Assignment 3

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1. Find the **vector equation** and the **parametric equation** for the line that joins $P(1,-1,2)$ and $Q(3,0,-2)$.

$$
\begin{aligned}
\vec{r}_0 &= \overrightarrow{OP} = \langle 1,-1,2\rangle \\
\vec{v} &= \langle 3-1, 0-(-1), -2-2\rangle = \langle 2,1,-4\rangle \\
\vec{r} &= \vec{r}_0 + t\vec{v} = \langle 1,-1,2\rangle + t\langle 2,1,-4\rangle = \langle 1+2t, -1+t, 2-4t\rangle \\
x &= 1+2t \qquad y = -1+t \qquad z = 2-4t \qquad (1+2t, -1+t, 2-4t)
\end{aligned}
$$

2. Given the vector function $r(t) = \langle t^4, t, t^2\rangle$, find the unit tangent vector $\vec{T}(1)$ and $\vec{r}'(t)\times\vec{r}''(t)$.

$$
\begin{aligned}
\vec{r}'(t) &= \left\langle 4t^3, 1, 2t\right\rangle \\
\vec{T}(t) &= \frac{\vec{r}'(t)}{|\vec{r}'(t)|} = \frac{\langle 4t^3, 1, 2t\rangle}{\sqrt{16t^6+1+4t^2}} \\
\vec{T}(1) &= \frac{\langle 4,1,2\rangle}{\sqrt{16+1+4}} = \frac{\langle 4,1,2\rangle}{\sqrt{21}} \\
\vec{r}''(t) &= \left\langle 12t^2, 0, 2\right\rangle \\
\vec{r}'(t)\times\vec{r}''(t) &= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 4t^3 & 1 & 2t \\ 12t^2 & 0 & 2 \end{vmatrix} \\
&= \left\langle (1)(2)-(2t)(0), (-1)\left(\left(4t^3\right)(2)-(2t)\left(12t^2\right)\right), \left(4t^3\right)(0)-(1)\left(12t^2\right)\right\rangle \\
&= \left\langle 2, 16t^3, -12t^2\right\rangle
\end{aligned}
$$