Homework Set 2

Arnav Patri

November 10, 2022

10 Graphs

10.1 Graphs and Graph Models

3. The graph has undirected edges and no loops, making it a simple graph.

4. The graph has multiple undirected edges and no loops, making it a multigraph.

5. The graph has multiple undirected edges and loops, making it a psuedograph.

6. The graph has multiple undirected edges and no loops, making it a multigraph.

7. The graph has directed edges and loops, making it a digraph.

8. The graph has multiple directed edges and loops, making it a directed multigraph.

9. The graph has multiple directed edges and loops, making it a directed multigraph.

10.2 Graph Terminology and Special Types of Graphs

1. $|V| = 6$, $|E| = 6$, $\deg a = 2$, $\deg b = 4$, $\deg c = 1$ (pendant), $\deg d = 0$ (isolated), $\deg e = 2$, $\deg f = 3$

2. $|V| = 5$, $|E| = 13$, $\deg a = 6$, $\deg b = 6$, $\deg c = 6$, $\deg d = 5$, $\deg e = 3$

3. $|V| = 9$, $|E| = 12$, $\deg a = 3$, $\deg b = 2$, $\deg c = 4$, $\deg d = 0$ (isolated), $\deg e = 6$, $\deg f = 0$ (isolated), $\deg g = 4$, $\deg h = 2$, $\deg i = 3$

5. A simple graph with 15 vertices each of degree 5 cannot exist, as all graphs must have an even number of vertices of odd degree.

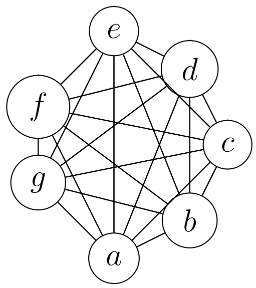
7. $|V| = 4$, $E = 7$, $\deg^- a = 3$, $\deg^+ a = 1$, $\deg^- b = 1$, $\deg^+ b = 2$, $\deg^- c = 2$, $\deg^+ c = 1$, $\deg^- d = 1$, $\deg^+ d = 3$

8. $|V| = 4$, $|E| = 8$, $\deg^- a = 1$, $\deg^- b = 3$, $\deg^- c = 2$, $\deg^- d = 1$, $\deg^+ a = 2$, $\deg^+ b = 4$, $\deg^+ c = 1$, $\deg^+ d = 1$

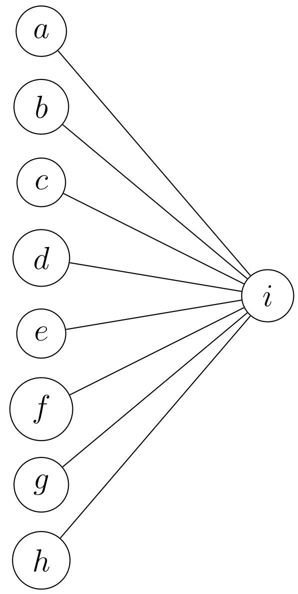
9. $|V| = 5$, $E = 13$, $\deg^- a = 6$, $\deg^+ a = 1$, $\deg^- b = 1$, $\deg^= b = 5$, $\deg^- c = 2$, $\deg^+ c = 5$, $\deg^- d = 4$, $\deg^= d = 2$, $\deg^- e = 0$, $\deg^= e = 0$

20.

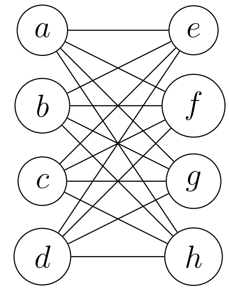
a)



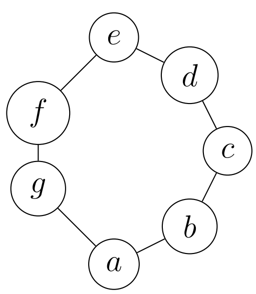
b)



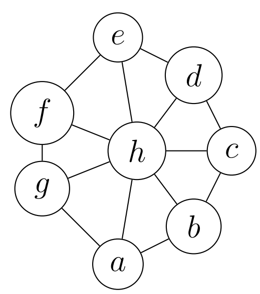
c)



d)

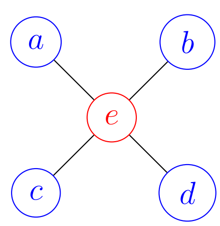


e)



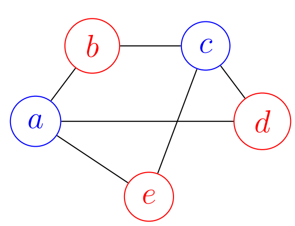
f)

21.



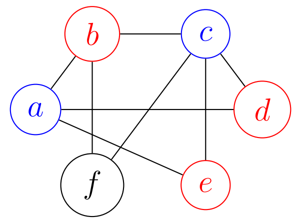
The graph is bipartite.

22.



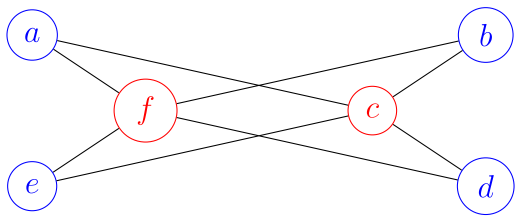
The graph is bipartite.

23.



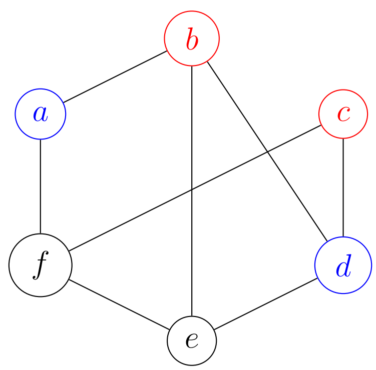
This graph is not bipartite, due to f.

24.



This graph is bipartite.

25.



This graph is not bipartite, due to e and f.

26.

a) K_1 and K_2 are bipartite, but K_n for $n \geq 3$ is not bipartite, as any 3 vertices are connected pairwise, so there is no way to partition them into 2 disjoint sets.

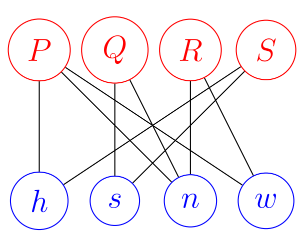
b) C_n is bipartite whenever n is even, as the vertices can simply alternate.

c) W_n is never bipartite, as every vertex is connected to the center of the wheel.

d) Q_n is always bipartite.

27.

a)



37.

a) $|V| = n, |E| = \binom{n}{2}$ b) $|V| = n, |E| = n$ c) $|V| = n+1, |E| = 2n$

d) $|V| = m+n, |E| = mn$ e) $|V| = 2^n, |E| = n2^{n-1}$

53. why

10.3 Representing Graphs and Graph Isomorphism

1.

Vertex	Adjacent Vertices
a	b, c, d
b	a, d
c	a, d
d	a, b, c

3.

Vertex	Terminal Vertices
a	a, b, c, d
b	d
c	a, b
d	b, c, d

5.

$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

7.

$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

a)

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

b)

$$\begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

c)

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

d)

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

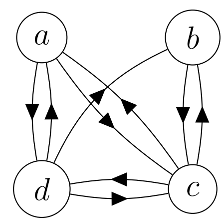
e)

$$\begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

f)

$$\begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

11.



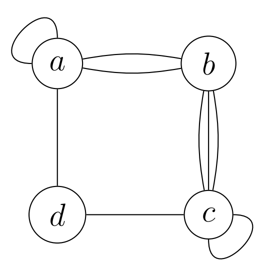
13.

$$\begin{array}{c c} & \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 \\ 1 & 1 & 0 & 1 \\ 0 & 2 & 1 & 0 \end{bmatrix} \end{array}$$

15.

$$\begin{array}{c c} & \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & 2 \\ 2 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix} \end{array}$$

17.



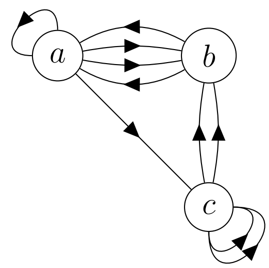
19.

$$\begin{array}{c c} & \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

21.

$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 1 & 1 & 2 & 1 \\ 1 & 0 & 0 & 2 \\ 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 0 \end{bmatrix} \end{array}$$

23.



31.

$$\mathbf{M}_{13} = \begin{array}{c} a \\ b \\ c \\ d \end{array} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{M}_{14} = \begin{array}{c} a \\ b \\ c \\ d \end{array} \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

$$\mathbf{M}_{15} = \begin{array}{c} a \\ b \\ c \\ d \end{array} \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

33. For an undirected graph, the sum of the entries in a column of the adjacency matrix is the number of edges that are connected to that column's vertex (loops only being counted once). For a digraph, it is the in-degree of the vertex.

35. For an undirected graph, the sum of the values of a column in the incidence matrix is equal to the number of nodes that the column's edge is incident to. This can only be 1 (if the edge is a loop) or 2.

36. a)

$$\begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & & \vdots \\ 1 & 1 & \cdots & 1 \end{bmatrix}$$

b)

$$\begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 1 \\ 1 & 0 & 1 & \cdots & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 1 & 0 & 0 & \cdots & 1 & 0 \end{bmatrix}$$

c)

$$\begin{bmatrix} 1 & 1 & 1 & \cdots & 1 & 1 & \\ 1 & 0 & 1 & 0 & \cdots & 0 & 1 \\ 1 & 1 & 0 & 1 & \cdots & 0 & 0 \\ 1 & 0 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \\ 1 & 0 & 0 & \cdots & 1 & 0 & \end{bmatrix}$$

d)

$$\begin{bmatrix} 0 & \cdots & 1 \\ \vdots & & \vdots \\ 1 & \cdots & 0 \end{bmatrix}$$

e)

$$] \; [$$

37. a)

$$\begin{bmatrix} a \end{bmatrix}$$

43. $|V_1| = |V_2| = 6, |E_1| = |E_2| = 9$