

Assignment 4

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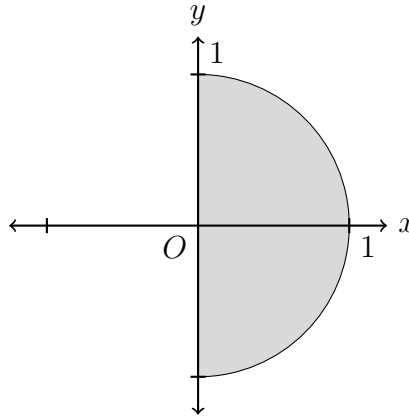
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- 1) Find and sketch the domain of the function $f(x, y) = \sqrt{x} + \sqrt{1 - x^2 - y^2}$.

Solution

$$\begin{aligned}\sqrt{x} &\implies x \geq 0 \\ \sqrt{1 - x^2 - y^2} &\implies x^2 + y^2 \leq 1\end{aligned}$$

The latter equation is that of a circle, so the domain is simply the region bounded by the positive half of the circle $x^2 + y^2 = 1$.



- 2) Let $f(x, y) = 4 - x^2 - 5y^2$. Find $f_x(1, 1)$ and $f_y(1, 1)$ and interpret these numbers as slopes.

Solution

$$\begin{aligned}f_x(x, y) &= -2x & f_y(x, y) &= -10y \\ f_x(1, 1) &= -2(1) = -2 & f_y(1, 1) &= -10(1) = -10\end{aligned}$$

At the point where $x = 1$ and $y = 1$, the slopes of the lines tangent to $f(x, y) = 4 - x^2 - 5y^2$ parallel to the x - and y -axes respectively are -2 and -10 .

- 3) Let $f(x, y) = x^3 + xy^2 - 3y^2$. Find f_x , f_y , f_{xx} , f_{yy} , and f_{xy} .

Solution

$$\begin{aligned}f_x(x, y) &= 3x^2 + y^2 & f_y(x, y) &= 2xy - 6y \\ f_{xx}(x, y) &= 6x & f_{yy}(x, y) &= 2x - 6 \\ f_{xy}(x, y) &= 2y\end{aligned}$$

- 4) Find the equation of the tangent plane for $z = 3x^2 + y^2$ at $P(1, 1, z_0)$.

Solution

$$z_0 = 3(1)^2 + (1)^2 = 4$$

$$\left. \frac{\partial z}{\partial x} \right|_{(x,y)=(1,1)} = 6x|_{(x,y)=(1,1)} = 6(1) = 6$$

$$\left. \frac{\partial z}{\partial y} \right|_{(x,y)=(1,1)} = 2y|_{(x,y)=(1,1)} = 2(1) = 2$$

$$z = 6(x - 1) + 2(y - 1) + 4$$

$$= 6x - 6 + 2y - 2 + 4$$

$$= 6x + 2y - 4$$