Assignment 2

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1. Find the angle between the vectors $\vec{a} = \hat{\imath} - 2\hat{\jmath} + \hat{\mathrm{k}}$ and $\vec{b} = 2\hat{\imath} + \hat{\jmath} - \hat{\mathrm{k}}$.

   Solution:

$$\begin{aligned} |\vec{a}||\vec{b}|\cos\theta &= \vec{a}\cdot\vec{b} \\ \sqrt{1^2+(-2)^2+(1)^2}\sqrt{2^2+1^2+(-1)^2}\cos\theta &= (1)(2)+(-2)(1)+(1)(-1) \\ \sqrt{6}\sqrt{6}\cos\theta &= -1 \\ \theta &= \arccos\left(-\frac{1}{6}\right) \approx 100^\circ \end{aligned}$$

2. Find the cross product of the vectors $\vec{a} = \langle 2, -1, 4\rangle$ and $\vec{b} = \langle 3, 0, -2\rangle$.

   Solution:

$$\begin{aligned} \vec{a}\times\vec{b} &= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{\mathrm{k}} \\ 2 & -1 & 4 \\ 3 & 0 & -2 \end{vmatrix} = \langle (-1)(-2)-(4)(0), -1((2)(-2)-(4)(3)), (2)(0)-(-1)(3)\rangle \\ &= \langle 2, 16, 3\rangle \end{aligned}$$

3. Find the equation of the plane $Ax + By + Cz = D$ through the points $P(1, -1, 2)$, $Q(-1, 0, 1)$, and $R(1, -1, 1)$.

   Solution

$$\begin{aligned} D &= A(1)+B(-1)+C(2) \\ D &= A-B+2C \\ -(D &= A-B+C) \\ 0 &= C \end{aligned}$$

$$\begin{aligned} D &= A(-1)+B(0)+C(1) \\ D &= -A+C \\ D &= -A+0 \\ A &= -D \end{aligned}$$

$$\begin{aligned} D &= A(1)+B(-1)+C(1) \\ D &= A-B+C \\ D &= -D-B+0 \\ B &= -2D \end{aligned}$$

$$\begin{aligned} -Dx-2Dy-0 &= D \\ 0 &= D(1+x+2y) \end{aligned}$$