Transform from a function of t to a function of s

Arnav Patri

November 27, 2022

1)

$$\mathcal{L}\{e^{3t}\} = \int_0^\infty e^{-st} e^{3t} dt = \int_0^\infty e^{-t(s-3)} dt = \left[-\frac{e^{-t(s-3)}}{(s-3)} \right]_0^\infty$$
$$= 0 + \frac{1}{s-3} = \frac{1}{s-3}$$

Using theorem 7.1.1,

$$\mathcal{L}\lbrace e^{3t}\rbrace = \frac{1}{s-3}$$

2)

$$\mathscr{L}{f(t)} = \int_0^\infty e^{-st} f(t) dt = F(s)$$

3) Using theorem 7.1.1,

$$\mathcal{L}\{\sin(3t)\} = \frac{3}{s^2 + 3^2} = \frac{3}{s^2 + 9}$$

4) Using theorem 7.1.1,

$$\mathscr{L}\{\cos^2(2t)\} = \mathscr{L}\left\{\frac{1+\cos(4x)}{2}\right\} = \frac{1}{2}\frac{1}{s} + \frac{1}{2}\frac{s}{s^2+4^2} = \frac{1}{2s} + \frac{s}{2s^2+32} = \frac{s^2+8}{s^3+16s}$$