Competitive Auctions for Cost-aware Cellular Traffic Offloading with Optimized Capacity Gain

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***Abstract*—Offloading part of cellular traffic through existing alternative wireless networks, such as femtocells and WiFi networks, is one promising solution to the severe traffic overload faced by cellular network providers (CSPs) nowadays. Most existing cellular offloading auction mechanisms assume the CSP has the knowledge of incoming overloaded traffic demand, and satisfy the demand by offloading. However, in practice, with the explosive growth of mobile device communications, the overloaded traffic demand at CSPs is very likely to pass over the total capability that third-party resource owners can provide. Then it is critical to enable CSPs to optimize the traffic handling capacity gain through offloading with budget constraints. In this paper, we propose two efficient Competitive Auction MEchanisms for mObile offloading, CAMEO-min and CAMEO-ws. Both mechanisms are proven to be non-budget-deficit, individually rational and incentive-compatible, and have guaranteed lower bounds on the ratio of the CSP's gain achieved in them to the maximum gain that the CSP could achieve in any omniscient auction (the auction with an omniscient auctioneer). Our extensive evaluations show that CAMEOs achieve very good performance in terms of the maximization of the CSP's gain especially when the global bidder dominance or the region dominance is big.**

***Index Terms*—cellular offloading, auction, mechanism design.**

I. INTRODUCTION

According to a recent report released by Cisco [5], global mobile data traffic grew 69 percent in 2014, and reached 2.5 exabytes per month at the end of 2014, up from 1.5 exabytes per month at the end of 2013. Such an massive amount of mobile traffic can deteriorate the quality of service in existing cellular networks. Compared with updating the cellular network's infrastructure or building more towers, which is both expensive and time-consuming, offloading part of cellular traffic through existing alternative wireless networks such as femtocells, WiFi, etc. could ease the burden of cellular networks and enhance users' experience in an economical manner [3, 6, 16].

In practice, femtocell devices or WiFi hotspots in the macrocells maintained by CSPs are usually owned by third-party entities . These owners include schools, restaurants, shopping malls, residences et. al. which spread over the macrocell. Shifting cellular traffic from the CSP to them requires to use their own resources (e.g. bandwidth, data quota, etc.), and they need to be well-motivated to do so. Therefore, it is of great importance to design incentive mechanisms that motivate the cooperation between the CSPs and these third-party owners, for putting cellular offloading into practical applications.

Most of the existing works on incentive issues for cellular traffic offloading [3, 6, 14, 18, 19] provide auction mechanisms in which CSPs purchase resources from third-party owners on demand. They either rely on the assumption that the offloading traffic demand is prior knowledge or can be predicted precisely.

However, there are some scenarios where CSPs' need for traffic offloading is bursty or overwhelming while the demand is difficult to be predicted accurately or within a short period of time. For example, in emergency cases, like Superstrom Sandy in 2012, 25 percents of cellular sites were out of service [15]. At the same time the potential mobile communication between survivors and the outside world is of high demand and can be life-saving. In these scenarios, it is ideal that the survived CSPs can quickly offload the overwhelming traffic demand as much as possible to some third-party resource owners. Unfortunately, existing works cannot provide effective incentive mechanisms to address this example problem.

More fundamentally, existing incentive frameworks for cellular traffic offloading consider that the CSP has a fixed quantitative goal to achieve, i.e., the traffic demand. However, in practice, with the explosive growth of mobile device communications, the overloaded traffic demand at CSPs is very likely to pass over the total capability that third-party resource owners can provide. Then no matter what incentive mechanisms are used in the system, the overloaded traffic demand could not be satisfied fully through offloading. Therefore, it is practical and critical to optimize the CSP's traffic handling capacity gain through precautionarily purchasing offloading resources, with a budget constraint.

To address the above issues, in this paper we seek alternative offloading auction solutions that is able to optimize CSP's network capacity gain with a limited budget. Designing such auction mechanisms has the following unique challenges.

- *Different optimization goals*. Usually the optimized capacity gain is considered as the (weighted) sum of offloading resources that are purchased. It makes sense when the cellular network itself has a reasonable amount of capacity of its own, since a CSP can address some issues like inter-regional fairness by controlling the traffic handled by its own infrastructure. However, when the cellular network's own capacity is very limited or negligible compared with the third-party resources, the only way that a CSP can address the inter-regional fairness issue is to control the amount of traffic offloaded to each region when purchasing the resource. In this case, the system's capacity optimization goal should not be the sum of purchased resource anymore, but one which also takes fairness into consideration. In this paper, we aim at both optimization goals.
- *Diverse spatial coverage and the shared budget limit*. Since the coverage of femtocells/hotspots is much smaller than the range of a CSP's coverage, the offloading resource provided by different femtocells/hotspots could be of use in different regions. Taking this into consideration, a CSP's coverage is divided into several small regions, or

microcells, such that each region is covered by a group of femtocells or hotspots. On the other hand, when the CSP is purchasing resources from different owners in different regions, it has a shared total budget. How the CSP distributes the budget to different microcells and different owners to achieve optimization is a challenging problem that we will address in this paper.

- *Performance lowerbound*. It is not known yet, with a budget constraint in this traffic offloading problem, whether it is possible to design an individually rational and truthful auction that can reach a guaranteed lower bound of performance. In this paper, the performance metric used is the competitive ratio, i.e., the ratio of CSP's capacity gain in practical cases where auction participants may lie about their types (e.g. offloading cost and capability) over the optimal capacity gain in an ideal case where the auctioneer knows all participants' true type. Even if it is feasible, designing a truthful incentive mechanism that can guarantee a lower bound of performance is challenging.

Specifically, in this paper we consider two general optimization goals of CSP traffic offloading: the *minimum model* to maximize the least amount of resource purchased among all regions, and the *weighted-sum model* that considers the CSP may have different preferences on different regions and represent the gain as the weighted sum of the amounts of resource that are purchased in all regions. For each model, we adopted a novel technique that we call Multi-region Profit Extract Partitioning (MPEP) to design a non-budget-deficit, individually rational, incentive-compatible auction mechanism that also has guaranteed performance lower bounds. Specifically, our auction mechanisms guarantee submitting its bid truthfully is every femtocell/hotspot owner's dominant strategy regardless of other owners' strategies. Furthermore, they are proven to have guaranteed lower bounds on the ratio of CSP's gain achieved in these auctions to the maximum gain that can be achieved in any omniscient auctions (i.e. the auction with an omniscient auctioneer).

Our main contributions can be summarized as follows.

- We study the incentive mechanism design problem for cellular offloading systems, and propose practical solutions that, for the first time, do not rely on knowledge or estimation of the cellular traffic.
- For the minimum model, we propose CAMEO-min, an efficient auction mechanism that is proved to be non-budget-deficit, individually rational, incentive compatible, and guarantees the CSP's expected gain in the auction is at least $0.25(\alpha-1)\beta/(m\alpha)OPT_{min}$. Here OPT_{min} equals the maximum gain that the CSP can get in any omniscient auction in the minimum model, m equals the total number of regions inside the macrocell, and α, β are the *global bidder dominance* and the *region dominance* respectively. (Please refer to Section III-C2 for detailed definitions of α and β.)
- For the weighted-sum model, we propose CAMEO-ws, an efficient auction mechanism that is proved to be non-budget-deficit, individually rational, incentive compatible, and guarantees the CSP's expected gain in the auction is at least $0.25(\alpha-1)w_{min}/\alpha OPT_{ws}$. Here OPT_{ws} equals the maximum gain that the CSP can get in any omniscient auction in the weighted-sum model, α is the *global bidder dominance* and w_{min} is the minimum of all regions' weights.
- We implement both mechanisms and evaluate their performance extensively. Experiment results show both mechanisms have good performance regarding the maximization of CSP's gain.

The rest of this paper is organized as follows. Section II introduces the background knowledge of our problem. Sections III and IV showcase our auction mechanisms. Section V presents evaluation results. Section VI discusses the related work. Finally, the paper is summarized in Section VII.

II. BACKGROUNDS AND PRELIMINARIES

A. Cellular offloading and capacity gain maximization

Consider a cellular network that consists of CSP owned macrocell base stations (MBSs), third-party owned femtocells (or WiFi hotspots), and mobile phone users. Cellular offloading techniques allow CSPs to offload part of MBSs' traffic to femtocells, thus mitigate MSBs' overloading issue.

In this paper, we focus on the *capacity gain maximization* problem within one macrocell sector. To maintain the quality of service under the pressure of the ever-increasing cellular data traffic, the CSP gains extra network capacity by precautionarily purchasing offloading resource from owners of the femtocells in this macrocell sector. To make our problem meaningful, we assume the CSP has a limited budget. We study how the CSP could maximize its network capacity gain under the constraint of this budget.

Note that femtocells have a much smaller coverage compared with MBSs, offloading resource purchased from one femtocell is not able to handle all traffic from the entire macrocell sector. To deal with this practical issue and provide a fine-grained optimization, we consider the entire sector has been divided into several non-overlapping regions each of which is fully covered by the femtocells that reside in it, and then model the network capacity gain to be jointly determined by the amounts of offloading resource in all regions.

Specifically, given the entire sector is divided into m regions and the amounts of offloading resource purchased in these regions equal $s_1, s_2, \ldots, s_m$, we consider two general models of the CSP's capacity gain in this paper:

The minimum model: We consider the CSP's capacity gain equals the minimum of the amounts of resource that are purchased in all regions, i.e.,

$$CG_{min} = \min_{i \in \{1,\ldots,m\}} s_i, \tag{1}$$

where CG_{min} denotes the capacity gain in the minimum model. In fact, the idea of minimum model is derived from the "Cask Effect"[1], which considers the overall performance of the entire sector to be determined by the region whose performance is the worst.

The weighted-sum model: We consider the CSP's capcity gain equals the weighted sum of the offloading resource's amounts purchased in all regions, i.e.,

$$\begin{aligned} CG_{ws} = \sum_{i=1}^{m} & w_i s_i, \\ & 0 \leq w_i \leq 1 \text{ for } i = 1, \ldots, m; \\ & w_1 + \ldots + w_m = 1, \end{aligned} \tag{2}$$

[1]The effect's name is derived from the fact that the maximum amount of liquid that a cask, or a barrel, can contain, is determined by the shortest stave of the cask, rather than the tallest one or the one of the average length.

where $(w_1, \ldots, w_m)$ is the weight vector that shows the relative importance of each region to the CSP and CG_{ws} denotes the CSP's capcity gain in the weighted-sum model. The weighted-sum model allows the CSP to introduce its preference and/or knowledge about each region into the weight vector, and thus into the optimization goal.

B. Economic model

To provide incentives to femtocell owners, we consider the following economic model.

Suppose there are n femtocell owners in the sector who provide offloading services and are willing to sell their offloading service (or resource), and each owner j has a private *valuation* $c_j \in \mathbb{R}^+$ of a single unit of its service, and a private *quota* $b_j \in \mathbb{R}^+$ which equals the maximum amount of service that it can provide. Same as most game theoretical literature, we refer to $B_j = (c_j, b_j)$ as the owner j's *type*. Note that here we assume the quota is "hard" in the sense that the seller would not be able to provide more offloading service than the quota under any circumstances.

As the *buyer*, the CSP runs a reverse auction with all owners. In the auction, as the *seller*, each owner submits a bid $B'_j = (c'_j, b'_j)$ to the buyer. Based on all sellers' bids, the buyer determines the auction's outcome using a predefined auction mechanism.

Specifically, the *outcome* of the auction is specified by two vectors $X = (x_1, \ldots, x_n)$ and $P = (p_1, \ldots, p_n)$, where $x_j \in \{0\} \cup \mathbb{R}^+$, and $p_j \in \{0\} \cup \mathbb{R}^+$ equal the amount of service that the buyer acquires from seller j, and the payment seller j would receive respectively. Conventionally, we call a seller j a *winner* if it sells some service to the buyer (i.e. $x_i > 0$) and receives a positive payment (i.e. $p_i > 0$) according to the auction's outcome.

As most game theoretical work, we assume the sellers are *rational* [13] or selfish in the sense that it always tries to maximize its *utility*. In our problem, seller j's utility U_j is defined as

$$U_j = \begin{cases} p_j - c_j x_j & \text{if } x_i \leq b_j, \\ -\infty & \text{otherwise.} \end{cases} \tag{3}$$

The value $-\infty$ in the definition above indicates that each user's constraint on the maximum amount of service is *hard*.

For the ease of presentation, we let $\mathcal{B} = (B_1, \ldots, B_n)$ and $\mathcal{B}' = (B'_1, \ldots, B'_n)$ all sellers' types and bids. In addition, we let $U_j(\mathcal{B}')$ denote seller j's utility given all sellers' bids are $\mathcal{B}'$, and let $U_j((c''_j, b''_j), \mathcal{B}'_{-j})$ denote seller j's utility given all sellers' bids are the same as the bids specified in $\mathcal{B}'$ except that seller j changes its bid to (c''_j, b''_j). Finally, let $C \in \mathbb{R}^+$ denote the buyer's limited budget.

To make the auction practical and effective, the auction mechanism adopted is often required to possess the following properties:

- the auction is *non-budget-deficit*, i.e., the buyer's total payment is no greater than its budget:

$$\sum_{j=1}^{n} p_j \leq C; \tag{4}$$

- the auction is *individual-rational*, i.e., for each seller j, it holds that:

$$c'_j x_j \leq p_j; \tag{5}$$

- the auction is *incentive-compatible* or *truthful*, i.e., the utility of each seller j is maximized by bidding its true type, regardless of the types reported by other sellers:

$$U_j(B_j, \mathcal{B}'_{-j}) \geq U_j(B'_j, \mathcal{B}'_{-j}). \tag{6}$$

C. Competitive auction mechanism designing problem

It would be perfect if we could design an auction that has the above properties, and meanwhile maximizes the capacity gain. However, it is unknown how to design such a perfect auction. Considering the above economical properties are essential for the auction to be practically applied, we set our goal to design auction mechanisms that possess properties (4), (5) and (6), and meanwhile enable the buyer to acquire a capacity gain that is as great as possible.

We measure the success of an auction by comparing the gain acquired in the auction with the optimal gain that the buyer could have acquired if all the true types of sellers are known in advance.

Specifically, let $\mathcal{J} = (J_1, \ldots, J_m)$ denote the topology information such that J_i $(1 \leq i \leq m)$ is the index set of all sellers who are in region i, and let $O(\mathcal{B}, \mathcal{J}, C)$ denote the optimal gain that the buyer could acquire in an omniscient auction. In addition, let $A(\mathcal{B}', \mathcal{J}, C)$ be the gain the buyer acquires in our auction.

We say our auction has a *competitive ratio* of ϵ if

$$\epsilon = A(\mathcal{B}', \mathcal{J}, C)/O(\mathcal{B}, \mathcal{J}, C) \tag{7}$$

holds.

D. Single-price auction versus multi-price auction

Note that for each winning seller j, we can compute a *clearing price* at which the buyer purchases its service:

$$q_j = p_j/x_j. \tag{8}$$

In this paper, we call an auction mechanism a *single-price auction* if it adopts a uniform clearing price for all winning sellers within the same region. Otherwise, we say the auction mechanism is a *multi-price auction*.

It has been proved in [1] that given the same budget, the maximum amount of service that a buyer can purchase from one region in multi-price omniscient auctions is no greater than twice of the maximum amount of service that a buyer can purchase from the same region in single-price omniscient auctions. It is not difficult to apply this result to show that the optimal gain that the buyer could acquire in multi-price omniscient auctions, in either the minimum model or the weight-sum model, is also no greater than twice of the optimal gain it could acquire in single-price omniscient auctions. Therefore, we consider the single-price auction mechanisms only, and use the optimal capacity gain achieved by single-price auctions as our benchmark for measuring our auctions' performance.

III. When CSP's capacity gain equals the minimum

In this section, we present our auction mechanism CAMEO-min that is designed for the minimum model. Before we present the details of CAMEO-min, we first show how to maximize CSP's gain in an omniscient auction in the minimum model.

A. Solving the optimal omniscient auction

Let $Q = (q_1, \ldots, q_m)$, where $q_i \in \{0\} \cup \mathbb{R}^+$ equals the clearing price at which the buyer purchases offloading service from sellers in region i, and let $O_{min}(\mathcal{B}, \mathcal{J}, C)$ denote the maximum capacity gain in the optimal omniscient auction. It is straightforward to see $O_{min}(\mathcal{B}, \mathcal{J}, C)$ equals the object function's value of the following optimization problem over all possible Q and X:

$$
\begin{aligned}
\max_{Q,X} \quad & \min_{i \in \{1,\ldots,m\}} \sum_{j \in J_i, c_j \leq q_i} x_j \\
s.t. \quad & \sum_{i=1}^{m} q_i \sum_{j \in J_i, c_j \leq q_i} x_j \leq C; \\
& 0 \leq x_j \leq b_j.
\end{aligned} \tag{9}
$$

We construct an efficient algorithm that solves the optimization problem above based on two key observations on the solution as follows. The first one is that the optimum can be always achieved with a solution (Q, X) such that all regions sell the same amount of service to the buyer (Please see Proposition III.1). This allows us to focus on solutions in which the same amount of service is sold to the buyer in all regions. The second one is that, among all solutions of (9) that sell the same amount of service in each region, there is always one solution such that the clearing price in each region equals the bidding price from one of the sellers in this region (Please see Proposition III.2). This allows us to search the optimal q_i in $\{c_j\}_{j \in J_i}$ only.

With the help of these two observations, we propose the OPT_m algorithm (please see Algorithm 1) which searches the optimal clearing prices in a greedy manner to the solve problem (9). It is easy to verify the correctness of the algorithm. In addition, due to the most computation-demanding operation is the sorting operation in the beginning, the complexity of the algorithm is $O(n \log n)$.

Proposition III.1. *Among all solutions of (9), there is always one solution in which all regions sell the same amount of service to the buyer.*

Proposition III.2. *among all solutions of (9) that sell the same amount of service in each region, there is always one solution such that the clearing price in each region equals the bidding price from one of the sellers in this region.*

It is easy to verify the above two Propositions are true. Due to the page limit, we omit the proofs.

B. CAMEO-min

In this section, we construct our auction mechanism CAMEO-min using a novel technique that we call Multi-region Profit Extract Partitioning (MPEP). CAMEO-min is able to achieve non-budget-deficit, individual rationality and incentive compatibility, and also has a theoretical lower bound on its competitive ratio for maximizing the capacity gain of the CSP.

The main idea of MPEP is as follows. First, MPEP allocates the budget to each region according to the specificity of the overall capacity gain function, and makes sure that, in an omniscient auction, the optimum of overall capacity gain can be jointly bounded by the optimal payoff of the allocated budget (i.e. the amount of service that is purchased with the budget) in each region. Then, in each region, MPEP randomly partitions the sellers into two groups, and performs two parallel fixed-price, fixed-budget sub-auctions on them. By carefully designing the parameters of the sub-auctions, MPEP guarantees that these sub-auctions achieves non-budget-deficit, individual rationality and incentive compatibility, and that the ratio of the final payoff in the non-omniscient case, to the optimal payoff in the omniscient case, has a lower bound in each region. With the help of all excellent properties that are achieved, we are able to prove the constructed mechanism achieves all our goals.

Algorithm 1 OPT-min($\mathcal{B}, \mathcal{J}, C$)

for each J_i in $\mathcal{J}$ **do**
    sort $\{c_j : j \in J_i\}$ in a non-decreasing order.
end for
let q_i be the minimum c_j in region S_i
compute $s_i = \sum_{j \in J_i, c_j \leq q_i} b_j$
compute $i_{min} = argmin_i s_i$
while $s_{i_{min}} \leq \frac{C}{\sum_i^m q_i}$ **do**
    $(s_{min}^{prev}, q^{prev}) = (s_{i_{min}}, \{q_i\})$
    $q_{i_{min}}$ increase to the next c_j in region $S_{i_{min}}$
    update s_i and i_{min}
end while
if $\frac{C}{\sum_i^m q_i} \geq s_{min}^{prev}$ **then**
    $(q^*, s_{min}) = (\{q_i\}, \frac{C}{\sum_i^m q_i})$
else
    $(q^*, s_{min}) = (q^{prev}, s_{min}^{prev})$
end if

Specifically, the sub-auctions that are performed by CAMEO-min (and also CAMEO-ws) are the ProfitExtract auction mechanism that is proposed in [8] and extended in [1]. Given a predefined purchasing goal s, a budget C and all sellers' bids B' (within one region) as inputs, ProfitExtract purchases at most s units of service in total from randomly chosen sellers whose claimed unit-cost is no greater than C/s. The ProfitExtract auction and its extension provide us a very good building block, however, we emphasize that it only guarantees the economic properties as well as the existence of the competitive ratio locally in each region. Since our problem focuses on the overall performance on the entire sector, it requires a lot of extra efforts and thoughts to achieve our goals regarding the entire sector. For example, we will see we achieve the lower bound on the competitive ratio in the entire sector by exploring the "non-increasing average back" property (Please see Propositon III.6) regarding the local optimal purchase in each region and allocating the budget evenly among each region. To be complete, here we present ProfitExtract in Algorithm 2.

To achieve the theoretical guarantee on the expected gain, CAMEO-min spends the buyer's budget C equally on $2m$ ProfitExtract Auctions. In each region i, CAMEO-min partitions all sellers into two groups g_{i0} and g_{i1} randomly, and computes the maximum amount of service that a buyer can purchase with a budget $0.5C/m$ in each group: s_{i0} and s_{i1}.

Note that the maximum amount of service purchased within one region can be computed by applying Algorithm 1 with $m = 1$. Let $o(B_J, J, C)$ denote the function that returns the maximum amount of service that the CSP can purchase with a single clearing price from sellers whose index are in J, given the all sellers' types B_J and a budget C. Applying Algorithm 1 with $m = 1$, it is easy to find $o(B, J, C)$ can be computed

Algorithm 2 $ProfitExtract_{(J)}$(s,C,B')

```
buyer plans to purchase s units of service.
buyer sorts sellers whose index are in J randomly.
for each sorted seller j do
    if the claimed unit-cost of seller j is no greater than C/s and s > 0 then
        buyer purchases x_j units of service, where x_j equals b'_j or s, whichever is smaller.
        buyer pays seller j with p_j which equals x_j C/s.
        buyer updates s with s - p_j.
    end if
end for
```

as

$$j^* = \arg\max_{j \in J} c_j \sum_{k=1}^{j-1} b_k \le C; \tag{10}$$

$$o(B_J, J, C) = \min(\sum_{k=1}^{j^*} b_k, C/c_{j^*}). \tag{11}$$

assuming $\{c_j\}$ has been sorted in a non-decreasing order.

Then CAMEO-min runs the ProfitExtract Auction with the purchasing goal equaling s_{i1} and the same budget $0.5C/m$ with all sellers in group g_{i0}, and runs the ProfitExtract Auction with the purchasing goal equaling s_{i0} and the budget $0.5C/m$ with all sellers in group g_{i1}.

Running the ProfitExtract Auction in each region as above, allows us to get a theoretical lower bound on the ratio of the total amount of service that the buyer purchases in this region to the optimal amount of service that it can purchase in this region with a budget of C/m. Finally, by exploring the non-increasing average payback property regarding the local optimal purchase in each region and allocating the budget evenly among each region, we are able to derive a lower bound on the competitive ratio regarding the buyer's expected gain.

Formally, we present our auction mechanism CAMEO-min in Algorithm 3 as follows:

Algorithm 3 CAMEO-min($\mathcal{B}', \mathcal{J}, C$)

```
for each region S_i do
    buyer partitions the index set J_i randomly into two sets: J_{i_0} and J_{i_1}.
    buyer computes s_{i0} = o(B'_{J_{i_0}}, J_{i0}, 0.5C/m) and s_{i1} = o(B'_{J_{i_1}}, J_{i1}, 0.5C/m)
    buyer runs ProfitExtract_{(J_{i0})}(s_{i1}, 0.5C/m, B'_{J_{i_0}}) on all sellers in J_{i1}.
    buyer runs ProfitExtract_{(J_{i1})}(s_{i0}, 0.5C/m, B'_{J_{i_1}}) on all hotspot owners in J_{i0}.
end for
```

C. Performance analysis

1) Non-budget-deficit, individual rationality, and incentive compatibility: We formally prove that CAMEO-min possesses all three properties: (4), (5) and (6).

Theorem III.3. *CAMEO-min is non-budget-deficit, individually rational, and incentive compatible.*

Proof. According to Algorithms 2 and 3, CAMEO-min runs ProfitExtract Auctions with sellers in each region, and the ProfitExtract Auction always stops buying service after using up the budget. Therefore, CAMEO-min is non-budget-deficit.

In addition, ProfitExtract only allows the buyer to purchase service from sellers whose claimed unit-price is no greater than the clearing price. Therefore, CAMEO-min is individually rational.

Finally, we prove the incentive compatibility of CAMEO-min.

For any seller $j \in J_{i0}$, its utility function is given by

$$U_j(B'_j = (c'_j, b'_j), \mathcal{B}'_{-j}) = \begin{cases} -\infty & x_j > b_j \\ 0 & c'j > q_i \\ (q_i - c_j)x_j & o.w. \end{cases}$$

where $x_j = min(b'_j, s)$ is the amount of service that the buyer purchases from seller j, $q_i = \frac{0.5C/m}{s_{i1}}$ is the clearing price that the buyer purchases service from sellers whose index is in J_{i0}. Apparently, s and q_i are invariant to B'_j.

In the case where $b'_j <= b_j$, $U_j(B'_j, \mathcal{B}'_{-j})$ is non-decreasing to b'_j;

In the case where $b'j > b_j$, $U_j(B'_j, \mathcal{B}'_{-j})$ is invariant to b'_j; Thus we can get

$$U_j(B'_j = (c'_j, b_j), \mathcal{B}'_{-j}) \ge U_j(B'_j = (c'_j, b'_j), \mathcal{B}'_{-j}) \tag{12}$$

Similarly,

$$U_j(B'_j = (c_j, b'_j), \mathcal{B}'_{-j}) \ge U_j(B'_j = (c'_j, b'_j), \mathcal{B}'_{-j}) \tag{13}$$

Combining (12) and (13), we know inequality(6) holds for sellers in Ji_0.

Symmetrically, the conclusion holds for sellers in J_{i_1}. □

2) Competitive ratio: Here we prove the lower bound of CAMEO-min's competitive ratio regarding the buyer's expected gain.

Same as [1], in each region i, we define b^i_{max} as the largest service quota from amongst the winning sellers in the optimal solution of $o(B, J_i, C/m)$, and define α as the lower bound for $o(B, J_i, C/m)/b^i_{max}$. Intuitively, $1/\alpha_i$ specifies an upper bound on the fraction of the maximum service purchased that any single seller contributes. Accordingly, we call α_i the *bidder dominance* in region i, and call

$$\alpha = \min\{\alpha_1, \ldots, \alpha_m\} \tag{14}$$

the *global bidder dominance*.

Before we prove the lower bound of CAMEO-min's competitive ratio, we define C_{max} as the largest part of the budget that the buyer spends in a single region in the optimal solution of $O_{min}(\mathcal{B}, \mathcal{J}, C)$, and define β as the lower bound for C/C_{max}. Intuitively, $1/\beta$ specifies an upper bound on the fraction of the entire budget that the buyer spends on any single region. Accordingly, we call β the *region dominance*.

In both CAMEO-min and CAMEO-ws, we would use function $o(B, J, C)$ to compute important parameters of the auction. To prove both auctions' competitive ratio's lower bound guarantees, first we make a few key observations on the $o(B_J, J, C)$.

Proposition III.4 (local competitive ratio bound)**.** *In CAMEO-min, s_i, the amount of service purchased for region S_i, is no less than* $\min(s_{i0}, s_{i1})$. *Furthermore,* $E(s_i) \ge 0.25(\alpha - 1)o(B, J_i, C/m)/\alpha$.

Proof. Since CAMEO-min performs the ProfitExtract auctions in each region using the same way as the Profit Extraction Partition Auction does, we get $s_i \geq \min(s_{i0}, s_{i1})$ and $E(s_i) \geq 0.25(\alpha_i - 1)o(B, J_i, C/m)/\alpha_i$ directly following the proof in [1]. Note that since the ProfitExtract auction is a randomized algorithm, we adopt $E(s_i)$, the expectation of the amount of service purchased.

In addition, it is easy to see $0.25(\alpha_i - 1)o(B, J_i, C/m)/\alpha_i$ is a non-decreasing function of α_i. Given (14), we know $E(s_i) \geq 0.25(\alpha - 1)o(B, J_i, C/m)/\alpha$. □

Proposition III.5 (non-decreasing payback)**.** *For any B and J, $o(B_J, J, C)$ is a non-decreasing function of C.*

Proposition III.6 (non-increasing average payback)**.** *For any B and J, $o(B_J, J, C)/C$ is a non-increasing function of C in $(0, +\infty]$.*

Due to the page limit, we omit proofs of the above propositions here.

Following Proposition (III.6), we immediately have the following lemma:

Lemma III.7. *For any C, B_J, J, and any $\gamma \in [0, 1]$, $o(B_J, J, \gamma C) \geq \gamma o(B_J, J, C)$ holds.*

Now, we are ready to prove our main theorem.

Theorem III.8 (CAMEO-min's competitive ratio lower bound)**.** *The competitive ratio of CAMEO-min has a lower bound of $0.25(\alpha - 1)\beta/(m\alpha)$, i.e. $E(A(\mathcal{B}', \mathcal{J}, C))/O_{min}(\mathcal{B}, \mathcal{J}, C) \geq 0.25(\alpha - 1)\beta/(m\alpha)$ always holds.*

Proof. Let s_i equal the amount of service purchased from sellers in region i in CAMEO-min. In addition, let C_i and S_i equal the amount of budget that the buyer spends in region i, and the amount of service purchased in region i in the optimal solution $O_{min}(\mathcal{B}, \mathcal{J}, C)$, respectively. Since CAMEO-min is incentive compatible, we know all sellers would truthfully bid, i.e. $\mathcal{B}' = \mathcal{B}$. According to the definition in (1), we have

$$\begin{aligned} E(A(\mathcal{B}', \mathcal{J}, C)) &= E(\min(s_1, \ldots, s_m)) \\ &\geq \min(E(s_1), \ldots, E(s_m)), \end{aligned} \tag{15}$$

and

$$\begin{aligned} O_{min}(\mathcal{B}, \mathcal{J}, C) &= \min(S_1, \ldots, S_m) \\ &\leq \min(o(B_{J_1}, J_1, C_1), \ldots, o(B_{J_m}, J_m, C_m)). \end{aligned} \tag{16}$$

According to Proposition III.4, for each i we have:

$$E(s_i) \geq 0.25(\alpha - 1)o(B_{J_i}, J_i, C/m)/\alpha. \tag{17}$$

In addition, we know

$$C_i \leq C/\beta \tag{18}$$

holds for $i = 1, \ldots, m$ according to the definition of β. Following Lemma III.7, we immediately know

$$\begin{aligned} o(B_{J_i}, J_i, C/m) &\geq \beta o(B_{J_i}, J_i, C/\beta)/m \\ &\geq \beta o(B_{J_i}, J_i, C_i)/m. \end{aligned} \tag{19}$$

Therefore, we know for all i

$$E(s_i) \geq 0.25(\alpha - 1)\beta o(B_{J_i}, J_i, C_i)/(m\alpha). \tag{20}$$

Putting (20) into (15), we know

$$\begin{aligned} &E(A(\mathcal{B}', \mathcal{J}, C)) \\ &\geq \min_i\{0.25(\alpha - 1)\beta o(B_{J_i}, J_i, C_i)/(m\alpha)\} \\ &\geq 0.25(\alpha - 1)\beta/(m\alpha) \min_i\{o(B_{J_i}, J_i, C_i)\} \\ &\geq 0.25(\alpha - 1)\beta/(m\alpha) O_{min}(\mathcal{B}, \mathcal{J}, C). \end{aligned} \tag{21}$$

Therefore

$$E(A(\mathcal{B}', \mathcal{J}, C))/O_{min}(\mathcal{B}, \mathcal{J}, C) \geq 0.25(\alpha - 1)\beta/(m\alpha). \tag{22}$$

□

IV. When CSP's capacity gain equals the weighted sum

In this section, we present CAMEO-ws that is designed for the weighted-sum model.

A. CAMEO-ws

Similar to CAMEO-min, CAMEO-ws also uses ProfitExtract auctions as its building blocks to achieve non-budget-deficit, individual rationality, ncentive compatibility and similar performance guarantees regarding the total service purchased in each single region. However, to possess the performance guarantee regarding the CSP's gain in the entire sector in the weighted-sum model, CAMEO-ws allocates the budget differently from CAMEO-min.

Let $C_1, \ldots, C_m$ denote the amounts of budget that are assigned to the m regions. In particular, CAMEO-ws sets each region's budget proportional to its weight, i.e.,

$$C_i = w_i C. \tag{23}$$

In addition, CAMEO-ws randomly divides the sellers in each region into two partitions, and performs two parallel ProfitExtract auctions with a budget of $0.5w_i C$ on each partition of sellers.

As we will see in Theorem IV.2, the above design would allow us to achieve a lower bound on the CAMEO-ws's competitive ratio.

In more detail, we present CAMEO-ws in Algorithm 4.

Algorithm 4 CAMEO-ws($\mathcal{B}', \mathcal{J}, C$)

for each region S_i **do**
  buyer partitions the index set J_i randomly into two sets: J_{i_0} and J_{i_1}.
  buyer computes $s_{i0} = o(B'_{J_i}, J_{i0}, 0.5w_i C)$ and $s_{i1} = o(B'_{J_i}, J_{i1}, 0.5w_i C)$
  buyer runs $ProfitExtract_{(J_{i0})}(s_{i1}, 0.5w_i C, B'_{J_{i0}})$ on all sellers in J_{i0}.
  buyer runs $ProfitExtract_{(J_{i1})}(s_{i0}, 0.5w_i C, B'_{J_{i1}})$ on all sellers in J_{i1}.
end for

B. Performance analysis

1) Non-budget-deficit, individual rationality, and incentive compatibility: CAMEO-ws and CAMEO-min differ in the way of allocating budget to each region. Since the budget allocation in CAMEO-ws is determined by $\{w_i\}$ only, it is easy to see CAMEO-ws is also non-budget-deficit, individually rational, and incentive compatible.

Theorem IV.1. *CAMEO-ws is non-budget-deficit, individually rational and incentive compatible.*

Proof. The proof for Theorem (IV.1) is basically the same as the proof of Theorem (III.3), thus we neglect the proof here. □

2) Competitive ratio: As for the competitive ratio of CAMEO-ws, we have the following theorem.

Theorem IV.2 (CAMEO-ws's competitive ratio lower bound)**.** $E(A(\mathcal{B}', \mathcal{J}, C))/O_{ws}(\mathcal{B}, \mathcal{J}, C) \geq 0.25 w_{min}(\alpha-1)/\alpha$*, where* w_{min} *equals the minimum of* $w_1, \ldots, w_m$*.*

Proof. Similarly, let s_i equal the amount of service purchased from sellers in region i in CAMEO-ws. In addition, let C_i and S_i equal the amount of budget that the buyer spends in region i, and the amount of service purchased in region i in the optimal solution $O_{ws}(\mathcal{B}, \mathcal{J}, C)$, respectively.

According to the definition in (2), we have

$$E(A(\mathcal{B}', \mathcal{J}, C)) = E(w_1 s_1 + \ldots + w_m s_m), \tag{24}$$

$$O_{ws}(\mathcal{B}, \mathcal{J}, C) = w_1 S_1 + \ldots + w_m S_m. \tag{25}$$

Due to the fact that sub-auctions performed in all regions are independent from each other, we know

$$E(A(\mathcal{B}', \mathcal{J}, C)) = w_1 E(s_1) + \ldots + w_m E(s_m). \tag{26}$$

In addition, due to the optimality of function $o(B_J, J, C)$, we have

$$O_{ws}(\mathcal{B}, \mathcal{J}, C) \leq w_1 o(B_{J_1}, J_1, C_1) + \ldots + w_m o(B_{J_m}, J_m, C_m). \tag{27}$$

According to Propositions III.5, III.6 and III.4, for each i we have:

$$\begin{aligned} E(s_i) &\geq 0.25(\alpha-1) o(B_{J_i}, J_i, w_i C)/\alpha \\ &\geq 0.25(\alpha-1) o(B_{J_i}, J_i, w_i C_i)/\alpha \\ &\geq 0.25 w_i (\alpha-1) o(B_{J_i}, J_i, C_i)/\alpha. \end{aligned} \tag{28}$$

Putting (28) and (27) into (26), we know

$$\begin{aligned} &E(A(\mathcal{B}', \mathcal{J}, C)) \\ \geq &0.25(\alpha-1)/\alpha \sum_{i=1}^{m} (w_i)^2 o(B_{J_i}, J_i, C_i) \\ \geq &0.25 w_{min}(\alpha-1)/\alpha \sum_{i=1}^{m} w_i o(B_{J_i}, J_i, C_i) \\ \geq &0.25 w_{min}(\alpha-1)/\alpha O_{ws}(\mathcal{B}, \mathcal{J}, C) \end{aligned} \tag{29}$$

Therefore

$$E(A(\mathcal{B}', \mathcal{J}, C))/O_{ws}(\mathcal{B}, \mathcal{J}, C) \geq 0.25 w_{min}(\alpha-1)/\alpha. \tag{30}$$

□

V. Evaluation

We have implemented CAMEO-min and CAMEO-ws, and performed extensive simulation experiments to evaluate our mechanisms. Results validate the effectiveness of our auctions' theoretical lower bounds on their competitive ratio. In addition, we can see both two mechanisms yield a relative stable and high gain to the CSP especially when the bid dominance or the region dominance is big.

A. Simulation Settings

A typical cell base station usually covers a 400-500m range in busy urban areas. Here we consider a cellular macrocell of 250m×250m in our simulation. Since the communication range of a femtocell is around 40m, we statically partition the macrocell into 6 (or more) regions. All femtocells are distributed randomly into these regions inside the macrocell.

We test our auctions using data that is generated from two different distributions: an unbiased one which is the uniform distribution, and a biased one which is the exponential distribution with the *ratio parameter* λ. In particular, when using the uniform distribution or the exponential distribution, femtocell owners' unit cost and service quota are both distributed in $[0, 1]$. All results are averaged over 100 runs of the experiment.

B. Simulation Results

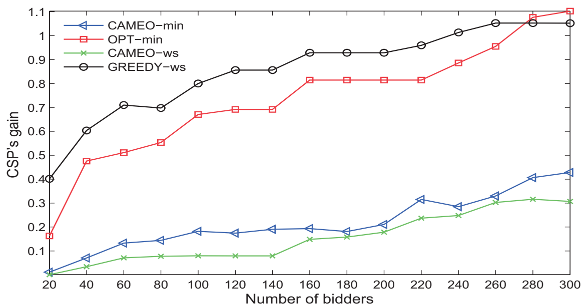
We have performed three sets of experiments.

Experiment Set 1: In the first set of experiments, we mainly test our auctions' performance given different numbers of bidders (20–300) and different amounts of budget ($C = 1, 10, 100$). Figure 1 shows CSP's gain in CAMEO-min, CAMEO-ws, OPT-min, and GREEDY-ws. Since we are unable to find an efficient algorithm that solves the optimal gain in the omniscient auction in the weighted-sum model as we find the OPT-min algorithm, we implement a greedy algorithm that always distributes GREEDY-ws to compare CAMEO-ws with.

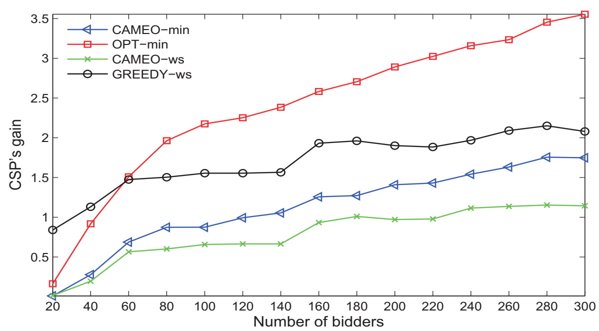
From the figure, we can see the CSP's gain increases as the number of femtocells increases in all four mechanisms. This is mainly because more sellers means the CSP has a greater probability to meet sellers with low unit-cost, and purchase cheap resources from them. In addition, we can see CAMEO-min (and CAMEO-ws resp.) brings the CSP a stable, relatively large portion of the gain that OPT-min (and GREEDY-ws resp.) brings the CSP. Especially when the budget is sufficient (C=100), CAMEO-min achieves a competitive ratio that is always above 60% and CAMEO-ws's performance is close to the performance of GREEDY-ws.

Experiment Set 2: In the second set of experiments, we mainly test the competitive ratios achieved by our mechanisms given different global bidder dominance and given different region dominance. We fix the number of femtocells to 200 and their types first, then change the value of α and β by adjusting the budget C from 1 to 100. Figures 2(a) and 2(b) show that CAMEO-min's competitive ratio and its lower bound increase as α or β increases, which validates the correctness of our results on the lower bound. In addition, we can see CAMEO-min's competitive ratio is almost twice of the expected lower bound that we have proved. This is mainly because we always neglect the service purchased in one of the two partitions. Therefore, the actual competitive ratios of our mechanisms are usually larger than their lower bounds proved by us in this paper.

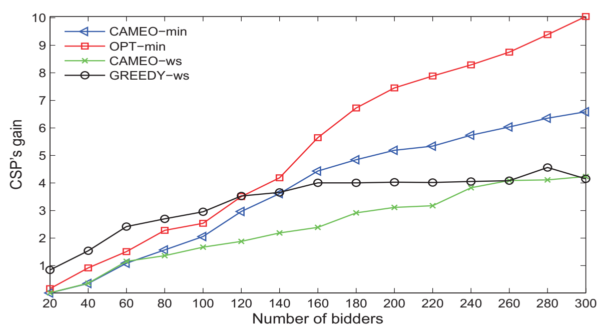
Experiment Set 3: In the last set of experiments, we test our mechanisms' performance using the testing data sampled from the exponential distribution. From Figure 3(a) and 3(b), we can see when λ is small ($\lambda \leq 1$) CAMEO-min (and CAMEO-ws resp.) achieves a large portion (around 50%) of the gain that the CSP gets in the omniscient auction with OPT-min (and GREEDY-ws resp.). However, the portions start to drop when λ grows. This is because when λ becomes large, heavy bias is introduced to the testing data. Accordingly, the probability that some "extreme" seller contributes most of the CSP's gain in one region and the probability that one region



(a) C = 1



(b) C = 10



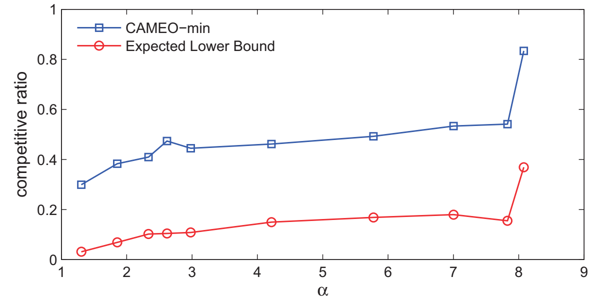
(c) C = 100

Fig. 1. The CSP's gain under different budget C with variable femtocell density.

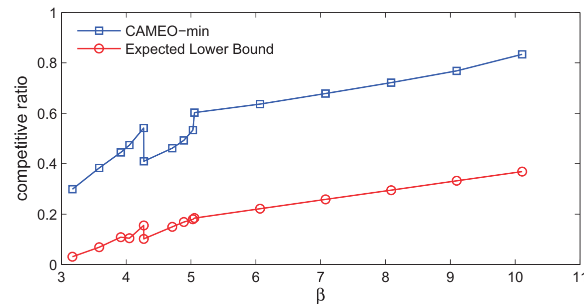
receives most of the CSP's budget become greater, which brings α close to 1 and β close to 0. This makes the lower bounds of our mechanisms approach 0, and deteriorates our mechanisms' performance.

VI. Related Work

The research on designing incentive mechanisms for cellular offloading is relatively new. So far, there have been some pioneering works e.g. [3, 6, 11, 14, 18, 19] in the area. While these pioneering works are very nice and interesting, most of them [3, 6, 14, 18, 19] rely on the assumption that the underlying traffic or offloading demands are known to the CSP or can be estimated precisely and efficiently. Furthermore, they have not studied the performance guarantee of their optimization results. In contrast, we do not make any assumptions regarding the knowledge or estimation of the traffic, and we provide a performance guarantee through rigorous analysis. The only exception among these pioneering works is [11], in which authors consider a very interesting issue that bidders may not know their true valuation accurately, and then propose an innovative solution. The problem studied in their work is quite different from ours.

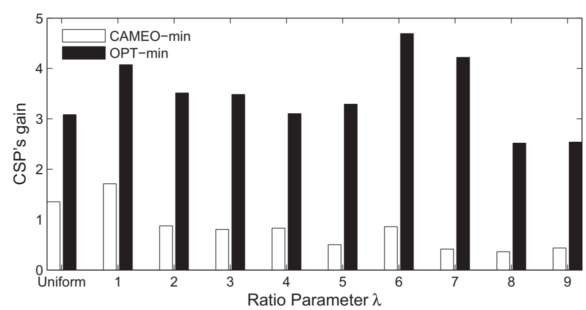


(a) CAMEO-min with variable α

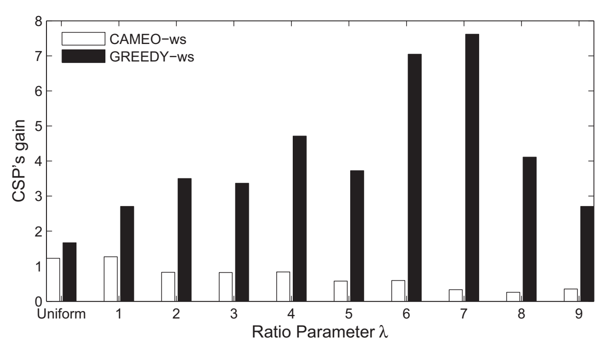


(b) CAMEO-min with variable β

Fig. 2. The competitive ratio of CSP under different (α, β) with 200 femtocells.



(a) CAMEO-min



(b) CAMEO-ws

Fig. 3. The CSP's gain under bids in different exponential distributions with 200 femtocells.

Since we aim to design auction mechanism that can maximize the CSP/auctioneer's gain, theoretically our work can be categorized into the general revenue-maximizing auction design problem. Most existing works (e.g. [4, 10, 12]) on this subject build their solutions in the Bayesian setting, which assumes that the probability distribution of the bidders' valuations are known to the auctioneer. A recent advance on the revenue maximization in multi-unit budget-constraint auctions that does not rely on the above assumption is [1]. In this work, Abrams proposes a randomized auction mechanism called *Profit Extract Partition Auction* (PEP), which is proved to have a theoretical lower bound on the auction's competitive ratio (i.e. the ratio of the auctioneer's expected revenue in the auction to the optimal revenue in an omniscient auction). The problem studied in [1] can be viewed as a special case of our problem in the weighted-sum model. However, directly applying PEP in our problem would immediately lose the performance guarantee due to the difference of our models and the author's. In fact, our problem is more complicated than the one studied in [1] due to the specificity of our underlying maximization of the object function. In particular, we aim to maximize the CSP's gain to which different individual regions may contribute differently. Furthermore, all regions share a limited budget, so optimizing CSP's revenue in one region could conflict with optimizing CSP's revenue in the other region. To deal with our problem, we construct our auctions based on a different technique that we call MPEP.

Finally, we note that the revenue-maximizing auction mechanism design problem has also been studied in the spectrum auction area. However, similar to works in the general revenue-maximizing auction design area, either their solutions are based on the Bayesian model (e.g. [2, 9]), or it is not known how to apply their results (e.g. [7, 17]) into our problem due to different specificities of the two areas.

VII. Conclusion

In this paper, we study competitive auction mechanism design problems for cellular offloading systems. We present two non-budget-deficit, individually rational, incentive-compatible auction mechanisms CAMEO-min and CAMEO-ws, and prove they both have theoretical performance guarantees on maximizing the CSP's gain in the auction. Experiments validate our theoretical results, and show both two mechanisms achieve a relatively good and stable performance, especially when the global bidder dominance or the region dominance is large.

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