Contemporary Physics

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Chapter 1

The Failures of Classical Physics

1.1 Review of Classical Physics

Mechanics

A particle of mass m and velocity v has *kinetic energy* K defined as

$$K = \frac{1}{2}mv^2 \tag{1.1}$$

and *linear momentum* $\vec{p}$ defined as

$$\vec{p} = m\vec{v} \tag{1.2}$$

Kinetic energy can be rewritten in terms of linear momentum as

$$K = \frac{p^2}{2m} \tag{1.3}$$

When particles collide, the two fundamental conservation laws are used to analyze the collision:

I. **Conservation of Energy.** The total energy of an isolated system remains constant if no external forces act upon it. In the case of a collision, the total energy of the particles must be the same both *before* and *after* they collide.

II. **Conservation of Linear Momentum.** The total linear momentum of an isolated system remains constant. In the case of a collision, the total linear momentum of the particles is the same both *before* and *after* the collision. As linear momentum is a vector, this law is generally applied for each component individually.

Another application of the principle of conservation of energy can be seen when a particle moves subject to an external force F. Such an external force often has a corresponding potential energy U, defined such that (for 1-D motion)

$$F = -\frac{\mathrm{d}u}{\mathrm{d}x} \tag{1.4}$$

The total energy E is the sum of the kinetic and potential energies:

$$E = K + U \tag{1.5}$$

As a particle moves, K and U may change, but E must remain constant.
When a particle with *linear momentum* $\vec{p}$ is at displacement $\vec{r}$ from the origin O, its angular momentum $\vec{L}$ about O is defined by

$$\vec{L} = \vec{r} \times \vec{p} \tag{1.6}$$

As is the case with linear momentum, angular momentum is conserved.

Velocity Addition

Let $\vec{v}_{AB}$ represent the velocity of A relative to B and $\vec{v}_{BC}$ be that of B relative to C. The velocity of A relative to C is then

$$\vec{v}_{AC} = \vec{v}_{AB} + \vec{v}_{BC} \tag{1.7}$$

Electricity and Magnetism

The electrostatic (Coulomb) force exerted by a charged particle q_1 on another charge q_2 has magnitude

$$F = \frac{1}{4\pi\varepsilon}\frac{|q_1||q_2|}{r^2} \tag{1.8}$$

The direction of the force is along the line that joins the particles. The Coulomb constant $k = 1/4\pi\varepsilon_0$ is

$$k \approx 8.99 \times 10^9\,\frac{\mathrm{N\,m^2}}{\mathrm{C^2}}$$

The corresponding potential energy is

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} \tag{1.9}$$

An electrostatic potential difference ΔV is established by a distribution of charges. When a charge q moves through a potential difference V the change in its electric potential energy is

$$\Delta U = q\Delta V \tag{1.10}$$

Charges are often measured in terms of the charge of the electron, which has magnitude

$$e \approx 1.602 \times 10^{-19}\,\mathrm{C}$$

The *electron-volt* (eV) is defined as the energy of a charge equal in magnitude to that of an electron passing through a potential difference of 1 V:

$$1\,\mathrm{eV} = 1.602 \times 10^{-19}\,\mathrm{J}$$

A magnetic field $\vec{B}$ can be produced by an electric current i. The magnitude of the magnetic field at the center of a circular current loop of radius r is

$$V = \frac{\mu_0 i}{2r} \tag{1.11}$$

The SI unit for magnetic field is the tesla (T), defined as

$$1\,\mathrm{T} = 1\,\frac{\mathrm{N}}{\mathrm{A\,m}}$$

The constant μ_0 is

$$\mu_0 \approx 4\pi \times 10^{-7} \, \frac{\mathrm{N\,s}^2}{\mathrm{C}^2}$$

The direction of the conventional (*positive*) current is opposite to the direction of travel of the negatively charged electrons, which are what typically produce the current in the wires. The direction of $\vec{B}$ is chosen by the right-hand rule.

The *magnetic moment* $\vec{\mu}$ of a current loop is defined as

$$|\vec{\mu}| = iA \tag{1.12}$$

where A is the geometric area enclosed by the loop. The direction of $\vec{\mu}$ is perpendicular to the plane of the loop, as determined by the right-hand rule.

When a current loop is placed in a uniform *external* magnetic field $\vec{B}_{\text{ext}}$, the torque $\vec{\tau}$ on the loop that tends to align $\vec{\mu}$ with $\vec{B}_{\text{ext}}$ is

$$\vec{\tau} = \vec{\mu} \times \vec{B}_{\text{ext}} \tag{1.13}$$

When the field is applied, $\vec{\mu}$ rotates such that its energy tends to a minimum, which occurs when $\vec{\mu}$ is parallel to $\vec{B}_{\text{ext}}$.

This interaction can also be described by

$$U = -\vec{\mu} \cdot \vec{B}_{\text{ext}} \tag{1.14}$$

Electromagnetic waves travel in free space at the speed of light c, which is

$$c = (\varepsilon_0 \mu_0)^{-1/2} \tag{1.15}$$

Chapter 2

The Special Theory of Relativity

2.1 Classical Relativity

A Galilean transformation

$$\vec{v}' = \langle v_x - u, v_y, v_z \rangle$$

where u is the speed of the frame O' relative to the frame O. Under this transformation, the time is the same for both reference frames.
A frame of reference moving at constant velocity is said to be an inertial frame.

2.2 Einstein's Postulates

The principle of