Module 9

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16.3.19. (p. 1152)

(a) Find a function f such that $\vec{F} = \nabla f$.

$$\begin{aligned}
\vec{F}(x,y) &= x^2y^3\hat{\imath} + x^3y^2\hat{\jmath} \\
f_x(x,y) &= x^2y^3 \\
f(x,y) &= \int \left[x^2y^3\right] \mathrm{d}x = \frac{x^3y^3}{3} + g(x) \\
f_y(x,y) &= x^3y^2 + g'(y) = x^3y^2 \\
g'(y) &= 0 \\
f(x,y) &= \frac{x^3y^3}{3}
\end{aligned}$$

(b) Use part (a) to evaluate $\int_C \vec{F} \cdot \mathrm{d}\vec{r}$ along the given curve C.

$$C : \vec{r}(t) = \left\langle t^3 - 2t, t^3 + 2t \right\rangle, 0 \le t \le 1$$

$$\int_C \vec{F} \cdot \mathrm{d}\vec{r} = f(\vec{r}(1)) - f(\vec{r}(0)) = f(-1,3) - f(0,0) = -9 - 0 = -9$$

16.4.7. (p. 1160) Use Green's Theorem to evaluate the line integral along the given positively oriented curve.

$$\int_C \left[x^2y^2 \,\mathrm{d}x + y \arctan y \,\mathrm{d}y\right] \text{ where } C \text{ is the triangle with vertices } (0,0),\ (1,0), \text{ and } (1,3)$$

$$\begin{aligned}
D &= \{(x,y) \mid 0 \le x \le 1, 0 \le y \le 3x\} \\
P &= x^2y^2 \implies P_y = 2x^2y \qquad Q = y \arctan y \implies Q_x = 0 \\
\int_C [P \,\mathrm{d}x + Q \,\mathrm{d}y] &= \iint_D \left[\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right] \mathrm{d}A = \int_0^1 \int_0^{3x} \left[0 - 2x^2y\right] \mathrm{d}y \,\mathrm{d}x = \int_0^1 \left[-x^2y^2\right]_0^{3x} \mathrm{d}x \\
&= \int_0^1 \left[-9x^4 - (0)\right] \mathrm{d}x = \left[-1.8x^5\right]_0^1 = -1.8 - (0) = -1.8
\end{aligned}$$