Chapter 1

Confidence Intervals

Chapter 2

Significance Tests

A **significance test** is a procedure that uses observed data to test between two claims, often made regarding parameters, about hypotheses.
The **null hypothesis** (***H*₀**) claims that the parameter is equal to a **null value**, what it was previously assumed to be, denoted by a subscript 0 on the parameter. It is often a statement of no change or difference.

$$H_0 : \text{parameter} = \text{null value}$$

The claim that is attempting to be supported is the **alternative hypothesis** (**H_a**). It can either be **one-sided**, claiming that the parameter is greater or less than the null value, or **two-sided**, claiming simply that the parameter is not equal to the null value.

$$H_a : \text{parameter} \gtrless \text{null value} \lor \text{parameter} \neq \text{null value}$$

A test's ***P*-value** is the probability of evidence being found for H_a when H_0 is true that is at least as strong as that observed.

$$P\text{-value} = P(\text{statistic supports } H_a \mid H_0)$$

The smaller the P-value, the lower the chances of receiving evidence of the alternative. A small P-value therefore supports the H_a.
If the P-value is less than the **significance level $\boldsymbol{\alpha}$**, H_0 can be rejected and it can be concluded that there is convincing evidence for H_a. If the P-value is greater than or equal to α, H_0 cannot be rejected, and it can be concluded that there is not convincing evidence for H_a.
The P-value is calculated using the **standardized test statistic** z, which is equal to the z score of the the sample statistic assuming the null hypothesis is true.

$$z = \frac{\text{statistic} - \text{null parameter}}{\text{standard error of statistic from null parameter}}$$

The P-value is equal to the probability of z satisfying the H_a assuming that H_0 is true. It can therefore be calculated as such, so long as Normality and independence are justified.

$$P\text{-value} = \begin{cases} P(\text{parameter} \gtrless \text{null parameter}) = P(Z \gtrless z) & H_a : \text{parameter} \gtrless \text{null parameter} \\ P(|\text{parameter}| < |\text{null parameter}|) = P(Z < -|z|) + P(Z > |z|) & H_a : \text{parameter} \neq \text{null parameter} \end{cases}$$

Conclusions should only ever be made regarding the rejection of H_0 and the convincing support of H_a. H_0 should never be supported and H_a should never be rejected.

When performing significance tests, two types of errors may occur:

- A **Type I error** occurs when H_0 is rejected despite H_a being false; the data provided convincing evidence for H_a despite it being incorrect.
- A **Type II error** occurs when H_0 is not rejected despite H_a being true; the data did not provide convincing evidence for H_a despite it being correct.

	H_a false	H_a true
H_0 rejected	Type I error	Correct conclusion
H_0 not rejected	Correct conclusion	Type II error

The probability of a Type I error occurring is equal to α.
As α increases, the probability of a Type I error increases but that of a Type II error decreases.
A confidence interval for $\hat{p}$ (using $s_{\hat{p}}$) can be used in tandem with a sample statistic to provide a set of plausible values for the true parameter, should the alternative hypothesis be convincingly supported.
A two-sided test of of H_0 at significance level α usually provides the same conclusion as a confidence level of the complement of α.

$$[P(Z < z) < \alpha] \approxeq [\text{null parameter} \in (\text{statistic} \pm ME)]$$

A test's **power** is the probability of convincing evidence being found that convincingly supports H_a given a value for the parameter being tested. This is also is equal to the probability of avoiding a Type II error.

$$\text{power} = 1 - P(\text{Type II Error})^C = P(\text{statistic convincingly supports } H_a \mid H_a \text{ is true})$$

Power can be increased in three ways:

1. Increasing the sample size
   - A large sample means that more data is collected and more information is given regarding the true population parameter. This also increases n, which decreases s, reducing z and therefore the P-value, making it more likely to fall below α.
2. Increasing the significance level
   - Increasing the significance level increases the probability of H_0 being rejected when H_a is true, as the maximum P-value for H_0 to be rejected increases.
3. Increasing the **effect size**, the minimum difference between the null parameter value and the alternative parameter value for the change to matter
   - Increasing the size of the difference that needs to be detected makes that difference more likely to be detected, as larger differences are easier to detect.

Significance Tests about Proportions

In order for a significance test of $H_0 : p = p_0$ to be performed, it must be verified that the distribution of $\hat{p}$ is approximately Normal assuming H_0 and that the standard error can be calculated, so Large Counts and the 10% condition (not for experiments) must be satisfied and interpreted for Normality and independence respectively.

To perform **1 proportion z-test**, a significance test about one proportion, z must be calculated.

$$z = \frac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}} = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

The P-value is the probability of p satisfying H_a when H_0 is true, which can be calculated using the cumulative probability function.

$$P\text{-value} = P(H_a) = \begin{cases} P(p \gtrless p_0) = P(Z \gtrless z) & H_a : p \gtrless p_0 \\ P(|p| < |p_0|) = P(Z < -|z|) + P(Z > |z|) & H_a : p \neq p_0 \end{cases}$$

When answering a question regarding a significance test about a proportion, a four step process can be used:

1. State State the hypotheses to be tested and the significance level and define any parameters.

2. Plan Identify the appropriate methods of inference and verify its conditions.

3. Do State the sample statistic(s), calculate the standardized test statistic(s), and calculate the P-value.

4. Conclude Make a conclusion regarding the hypotheses within the problem's context.

Significance Tests about Differences in Proportions

A **2 proportion *z*-test** can be performed to compare the proportions for two populations is based on the difference between sample proportions.

$$H_0 : p_1 - p_2 = p_0 \qquad H_a : p_1 - p_2 \gtrless p_0 \qquad H_a : p_1 - p_2 \neq p_0$$

Typically, p_0 is 0, so these hypotheses can be rewritten.

$$H_0 : p_1 = p_2 \qquad H_a : p_1 \gtrless p_2 \qquad H_a : p_1 \neq p_2$$

A significance test first assumes that the null hypothesis $H_0 : p_1 = p_2$ is true. This common value is referred to as p.
The **combined sample proportion** is denoted $\hat{p}_C$ and is equal to the total successes divided by the total sample size, making it effectively a weighted average. It is the sample proportion that assumes that the parameter values are equal.

$$\hat{p}_C = \frac{x_1 + x_2}{n_1 + n_2} = \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2}$$

The Large Counts condition must be met with $\hat{p}_C$.

$$n_1\hat{p}_C, n_1(1 - \hat{p}_C), n_2\hat{p}_C, n_2(1 - \hat{p}_C) \geq 10$$

For a significance test to be run about a difference of proportions, the randomness, independence (10%) (for each proportion), and Large Counts conditions must be met.

The standardized test statistic is the z score, using the difference in proportions and its standard error assuming the mean to be 0 (H_0 to be true).

$$z = \frac{\hat{p}_1 - \hat{p}_2 - \mu_{\hat{p}_1 - \hat{p}_2}}{s_{\hat{p}_1 - \hat{p}_2}} = \frac{\hat{p}_1 - \hat{p}_2 - 0}{\sqrt{\frac{\hat{p}_C(1-\hat{p}_C)}{n_1} + \frac{\hat{p}_C(1-\hat{p}_C)}{n_2}}} = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}_C\left(1 - \hat{p}_C\right)\left(\frac{1}{n_1} - \frac{1}{n_2}\right)}}$$

Significance Tests about Means

Chapter 3

Chi-Square Tests

Chapter 4

Slopes