Assignment 2

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May 25, 2022

1. Find the angle between the vectors $\vec{a} = \hat{\imath} - 2\hat{\jmath} + \hat{\mathrm{k}}$ and $\vec{b} = 2\hat{\imath} + \hat{\jmath} - \hat{\mathrm{k}}$.

   Solution:

$$\begin{aligned} |\vec{a}||\vec{b}|\cos\theta &= \vec{a}\cdot\vec{b} \\ \sqrt{1^2+(-2)^2+(1)^2}\sqrt{2^2+1^2+(-1)^2}\cos\theta &= (1)(2)+(-2)(1)+(1)(-1) \\ \sqrt{6}\sqrt{6}\cos\theta &= -1 \\ \theta &= \arccos\left(-\frac{1}{6}\right) \approx 100^\circ \end{aligned}$$

2. Find the cross product of the vectors $\vec{a} = \langle 2, -1, 4\rangle$ and $\vec{b} = \langle 3, 0, -2\rangle$.

   Solution:

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{\mathrm{k}} \\ 2 & -1 & 4 \\ 3 & 0 & -2 \end{vmatrix} = \langle (-1)(-2)-(4)(0), -1((2)(-2)-(4)(3)), (2)(0)-(-1)(3)\rangle = \langle 2, 16, 3\rangle$$

3. Find the equation of the plane $Ax + By + Cz = D$ through the points $P(1, -1, 2)$, $Q(-1, 0, 1)$, and $R(1, -1, 1)$.

   Solution

$\overrightarrow{PQ} = \langle -1-1, 0-(-1), 1-2\rangle = \langle -2, 1, -1\rangle$

$\overrightarrow{PR} = \langle 1-1, -1-(-1), 1-2\rangle = \langle 0, 0, -1\rangle$

$$\vec{n} = \overrightarrow{PQ}\times\overrightarrow{PR} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{\mathrm{k}} \\ -2 & 1 & -1 \\ 0 & 0 & -1 \end{vmatrix} = \langle (1)(-1)-(-1)(0), -1((-2)(-1)-(-1)(0)), (-2)(0)-(-1)(0)\rangle = \langle -1, -2, 0\rangle$$

$D = n_x x + n_y y + n_z z = -1(1) + (-2)(-1) + 0(2) = -1 + 2 + 0 = 1$

$1 = -x - 2y$