Homework Set 3

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7 Discrete Probability

1. Boys and girls being equally likely implies that the probability of each is $1/2 = 0.5$ (assuming them to be the only 2 possible outcomes). The number of trials (the number of children) and the probability of each child being a boy are then fixed at 3 and 0.5 respectively, meaning that this situation follows a binomial setting with probability of success $p = 0.5$ and number of trials $n = 3$. As such,

$$\mu_X = np = 0.5 \times 3 = 1.5$$
$$\sigma_X = \sqrt{np(1 - p)} = \sqrt{0.5(3)(1 - 0.5)} = \sqrt{0.75} = 0.5\sqrt{3} \approx 0.866$$

The mean is 1.5 boys while the standard deviation is about 0.866 boys.

2. X is a discrete random variable that follows the given probability distribution

x	0	1	2	3	4
$P(x)$	0.3	0.4	0.2	0.06	0.04

As such,

$$\mu_X = \sum x_i P(x_i) = 1.14$$
$$\sigma_X = \sqrt{\sum (x_i - \mu_X)^2 P(x_i)} \approx 1.039$$

The mean is 1.14 toppings while the standard deviation is about 1.039 toppings.

3. One ticked being randomly selected from $1,000$ means that the probability of any one ticket being selected is $1/1,000 = 0.001$. This describes a discrete random variable X with probability distribution

x	0	387
$P(x)$	$1 - 0.001 = 0.999$	0.001

The expected value of this random variable is then

$$\mathbb{E}[X] = \sum x_i P(x_i) = 0.387$$

X represents the money gained from the drawing rather than the profit, though. The amount of money spent on the ticket is a constant 2, so 2 can simply be subtracted from this result to yield the expected profit of entering the drawing, which is $-\$1.613$

4. The number of defective computers is a discrete random variable X with probability distribution

x	0	1	2	3	4
$P(x)$	0.4305	0.4039	0.1421	0.0222	0.0013

As such,

$$\mu_X = \sum x_i P(x_i) = 0.7599$$

The mean number of defective computers in a batch of 4 is 0.7599.

5. The number of defective computers is a discrete random variable X with probability distribution

x	0	1	2	3	4
$P(x)$	0.5220	0.3685	0.0975	0.0115	0.0005

As such,

$$\sigma_X = \sqrt{\sum (x_i - \mu_X)^2 P(x_i)} \approx 0.714$$

6. The number of trials (days) and the probability of success (rain) are fixed at 3 and 0.4 respectively. This situation therefore follows a binomial setting with probability of success $p = 0.4$ and number of trials $n = 3$. Let X be a random variable following this binomial distribution. The only possible values of X are 0, 1, 2, and 3. As such, the complement of $X = 0$ is equivalent to $X \geq 1$. Therefore,

$$P(X \geq 1) = 1 - P(X = 0) = 1 - \binom{3}{0} 0.4^0 (1 - 0.4)^{3-0} = 1 - 0.216 = 0.784$$

The probability of it raining on at least 1 of the 3 days is 0.784.

7. The probability of a given student owning a credit card given that they are a freshman is the relative frequency of owning one among freshman. This is simply the proportion of freshman that carry credit cards:

$$P(\text{carries a credit card} \mid \text{freshman}) = \frac{\# \text{ of freshman credit card carriers}}{\# \text{ of freshman}}$$
$$= \frac{40}{60} = \frac{2}{3} \approx 0.667$$

The probability of a randomly selected freshman carrying a cred card is about 0.667.

8. The probability of a given student being a freshman given that they carry a credit card is simply the relative frequency of being a freshman among those that own one. This is the proportion of credit card carriers that are freshman:

$$P(\text{freshman} \mid \text{carries a credit card}) = \frac{\# \text{ of freshman credit card carriers}}{\# \text{ of credit card carriers}}$$
$$= \frac{49}{61} \approx 0.803$$

The probability of a randomly selected credit card owner being a freshman is about 0.803.

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