Satellite Tracking using Kalman Filter

Digital Signal Processing Report

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Introduction

In the realm of satellite tracking, accurate and reliable orbital predictions are crucial for a variety of applications, including communication, Earth observation, and scientific research. The dynamic nature of space environments, influenced by gravitational forces and atmospheric drag, makes real-time tracking and precise trajectory estimation challenging. To address this complexity, the project aims to implement a Kalman Filter-based approach to enhance the accuracy of satellite tracking systems.

Objective

The primary goal of this project is to implement the Kalman Filter as a predictive tool to continually enhance the precision of satellite tracking. This involves leveraging the filter's adaptive nature to iteratively update and correct orbital predictions based on real-time measurements. By dynamically adjusting predictions, the project seeks to mitigate errors caused by environmental factors, gravitational perturbations, and other dynamic influences on satellite trajectories. Various noises that get added to the system are filtered out by the Kalman filter. We simulate various parameters as measured by the satellite sensors and then add noise to it. We proceed to remove the noise using the Kalman Filter.

Approach

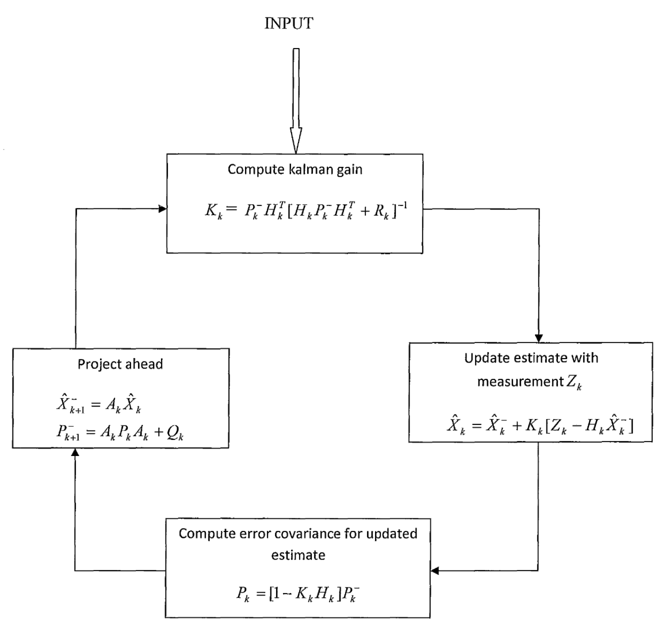
1. **Kalman Filter Application:**
   The project integrates the Kalman Filter into the satellite tracking system to optimize the estimation of the satellite's state (position, velocity, and other relevant parameters) by combining predictions from mathematical models with observed measurements.
2. **Orbital Dynamics Modeling:**
   Accurate modeling of the complex orbital dynamics is a critical component. Mathematical models consider gravitational forces, atmospheric drag, and other factors affecting the satellite's movement, providing a foundation for the Kalman Filter to refine predictions.

3. **Real-Time Data Integration:**
   Various sensors, such as GPS, radar, and inertial sensors, contribute real-time data on the satellite's position and velocity. The Kalman Filter processes this information dynamically, allowing for continuous adjustment of the predicted trajectory.
4. **Adaptive Correction Mechanism:**
   The Kalman Filter's adaptive nature enables it to learn from past predictions and observations, adjusting the estimation process to account for uncertainties and inaccuracies in the data. This correction mechanism contributes to improved tracking precision.

Theory

Kalman Filter

The Kalman filter has several uses in technology. Guidance, navigation, and control of vehicles—especially ships, airplanes, and spacecraft positioned dynamically—are often used applications. In addition, Kalman filtering is a widely utilized concept in time series analysis for econometrics and signal processing. One of the key concepts in robotic motion planning and control is Kalman filtering, which is also useful for optimizing trajectories. The movement control system of the central nervous system can also be modeled using Kalman filtering. The use of Kalman filters offers a realistic model for estimating the current state of a motor system and issuing updated commands because of the time lag between issuing motor commands and receiving sensory feedback.



The Algorithm operates in two stages, one for prediction and the other for updating. The Kalman filter generates estimates of the current state variables and their uncertainty for the prediction phase. The following measurement's result, which is inevitably tainted by error and random noise, is then observed, and these estimates are updated using a weighted average, where estimates with higher certainty are assigned more weight. The process is iterative. It does not require any more historical data to work in real time; all that is needed are the current input measurements, the previously calculated state, and its uncertainty matrix.

Common Satellite Tracking Parameters

For the situation presented, let

V= the satellite velocity

X_0= the horizontal distance

Z_0= the altitude above the horizon

$$a = \frac{V}{X_0}$$

$$b = \frac{Z_0}{X_0}$$

$$\theta_{AZ} = \tan^{-1} at$$

$$\theta_{EL} = \tan^{-1} \frac{b}{\sqrt{1+(at)^2}}$$

The dynamic equations of motions in the antenna reference frame are

$$\ddot{\theta} = -a\dot{\theta}_{AZ} \sin 2\theta_{AZ} + w_1(t)$$

$$\ddot{\theta}_{EL} = \dot{\theta}_{EL} \{\frac{1}{t} + \frac{\dot{\theta}_{El}}{\cos^2 \theta_{Elt}} [\frac{3}{\tan \theta_{El}} - \sin 2\theta_{EL}]\} + w_2(t)$$

Assigning the states to be $\theta_{AZ} = X_1, \dot{\theta}_{AZ} = X_{2,} \theta_{EL} = X_4, \dot{\theta}_{EL} = X_5$ and adding two more states for the acceleration noise, the system equations are

$$\dot{X} = X_2$$

$$\dot{X}_2 = -aX_2 \sin 2X_1 + X_3$$

$$\dot{X}_3 = -\frac{1}{\beta_{AZ}} X_3 + G_{AZ} W_1(t)$$

$$\dot{X}_4 = X_5$$

$$\dot{X}_5 = -\frac{a^2}{b^2} \cos^2 X_4 \tan^3 X_4 - \frac{a}{b^2} \tan X_1 \tan^2 x_4 [3 - 2\sin^2 X_4] X_5 + X_6$$

$$\dot{X}_6 = -\frac{1}{\beta_{EL}} X_6 + G_{EL} W_2(t)$$

The dynamic equations requires for the system are now in the form

$$\dot{X}(t) = f(X(t), t) + G(X(t), t) W(t)$$

With each measurement, the Kalman filter gains, error covariance's and state estimates are recalculated. New estimates of the states are produced from

$$f_1 = \hat{X}_2$$

$$f_2 = -a\hat{X}_2 \sin 2\hat{X}_1 + \hat{X}_3$$

$$f_3 = -\frac{1}{\beta_{AZ}} \hat{X}_3$$

$$f_4 = \hat{X}_5$$

$$f_5 = -\frac{a^2}{b^2} \cos^2 \hat{X}_4 \tan^3 \hat{X}_4 - \frac{a^2}{b^2} \tan \hat{X}_1 \tan^2 \hat{X}_4 [3 - 2\sin^2 \hat{X}_4] \hat{X}_5 + \hat{X}_6$$

$$f_6 = -\frac{1}{\beta_{EL}} \hat{X}_6$$

By using these previous physics equations, we apply the discrete linearised Kalman filter on these equations and the matrix elements are determined as per the code. We simulate input and white noise sequence and filter out the noise.

Noise

The optimality of Kalman filtering is based on the assumption that errors have a normal (Gaussian) distribution. Regardless of Gaussianity, if the process and measurement covariances are known, the Kalman filter is the best linear estimator in terms of least mean-square-error.

How accurately the system's state can be ascertained is limited by noisy sensor data, approximations in the equations that describe the system evolution, and unaccounted-for external influences. The uncertainty caused by noisy sensor data and, to some extent, random external influences is successfully handled by the Kalman filter. By averaging the system's anticipated state and the new measurement using a weighted average, the Kalman filter generates an estimate of the system's state. Values with greater (i.e., smaller) estimated uncertainty are "trusted" more, according to the weights. The covariance, which is a gauge of the predicted uncertainty of the system's state prediction, is used to compute the weights. A new state estimate that falls between the measured and anticipated states and has a better estimated uncertainty than either one alone is the outcome of the weighted average. Every time step, this process is repeated, with the new estimate and its covariance influencing the forecast made in the subsequent iteration. This indicates that the Kalman filter is recursive in nature and that it just needs the most recent "best guess"—rather than the complete history of a system's state—to determine a new state.

We use the standard equations for noise covariance matrix and noise spectral density matrix for discrete time Kalman filter.

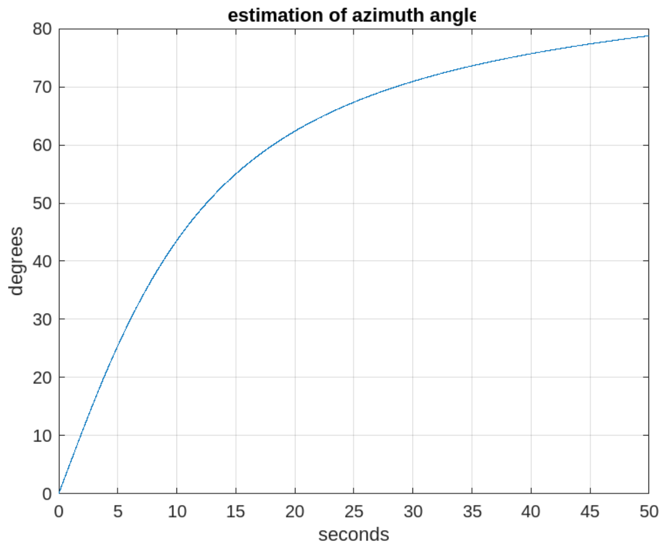
$$Q_K = G(t)Q(t)G^T(t)$$

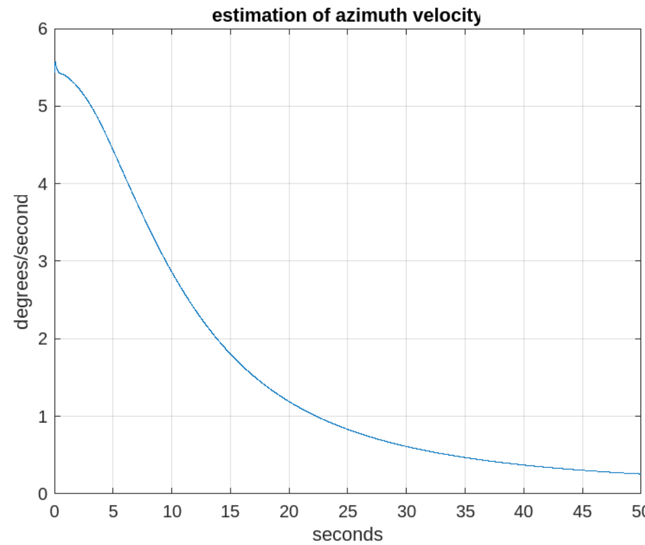
Noise covariance matrix Qk.

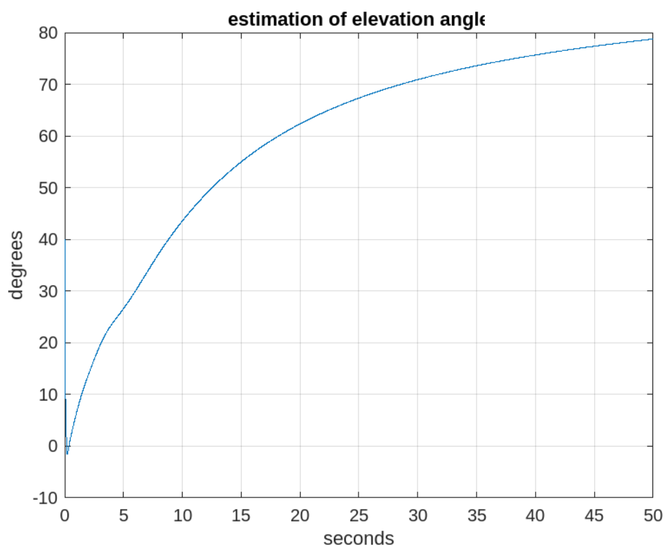
$$R_K = \begin{bmatrix} \sigma_{AZ}^2 & 0 \\ 0 & \sigma_{EL}^2 \end{bmatrix}$$

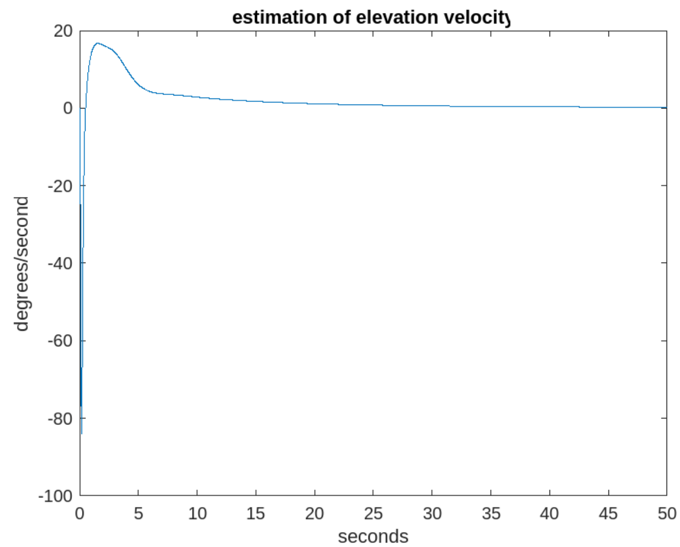
Noise spectral density matrix

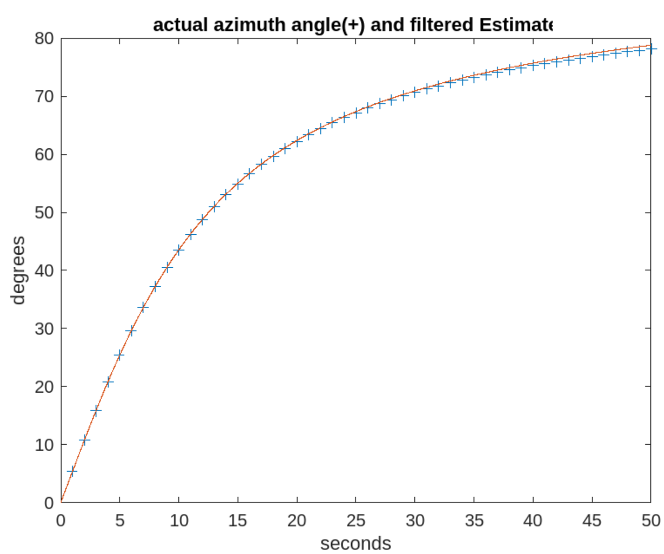
Observations

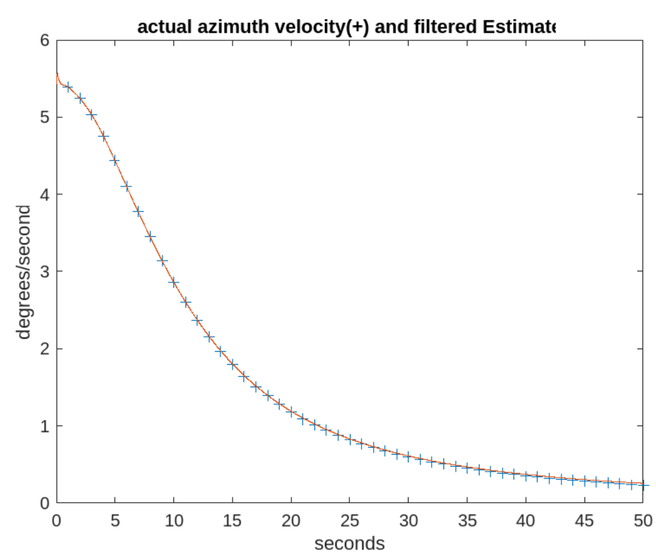


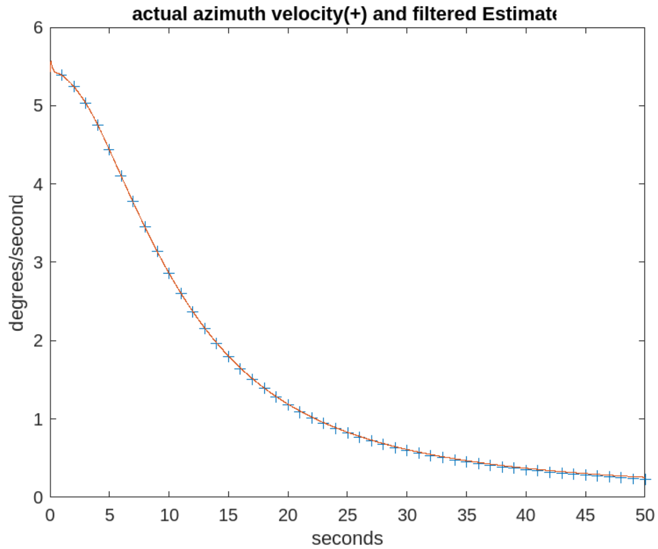












Here, we can see the various parameters comparison with true values and filtered values. The true values are added with white noise mentioned above and then filtered. The output from the discrete filter is plotted alongside true values in the last three graphs.

Benefits

1. Precision Improvement: By integrating the Kalman Filter, the project aims to significantly improve the precision of satellite tracking, providing more accurate and reliable position information.
2. Dynamic Adaptability: The system's ability to adapt to changing conditions in real-time ensures that it remains effective in various environmental scenarios, contributing to robust and resilient satellite tracking.
3. Versatility: The proposed solution is versatile and can be applied to a range of satellite types and orbital configurations, making it a valuable asset for diverse satellite-based applications.

Conclusion

In conclusion, the integration of the Kalman Filter into satellite tracking systems presents a substantial leap forward in enhancing the precision, reliability, and adaptability of orbital predictions. The project successfully demonstrated the efficacy of the Kalman Filter in refining satellite trajectory estimations by dynamically incorporating real-time sensor data and iteratively adjusting predictions. The key components of the project, including the Kalman Filter application, orbital dynamics modeling, real-time data integration, and the adaptive correction mechanism, collectively contribute to a more robust and accurate satellite tracking solution.