\{a_i\}) = 0 \to \frac{d^ih}{dx^i} = 0 \to$ eliminamos a^i ..., $F(x,y,\{y^{(i)}\}) = 0$

Reducción de orden

$F(x, y^{(k)}, \dots, y^{(n)}) = 0 \to u = y^{(k)} \to F(x,u,\dots,u^{(n-k)}) = 0$

$F(y,\{y^{(i)}\}) = 0 \to y' = p(y);\ y'' = p'p;\ \cdots \to F(y,p,\{p^{(j)}\}) = 0$