15.3.27. (pg. 1068)

Arnav Patri

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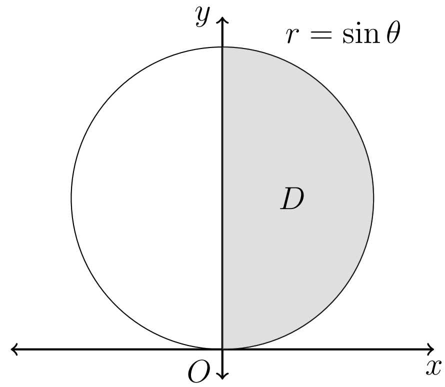
$f(x, y) = x$



(a) Set up an iterated integral in polar coordinates for the volume of the solid under the graph of the given function and above the region D.

$$\iint_D f(x, y)\,\mathrm{d}A = \int_0^{\frac{\pi}{2}} \int_0^{\sin\theta} f(r\cos\theta, r\sin\theta) r\,\mathrm{d}r\,\mathrm{d}\theta$$
$$= \int_0^{\frac{\pi}{2}} \int_0^{\sin\theta} r^2 \cos\theta\,\mathrm{d}r\,\mathrm{d}\theta$$

(b) Evaluate the iterated integral to find the volume of the solid.

$$\int_0^{\frac{\pi}{2}} \int_0^{\sin\theta} r^2 \cos\theta\,\mathrm{d}r\,\mathrm{d}\theta = \int_0^{\frac{\pi}{2}} \left[\frac{r^3\cos\theta}{3}\right]_0^{\sin\theta} \mathrm{d}\theta$$
$$= \int_0^{\frac{\pi}{2}} \left[\frac{\sin^3\theta\cos\theta}{3}\right] \mathrm{d}\theta$$
$$u = \sin\theta \qquad \theta_1 = \sin(0) = 0 \qquad \theta_2 = \sin\left(\frac{\pi}{2}\right) = 1$$
$$\mathrm{d}u = \cos\theta\,\mathrm{d}\theta$$
$$\int_0^{\frac{\pi}{2}} \left[\frac{\sin^3\theta\cos\theta}{3}\right] \mathrm{d}\theta = \int_0^1 \left[\frac{u^3}{3}\right] \mathrm{d}u$$
$$= \left[\frac{u^4}{12}\right]_0^1$$
$$= \frac{1}{12}$$