

Differential Equations Project: Phase 3

Arnav Patri and Shashank Chidige

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This is the dependent variable Stock Price of Google (S) and the independent variable Days (t). The data is taken from Yahoo Finance from the ticker GOOG.

Table 1	
Stock Price (USD)	Time (Days from start date of 9/27/22)
141.501007	0
136.184006	1
134.520996	2
133.265503	3
136.462494	4
133.764999	5
136.177002	6
137.354004	7
139.185501	8
140.056	9
138.847504	10
136.712997	11
137.899994	12
141.412003	13
141.675003	14
142.960495	15
143.822006	16
142.414993	17
142.780502	18
138.625	19
138.772995	20
139.671997	21
146.427505	22
146.128998	23
148.270493	24
143.774002	25
145.863007	26
146.789993	27
148.682999	28

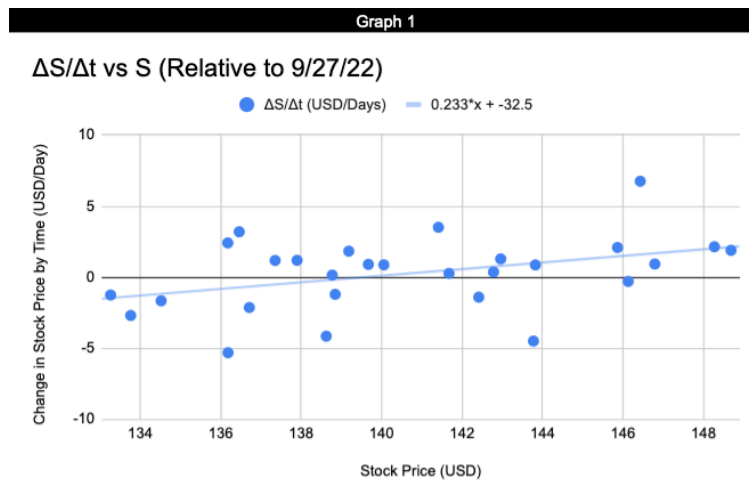
This table calculates the changes in each variable.

Table 2					
Stock Price (USD)	Time (Days from start date of 9/27/22)	ΔS^* (USD)	Δt^{**} (Days)	$\Delta S/\Delta t$ (USD/Days)	Stock Price (USD)
141.501007	0				141.501007
136.184006	1	-5.317001	1	-5.317001	136.184006
134.520996	2	-1.66301	1	-1.66301	134.520996
133.265503	3	-1.255493	1	-1.255493	133.265503
136.462494	4	3.196991	1	3.196991	136.462494
133.764999	5	-2.697495	1	-2.697495	133.764999
136.177002	6	2.412003	1	2.412003	136.177002
137.354004	7	1.177002	1	1.177002	137.354004
139.185501	8	1.831497	1	1.831497	139.185501
140.056	9	0.870499	1	0.870499	140.056
138.847504	10	-1.208496	1	-1.208496	138.847504
136.712997	11	-2.134507	1	-2.134507	136.712997
137.899994	12	1.186997	1	1.186997	137.899994
141.412003	13	3.512009	1	3.512009	141.412003
141.675003	14	0.263	1	0.263	141.675003
142.960495	15	1.285492	1	1.285492	142.960495
143.822006	16	0.861511	1	0.861511	143.822006
142.414993	17	-1.407013	1	-1.407013	142.414993
142.780502	18	0.365509	1	0.365509	142.780502
138.625	19	-4.155502	1	-4.155502	138.625
138.772995	20	0.147995	1	0.147995	138.772995
139.671997	21	0.899002	1	0.899002	139.671997
146.427505	22	6.755508	1	6.755508	146.427505
146.128998	23	-0.298507	1	-0.298507	146.128998
148.270493	24	2.141495	1	2.141495	148.270493
143.774002	25	-4.496491	1	-4.496491	143.774002
145.863007	26	2.089005	1	2.089005	145.863007
146.789993	27	0.926986	1	0.926986	146.789993
148.682999	28	1.893006	1	1.893006	148.682999

$$\Delta S_i = S_i - S_{i-1} \quad (\text{Equation 1}^*)$$

$$\Delta t_i = t_i - t_{i-1} \quad (\text{Equation 2}^{**})$$

This is the graph of $\Delta S/\Delta t$ (in USD/day) vs S (in USD) with a linear regression performed.



The DE being used to model stock price is

$$\frac{dS_t}{dt} = \mu S_t + \sigma S_t \frac{dW_t}{dt}$$

6 where S_t is stock price as a function of time t , μ is the (constant) drift, σ is the (constant) volatility, and W_t is a standard Wiener process. The dependent variable t is not present, making this an autonomous DE. The only information needed for the model are μ , σ , and the initial stock price S_0 .

μ is the amount that $\mathbb{E}(S)$ changes, making it the coefficient of the linear regression of $\Delta S/\Delta t$ against S , so $\mu \approx 0.233$.

σ is simply the standard deviation of the stock price in the sample, so $\sigma \approx 4.356$.

The model then becomes

$$\boxed{\frac{dS_t}{dt} = 0.233S_t + 4.356S_t \frac{dW_t}{dt}}$$