AP Physics C: Electricity and Magnetism

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Chapter 1

Electrostatics and Gauss' Law

1.1 Electrostatics

1.1.1 Electric Charge

An electron is negatively charged.
Two particles of the same polarity repel each other while those of opposite polarity are attracted.
Conductors are materials in which electrons are able to move relatively freely. **Nonconductors/Insulators** are the opposite, limiting electron movement.
Semiconductors are materials that are between conductors and insulators in terms of conductivity.
Superconductors are perfect conductors.
Atoms are comprised of positively charged protons, negatively charged electrons, and neutral (though very slightly negatively charged) neutrons. In conductors, the outermost electrons are able to move relatively freely. These mobile electrons are called **conduction electrons**. **Induction** describes the phenomenon of neutral conductors being attracted to charged ones.
Coulomb's law describes the electrostatic force between two particles of charges q_1 and q_2 as

$$F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{r^2} \qquad \text{(Coulomb's Law)}$$

where $\varepsilon_0 \approx 8.85 \times 10^{-12}\,\mathrm{C^2/N^2\,m^2}$, the **vacuum permittivity constant**. This is often rewritten as

$$F = k\frac{|q_1||q_2|}{r^2} \qquad \text{(Coulomb's Law)}$$

where $k = \frac{1}{4\pi\varepsilon_0} \approx 8.99 \times 10^9\,\mathrm{N\,m^2/C^2}$ is the **electrostatic constant** or the **Coulomb constant**.
The electrostatic force is pointed either directly towards or away from the other particle. If multiple are acting on the same particle, the net force is the *vector* sum.
Particles that interact through the electrostatic force form a *third-law pair*.
One **shell theories** hypothesizes that a shell with uniform charge density acts like a single particle at its center from the perspective of a particle outside the shell while another claims that it cancels out, providing no net force to a particle within the shell.
Electric charge is **quantized**, meaning that it can only take on certain (*discrete*) values.
A particle's charge q can be written as ne, where n is a nonzero integer and $e \approx 1.609 \times 10^{-19}\,\mathrm{C}$ is the elementary charge

$$q = ne$$

The proton, neutron, and electron, denoted p, n, and e (or e-) respectively, have the corresponding charges e, 0, and $-e$.
The net charge of an isolated system is always conserved.

1.1.2 Electric Fields

The **electric field $\vec{E}$** is the vector field of the electric charge on every point in a region surrounding a charged object. It is measured in N/C To measure it, a **positive** charge q_0, called a *test charge* is placed at a point. The electrostatic force F is then measured on the test charge. The electric field at this point is defined to be

$$\vec{E} = \frac{\vec{F}}{q_0}$$

The magnitude of electric field due to a point charge q at any point of distance r from said point charge is

$$E = \frac{F}{q_0} = k\frac{|q|}{r^2}$$

The direction vector $\vec{d}$ of a dipole typically goes from the negative end to the positive.
The **dipole moment $\vec{p}$** is defined as

$$\vec{p} = q\vec{d}$$

The dipole moment always attempts to align with the direction of the field, making it simple to see the direction of rotation of the dipole. The torque $\vec{\tau}$ on a dipole in an electric field is

$$\vec{\tau} = \vec{p} \times \vec{E}$$

Linear charge density is denoted as λ as is found as

$$\mathrm{d}q = \lambda \, \mathrm{d}s$$

for a curved rod of charge Q and length s. It is generally useful to use θ, where

$$\theta = \frac{s}{r} \implies \mathrm{d}s = r \, \mathrm{d}\theta$$

A vector into the paper is denoted on one end by $\otimes$ while one pointing out is denoted by $\odot$.
Work W is the integral of force with respect to displacement, making it

$$W = \int_C \vec{F} \cdot \mathrm{d}\vec{r}$$

The work done by a conservative force is denoted by W_c, a change in potential energy by ΔU, and the gravitational force by F_g. It should be noted that gravity is a conservative force and that

$$W_c = -\Delta U$$

As such,

$$\Delta U_g = mg(\Delta h)$$

1.2 Gauss' Law

The **area vector** $\mathbf{d}\vec{\boldsymbol{A}}$ for an area element on a surface is a vector with magnitude equal to area $\mathrm{d}A$ of the element that is perpendicular to the surface pointing outwards.
The **electric flux** $\mathbf{d}\boldsymbol{\Phi}_E$ is given by

$$\mathrm{d}\Phi_E = \vec{E} \cdot \mathrm{d}\vec{A} \qquad \text{(electric flux)}$$

with units $\mathrm{N/C\,m^2}$. The **total flux** through a surface is found by the surface integral

$$\Phi_E = \iint_S \vec{E} \cdot \mathrm{d}\vec{A} \qquad \text{(total flux)}$$

Through a **closed surface** (as used in Gauss' law),

$$\Phi_E = \oiint_S \vec{E} \cdot \mathrm{d}\vec{A} = \oiint_S E \,\mathrm{d}A \cos\varphi \qquad \text{(net flux)}$$

where φ is the angle between the electric field and the surface.
For a uniform electric field,

$$\Phi_E = \oiint_S E \,\mathrm{d}A \cos\varphi = E\cos\varphi \oiint_S \mathrm{d}A = EA\cos\varphi$$

The relationship between the surface and the field can be described by flux as

Φ_E	< 0	0	> 0
φ	$< 90°$	$90°$	$> 90°$

Gauss' law relates the net flux Φ_E of an electric field through a closed (Gaussian) surface to the net charge q_{enc} enclosed by the surface as

$$\varepsilon_0 \Phi_E = q_{\text{enc}} \qquad \text{(Gauss' law)}$$

If excess charge is placed on a conductor, the charge will move to the surface.
Everywhere inside a conductor, $E_{\text{net}} = 0$.
A **(uniform) surface charge density** $\boldsymbol{\sigma}$ is equal to

$$\sigma = \frac{q}{A} \qquad \text{(uniform surface charge density)}$$

The magnitude of the electric field outside of a conductor with uniform surface charge density σ is

$$E = \frac{\sigma}{\varepsilon_0} \qquad \text{(conducting surface)}$$

That outside of an insulator is

$$E = \frac{\sigma}{2\varepsilon_0} \qquad \text{(insulator)}$$

The magnitude of the electric field produced by a uniform spherical shell of radius R is

$$E = \begin{cases} k\dfrac{q}{r^2} & r \geq R \\ 0 & r < R \end{cases}$$

A **(uniform) volume charge density** $\boldsymbol{\rho}$ is equal to

$$\rho = \frac{q}{V} \qquad \text{(uniform volume charge density)}$$

Within a sphere of radius R with uniform volume charge density, the magnitude of the field is radial:

$$E = \left(k\frac{Q}{R^3}\right) r \qquad \text{(uniform charge, field at } r \leq R\text{)}$$

Chapter 2

Conductors, Capacitors, and Dielectrics

2.1 Electric Potential

Electrostatic forces are **conservative**, so $W_C = -\Delta U$. It can then be seen that

$$\Delta U = -\int_C \vec{F} \cdot \mathrm{d}\vec{s} = -q \int_C \vec{E} \cdot \mathrm{d}\vec{s}$$

The change in **electric potential $\boldsymbol{V}$** (measured in volts (V)) is found as

$$\Delta V = \frac{\Delta U}{q} = -\frac{q \int_C \vec{E} \cdot \mathrm{d}\vec{s}}{q} = -\int_C \vec{E} \cdot \mathrm{d}\vec{s}$$

If the initial potential is set to 0, then

$$V = -\int_C \vec{E} \cdot \mathrm{d}\vec{s}$$

Adjacent points with the same electric potential form an **equipotential surface**, which can be imaginary or real.
The electric potential from a point charge can be found as

$$V_f - V_i = -\int_C \vec{E} \cdot \mathrm{d}s = -\int_r^\infty E \,\mathrm{d}r = k\frac{q}{r}$$

Setting V_f to 0 (at ∞),

$$V_i = V = k\frac{q}{r}$$

The potential due to a collection of n charged particles is simply the sum of the individual potentials:

$$V = \sum_{i=1}^{n} V_i = \sum_{i=1}^{n} k\frac{q_i}{r_i} \qquad (n \text{ charged particles})$$

Note that direction is not considered.
As a convention, positively charged particles produce positive potentials while negative ones produce negative potentials.
The potential energy of a system of particles is the sum of the potential energies of every pair of

particles in the system. It is equal to the work required to assemble the system with particles that are initially at rest and infinitely far apart. For two particles of distance r,

$$U = k\frac{q_1 q_2}{r} \qquad \text{(2-particle system)}$$

The x component of an electric field can be found from potential as

$$E_x = -\frac{\mathrm{d}V_x}{\mathrm{d}x}$$

For a continuous charge distribution over an extended object, the net potential can be found as

$$V = \int_C \mathrm{d}V = k\int_C \frac{\mathrm{d}q}{r}$$

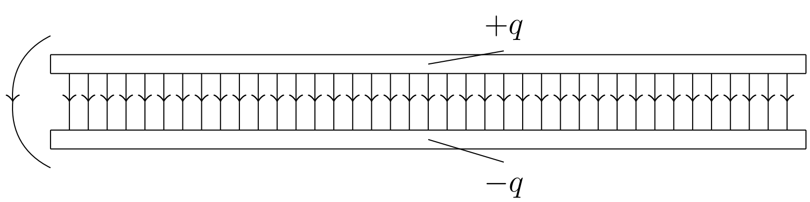
A substitution can then be made using the appropriate charge density.

2.2 Capacitance

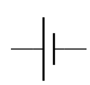
A capacitor is comprised of 2 isolated conductors with charges $+q$ and $-a$. Its **capacitance *C*** is defined as

$$C = \frac{|q|}{V}$$

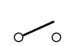
where V is the potential difference between the plates. It is measured in C/V or **Farads (F)**. By its definition, capacitance is always positive.
A capacitor's capacitance is a constant inherent to its physical attributes.
A parallel-plate capacitor is comprised of 2 parallel plates of area A separated by a distance d. The charges on the faces of the plates facing each other are of magnitude q and opposite signs.
The electric field due to a parallel-plate capacitor is uniform only between the plates.



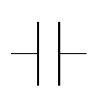
A battery is denoted by



where the larger side is positive and the shorter negative.
An open switch is denoted by



A capacitor is denoted by



When a circuit with a battery, an open switch, and an uncharged capacitor is completed by closing the switch, conduction electrons shift, resulting in the capacitor plates being of opposite charges.

Gauss' law can be used to relate the electric field between a capacitor's plates to the charge q on either plate as

$$V = -\int_C \vec{E} \cdot \mathrm{d}\vec{s} = -\int_-^+ E\,\mathrm{d}s$$

It is assumed that the plates of the capacitor are large and close enough for fringing to be negligible, making $\vec{E}$ constant between the plates. Using a Gaussian surface that encloses just the charge q on the positive plate,

$$\oiint \vec{E} \cdot \mathrm{d}A = EA = \frac{q_{\mathrm{enc}}}{\varepsilon_0} = \frac{q}{\varepsilon_0}$$

so

$$E = \frac{q}{A\varepsilon_0} \qquad \text{and} \qquad q = EA\varepsilon_0$$

Where A is the plate's area. Therefore

$$V = \int_-^+ E\,\mathrm{d}s = E\int_0^d \mathrm{d}s = Ed$$

Substituting CV for q yields

$$C = \frac{q}{V} = \frac{EA\varepsilon_0}{Ed} = \frac{A\varepsilon_0}{d} \qquad \text{(parallel-plate capacitor)}$$

Consider a cylindrical capacitor of length L formed by 2 coaxial cylinders of radii a and b. Assume that $L \gg b$ so that fringing may be neglected. Each plate has charge q, so

$$q = EA\varepsilon_0 = E\varepsilon_0(2\pi rL)$$

Using Gauss' law,

$$\oiint \vec{E} \cdot \mathrm{d}A = E(2\pi rL) = \frac{q_{\mathrm{enc}}}{\varepsilon_0} = \frac{q}{\varepsilon_0}$$

so

$$E = \frac{1}{2\pi\varepsilon_0}\frac{q}{rL}$$

and

$$V = \frac{q}{2L\pi\varepsilon_0}\int_a^b \frac{\mathrm{d}r}{r} = \frac{q}{2L\pi\varepsilon_0}\ln\left(\frac{b}{a}\right)$$

The capacitance is then

$$C = \frac{q}{V} = 2\pi\varepsilon_0\frac{L}{\ln(b/a)} \qquad \text{(cylindrical capacitor)}$$

Capacitors connected in parallel can be replaced by a single capacitor with the same total charge q_{eq} and potential difference V as the original capacitors.
When a potential difference V is applied across several parallel capacitors, that potential difference V is applied to each capacitor. The total charge q_{eq} is the the sum of the charges of each individual capacitor.

$$q_{\mathrm{eq}} = \sum q_i$$

The equivalent capacitance C_{eq} is then simply

$$C_{\mathrm{eq}} = \sum C_i \qquad \text{(capacitors in parallel)}$$

Capacitors connected in series can be replaced by a single capacitor with the same total charge and potential difference.
When a potential difference V is applied across several series capacitors, the capacitors all have the same charge q. The sum of the potential differences across all capacitors is equal to the applied potential difference V.

$$V_{\text{eq}} = \sum q \left(\frac{1}{C_i}\right)$$

The reciprocal of the equivalent capacitance C_{eq} is then

$$\frac{1}{C_{\text{eq}}} = \sum \frac{1}{C_i} \qquad \text{(capacitors in series)}$$

The **electric potential energy** $\boldsymbol{U_c}$ of a charged capacitor is

$$U_c = \frac{q^2}{2C} = \frac{1}{2}CV^2 \qquad \text{(potential energy)}$$

This is equal to the work required to charge the capacitor. This energy can be viewed as being stored in the electric field between the plates.
Every electric field has an associated stored energy. In a vacuum, the **energy density** $\boldsymbol{u}$ in a field of magnitude E is

$$u = \frac{1}{2}\varepsilon_0 E^2 \qquad \text{(energy density)}$$

A **dielectric** is an insulating material placed between the plates of a capacitor. This increases the structural integrity of the capacitor while increasing its capacitance.
The **dielectric constant** $\boldsymbol{\kappa}$ is a unitless constant that is the ratio of the final capacitance to the initial capacitance.

$$C = \kappa C_0$$

Chapter 3

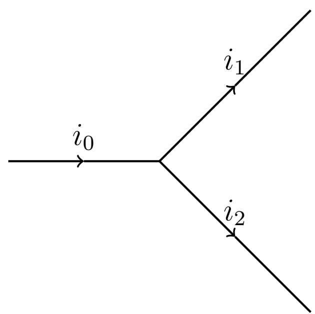
Circuits

3.1 Current and Resistance

If charge $\mathrm{d}q$ passes through a hypothetical plane in time $\mathrm{d}t$, the **current $\boldsymbol{i}$** is defined as

$$i = \frac{\mathrm{d}q}{\mathrm{d}t} \qquad \text{(definition of current)}$$

It is measured in units of C/S (**Amperes A**). Consider the following junction:



As the charge is conserved,

$$i_1 + i_2 = i_0$$

The current i and the **current density $\vec{J}$** are related by

$$i = \iint_S \vec{J} \cdot \mathrm{d}\vec{A}$$

where $\mathrm{d}\vec{A}$ is a vector that is orthogonal to a surface element of area (in the direction of the current density by convention) and the integral is taken over any surface that cuts across the conductor. The direction of $\vec{J}$ is the same as that of the velocity the moving charges if they are positive and the opposite direction if they are negative.
Current density is measured in units of $\mathrm{A/m^2}$.
Charges move near the speed of light, ricocheting along the sides of the wire. The net movement along the wire is the **drift velocity $\vec{v}_d$**. Positive charge carriers drift at this speed in the direction of $\vec{E}$. By convention, the directions of the drift speed, current density, and current are drawn in the same direction.
The drift velocity is related to the current density as

$$\vec{J} = ne\vec{v}_d$$

where e is the charge of an electron and n is the number of charge carriers divided by the volume. The product ne is the **carrier charge density** in $\mathrm{C/m^e}$.
The volume of the cross section of a wire is the product of the length of the region Δx and the cross-sectional area A. The length considered is the product of the drift velocity and the change in time, so

$$V = A\Delta x = v_d A\Delta t$$

The total charge ΔQ is the product of the number of charge carriers and their individual charge q. The number of charges is simply equal to the product of n and the volume, so

$$\Delta Q = q(nV) = nqv_d A\Delta t$$

When the charge carriers are electrons,

$$\Delta Q = nev_d A\Delta t$$

Dividing both sides by Δt,

$$i_{\text{avg}} = nev_d A$$

The **electrical resistance $\boldsymbol{R}$** is defined as

$$R = \frac{V}{i} \qquad \text{(definition of } R\text{)}$$

where V is the potential difference across the conductor and i is the current through the conductor. It is measured in units of V/A (Ω). **Resistivity $\boldsymbol{\rho}$** (measured in $\Omega \cdot \mathrm{m}$) is defined as

$$\rho = \frac{E}{J} \qquad \text{(definition of } \rho\text{)}$$

The reciprocal of this the conductivity **conductivity $\boldsymbol{\sigma}$** (measured in Siemens per meter (S/m))

$$\sigma = \frac{1}{\rho} \qquad \text{(definition of } \sigma\text{)}$$

Resistance is a property of an *object* while resistivity is a property of a *material*. The resistance of a conducting wire of length L with uniform cross sectional area A is

$$R = \rho\frac{L}{A}$$

Ohm's law states that the current through a device is always directly proportional to the potential difference that is applied to it:

$$I \propto V \qquad \text{or} \qquad I = \frac{V}{R} \qquad \text{(Ohm's law)}$$

A material that obeys Ohm's law is said to be **Ohmic**. A conducting device is Ohmic when the resistance of the device is independent of the magnitude and polarity of the applied potential difference. A conducting material is Ohmic when its resistivity is independent of the magnitude and direction of the applied electric field.
The **power $\boldsymbol{P}$** (rate of energy transfer) in an electrical device across which a potential difference of V is maintained is

$$P = iV \qquad \text{(rate of electrical energy transfer)}$$

If the device is a resistor, this can be written as

$$P = i^2 R = \frac{V^2}{R} \qquad \text{(resistive dissipation)}$$

3.2 Circuits

In order for a steady flow of charge to be maintained, a device is required to do work on the charge carriers. Such a device is called an **emf device**, which is said to provide emf $\mathcal{E}$.
One terminal of an emf device, called the positive terminal, is kept at a higher potential than the other. This can be represented on a diagram by an arrow pointing from the negative to the positive terminal. (This arrow has a circle on the negative end to distinguish it from that denoting current direction.)
An emf device does work on charges to maintain the potential difference between its terminals. If work $\mathrm{d}W$ is done on positive charge $\mathrm{d}q$ to force it from the negative to the positive terminal, then the emf is

$$\mathcal{E} = \frac{\mathrm{d}W}{\mathrm{d}q} \qquad \text{(definition of } \mathcal{E}\text{)}$$

An **ideal emf device** is one without any internal resistance. The potential difference between its terminals is equal to the emf.
A **real emf device** has internal resistance, denoted r. The potential difference between its terminals is equal to the emf only when no current is flowing through the device.
Because $P = i^2R$, in time interval $\mathrm{d}t$, an amount of energy $i^2R\,\mathrm{d}t$ will appear in the resistor as thermal energy. Said energy is said to be **dissapated**.
Over the same amount of time, a charge $\mathrm{d}q = i\,\mathrm{d}t$ will have moved through the battery, and the work done on this charge is

$$\mathrm{d}W = \mathcal{E}\,\mathrm{d}q = \mathcal{E}i\,\mathrm{d}t$$

By conservation of energy, the work done by the ideal battery must be equal to the thermal energy that appears in the resistor:

$$\mathcal{E}i\,\mathrm{d}t = i^2R\,\mathrm{d}t$$

which gives

$$i = \frac{\mathcal{E}}{R}$$

The **loop rule** states that the algebraic sum of changes in potential encountered in the complete traversal of a given loop in a circuit is 0.
The **resistance rule** states that for a move through a resistance in the direction of current, the change in potential is $-iR$. In the direction opposite of current, it is iR.
The **emf rule** states that a move through an ideal emf device in the direction of the emf arrow yields a change in potential of $\mathcal{E}$. In the opposite direction, it is $-\mathcal{E}$.
To ground a circuit is to connect one point of it to Earth's surface.
The rate P_{emf} at which an emf device transfers energy to both the charge carriers and to the internal thermal energy is

$$P_{\text{emf}} = \mathcal{E} \qquad \text{(power of emf device)}$$

For resistors placed in series, the equivalent resistance can be found by the loop rule, which yields

$$R_{\text{eq}} = \sum R_i \qquad \text{(resistances in series)}$$

Using the loop rule,

$$\mathcal{E} - iR_{\text{eq}} = 0$$

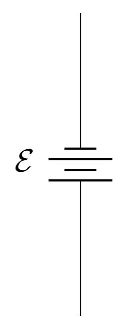
so

$$i = \frac{\mathcal{E}}{R_{\text{eq}}}$$

When a potential difference V is applied to resistors in series, the currents i are the same across each resistor. The sum of the potentials of each resistor is equal to the total applied potential difference V. As such, resistors in series with can be replace by an equivalent resistance R_{eq} with the same current i and same total potential difference V as the original resistances.
The **junction rule** states that the sum of the currents entering a given junction must be equal to the sum of those leaving that junction.
For parallel resistors, the equivalent resistance is

$$\frac{1}{R_{\text{eq}}} = \sum \frac{1}{R_i} \qquad \text{(resistances in parallel)}$$

Parallel resistors have the same potential difference across every resistor.
An **ammeter** is an instrument used to measure currents. It is used by breaking/cutting part of the wire and substituting the ammeter.
In order for an ammeter to be accurate, its resistance must be much smaller than those of other resistances in the circuit.
A **voltmeter** is an instrument used to measure potential differences. Connecting its terminals to any two points in a circuit yields their potential difference.
In order for a voltmeter to provide an accurate reading, its resistance must be much larger than those of any circuit element across which it is connected. This ensures that negligible current passes through the voltmeter, preventing it from affecting the potential difference that is to be measured.



A capacitor of capacitance C is initially uncharged. To charge is, a switch is closed. This completes an RC series circuit consisting of the capacitor, the ideal battery of emf $\mathcal{E}$, and a resistance R.
Once the circuit is completed, the charge begins to flow. This current increases the charge q on the plates and the potential difference $V_C = q/C$ across the capacitor. When that potential difference is equal to the emf, the current is 0. The equilibrium (final) charge on the fully charged capacitor is then $q = CV = C\mathcal{E}$.
Applying the loop rule yields

$$\mathcal{E} - iR - \frac{q}{C} = 0$$

Substituting and rearranging,

$$R\frac{\mathrm{d}q}{\mathrm{d}t} + \frac{q}{C} = \mathcal{E}$$

Solving this yields

$$q = C\mathcal{E}\left(1 - \mathrm{e}^{-\frac{t}{RC}}\right) \qquad \text{(charging a capacitor)}$$

Differentiating yields

$$i = \frac{\mathrm{d}q}{\mathrm{d}t} = \frac{\mathcal{E}\mathrm{e}^{-\frac{t}{RC}}}{R} \qquad \text{(charging a capacitor)}$$

Chapter 4

Magnetism

A magnetic field $\vec{B}$ is defined as a vector quantity that exists when it exerts a force $\vec{F}_B$ on a charge moving with velocity $\vec{v}$. The magnitude of this force can be measured when $\vec{v}$ is perpendicular to it. The magnitude of $\vec{B}$ can then be defined as

$$B = \frac{F_B}{|q|v} \qquad \text{(magnitude of magnetic field)}$$

where q is the charge of the particle. It is measured in units of Teslas (T). These can be summarized by the vector equation

$$\vec{F}_B = q\vec{v} \times \vec{B} \qquad \text{(magnetic force)}$$

It should be noted that a motionless charge does not have any force on it due to a magnetic field.
$\vec{v}$ and $\vec{B}$ must not be perpendicular in order for there to be a nonzero resultant force.
As with electric fields, magnetic fields can be represented by field lines, the direction of the tangent at a given point giving the direction of $\vec{B}$ at that point and the spacing of the lines representing its magnitude.
Field lines enter a magnet at one end (the south pole) and exit from the opposite side (the north pole). As such, magnets are called **magnetic dipoles**.
Opposite poles attract while the like poles repel.
Electric field lines begin and terminate at a charge while magnetic field lines are closed loops.
A compass needle aligns itself with magnetic field lines.
A charged particle moving through a region subject to both electric and magnetic fields is subject to the forces exerted by both.
Two perpendicular fields are said to be **crossed fields**.
If the forces exerted by the fields oppose each other, a particular speed will result in no deflection of the particle.
The drifting conduction electrons in a copper wire can still be deflected by a magnetic field. This phenomenon is called the **Hall effect**, which enables the polarity of the charge carriers in a conductor to be found. The number of such charge carriers per unit volume can also be found.
Suppose a uniform magnetic field $\vec{B}$ is applied perpendicular to the direction of current i. A Hall-effect potential difference V is set up. The forces exerted by the electric and magnetic fields are then balanced. The number density n of the charge carriers can be determined as

$$n = \frac{Bi}{v\ell e}$$

where

$$\ell = \frac{A}{d}$$

is the thickness of the strip.
A conductor moving through a uniform magnetic field $\vec{B}$ at speed v has a Hall-effect potential difference is

$$V = vBD \qquad \text{(Hall-effect potential difference)}$$

where d is the width orthogonal to both $\vec{v}$ and $\vec{B}$.
Consider a beam of electrons projected into a chamber. They enter with speed v, moving into a region with uniform magnetic field $\vec{B}$ directed out of the plane. A magnetic force continuously deflects the electrons, as the velocity and magnetic field are always perpendicular. This deflection results in the electrons following a circular path. Applying Newton's second law to the circular motion yields

$$|q|vB = \frac{mv^2}{r}$$

Therefore

$$r = \frac{mv}{|q|B}$$

The period is

$$T = \frac{2\pi r}{v} = \frac{2\pi mv}{|q|Bv} = \frac{2\pi m}{|q|B}$$

A cyclotron is comprised of two hollow conducting D-shaped shells. These shells are part of an electric oscillator that alternates the potential difference across the gap between the shells. The electric polarities of the shells are alternated so that the electric field in the gap alternates in direction between the shells. The shells are immersed in a uniform magnetic field in the direction of the rectangular cross section (in the plane of the gap). Protons circulate within the cyclotron.
The key to operating a cyclotron f at which the proton circulates in the magnetic field must be equal to the fixed frequency f_{osc} of the oscillator.

$$f = f_{\text{osc}} \qquad \text{(resonance condition)}$$

The resonance condition means that if the energy of a circulating proton increases, energy must be fed to it at a frequency f_{osc} that is equal to the frequency f at which the proton naturally circulates in the magnetic field.
A synchrotron is like a cyclotron except that the magnetic field and oscillating frequency vary with time during the accelerating cycle. When done properly, the frequency of the circulating protons remains in step with the oscillator at all times, and the protons follow a circular path rather than a spiraling one. As such, the magnet need only extend over said circular path. If high energies are to be achieved, though, the path must still be large.
A straight wire carrying a current i in a uniform magnetic field experiences a force

$$\vec{F}_B = i\vec{\ell} \times \vec{B} \qquad \text{(magnetic force on a current)}$$

where $\vec{\ell}$ is a length vector with magnitude ℓ and direction following the conventional direction of current.
If a wire is not straight or a magnetic field not uniform, the wire can be broken into differentials. The force on the wire is then the vector sum of all the forces on the differential elements:

$$\vec{F}_B = \int i \times \mathrm{d}\vec{\ell} \times \vec{B} \qquad \text{(magnetic force on a current)}$$

where $\mathrm{d}\vec{\ell}$ is in the direction of i.
Consider a rectangular loop of current-carrying wire in the plane of a uniform magnetic field that is free to rotate in the plane. The magnetic forces on the wire produce a torque that rotates it. A commutator can reverse the direction of the magnetic field every half-revolution so that the torque always acts in the same direction.
The net force on the loop is the vector sum of the forces on each of its four sides and is equal to 0. The net torque on the coil has magnitude

$$\tau = NiAB\sin\theta$$

where N is the number of turns of the coil, A is the area of each turn, i is the current, and B is the magnitude of the magnetic field, and θ is the angle between the vector normal to the coil in the plane of the magnetic field and the magnetic field.
A coil in a uniform magnetic field will experience a torque given by

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

where $\vec{\mu}$ is the **magnetic dipole moment** of the coil, with magnitude

$$\mu = NiA \qquad \text{(magnetic dipole moment)}$$

and direction is given by the right-hand rule.
The orientation energy of a magnetic dipole in a magnetic field is

$$U(\theta) = -\vec{\mu} \cdot \vec{B}$$

The magnetic moment vector wants to be aligned with the magnetic field.
The magnitude of a magnetic field $\mathrm{d}\vec{B}$ produced at a point at distance r from a current-length element $\mathrm{d}\vec{s}$ is

$$|\,\mathrm{d}B\,| = \frac{\mu_0}{4\pi}\frac{i\,\mathrm{d}s\sin\theta}{r^2} \qquad \text{(law of Biot and Savart)}$$

where θ is the angle between $\mathrm{d}\vec{s}$ and $\hat{r}$, a unit vector pointing from the $\mathrm{d}\vec{s}$ to the point. μ_0 is the vacuum permeability constant:

$$\mu_0 \approx 4\pi \times 10^{-7}\,\mathrm{T}\,\frac{\mathrm{m}}{\mathrm{A}} \approx 1.26 \times 10^{-6}\,\mathrm{T}\,\frac{\mathrm{m}}{\mathrm{A}} \qquad \text{(vacuum permeability constant)}$$

The vector form of the law of Biot and Savart is

$$\mathrm{d}\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,\mathrm{d}\vec{s} \times \vec{r}}{r^3} \qquad \text{(law of Biot and Savart)}$$

The magnitude of the magnetic field *at the center of a circular arc* of radius R and central angle φ (in radians) carrying current i is

$$B = \frac{\mu_0 i\varphi}{4\pi R}$$

The magnitude of the magnetic field at a perpendicular distance R from a *long-straight wire* carrying a current i is

$$B = \frac{\mu_0 i}{2\pi r}$$

To find the force on one current-carrying wire due to another, the field due to the second wire at the first wire should be found before the force on the first wire due to that field.

Parallel wires carrying currents *in the same direction* attract each other, while those carrying currents *in opposite direction* repel. The magnitude of the force on either wire of length ℓ is

$$F_{ba} = \frac{\mu_0 L i_a b_a}{2\pi d}$$

where d is the separation of the wires and i_a and i_b are the currents in each wire. **Ampere's law** states that

$$\oint \vec{B} \cdot \mathrm{d}\vec{s} = \mu_0 i_{\text{enc}} \qquad \text{(Ampere's law)}$$