Truthful Auctions for Continuous Spectrum with Variable Bandwidths

Tingting Chen and Sheng Zhong

***Abstract*—Dynamic spectrum auctions have been considered a promising approach to effectively re-distribute spectrum resources in the secondary spectrum market. However, the existing spectrum auctions are limited to allocating spectrum in units of channels. Recently software defined radio technologies make exciting progress in operating radios with variable bandwidths. They push the need for designing more flexible spectrum auction frameworks that allow to allocate spectrum with variable bandwidth to the secondary user. In this paper, we design truthful spectrum auction frameworks in which secondary users can bid for, and then be actually allocated spectra with variable bandwidths. We first present a truthful framework for auctions of variable-bandwidth spectra in single collision domains, which can achieve system efficiency. Then, we propose a similar framework for multiple collision domains and rigorously show that it is also truthful. Results of extensive evaluations demonstrate that both of our spectrum auction frameworks for variable bandwidth are effective.**

***Index Terms*—Dynamic wireless spectrum allocation, secondary spectrum market, spectrum auction.**

I. INTRODUCTION

WITH the fast-growing popularity of wireless network technologies, the wireless spectrum is becoming increasingly crowded. At the same time, measurements show that the spectrum allocated through static auctions is under utilized [20]. In order to utilize wireless spectrum more efficiently, recently a lot of attention has been given to building a secondary spectrum market, in which primary spectrum users can dynamically sell available channels to secondary users, through real-time auctions. Some spectrum auction frameworks have been proposed, with the goals of truthfulness (the property that secondary users are auctioning for the spectrum that they truly desire), social efficiency (optimal distribution of spectrum in the system in terms of total valuation), maximum revenue and/or fairness (e.g., [33], [14], [10]).

Dynamic spectrum auctions are effective in re-allocating spectrum resources and providing incentives for primary users to re-distribute their spectrum. However, we notice that the existing auction frameworks have the following limitation: In existing auction frameworks, available spectrum can only be auctioned in units of channels, i.e., each secondary user bids for one or multiple channels and the payments due are calculated based on the allocated channels. In contrast, recently advanced software defined radio technologies push the need for more flexible spectrum auction frameworks that allow to allocate spectra with variable bandwidths to secondary users. Specifically, these technologies enable the communication devices to operate with variable bandwidths, e.g., the 802.22 draft already includes the support for variable bandwidth [2]. Moreover, recent studies (e.g., [32], [3]) have focused on building wireless networks in which the channel bandwidth can be adaptively changed. All the exciting progress in operating radios in variable bandwidths becomes an outcry for spectrum auction frameworks that allow primary users to allocate spectra in variable bandwidths, not just channels with a fixed bandwidth.

To the best of our knowledge, no existing work has studied variable-bandwidth spectrum auctions. In this paper, we aim to design a truthful auction framework that supports allocating variable bandwidths of spectra. Moreover, since system efficiency is a natural requirement when considering a spectrum auction, we also require our auction framework to be system efficient. Here system efficiency means that the system-wide total valuation should be maximized (see Section II for the precise definition).

The major challenges in designing such auction frameworks come from the nature of variable bandwidth. First, bidding for variable bandwidths makes it more complicated to describe a secondary user's demand for, and valuation of, the spectrum. Second, when allocating variable-bandwidth spectrum, we need to be more careful in order to avoid interference. Different from the channel allocation, where interference happens only when neighbors are assigned to exactly the same channel, in the variable-bandwidth spectrum allocation, interference exits as long as the overlap of neighbors' assigned spectra is not zero. Third, in order to guarantee truthfulness, it is more challenging to determine the amount of charge for a variable bandwidth spectrum than for a channel with a fixed bandwidth.

In addition to the above challenges, there is another factor that contributes to the hardness of the problem we try to solve: In this paper, we allow each secondary user to have multiple devices located at different positions. To bid for spectrum, each secondary user submits his valuation of spectrum for each device he owns. This further complicates the spectrum auction problem, because a secondary user can cheat in his claim of valuation on one device, which may benefit his other devices, so that he can obtain higher overall utilities. We will show how to tackle this difficulty in Section IV.

Manuscript received June 14, 2013; revised October 4, 2013; accepted November 7, 2013. The associate editor coordinating the review of this paper and approving it for publication was Y.-C. Ko.

This work was supported by RPGE, NSFC-61021062, and NSFC-61300235.

Part of the work was done while both authors were at the University at Buffalo and supported by NSF-0845149.

T. Chen is with the Computer Science Department, Oklahoma State University (e-mail: tingting@cs.okstate.edu).

S. Zhong, the corresponding author, is with the State Key Laboratory for Novel Software Technology, Nanjing University, and also with the Department of Computer Science and Technology, Nanjing University (e-mail: zhongsheng@nju.edu.cn).

Digital Object Identifier 10.1109/TWC.2013.120413.131072

In order to support variable bandwidth spectrum auctions, we need to enable secondary users to submit their valuations for each possible bandwidth of spectrum on each device. So, unlike existing dynamic spectrum auction frameworks, our frameworks use a valuation function to describe each secondary user's valuation of all possible allocations of spectrum on each device. Hence, in an auction, each secondary user submits a set of valuation functions, not a set of numbers, to the primary user, as his bid. Valuation functions promise a more flexible form of bidding, by which variable bandwidth of spectrum can be auctioned.

Our main results in this paper are two spectrum auction frameworks for allocating variable bandwidths of spectra, in different settings. Our first framework, called VSA-S, is for Variable bandwidth Spectrum Auction in Single collision domains. We rigorously show that VSA-S is truthful. In addition, we show that VSA-S is *system efficient.* Our second framework, called VSA-M, is for Variable bandwidth Spectrum Auction in Multiple collision domains. We show that it is truthful just like VSA-S.

Our contributions can be summarized as follows:

- We are the *first* to study spectrum auctions for allocating variable bandwidth spectrum to secondary users.
- For variable bandwidth spectrum auction in single collision domains, we present an auction framework VSA-S. We rigorously show that VSA-S is truthful and system efficient.
- For variable bandwidth spectrum auction in multiple collision domains, we present a framework VSA-M that can be shown to be truthful.
- We have done extensive experiments to evaluate our spectrum auction frameworks and the results demonstrate that they have good performance.

The rest of this paper is organized as follows. In Section II, we present the technical preliminaries. In Sections III, we propose VSA-S and in Section IV we propose VSA-M. In Section V, we discuss the implementation of virtual money. Section VI is dedicated to evaluation results. Finally, we briefly review related literature in Section VII and then conclude in Section VIII.

II. Technical Preliminaries

We consider spectrum auctions in which a primary user sells spectra with variable bandwidths to K secondary users. Suppose that each secondary user i has D_i devices that need to use a bandwidth of spectrum. We use an interference graph to model the interference among the devices. If and only if two devices are within the interference range of each other, they become neighbors (i.e., connected by an edge) in the interference graph. (Throughout the paper, we use "neighbor" to refer to a neighbor in the interference graph.)

We consider a spectrum auction setting where the primary spectrum owner can sell continuous wireless spectrum to secondary users. A centralized auctioneer can obtain the information about available spectrum stored in an occupancy database. According to their needs, secondary users submit their bids to the auctioneer, and the auctioneer runs the auction to dynamically sell the continuous spectrum to the winning secondary user. All transactions are completed at the auctioneer.

Assume that the information about available spectrum stored in an occupancy database as required by FCC [1] changes slowly over time.[1] Suppose that the frequency spectrum held by the primary user available for auction is (f_l, f_h), where $f_h - f_l = W$. Unlike the previous works where secondary users submit numbers to represent their valuations in order to bid for channels, in our auctions each secondary user i submits a valuation function set V_i to the primary user. In particular, for each of his device d, i submits a valuation function $v_{i,d}()$. The input of this valuation function is the bandwidth that is available to the device (i, d). The output of this valuation function is the secondary user i's valuation on device d of using this assigned bandwidth. Intuitively, $v_{i,d}()$ represents the i's satisfaction levels on device d of using different bandwidths of spectra. We adopt the standard assumption from the literature of economics [17] that every valuation function $v_{i,d}()$ is strictly increasing and quasi-concave.

Upon receiving the valuation functions from the secondary users, the primary user first assigns spectrum with variable bandwidth to each winning device according to the valuation functions, and then computes the corresponding prices for using the spectra charged to the secondary users. Denote by $w_{i,d}$ the bandwidth of spectrum that is allocated to device (i, d) in the auction.

A. Truthfulness

In the spectrum auction, the utility for each secondary user i is decided by the profile of valuation function sets of all secondary users. We denote this profile by V. Formally, we have:

$$\mathsf{u}_i(\mathsf{V}) = \sum_{1 \le d \le D_i} v_{i,d}(w_{i,d}) - p_i, \tag{1}$$

Intuitively, this means that the utility of secondary user i is equal to i's total valuation of the spectra assigned to his devices minus the payment he needs to make for his use of spectra.

Given this game theoretic model, a spectrum auction in our scenario is truthful if and only if it is a *dominant strategy equilibrium* (DSE) [24] for all secondary users to submit their true valuation functions. Intuitively, a DSE guarantees that every player of the game has incentives to play the strategies specified by the DSE regardless of other players' behavior.

Definition 1. *A spectrum auction is said to be truthful if it is a DSE for all secondary users to submit their true valuation function sets, i.e., for any secondary user i, assuming $V_i^T()$ is the true valuation function set of secondary user i, for any valuation function set $V_i^A()$ submitted by secondary user i, for any profile V_{-i} of valuation functions submitted by all secondary users other than i,*

$$\mathbf{u}_i(V_i^T(), \mathsf{V}_{-i}) \ge \mathbf{u}_i(V_i^A(), \mathsf{V}_{-i}).$$

[1]The technologies of detecting primary users and accessing the spectrum over small timescales have been proposed by some existing works, e.g., [19].

Another goal of our design is to achieve system efficiency for the spectrum allocation. System efficiency is defined as follows.

Definition 2. *A spectrum auction is system efficient if the spectrum allocation achieves the maximum system-wide total utility, i.e.,*

$$(w_{1,1}, \cdots, w_{i,d}, \cdots) = \mathit{arg}\ \max_{w_{i,d}} \sum_i \sum_d v_{i,d}(w_{i,d}).$$

III. Auction Framework for Single Collision Domain

In this section, we design and analyze a variable bandwidth spectrum auction framework, VSA-S, for the situation in which all devices are in a single collision domain. The design goal is to achieve the maximum system-wide total valuation and to guarantee truthfulness as well. Although the major contribution of this paper is the design of the spectrum auction framework for multiple-collision domains, thoroughly studying a simpler case allows us to better understand the nature of spectrum auction for variable bandwidth, and develop the key technique also used in multiple collision domains.

A. Design of Auction Framework

Now we first formalize the spectrum allocation problem in a single collision domain. Suppose that after the auction, each device (i, d) has been assigned a bandwidth $w_{i,d}$ of spectrum. Since all devices are in a same collision domain, to avoid interference, we must have $\sum_i \sum_d w_{i,d} = W$. To maximize the system-wide total valuation, the spectrum allocation problem becomes the optimization problem

$$\text{Maximize} \sum_i \sum_d v_{i,d}(w_{i,d})$$

$$\text{Subject to:} \sum_i \sum_d w_{i,d} = W, \text{and } w_{i,d} \geq 0, \forall i, \forall d.$$

One straightforward solution is to use the method of Lagrange multipliers [4] to find the maximum total valuation and the bandwidth of spectrum that each device should be allocated. In the payment step, for each winning device, the price of the bandwidth can be calculated in the fashion similar to the well-known VCG mechanism[27]. In particular, after temporarily removing the winning device (i, d), the above spectrum allocation algorithm is executed again without (i, d). Then the price for (i, d) is calculated as the sum of the valuation increases of all other devices.

However, when the number of devices is large, to find the maximum total valuation and compute the payments will take a very long time, which is not desirable for real-time spectrum auctions. To remedy this problem, we design a method to enable allocating the spectrum with maximum total valuation with low computation cost and computing the payments for all devices in one round. In particular, we discretize the spectrum into slices. Assume that there is a constant ϵ such that all assigned bandwidths must be multiples of ϵ.[2]

[2]Since the precision of involved computing is limited, there must be such a small constant ϵ.

We design VSA-S, the details of which are shown in Algorithm 1. We adopt the well-known multi-unit VCG auction mechanism to achieve system efficiency and truthfulness. The key ideas of multi-unit VCG are greedy spectrum allocation and opportunity-cost-based payment. In particular, we rank all valuations of spectrum slices (from all secondary users) and pick the N largest. Our algorithm assigns the N spectrum slices corresponding to N highest slice valuations. A winning secondary user i is charged the total valuation of all the newly assigned slices that replace the slices originally assigned to i's devices, when the system is without i.

Algorithm 1 VSA-S: Variable bandwidth spectrum auction framework for single collision domain

1: **INPUT:** Available spectrum (f_l, f_h); valuation function set V_i from each secondary user i, and a small number ϵ^3.
2: **OUTPUT:** Allocated spectrum for device d of each secondary user i: $(L_{i,d}, H_{i,d})$, and price p_i for each i.
3: $N = \frac{W}{\epsilon}$.
4: **for** each secondary user i **do**
5:     $p_i = 0$.
6:     **for** each device d **do**
7:         $(L_{i,d}, H_{i,d}) = (0, 0)$.
8:     **end for**
9: **end for**
10: Compose a sequence B, using $v'_{i,d}(j\epsilon)$ for all $i, d, 0 \leq j \leq N-1$.
11: $B' = sort(B)$. //ordering from largest to smallest.
12: $A = (B'(1), B'(2), \cdots, B'(N))$.
13: $P = (B'(N+1), B'(N+2), \cdots, B'(2N))$.
14: **for** each i **do**, $P_i = \{\epsilon \cdot v'_{i',d}(q\epsilon) | v'_{i',d}(q\epsilon) \in P \text{ and } i' \neq i\}$ **end for**.
15: $s = f_l$.
16: **for** each i **do**
17:     $n_i = |\{v'_{i,d}(q\epsilon) | \forall d, \forall q, \text{s.t.} v'_{i,d}(q\epsilon) \in A\}|$.
18:     **if** $n_i > 0$ **then**
19:         **for** each d **do**
20:             $n_{i,d} = |\{v'_{i,d}(q\epsilon) | \forall q, \text{s.t.} v'_{i,d}(q\epsilon) \in A\}|$
21:             $w_{i,d} = n_{i,d} \cdot \epsilon$.
22:             $(L_{i,d}, H_{i,d}) = (s, s + n_{i,d} \cdot \epsilon)$
23:             $s = s + n_{i,d} \cdot \epsilon$.
24:         **end for**
25:     **end if**
26:     $p_i = \sum_{m=1}^{m=n_i} P_i(m)$.
27: **end for**

To be more specific, from line 4 to line 9 in Algorithm 1, the charging price and allocated spectrum bandwidth for each secondary user are initiated as 0. In line 10, $v'_{i,d}(j\epsilon)$ is the derivative value of the submitted valuation function of (i, d) at $j\epsilon$. For each valuation function and each different non-negative integer j, we compute such a $v'_{i,d}(j\epsilon)$, and let all these $v'_{i,d}(j\epsilon)$ form a sequence B. We then sort the sequence as shown in line 11 in the decreasing order and the sorted sequence is named B'. We pick two parts from the sequence B' to form two new sequences, i.e., a sequence A which consists of the first N elements in B' and a sequence P with N elements ranked from $N+1$ to $2N$ (as shown in line 12 and 13). In line 14, we exclude the possibility that one entity will be charged using the valuation submitted by itself. Line 19 to 24 are to allocate bandwidth to each device of the winning entities. For each entity i, n_i denotes the number of ϵ slices that it wins, i.e., ranked among the top N. The $n_i \times \epsilon$ bandwidth will be charged using the top n_i evaluation in P, that are not being

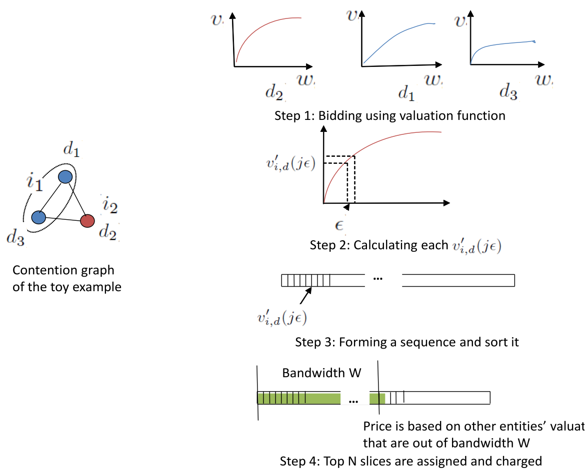


Fig. 1. Example for single collision domain auction, VSA-S.

submitted by i.

We illustrate the auction process using an example illustrated in Fig. 1. In the example, there are two entities i_1 and i_2, i_1 has two devices, d_1 and d_3, and i_2 has one device d_2. Fig. 1 shows the four steps of VSA-S.

In VSA-S, $B'(1)$ denotes the 1st element of a sequence B'. Similar notations are used throughout this paper. Note that when we write the details of VSA-S, each element of B should not only be $v'_{i,d}(j\epsilon)$; it should also contain the corresponding index (i, d, j). Since the elements of B', A, P, P_i all originate from B, they should similarly contain the indices.

B. Analysis of Framework

1) Truthfulness: Now we formally analyze VSA-S. We first prove the truthfulness of VSA-S. Then, we prove its optimality.

Theorem 3. *In single collision domain, VSA-S is truthful.*

Proof: Consider an arbitrary agent i. Given V_{-i}, the profile of valuation function sets submitted by all entities other than i, consider two possible strategies of agent i: The first strategy is that agent i submits its true valuation function set V_i^T, while the second is that agent i submits an arbitrary valuation function set V_i^A. Clearly, these two strategies may lead to different values of variables in our VSA-S mechanism, and thus different payoffs of agent i. For convenience, we use superscript T to denote the value of a variable when V_i^T is submitted, e.g., n_i^T is the value of n_i when V_i^T is submitted; correspondingly, we use superscript A to denote the value of a variable when V_i^A is submitted, e.g., n_i^A is value of n_i when V_e^A is submitted. Those values that are not affected by agent i's submitted valuation function set remain without either superscript, e.g., $b_{i',d,j}$ for $i' \neq i$. It is easy to get that

$$
\begin{aligned}
&\mathsf{u}_i(V_i^T, V_{-i})\\
&= \sum_{d=1}^{D_i} v_{i,d}^T(w_{i,d}^T) - p_i^T\\
&= \sum_{d=1}^{D_i} \int_0^{(n_{i,d}^T-1)\epsilon} v_{i,d}^{T'}(x)dx - \sum_{m=1}^{n_i^T} P_i^T(m)\\
&= \sum_{d=1}^{D_i} \sum_{m=0}^{n_{i,d}^T-1} \epsilon \cdot v_{i,d}^{T'}(m\epsilon) - \sum_{m=1}^{n_i^T} P_e^T(m)
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
\mathsf{u}_i(V_i^A, V_{-i}) &= \sum_{d=1}^{D_i} v_{i,d}^T(w_{i,d}^A) - p_i^A = \ \ldots\\
&= \sum_{d=1}^{D_i} \sum_{m=0}^{n_{i,d}^A-1} \epsilon \cdot v_{i,d}^{T'}(m\epsilon) - \sum_{m=1}^{n_i^A} P_e^A(m)
\end{aligned}
$$

Hence,

$$
\begin{aligned}
&\mathsf{u}_i(V_i^T, V_{-i}) - \mathsf{u}_i(V_i^A, V_{-i})\\
&= (\sum_{d=1}^{D_i} \sum_{m=0}^{n_{i,d}^T-1} \epsilon \cdot v_{i,d}^{T'}(m\epsilon) - \sum_{d=1}^{D_i} \sum_{m=0}^{n_{i,d}^A-1} \epsilon \cdot v_{i,d}^{T'}(m\epsilon))\\
&\quad - (\sum_{m=1}^{n_i^T} P_i^T(m) - \sum_{m=1}^{n_i^A} P_i^A(m)).
\end{aligned}
$$

Let ${B'}_i^T$ (resp., ${B'}_i^A$) be the subsequence of ${B'}^T$ (resp., ${B'}^A$) that consists of all elements $b_{i,d,j}$ for all d and all j. By our VSA-S mechanism, clearly we have that

$$
\sum_{d=1}^{D_i} \sum_{m=0}^{n_{i,d}^T-1} v_{i,d}^{T'}(m\epsilon) = \sum_{m=1}^{n_i^T} {B'}_i^T(m). \tag{2}
$$

On the other hand, $\sum_{d=1}^{D_i} \sum_{m=1}^{n_{i,d}^A} b_{i,d,m}^T$ is the sum of n_i^A elements of ${B'}_i^T$. Let M_A be the set of indices for these elements. Then, we have $|M_A| = n_i^A$ and that

$$
\sum_{d=1}^{D_i} \sum_{m=0}^{n_{i,d}^A-1} v_{i,d}^{T'}(m\epsilon) = \sum_{m \in M_A} {B'}_i^T(m). \tag{3}
$$

Now let B'_{-i} be the sequence we obtain by removing all elements of ${B'}_i^T$ from ${B'}^T$. Note that when we remove all elements of ${B'}_i^A$ from ${B'}^A$, we get the same sequence B'_{-i}. It is not hard to see

$$
\sum_{m=1}^{n_i^T} P_i^T(m) = \sum_{m=N-n_i^T+1}^{N} \epsilon \cdot B'_{-i}(m); \tag{4}
$$

$$
\sum_{m=1}^{n_i^A} P_i^A(m) = \sum_{m=N-n_i^A+1}^{N} \epsilon \cdot B'_{-i}(m). \tag{5}
$$

Combining (2)(3)(4)(5), we get that

$$\begin{aligned}&\mathsf{u}_i(V_i^T, V_{-i}) - \mathsf{u}_i(V_i^A, V_{-i})\\ &= \epsilon \cdot (\sum_{m=1}^{n_i^T} {B'}_i^T(m) - \sum_{m \in M_A} {B'}_i^T(m))\\ &\quad - \epsilon \cdot (\sum_{m=N-n_i^T+1}^{N} B'_{-i}(m) - \sum_{m=N-n_i^A+1}^{N} B'_{-i}(m))\end{aligned}$$

We distinguish two cases:
Case A: $n_i^T \geq n_i^A$.

$$\begin{aligned}&\mathsf{u}_i(V_i^T, V_{-i}) - \mathsf{u}_i(V_i^A, V_{-i})\\ &= \epsilon \cdot (\sum_{m=1}^{n_i^A} {B'}_i^T(m) - \sum_{m \in M_A} {B'}_i^T(m))\\ &\quad + \epsilon \cdot \sum_{m=n_i^A+1}^{n_i^T} {B'}_i^T(m) - \epsilon \cdot \sum_{m=N-n_i^T+1}^{N-n_i^A} B'_{-i}(m)\\ &\geq 0 + \epsilon \cdot (n_i^T - n_i^A){B'}_i^T(n_i^T)\\ &\quad - \epsilon \cdot (n_i^T - n_i^A)B'_{-i}(N - n_i^T + 1)\\ &= \epsilon \cdot (n_i^T - n_i^A)({B'}_i^T(n_i^T) - B'_{-i}(N - n_i^T + 1))\end{aligned}$$

The inequality above is due to the fact that ${B'}_i^T$ and ${B'}_{-i}^T$ are both sorted from the largest to the smallest. From VSA-S, we can see that ${B'}_i^T$ has n_i^T elements in A^T, i.e., in the top N elements of ${B'}^T$. Hence,

$${B'}_i^T(n_i^T) \geq {B'}^T(N).$$

This implies that B'_{-i} has $N - n_i^T$ elements in the top N elements of ${B'}^T$. Hence,

$$B'_{-i}(N - n_i^T + 1) \leq {B'}^T(N).$$

Combining all the above three inequalities, we get that

$$\mathsf{u}_i(V_i^T, V_{-i}) - \mathsf{u}_i(V_i^A, V_{-i}) \geq 0.$$

Case B: $n_i^T < n_i^A$. We partition M_A into two subsets $M_{A,1}$ ($|M_{A,1}| = n_i^T$) and $M_{A,2}$ ($|M_{A,2}| = n_i^A - n_i^T$), such that the elements with indices in $M_{A,1}$ are the largest n_i^T elements with indices in M_A.

$$\begin{aligned}&\mathsf{u}_i(V_i^T, V_{-i}) - \mathsf{u}_i(V_i^A, V_{-i})\\ &= (\sum_{m=1}^{n_i^T} {B'}_i^T(m) - \sum_{m \in M_{A,1}} {B'}_i^T(m))\\ &\quad - \sum_{m \in M_{A,2}} {B'}_i^T(m) + \sum_{m=N-n_i^A+1}^{N-n_i^T} B'_{-i}(m)\\ &\geq 0 - \sum_{m \in M_{A,2}} {B'}_i^T(m) + (n_i^A - n_i^T)B'_{-i}(N - n_i^A + 1)\end{aligned}$$

Again, the inequality above is due to the fact that ${B'}_i^T$ and ${B'}_{-i}^T$ are both sorted from the largest to the smallest. Recall that ${B'}_i^T$ has n_i^T elements in the top N elements of ${B'}^T$. Hence, $\{{B'}_i^T(m)|m \in M_A\}$ has at most n_i^T elements in the top N elements of ${B'}^T$. Consequently, $\{{B'}_i^T(m)|m \in M_{A,2}\}$ has no element in the top N elements of ${B'}^T$, which implies that, for all $m \in M_{A,2}$,

$${B'}_i^T(m) \leq {B'}^T(N).$$

On the other hand, similar to Case A, B'_{-i} has $N - n_i^T$ elements in the top N elements of ${B'}^T$. Since $N - n_i^A + 1 < N - n_i^T + 1$,

$$B'_{-i}(N - n_i^A + 1) \geq {B'}^T(N).$$

Combining all the above three inequalities, we get that

$$\mathsf{u}_i(V_i^T, V_{-i}) - \mathsf{u}_i(V_i^A, V_{-i}) \geq 0.$$

To summarize, for both Case A and Case B we have shown that

$$\mathsf{u}_i(V_i^T, V_{-i}) \geq \mathsf{u}_i(V_i^A, V_{-i}).$$

Hence, all entities submitting true valuation function sets is a DSE, which means VSA-S is truthful. ■

2) System Efficiency:

Theorem 4. *In single collision domain, VSA-S achieves maximum total valuation.*

Proof:

Let $\Psi = \{w_{i,d}|\forall i, \forall d\}$ be the spectrum assignment result of VSA-S. Let $\Psi' = \{w'_{i,d}|\forall i, \forall d\}$ be the spectrum assignment result of an arbitrary different mechanism. It is easy to get that

$$\begin{aligned}&\sum_i v_i(w_i) - \sum_i v_i(w'_i)\\ &= \sum_i (\sum_{\substack{1 \leq d \leq D_i\\ w'_{i,d} = w_{i,d}}} (v_{i,d}(w_{i,d}) - v_{i,d}(w'_{i,d}))\\ &\quad + \sum_{\substack{1 \leq d \leq D_i\\ w'_{i,d} < w_{i,d}}} (v_{i,d}(w_{i,d}) - v_{i,d}(w'_{i,d}))\\ &\quad + \sum_{\substack{1 \leq d \leq D_i\\ w'_{i,d} > w_{i,d}}} (v_{i,d}(w_{i,d}) - v_{i,d}(w'_{i,d})))\\ &= \sum_i (\sum_{\substack{1 \leq d \leq D_i\\ n'_{i,d} < n_{i,d}}} \int_{(n'_{i,d}+1)\epsilon}^{n_{i,d}\epsilon} v'_{i,d}(x)dx\\ &\quad - \sum_{\substack{1 \leq d \leq D_i\\ n'_{i,d} > n_{i,d}}} \int_{(n_{i,d}+1)\epsilon}^{n'_{i,d}\epsilon} v'_{i,d}(x)dx)\end{aligned}$$

It is also clear that,

$$\sum_i \sum_{\substack{1 \leq d \leq D_i\\ n'_{i,d} < n_{i,d}}} \sum_{m=n'_{i,d}+1}^{n_{i,d}} B'(N) = \sum_i \sum_{\substack{1 \leq d \leq D_i\\ n'_{i,d} > n_{i,d}}} \sum_{m=n_{i,d}+1}^{n'_{i,d}} B'(N) \tag{6}$$

From VSA-S, we can see that

$$\sum_i \sum_{\substack{1\le d\le D_i\\ n'_{i,d}<n_{i,d}}} \int_{(n'_{i,d}+1)\epsilon}^{n_{i,d}\epsilon} v'_{i,d}(x)dx \tag{7}$$

$$=\sum_i \sum_{\substack{1\le d\le D_i\\ n'_{i,d}<n_{i,d}}} \sum_{m=n'_{i,d}}^{n_{i,d}-1} \epsilon\cdot v'_{i,d}(m\epsilon) \tag{8}$$

$$\ge\sum_i \sum_{\substack{1\le d\le D_i\\ n'_{i,d}<n_{i,d}}} \sum_{m=n'_{i,d}+1}^{n_{i,d}} \epsilon\cdot B'(N), \tag{9}$$

and that

$$\sum_i \sum_{\substack{1\le d\le D_i\\ n'_{i,d}>n_{i,d}}} \int_{(n_{i,d}+1)\epsilon}^{n'_{i,d}\epsilon} v'_{i,d}(x)dx \le \sum_i \sum_{\substack{1\le d\le D_i\\ n'_{i,d}>n_{i,d}}} \sum_{m=n_{i,d}+1}^{n'_{i,d}} \epsilon\cdot B'(N). \tag{10}$$

From equations (6) (9) and (10), we can obtain that

$$\begin{aligned}&\sum_i \sum_{d=1}^{D_i} v_{i,d}(w_{i,d}) - \sum_i \sum_{d=1}^{D_i} v_{i,d}(w'_{i,d})\\ =\ &\sum_i\Big(\sum_{\substack{1\le d\le D_i\\ n'_{i,d}<n_{i,d}}} \int_{(n'_{i,d}+1)\epsilon}^{n_{i,d}\epsilon} v'_{i,d}(x)dx\\ &- \sum_{\substack{1\le d\le D_i\\ n'_{i,d}>n_{i,d}}} \int_{(n_{i,d}+1)\epsilon}^{n'_{i,d}\epsilon} v'_{i,d}(x)dx \ \ge\ 0.\end{aligned}$$

Therefore, VSA-S achieves maximum total valuation. ■

IV. Auction Framework for Multiple Collision Domains

In the previous section, we have proposed a spectrum auction framework for single collision domain that guarantees truthfulness and system efficiency. However, if we consider multiple collision domains, the problem becomes very challenging. In particular, given the valuation function sets submitted by the secondary users, (even if they are all true), it is still very difficult to design a spectrum allocation algorithm that maximizes the total valuation in the system. Even one simplified version of our problem is already NP-Hard: In fact, when we simplify our problem to require that each device is allocated with a fixed bandwidth of spectrum, rather than variable bandwidth, the spectrum allocation problem in multiple collision domains becomes equivalent to the well-studied graph coloring problem[32]. Therefore, designing a system efficient spectrum allocation for variable bandwidths is NP-hard.

Given this fact of hardness, we have to weaken our objective for multiple collision domains. We will focus on designing a truthful and feasible spectrum auction framework, which is also challenging.

A. Challenges in Design

Our goal is to design a truthful spectrum auction framework that allocates spectra with variable bandwidths to the devices of secondary users without interference.

There are many challenges in designing such a spectrum auction framework. The first challenge is that we need to figure out how to charge the use of spectra with variable bandwidths. In most existing works (e.g., [33], [10], [14]) one key idea for ensuring truthfulness is to charge a winning secondary user a *critical value* of payment[22]. If a secondary user bids higher than his critical value, then he wins; If a secondary user bids lower than his critical value then he loses in the auction. For a secondary user, the existing works use a neighbor bidder's valuation on a channel as the critical value, which satisfies certain conditions.The neighbor is called critical neighbor or threshold neighbor[33], [10]. However, it is difficult to apply this idea directly to our problem, because for each allocated variable-bandwidth spectrum it is more complicated to find its corresponding critical value. To tackle this difficulty, we continue to adopt the idea used above of discretizing the spectrum into small intervals, to facilitate the spectrum allocation and price calculation.

Second, to avoid interference, most existing spectrum auction systems assign channels to secondary users sequentially, to make sure that the already assigned channels do not interfere with the allocation in the next steps. As a result, to compute the price for *each* winning secondary user, it needs to re-run the algorithm once. This is too expensive to be feasible for our scenario, since ϵ is supposed to be very small. In our design, instead of sequentially allocating spectra, we simultaneously allocate spectra to all devices and thus it only needs to run the algorithm once in total to compute the prices for all the winning secondary users. At the same time, we guarantee no interference in the system.

The third challenge comes from multiple devices that each secondary user may have. Suppose that a secondary user cheats in the valuation function on one of his devices. Even though this particular device does not obtain more spectrum by this cheating, it may still affect the spectrum allocation for the other devices owned by the cheating secondary user. The overall utility of the cheating secondary user, calculated as the sum of spectrum valuations from all his devices, may turn out higher eventually. This situation is difficult to deal with in order to guarantee truthfulness. We will show how we solve this problem later in this section.

B. Design of Auction Framework

Now we introduce our spectrum allocation algorithm, which consists of two steps. As the first step of our algorithm, to enable simultaneous allocation, we divide the spectrum (f_l, f_h) into Δ intervals, and assign the center frequency of each device to one of the interval centers. Neighbor devices in the interference graph are not assigned in the same intervals, and the devices belonging to the same secondary user are not assigned in adjacent spectrum intervals (in order to prevent secondary users from cheating by manipulating valuation of multiple adjacent devices). In our allocation algorithm, the spectrum of each device can grow from its center frequency to gradually farther spectrum slices, but not beyond the center frequency in the adjacent intervals. In this way, each device is competing for spectrum with the devices in the adjacent spectrum intervals. Consequently, its critical neighbor can be fixed by one run of the allocation algorithm.

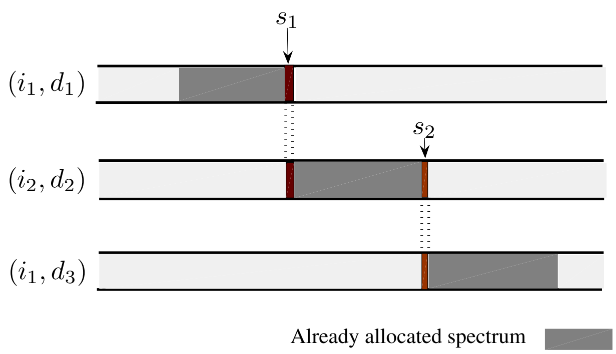


Fig. 2. Example for the necessity of symmetric allocation.

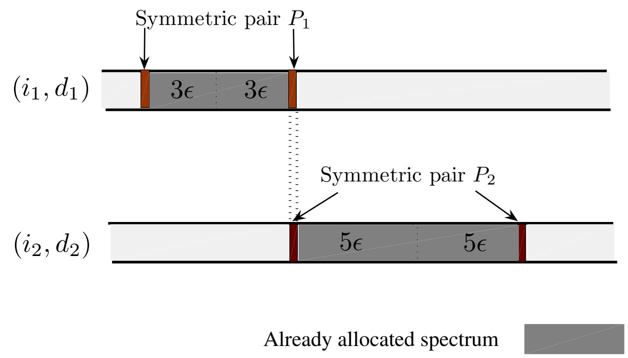


Fig. 3. Example for the 3rd restriction.

As the second step of our algorithm, we assign spectrum slices to devices in a greedy fashion. In particular, VSA-M "grows" each device's spectrum following the rules below:

(1) No Overlaps. The growth of a device's spectrum must start from the spectrum slices closest to its center frequency and gradually go to farther spectrum slices, but not beyond the center frequency in the adjacent intervals. Once VSA-M decides not to assign a symmetric pair to the device, the growth must terminate.

(2) Symmetric Allocation. The spectrum of a device grows in *symmetric pairs* of spectrum slices. Specifically, suppose that, in the first step, VSA-M has assigned the center frequency of device (i, d) as $\mathsf{CF_{i,d}}$. Then, in the second step, after assigning symmetric pairs of spectrum slices to this device, the device's spectrum can only grow to $(\mathsf{CF_{i,d}} - n\epsilon, \mathsf{CF_{i,d}} + n\epsilon)$, where $n \geq 0$ is an integer.

The fundamental reason of using symmetric allocation is to guarantee truthfulness when secondary users have multiple devices. Suppose that we allow devices to grow spectrum in single slices. Without loss of generality, we assume that starting from the center frequency, the spectrum slice on the left side is always first considered to be assigned, and then the slice on the right side. A toy example scenario is shown in Fig. (2). Suppose that in the allocation algorithm, device (i_1, d_1) is competing for slice s_1 with device (i_2, d_2), and device (i_1, d_3) is competing s_2 with device (i_2, d_2), where d_1 and d_3 are from the same secondary user. If each secondary user bids truthfully, slice s_1 will be assigned to device (i_1, d_1) and s_2 to (i_2, d_2). Nevertheless, i_1 can submit a lower valuation on slice s_1, so that device (i_2, d_2) will win s_1, and then the price for slice s_2 (valuation of i_2 on s_2) becomes lower for i_1. In this way, secondary user i_1 manipulates the price for s_2 by changing his valuation functions. Due to characterization of truthful auctions [22], we can see that this is not truthful. Therefore, we use the symmetric spectrum allocation, i.e., growth of spectrum in slice pairs, to prevent the secondary users from cheating.

(3) Greedy Assignment. A symmetric pair is assigned to a device only if the device's valuation of this pair of spectrum slices is the highest among all its neighbors' valuations of the *conflicting pairs*. Here a conflicting pair is a symmetric pair of spectrum slices for a neighbor (not for this device) that overlaps with this symmetric pair.

In Figure 3, we illustrate the rule (3) using an example. Here secondary user i_1's device d_1 and i_2's device d_2 are adjacent. For spectrum symmetric slice pair P_1, P_2 is the only conflicting pair. i_1's valuation on P_1 is $v_{e_1,d_1}(8\epsilon) - v_{e_1,d_1}(6\epsilon)$ and i_2's valuation on P_2 is $v_{e_2,d_2}(12\epsilon) - v_{e_2,d_2}(10\epsilon)$. If $v_{e_1,d_1}(8\epsilon) - v_{e_1,d_1}(6\epsilon) > v_{e_2,d_2}(12\epsilon) - v_{e_2,d_2}(10\epsilon)$, then P_1 is assigned to (v_1, d_1).

Once the growths of all devices' spectra are completed, VSA-M calculates the payment each secondary user needs to make for his use of spectra: This payment is equal to the sum of payments the secondary user needs to make for each symmetric pair of spectrum slices assigned to each of his devices. For each pair assigned to a device, the amount of payment due is determined by the device's neighbors belonging to other secondary users. We consider the valuations of conflicting pairs from such neighbors and use the highest such valuation as the payment.

In the example illustrated in Figure 3, assume that d_2 is the only neighbor of d_1 and that $e_1 \neq e_2$. Then the payment due for usage of P_1 is the valuation of P_2: $v_{e_2,d_2}(B_\ell + 12\epsilon) - v_{e_2,d_2}(B_\ell + 10\epsilon)$.

The entire VSA-M framework is shown in Algorithm 2. In Algorithm 2, the whole spectrum is divided into Δ equal pieces. In line 4 to line 11, a spectrum frequency center (one of the Δ pieces) is assigned to each device of each secondary user. The rules of assignments as shown in line 6 are that 1) any neighboring devices in the contention graph is not assigned with the same center frequency in order to avoid interference; 2) the devices belonging to the same secondary user are not assigned in adjacent spectrum, in order to avoid the cheating behavior of entities with multiple devices. As shown in line 14, if a device's valuation for a certain ϵ slice is greater than all its competitors' valuations, then this slice is allocated to this device, with the price as the maximum valuation if its competitors. The algorithm stops to allocate spectrum to this device if the device cannot win all its competitors at some slice (line 16). Line 20 indicates that the spectrum allocation starts from the frequency center in the units of ϵ slices.

C. Analysis of Auction Framework

1) Truthfulness: We have the following theorems regarding the truthfulness of VSA-M.

Algorithm 2 VSA-M: Truthful variable bandwidth spectrum auction framework for multiple collision domains

INPUT: Available Space (f_l, f_h); valuation function set V_i from each secondary user i; the set of neighbors of (i,d): $\mathsf{Neighbr(i,d)}$; ϵ, Δ.
OUTPUT: Allocated spectrum for each device d of each secondary user i: $(L_{i,d}, H_{i,d})$, and price p_i for each i.
$s = \frac{W}{\Delta}$. $N = \frac{W}{2\epsilon}$.
for each device (i,d) **do**
  for each integer x s.t., $1 \le x \le \Delta$ **do**
    if $\forall (i',d') \in \mathsf{Neighbr(i,d)}$, $f_l + (x - \frac{1}{2}) \cdot s \neq \mathsf{CF_{i',d'}}$ **and** $\forall d'' \neq d$, s.t., $|f_l + (x - \frac{1}{2}) \cdot s - \mathsf{CF_{i,d''}}| \neq \mathsf{s}$ **then**
      $\mathsf{CF_{i,d}} = f_l + (x - \frac{1}{2}) \cdot s$.
      Break.
    end if
  end for
end for
for each device (i,d) **do**
  for $(t = 0; t \le \frac{s}{\epsilon} - 1; t{+}{+})$ **do**

$$\textbf{if } v'_{i,d}(2t\epsilon) > \max_{\substack{i' \neq i \\ (i',d') \in \mathsf{Neighbr(i,d)} \\ \& |\mathsf{CF_{i,d}} - \mathsf{CF_{i',d'}}| = \mathsf{s}}} v'_{i',d'}(2(\frac{s}{2\epsilon} - t + 1)\epsilon)$$

$$\textbf{then } p_{i,d,t} = \max_{\substack{i' \neq i \\ (i',d') \in \mathsf{Neighbr(i,d)} \\ \& |\mathsf{CF_{i,d}} - \mathsf{CF_{i',d'}}| = \mathsf{s}}} \epsilon \cdot v'_{i',d'}(2(\frac{s}{2\epsilon} - t + 1)\epsilon).$$

      else Break.
      end if
  end for
  $n_{i,d} = t - 1$.
  $L_{i,d} = \mathsf{CF}_{i,d} - n_{i,d}\epsilon$; $H_{i,d} = \mathsf{CF}_{i,d} + n_{i,d}\epsilon$.
end for
for each secondary user i **do**
  $p_i = \sum_{d=1}^{D_i} \sum_{m=1}^{n_{i,d}} p_{i,d,m}$.
end for

Theorem 5. *In multiple collision domains, VSA-M is truthful.*

Proof: Consider an arbitrary entity i. Given V_{-i}, the profile of valuation function sets submitted by all secondary users other than i, consider two possible strategies of i, i.e., submitting his true valuation function set V_i^T and submitting an arbitrary valuation function set V_i^A. Notations such as V_i^T, n_i^T, V_i^A and n_i^A are defined similarly to in the proof of Theorem 3. (Recall that subscript T means the value is for the scenario that i submits V_i^T and subscript A means the value is for the scenario that i submits V_i^A.) Those values that are not affected by i's submitted valuation function set remain without either superscript, e.g., $v'_{i',d}(j\epsilon)$ for $i' \neq i$.

We can easily get that

$$\begin{aligned}
& \mathsf{u}_i(V_i^T, V_{-i}) \\
= \ & \sum_{d=1}^{D_i} v_{i,d}^T(w_{i,d}^T) - p_i^T \\
= \ & \sum_{d=1}^{D_i} \int_0^{\epsilon\cdot(n_{i,d}^T - 1)} v^{T'}(x)dx - \sum_{d=1}^{D_i} \sum_{m=1}^{n_{i,d}^T} p_{i,d,m}^T \\
= \ & \sum_{d=1}^{D_i} \sum_{m=0}^{n_{i,d}^T - 1} \epsilon \cdot v_{i,d}^{T'}(m\epsilon) - \sum_{d=1}^{D_i} \sum_{m=1}^{n_{i,d}^T} p_{i,d,m}^T.
\end{aligned}$$

Similarly, we can obtain that

$$\mathsf{u}_i(V_i^A, V_{-i}) = \sum_{d=1}^{D_i} \sum_{m=0}^{n_{i,d}^A - 1} \epsilon \cdot v_{i,d}^{T'}(m\epsilon) - \sum_{d=1}^{D_i} \sum_{m=1}^{n_{i,d}^A} p_{i,d,m}^A.$$

Hence,

$$\begin{aligned}
& \mathsf{u}_i(V_i^T, V_{-i}) - \mathsf{u}_i(V_i^A, V_{-i}) \\
= \ & \sum_{d=1}^{D_i} (\sum_{m=0}^{n_{i,d}^T - 1} \epsilon \cdot v_{i,d}^{T'}(m\epsilon) - \sum_{m=0}^{n_{i,d}^A - 1} \epsilon \cdot v_{i,d}^{T'}(m\epsilon) - (\sum_{m=1}^{n_{i,d}^T} p_{i,d,m}^T \\
& - \sum_{m=1}^{n_{i,d}^A} p_{i,d,m}^A))
\end{aligned}$$

Our algorithm states that each

$$p_{i,d,m} = \max_{\substack{i' \neq i \\ (i',d') \in \mathsf{Neighbr(i,d)} \\ \& |\mathsf{CF_{i,d}} - \mathsf{CF_{i',d'}}| = \mathsf{s}}} \epsilon \cdot v'_{i',d'}(2(\frac{s}{2\epsilon} - m + 1)\epsilon)$$

Since for each m, , $\frac{s}{2\epsilon} - m + 1$ is a fixed number and not affected by i's submitted valuation function set, $p_{i,d,m}$ remains the same regardless of whether i submits V_i^T or V_i^A.

Then $\forall m \ s.t. \ 1 \le m \le \min(n_{i,d}^T, n_{i,d}^A)$ we have

$$p_{i,d,m}^T = p_{i,d,m}^A$$

. We distinguish two cases.

Case A. $n_{i,d}^T \ge n_{i,d}^A$.

$$\begin{aligned}
& \mathsf{u}_i(V_i^T, V_{-i}) - \mathsf{u}_i(V_i^A, V_{-i}) \\
= \ & \sum_{d=1}^{D_i} (\sum_{m=n_{i,d}^A}^{n_{i,d}^T - 1} \epsilon \cdot v_{i,d}^{T'}(m\epsilon) - \sum_{m=n_{i,d}^A + 1}^{n_{i,d}^T} p_{i,d,m}^T) \\
= \ & \sum_{d=1}^{D_i} (\sum_{m=n_{i,d}^A}^{n_{i,d}^T - 1} (\epsilon \cdot v_{i,d}^{T'}(m\epsilon) - p_{i,d,m}^T)) \\
\ge \ & 0
\end{aligned}$$

The inequality above is due to the fact that, for any $m \le n_{i,d}^T$, we must have $b_{i,d,m}^T \ge p_{i,d,m}^T$.

Case B. $n_{i,d}^T < n_{i,d}^A$.

$$\begin{aligned}
& \mathsf{u}_i(V_i^T, V_{-i}) - \mathsf{u}_i(V_i^A, V_{-i}) \\
= \ & \sum_{d=1}^{D_i} (- \sum_{m=n_{i,d}^T}^{n_{i,d}^A - 1} \epsilon \cdot v_{i,d}^{T'}(m\epsilon) + \sum_{m=n_{i,d}^T + 1}^{n_{i,d}^A} p_{i,d,m}^A) \\
= \ & \sum_{d=1}^{D_i} \sum_{m=n_{i,d}^T}^{n_{i,d}^A - 1} (\max_{\substack{i' \neq i \\ (i',d') \in \mathsf{Neighbr(i,d)} \\ \& |\mathsf{CF_{i,d}} - \mathsf{CF_{i',d'}}| = \mathsf{s}}} \epsilon \cdot v'_{i',d'}(2(\frac{s}{2\epsilon} - m + 1)\epsilon)) \\
& - \epsilon \cdot v_{i,d}^{T'}(m\epsilon)) \\
\ge \ & 0.
\end{aligned}$$

This inequality holds because $\forall m$ s.t., $m > n_{i,d}^T$,

$$v_{i,d}^{T'}(m\epsilon) \le \max_{\substack{i' \neq i \\ (i',d') \in \mathsf{Neighbr(i,d)} \\ \& |\mathsf{CF_{i,d}} - \mathsf{CF_{i',d'}}| = \mathsf{s}}} v'_{i',d'}(2(\frac{s}{2\epsilon} - m + 1)\epsilon)$$

Therefore, it is a DSE for all secondary users to submit their true valuation function sets. ■

For the multiple collision domain scenarios, according to our design, interference can be avoided among neighboring secondary users as the spectrum that they are allocated does not overlap. For the hidden terminal problem, since in our system model we assume that the auctioneer has the system-wide contention graph, so the hidden terminal problem can be avoided at the level of secondary users. At the level of devices, we rely the MAC layer protocol such as CSMA/CA to solve the hidden terminal problem.

2) Fairness: Our goal of this paper is focusing on social welfare. We have not considered fairness issue when designing our auctions. Indeed, in wireless spectrum auctions, fairness could be an important issue to consider. For example, if a secondary user who always bids lower than other secondary users, i.e., he has lower valuations of the spectrum usage, he may never be allocated spectrum bandwidth. It may cause this secondary user to leave the spectrum market. In a long run, it may harm the revenue of the primary user.

To address the starvation issue in continuous spectrum auction, we can allocate a minimum bandwidth of spectrum to each participating secondary user and charge for a flat rate. Of course, there will always be a tradeoff between fairness and system-wide valuation. For example, if due to fairness, some spectrum bandwidth (e.g., 4 MHz) is allocated to a secondary user with lower valuation (e.g., 20) compared with another user with higher valuation (e.g., 40), then the system-wide total valuation will decrease by 20. For spectrum allocations on channels, there are some existing works focusing on the tradeoff between fairness and social welfare, e.g.,[10]. We leave the specific design of max-min fair continuous spectrum allocation in our future work.

V. Implementation of Virtual Money

In order to fully implement dynamic spectrum auctions in reality, a comprehensive system need to be in place to realize virtual money. In this section, we present an efficient implementation of virtual money with dynamic spectrum allocation devices.

Indeed, implementing virtual money in the auction framework does not fundamentally change the problem of truthful spectrum allocation with variable bandwidth. It makes the auction framework complete and more flexible. If real money or traditional virtual credits are used in the market, all the transactions must be initiated and cleared at the centralized auctioneer. In addition, the centralized auctioneer must work with a centralized authority (e.g., central bank) who keeps all the transaction records and current balance of each secondary user. It adds more overhead to the system.

Compared with this method, our implementation of virtual money using cryptographic techniques provides a secure and more distributed way of currency circulation. With our implementation, the current balance of each secondary user does not have to be kept at any centralized authority. The balance information is securely carried at each secondary user's devices. Moreover, each user cannot forge the virtual money by itself, due to the one-way difficulty property of Hash functions. Of course, the virtual currency in our system can be purchased using real money.

We propose that each DSA device is preloaded with a constant amount of virtual money. This amount of virtual money should be sufficient for a typical user in a period. If the device owner needs to access more spectrum than a typical user, she can choose to purchase additional virtual money using real money.

There are many possible ways to implement virtual money, with different computational overheads and different guarantees of security. In this section, we present an approach to implement virtual money using reverse hash chains, which has low computational overheads but can provide a reasonable security guarantee. After a brief description of the entire approach, we give a method for fast computing of hash-chain tails, which is needed in our approach. In addition, we also present a method that can further improve the efficiency.

Approach Using Reverse Hash Chains The main idea of our approach is to use a reverse hash chain. Recall that, when each device is manufactured, it be preloaded with an amount of virtual money. Suppose the amount of preloaded money is m. Let (k_{prv}, k_{pub}) be a pair of keys for a central bank of virtual money, where k_{prv} is the private key and k_{pub} is the public key. (We assume that k_{pub} is known to every device.) To implement the preloading of this amount of virtual money, we store a tuple $< r_i, H^m(r_i), S_{k_{prv}}() >$ in each device i, where r_i is a random number only known to the device, $H()$ is a well-known cryptographic hash function (e.g., SHA-512), and $S()$ is a digital signature algorithm. Intuitively, (r_i, $H(r_i)$, $H^2(r_i)$, …, $H^m(r_i)$) forms a hash chain of length m, which represents the amount of preloaded virtual money. The device i does not need to keep the entire hash chain; in stead, it only keeps the head r_i, the tail $H^m(r_i)$, and the central bank's signature on the tail.

The first time a device i uses DSA, it broadcasts $< H^m(r_i), S_{k_{prv}}(H^m(r_i)) >$, i.e., the tail of the hash chain and the signature on the tail, to all other devices. Upon receipt of this message, each device verifies that the signature is valid using k_{pub}, and then makes a record of the received tail. Note that each device maintains a record of the current tail for each other device.

When device i needs to make a payment of μ ($\mu \leq m$), it does so by revealing to the public the subchain of length μ at the current tail, and then removing this subchain from its current chain. To be more precise, assuming the current tail kept by device i is $H^{m'}(r_i)$, device i simply broadcasts $H^{m'-\mu}(r_i)$ to all devices, and then replaces the current tail in its record with $H^{m'-\mu}(r_i)$.

Correspondingly, when device j receives hash value φ_i from device i for a payment of amount μ, assuming the current tail for device i in AP j's record is ϑ_i, device j needs to verify that $\vartheta_i = H^{\mu}(\varphi_i)$. After the verification, device j replaces the current tail for device i in its own record with φ_i.

Fast Computing of Hash-Chain Tail The above approach for implementing virtual money requires frequent computing of hash-chain tails. For example, when device i needs to make a payment of μ, it needs to compute $H^{m'-\mu}(r_i)$. If $m'-\mu$ is large, it may take some time to compute $H^{m'-\mu}(r_i)$ from r_i. Consequently, we propose a very simple way to expedite this computation: For a constant LEN, device i should also be preloaded with $H^{\mathsf{LEN}}(r_i)$, $H^{2\cdot\mathsf{LEN}}(r_i)$, $H^{3\cdot\mathsf{LEN}}(r_i)$,…, in addition to the head and tail of the hash chain and the signature on the tail. In this way, when device i needs to compute $H^{m'-\mu}(r_i)$, the device only needs to compute it from $H^{\mathsf{MLEN}}(r_i)$, where MLEN is the largest multiple of LEN less than or equal to $m' - \mu$. Consequently, device i needs much less time to compute $H^{m'-\mu}(r_i)$.

Further Improvement of Efficiency When we need to handle very large amounts of virtual money, we have a method to further improve the efficiency: In stead of using one long hash chain of length m to represent virtual money of amount m, we use several short hash chains. Each of these shorter hash chains corresponds to virtual money counted in a distinct denomination. For example, to represent virtual money of amount $m = 123, 456, 789$, we can use three hash chains, where the first has a length of 123, the second has a length of 456, and the third has a length of 789. So each hash value in the first chain is worth one million, each hash value in the second chain is worth one thousand, and each hash value in the third chain is worth one unit of virtual money. Of course, we need to slightly revise our approach described above in order to support this multi-hash-chain method. However, this method can improve the efficiency significantly if very large amounts of virtual money are needed.

VI. Evaluations

We carry out three sets of experiments with different objectives.

- The first set of experiments evaluate how the payoff of an entity is affected by its possible cheating actions (in its valuation function set). The results demonstrate that, when either VSA-S or VSA-M is used, entities' cheating actions never increase their own payoffs. Hence, the truthfulness of VSA-S and VSA-M is verified.
- The second set of experiments focus on the total valuation of allocated spectra in the system, for both VSA-S and VSA-M.
- The third set of experiments are on the overheads introduced by the payment scheme. The results have confirmed the efficiency of our scheme.

A. Experiments Setup

The experiments are performed using GloMoSim [18] on a laptop with 2.0GHz Centrino CPU and 1.96GB RAM. We modify GloMoSim to enable the use of variable spectra bandwidths, by setting the MAC layer parameters described in [5].

Unless specified otherwise, we assume that 3 secondary users, each of whom has 2 devices, are randomly located in an area of 300×300 m^2 (for single collision domain experiments), or 600×600 m^2 (for multiple collision domains experiments). The transmission power of each device is 16dBm. The path loss is set to free space. In all experiments except those in Section VI-C, we assume that the available band is 48MHz in DTV whitespace (644MHz-692MHz). All traffic is single hop UDP flows that are always backlogged. We set the packet size to 1500 Bytes.

In our experiments, we assume each valuation function is in one of the following two forms:

$$v_{i,d}(w_{i,d}) = \begin{cases} \beta_{i,d} \log(1 + \gamma_{i,d} \cdot w_{i,d}) & \text{if } w_{i,d} < 1/\gamma_{i,d} \\ \beta_{i,d} \log 2 & \text{if } w_{i,d} \geq 1/\gamma_{i,d}. \end{cases} \tag{9}$$

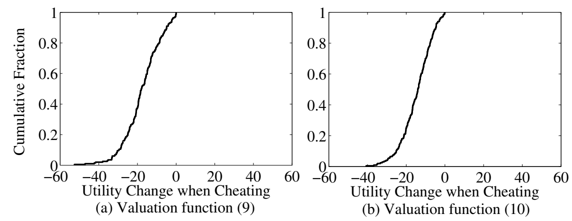


Fig. 4. Payoff change for VSA-S. It shows that the payoff change when cheating is never positive when VSA-S is used.

$$v_{i,d}(w_{i,d}) = \begin{cases} \beta_{i,d}\sqrt{\gamma_{i,d} \cdot w_{i,d}}, & \text{if } w_{i,d} < 1/\gamma_{i,d} \\ \beta_{i,d}\sqrt{2} & \text{if } w_{i,d} \geq 1/\gamma_{i,d}. \end{cases} \tag{10}$$

The difference in devices' valuation functions is reflected in the difference in the values of $\beta_{i,d}$ and $\gamma_{i,d}$. When secondary users submit their valuation function sets, they may cheat by changing their values of $\beta_{i,d}$ and $\gamma_{i,d}$. When a secondary user is truthful, it should use the true values of $\beta_{i,d}$ and $\gamma_{i,d}$, denoted by $\beta^\star_{i,d}$ and $\gamma^\star_{i,d}$. We assume $\gamma^\star_{i,d} = 1/(n_{i,d} * 1M)$. In experiments, we randomly set each $n_{i,d}$ as an integer in $[1, 20]$. We set $\epsilon = 1$MHz.

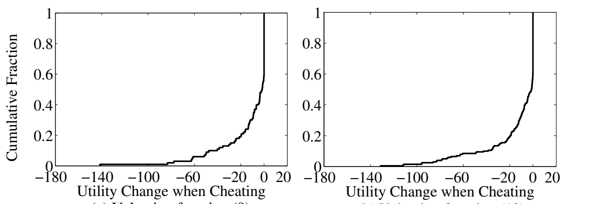
B. Truthfulness and Payoffs

In this set of experiments, we study the truthfulness of our mechanisms. In particular, we evaluate how the cheating behavior of entities affects their own payoffs. In each experiment, one random entity is picked to be the cheater; its claimed valuation function set has each $\beta_{e,d}$ (resp., $\gamma_{e,d}$) randomly chosen between 0 and $3\beta^\star_{e,d}$ (resp., $3\gamma^\star_{e,d}$).[4] We measure the payoff of the cheating entity in each experiment and also the same entity's payoff when the entity behaves honestly. The difference is the entity's payoff change for cheating. If the change is positive, then cheating benefits the entity; otherwise, cheating does not benefit.

Payoffs in VSA-S We perform the above experiments on VSA-S, with 1000 runs using valuation functions in the form of (9) and another 1000 runs using valuation functions in the form of (10). From Fig. 4 (a) we can observe that the payoff change when cheating is never positive. In other words, entities never benefit from, and usually lose for, cheating. The average payoff loss when cheating is 18.94. Similar observations can be made from Fig. 4 (b). In this case, the average payoff loss when cheating is 14.62. Overall, the truthfulness of VSA-S is verified.

Payoffs in VSA-M We also perform similar experiments on mechanisms for multiple collision domains. Fig. 5 shows the results for VSA-M, We can see that, if VSA-M is used, an entity's cheating can never benefits itself (i.e., there is no positive payoff change for cheating). Consequently, the truthfulness of VSA-M is verified.

[4]We have this random choice of cheater and cheater's action because it is hard to predict who will be the cheater and how the cheater will behave in reality. By repeating this experiment for many times, we hope that at least some of the randomly picked cheating actions will be consistent with real cheaters' actions in reality.



(a) Valuation function (9) (b) Valuation function (10)

Fig. 5. Payoff change for VSA-M. It shows that there is no positive payoff change for cheating when VSA-M is used.

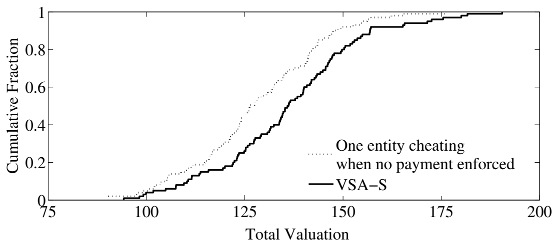
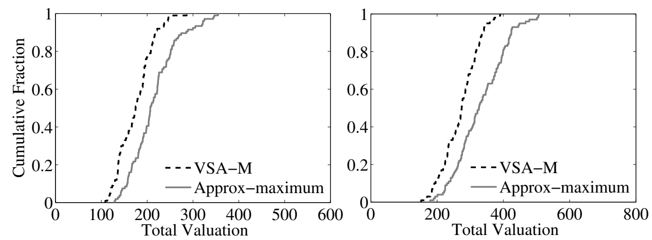


Fig. 6. Total valuation of allocated spectra in single collision domains.



(a) 2.4G ISM band; 80MHz available. (b) DTV Whitespace;48MHz available

Fig. 7. Total valuation of allocated spectra in multiple collision domains.

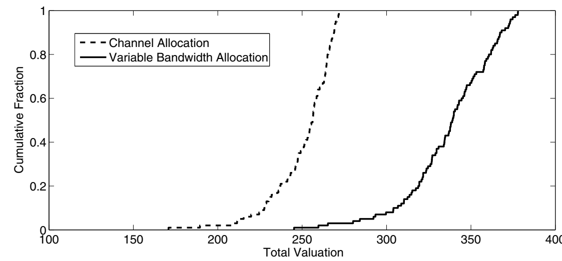


Fig. 8. Comparison of VSA-S and channel allocation on total valuation of allocated spectra in single collision domain.

C. Total Valuation

The second set of experiments is to evaluate our two auction frameworks in terms of satisfying spectrum demands.

For the single collision domain, we measure the total valuation of allocated spectra for the case that all secondary users bid truthfully and compare it with the case that there is no payment scheme enforced in the system and one secondary user cheats in his submission of valuation functions. The result distributions shown in Fig. 6 demonstrate that VSA-S, which guarantees the truthfulness and system efficiency, can significantly increase (7.62% on average) the total valuation of allocated spectra with the presence of one cheating secondary user.

For multiple collision domains, we measure the total valuation of allocated spectra for VSA-M in two different bands, the 2.4GHz ISM band and the DTV whitespaces, respectively. We assume that there are 80MHz available bandwidth in the 2.4GHz ISM band, and 48 MHz available bandwidth (644MHz-692MHz) in DTV whitespaces. Fig. 7 shows the distributions of total valuation of allocated spectra of 100 runs, for our auction framework VSA-M, and a spectrum allocation algorithm that achieve approximate maximum total valuation using [25]. In the figure, we can see that, for both the 2.4GHz ISM band and the DTV whitespace, the total valuation of allocated spectra in the system remains at a high level, compared with the approximate algorithm. Since the there are more bandwidth available in 2.4GHz ISM band, system-wide total valuation is higher than that of DTV whitespace.

We also compare our VSA-S with spectrum auctions on units of channels using [33] on system-wide total valuation. In particular, we run the two auction algorithms respectively to allocate spectrum of 48MHz in DTV whitespace (644MHz-692MHz). For VSA-S we use valuation functions in the form of Eqn (9). We randomize $\beta_{i,d}$ and $\gamma_{i,d}$ in each run. For spectrum auction on channels, each channel has bandwidth 20MHz. Fig. 8 shows the result of system-wide total valuation in the form of empirical cumulative distribution function from 100 runs. As we can see that the system-wide total valuation of VSA-S is much higher (34.28% on average) than the spectrum auction on channels. The reason is that our spectrum auction with variable bandwidth enables much more fine-grained valuation and utilization of the available spectrum than channel allocation. Moreover, for our experiment, the 48MHz can only support 2 channels (40MHz) while our VSA-S auction can make the full use of the available spectrum. Since the advantage of VSA-S in total valuation is rooted in the more fine-grained spectrum utilization opposed to channels, the results in the single collision domains can be extended to multiple collision domains.

D. Computational Overhead

In this set of experiments, we evaluate the computational overhead introduced by our payment scheme. In particular, we distinguish two types of computational overhead, namely the overhead for computing the hash value in order to make a payment and the overhead for verifying a payment, and evaluate both of them. In these experiments, the amount of preloaded virtual money is 1000; $\mathsf{LEN} = 100$ for fast computing of hash-chain tails; the key length is 1024 bits.

For the first type of overhead, we measure the average amounts of time for a device to compute a payment using different methods: the basic method of directly computing the hash value from the head of the hash chain, and the fast hash-chain tail computation method given in Section V. The results are shown in Fig. 9. In the figure, we can see that the basic method is pretty fast, but the fast hash-chain tail computation method is even faster. We also observe that for the basic method, making later payments is faster than making earlier payments. The reason is that the length of the hash chain decreases over time, and thus making later payments requires fewer numbers of hashing.

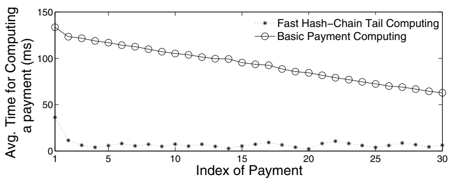


Fig. 9. Average computational overhead for making a payment.

We also evaluate the overhead for verifying a payment. From the results of 100 runs, we find that the average time for verifying a payment is 3.50 ms, and the standard deviation is about 0.32ms.

VII. Related Work

Dynamic spectrum allocation with variable bandwidth has been studied extensively(e.g., [15], [32], [5], [11], [21], [3]). In the KNOWS project [32], [31], [23], the concept of time-spectrum block is introduced and close-to-optimal central and distributed spectrum allocation algorithms [32] are proposed. In [21], Moscibroda et al. design algorithms to assign dynamic channel width that matches the traffic load. Their results show that load-aware dynamic spectrum allocation can significantly improve the spectrum utilization. Another important recent contribution [6] in DSA is on DTV whitespaces. In [6], in addition to providing some basic design rules and an architecture, Deb et al. also present a demand-based dynamic spectrum allocation algorithm that achieves high performance. These works on dynamical spectrum allocation consider the scenario that secondary users are not coordinated. The secondary users need to perform accurate and complicated spectrum sensing to avoid interfering with the primary user. In contrast, our work studies dynamic spectrum auctions, in which primary users can better control the usage of the available spectrum. Hence compared with these works above, we focus on a different system setting.

There is a considerable number of existing works on dynamic spectrum auctions (e.g., [13], [34], [33], [16], [14], [35], [10], [9]). Researchers produce nice and elegant spectrum auction frameworks with the goals of truthfulness, system efficiency, maximum revenue or fairness. For example, in [33], Zhou et al. propose a truthful and computationally efficient auction scheme; in [35], Zhou and Zheng make an important improvement by considering the incentives of the spectrum seller. Another truthful spectrum auction scheme is presented in [14] for generating more revenue from the auctions. As we have have mentioned, all these works on dynamic spectrum auctions only sell spectrum in units of channels. Our work provides more flexibility in selling unused spectrum with variable size of units.

There are also a number of works on non-cooperative channel assignment problem in wireless networks [12], [26], [7], [29], [8], [28]. For multiple radio devices, Felegyhazi et al. [7] introduce a strategic game model and obtain elegant theoretical results. After this work, Wu et al. [29] propose a solution based on strictly dominant strategies, and Gao et. al. [8] obtain interesting results in multi-hop networks. All these works are on assignment of fixed-width channels, rather than on allocation of spectra with variable bandwidths.

TABLE I
Comparison of Our Work With Some Existing Works

	[6]	[33]	[30]	[10]	Our work
Truthfulness	×	√	√	√	√
Max System Valuation	√	√	√	×	√
Spectrum Reuse	√	√	×	√	√
Variable Bandwidth	√	×	×	×	√

In a recent work [28], Wu et al. consider the non-cooperative channel allocation problem, when the channel width is adaptive. They model the adaptive width channel allocation problem as a strategic game and design a payment scheme to guarantee the system converges to a dominant strategy equilibrium and achieve system optimality. Although this work makes good contributions to non-cooperative adaptive-width channel allocation in general, our focus is in a different setting of secondary spectrum market and we allow more fine-grained spectrum allocation with the consideration of primary and secondary users' incentives.

We summarize the comparison of our work and some of the typical existing works in Table I. As we can see that these existing spectrum allocation mechanisms either cannot achieve truthfulness (e.g., [6]), maximum system-wide valuation (e.g., [10]), spectrum reuse in multiple collision domains (e.g., [30]), or allocation to variable bandwidth (e.g., [10], [30], [33]). In contrast, our spectrum auctions proposed in this paper can satisfy the four properties above.

We emphasize that our work is the first wireless spectrum auction that allows to allocate variable bandwidth instead of fixed channels. One of the major contribution of this paper is that in our auction, the secondary users are submitting valuation functions rather than bidding prices in the traditional bidding. This technical novelty has not appeared in any other existing works of spectrum auctions.

VIII. Conclusion and Future Work

Dynamic spectrum auctions are considered promising in utilizing the unused spectrum more efficiently in the secondary spectrum market. In this paper, we consider a more flexible form of dynamic spectrum auction, i.e., the spectrum can be sold in variable bandwidths. To solve this problem, we propose two spectrum auction frameworks with proved truthfulness and system efficiency properties, for single collision domain and multiple collision domains, respectively.

There are many possible ways to further improve our auction frameworks. For example, our auction framework for multiple collision domains can be further extended to achieve better approximation of system efficiency. Moreover, other design goals such as maximum revenue are also desirable for variable bandwidth spectrum auction. We leave these topics to future work.

References

[1] F. 08-260, "Second rep. and order and memorandum opinion and order," 2008.

[2] I. 802.22, "IEEE standards for wireless regional area network (WRAN) using white spaces," http://www.ieee802.org/22/.

[3] P. Bahl, R. Chandra, T. Moscibroda, R. Murty, and M. Welsh, "White space networking with Wi-Fi like connectivity," in *Proc. 2009 ACM SIGCOMM*.

[4] D. P. Bertsekas. *Nonlinear Programming*, 2nd ed. Athena Scientific, 1999.

[5] R. Chandra, R. Mahajan, T. Moscibroda, R. Raghavendra, and P. Bahl, “A case for adapting channel width in wireless networks,” in *Proc. 2008 ACM SIGCOMM*.

[6] S. Deb, V. Srinivasan, and R. Maheshwari, “Dynamic spectrum access in DTV whitespaces: design rules, architecture and algorithms,” in *Proc. 2009 ACM MOBICOM*.

[7] M. Felegyhazi, M. Cagalj, S. S. Bidokhti, and J.-P. Hubaux, “Noncooperative multi-radio channel allocation in wireless networks,” in *Proc. 2007 IEEE INFOCOM*.

[8] L. Gao and X. Wang, “A game approach for multi-channel allocation in multi-hop wireless networks,” in *Proc. 2008 ACM MOBIHOC*.

[9] A. Gopinathan and Z. Li, “A prior-free revenue maximizing auction for secondary spectrum access,” in *Proc. 2011 IEEE INFOCOM*.

[10] A. Gopinathan, Z. Li, and C. Wu, “Strategyproof auctions for balancing social welfare and fairness in secondary spectrum markets,” in *Proc. 2011 IEEE INFOCOM*.

[11] R. Gummadi and H. Balakrishnan,” “Wireless networks should spread spectrum based on demands,” in *Proc. 2008 ACM HotNets-VII*.

[12] M. M. Halldorsson, J. Y. Halpern, and L. E. L. etc., “On spectrum sharing games,” in *Proc. 2004 ACM PODC*.

[13] J. Huang, R. Berry, and M. Honig, “Auction-based spectrum sharing,” *Mobile Networks and Applications*, vol. 11, no. 3, 2006.

[14] J. Jia, Q. Zhang, Q. Zhang, and M. Liu, “Revenue generation for truthful spectrum auction in dynamic spectrum access,” in *Proc. 2009 ACM MOBIHOC*.

[15] T. Kamakaris, M. Buddhikot, and R. Iyer, “A case for coordinated dynamic spectrum access in cellular networks,” in *Proc. 2005 IEEE DySPAN*.

[16] G. Kasbekar and S. Sarkar, “Spectrum auction framework for access allocation in cognitive radio networks,” in *Proc. 2009 ACM MOBIHOC*.

[17] D. M. Kreps, *A Course in Microeconomic Theory*. Princeton University Press, 1990.

[18] U. P. C. Laboratory “Information systems simulation library,” http://pcl.cs.ucla.edu/projects/glomosim/.

[19] X. Liu, S. Huang, and Z. Ding, “Opportunistic spectrum access in cognitive radio networks,” in *Proc. 2008 IEEE INFOCOM*.

[20] M. McHenry, P. Tenhula, D. McCloskey, D. Roberson, and C. Hood, “Chicago spectrum occupancy measurements and analysis and a long-term studies proposal,” in *Proc. 2006 International Workshop on Technology and Policy for Accessing Spectrum*.

[21] T. Moscibroda, R. Chandra, Y. Wu, S. Sengupta, P. Bahl, and Y. Yuan, “Load-aware spectrum distribution in wireless LANs,” in *Proc. 2008 IEEE ICNP*.

[22] R. Myerson, “Optimal auction design,” *Mathematics of Operations*, 1981.

[23] S. Narlanka, R. Chandra, P. Bahl, and J. I. Ferrell, “A hardware platform for utilizing the TV bands with a Wi-Fi radio,” in *Proc. IEEE LANMAN*.

[24] M. J. Osborne, *A Course in Game Theory*. The MIT Press, 1994.

[25] M. T. S. Sakai and K. Yamazaki, “A note on greedy algorithms for the maximum weighted independent set problem,” *Discrete Applied Mathematics*, 2003.

[26] R. W. Thomas, R. S. Komali, L. A. DaSilva, and A. B. MacKenzie, “Joint power and channel minimization in topology control: a cognitive network approach,” in *Proc. 2007 IEEE CogNet Workshop*.

[27] W. Vickrey, “Counterspeculation, auctions, and competitive sealed tenders,” *J. of Finance*, 1961.

[28] F. Wu, N. Singh, N. Vaidya, and G. Chen, “On adaptive-width channel allocation in non-cooperative, multi-radio wireless networks,” in *Proc. 2011 IEEE INFOCOM*.

[29] F. Wu, S. Zhong, and C. Qiao, “Globally optimal channel assignment for non-cooperative wireless networks,” in *Proc. 2008 IEEE INFOCOM*.

[30] T. Yamada, D. Burgkhardt, I. Cosovic, and F. Jondral, “Resource distribution approaches in spectrum sharing systems,” *EURASIP J. Wireless Commun. Netw.*, 2008.

[31] Y. Yuan, P. Bahl, R. Chandra, P. A. Chou, J. I. Ferrell, T. Moscibroda, S. Narlanka, and Y. Wu, “Knows: cognitive networking over white spaces,” in *Proc. 2007 IEEE DySPAN*.

[32] Y. Yuan, P. Bahl, R. Chandra, T. Moscibroda, and Y. Wu, “Allocating dynamic time-spectrum blocks in cognitive radio networks,” in *Proc. 2007 ACM MobiHoc*.

[33] X. Zhou, S. Gandhi, S. Suri, and H. Zheng, “eBay in the sky: strategy-proof wireless spectrum auctions,” in *Proc. 2008 ACM MOBICOM*.

[34] X. Zhou, S. Mettu, H. Zheng, and E. M. Belding, “Traffic-driven dynamic spectrum auctions,” in *Proc. 2008 IEEE Workshop SDR*.

[35] X. Zhou and H. Zheng, “Trust: a general framework for truthful double spectrum auctions, in *Proc. 2009 IEEE INFOCOM*.

Tingting Chen is an assistant professor in Computer Science Department at Oklahoma State University, U.S. She received her Ph.D. degree from Computer Science and Engineering Department, at State University of New York at Buffalo, in 2011. She obtained her B.S. and M.S. degrees in computer science from the department of computer science and technology, Harbin Institute of Technology, China, in 2004 and 2006 respectively. Her research interests include data privacy and economic incentives in wireless networks.

Sheng Zhong received his BS (1996), MS (1999) from Nanjing University, Ph.D. (2004) from Yale University, all in computer science. He is interested in incentives, security, and privacy issues.