Chapter 9 Homework

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9.1 Significance Tests: The Basics

1. No homework? $H_0 : p = 0.75$ and $H_a : p < 0.75$ where p is the true proportion of students at Mr.Tabor's school that completed their math homework last night.

3. How about juice? $H_0 : \mu = 180$ ml and $H_a : \mu \neq 180$ ml where μ is the true mean amount of liquid in a bottle (dispensed by the machine) in milliliters.

4. Attitudes $H_0 : \mu = 115$ and $H_a : \mu > 115$ where μ is the true mean score on the SSHA for students at the teacher's college that are over the age of 30.

5. Cold cabin? $H_0 : \sigma = 3$°F and $H_a : \sigma > 3$°F where σ is the true standard deviation of the temperature allowed by the thermostat in degrees Fahrenheit.

7. Stating hypotheses

a. The null hypothesis must be a statement of equality while the alternative hypothesis must be an inequality; $H_0 : p = 0.37$; $H_a : p > 0.37$.

b. Hypotheses must always make predictions regarding a population parameter rather than a sample statistic; $H_0 : \mu = 3000$ grams; $H_a : \mu < 3000$ grams.

9. No homework?

a. If $H_0 : p = 0.75$ is true, then 75% of all students at Mr.Tabor's school completed their math homework last night.

b. Assuming that $H_0 : p = 0.75$ is true, the probability that $\hat{p} \leq 0.68$ for a random sample is 12.65%.

10. Attitudes

a. If $H_0 : \mu = 115$ is true, then the true mean score on the SSHA for students at the teacher's college that are over the age of 30 is 115.

b. Assuming that $H_0 : p = 115$ is true, then the probability that $\hat{p} \geq 125.7$ due to sheer random chance, as is the case in this sample, is 1.01%.

13. Interpreting a P-value The interpretation did not include the assumption that $H_0 : \mu = 100$ is true or the inequality $\mu > 100$.

15. No homework At a confidence level of $\alpha = 0.05$, there is not satisfactory evidence supporting $H_a : p < 0.75$, as the P-level of 0.1265 is greater than α, so the null hypothesis $H_0 : p = 0.75$ cannot be disregarded.

16. Attitudes At a confidence level of $\alpha = 0.05$, there is satisfactory evidence supporting the claim that the average SSHA score for students above the age of 30 is higher, as the P-level of 0.0101 is less than α, and the null hypothesis $H_0 : p = 115$ can be disregarded.

19. Making conclusions It was not specified that the P-value was greater than the α, simply that it was large. Additionally, a P-value greater than the significance level does not support H_0, instead not supporting H_a.

21. Heavy bread?

a. μ = true mean weight of a loaf of bread produced at the bakery (in pounds); $H_0 : \mu = 1$; $H_a : \mu < 1$.

b. The sample mean is less than that predicted by the H_0, which would support H_a.

c. Assuming that H_0 is true, there is an 8.06% chance of this sample's outcome occurring in a random sample.

d. Because the P-level of H_0 against H_a is greater than the $\alpha = 0.01$ significance level, the data does not provide convincing evidence for the hypothesis that the true mean weight of a loaf of bread produced at the bakery is less than 1 lbs, and H_0 cannot be rejected.

23. Opening a restaurant A Type I error would be finding convincing evidence for the true mean income of those living near the potential location being greater than \$85,000 when such is not the case. This would result in the restaurant being opened in a place where the people in the vicinity are unable to afford to eat there, meaning that the restaurant would have to either reduce its prices (by cutting either margins or costs) or close and relocate.
A Type II error would be failing to find convincing evidence of the true mean income of those living close to the potential location being at least \$85,000. This would result in the location being passed up despite being suitable.

25. Awful accidents

a. A Type I error would occur if convincing evidence of the true proportion of calls involving life-threatening injuries over this 6-month period for which emergency personnel took over 8 minutes to arrive being less than 0.22 was found despite this hypothesis being false.
A Type II error would occur if convincing evidence of the true proportion of calls involving life-threatening injuries over this timeframe for which it took an excess of 8 minutes for emergency personnel to arrive being less than 0.22 was not found despite this hypothesis being true.

b. In this case, a Type I error would be more harmful, as it would make it seem as though there was less room for improvement than there really actually is, which will likely result in a reduced drive to improve, potentially resulting in the proportion staying the same or even increasing, resulting in more deaths due to wasted time.

c. As the probability of a Type I error occurring is equal to α and that a Type I error would be more serious than a Type II one, the significance level should be lower than $\alpha = 0.05$.

27. More lefties?

a. $p = 0$ the true proportion of students at Simon's school that are left-handed; $H_0 : p = 0.1$; $H_a : p > 0.1$.

b. The P-value of H_0 against the result of the sample is 24/200, equal to 12%. This means that the probability of receiving the observed results due to sheer chance assuming, that H_0 is true.

c. The P-value is greater than the the assumed confidence level of $\alpha = 0.05$, so the data provided by the survey is not enough to warrant disregarding H_0 and convincingly support the conclusion that the true proportion of students at Simon's school that are left-handed is greater than 0.1.

9.2 Tests About a Population Proportion

35. Home computers The randomness condition is met, as there are Jason's school is large, so there are likely over 600 students at school making the sample size n of 60 less than a tenth of the population, so independence can be assumed, and the sample itself is random. The Large Counts condition is also met, as np_0 and $n(1 - p_0)$ are both greater than 10, at 48 and 12 respectively. A Normal distribution can therefore be used to approximate the sampling distribution of $\hat{p}$.

37. The chips project

a. There are 400 students in the population, so the sample size n of 50 is over 10% of the population, so independence cannot be assumed.

b. As np_0 and $n(1 - p_0)$ are both 25, which is greater than 10, the Large Counts condition is met, so the sampling distribution of $\hat{p}$ is approximately Normal.

39. Home computers

a. The sample proportion is 41/60, which is about 0.6833, which is less than 0.8.

b.

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{\frac{41}{60} - 0.8}{\sqrt{\frac{0.8(1-0.8)}{60}}} \approx -2.27$$

$$P\text{-value} = \texttt{normalcdf}\,(\text{lower} : -\infty, \text{upper} : z \approx -2.27, \mu : 0, \sigma : 1) \approx 0.0116$$

c. As the P-value is less than α, H_0 can be rejected, as there is convincing evidence that the true proportion of all students at Jason's school that own computers is less than 0.8.

41. Significance tests

a.

$$P\text{-value} = \texttt{normalcdf}\,(\text{lower} : z \approx 2.19, \text{upper} : \infty, \mu : 0, \sigma : 1) \approx 0.0143$$

Assuming that $H_0 : p = 0.5$ is true, there is about a 1.43% chance of having received this result from a random sample.

b. The P-level of 0.0143 is greater than α, so H_0 cannot be rejected, as there is not convincing evidence of the true proportion of being greater than 0.5.

c.

$$\hat{p} = p_0 + z\sqrt{\frac{p_0\,(1-p_0)}{n}} = 0.5 + 2.19\sqrt{\frac{0.5\,(1-0.5)}{200}} \approx 0.5774$$

43. Bullies in middle school

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{\frac{445}{558} - 0.75}{\sqrt{\frac{0.75(1-0.75)}{558}}} \approx 2.591$$

$$P\text{-value} = \texttt{normalcdf}\,(\text{lower} : z \approx 2.591, \text{upper} : \infty, \mu : 0, \sigma : 1) \approx 0.005$$

The probability of evidence of the sample proportion of middle school students that engage in bullying being at least 445/558 is about 0.005 for a random sample assuming that the true population proportion p is equal to 0.75, which is less than the significance level $\alpha = 0.05$, so there this data provides convincing evidence that p is greater than 0.75.

59. Potato chips If the true proportion p of potatoes with blemishes in a shipment is 0.11, there is a 76.5% chance of convincing evidence being found for $H_a : p > 0.08$.

60. Upscale Restaurant If the true mean income of people living near the location

61. Powerful potatoes

a. Increasing the significance level also increases the test's power.

b. Decreasing the sample size also decreases the test's power.

c. Decreasing the difference between p_a and p, the effect size, decreases makes it less likely that a significant difference will be detected, decreasing the power.

62. Restaurant power

a. Decreasing the sample size decreases the standard deviation, decreasing the range within which values are significant, making it harder to reject H_0, increasing the chances of a Type II error occurring, reducing the power.

b. Reducing the effect size makes a difference in the parameter more difficult to detect, increasing the chances of a Type II error occurring, reducing the power.

c. Increasing the significance level makes it easier to reject H_0, decreasing the chances of a Type II error, increasing the power.

63. Potato power problems

a. Using a higher significance level increases the chance of the null hypothesis being rejected despite the alternative hypothesis being false.

b. Increasing the sample size makes the study more costly to perform.

64. Restaurant power problems

a. Increasing the significance level also increases the probability of a Type I error.

b. Increasing the sample size makes the study more expensive to carry out.

65. Better parking

a. At a significance level of $\alpha = 0.05$, there is a 75% chance of convincing evidence for more than 37% of students at the school approving of the provided parking being found by a random sample if the true population proportion is 0.45.

b.

$$P(\text{Type I Error}) = \alpha = 0.05 \qquad P(\text{Type II Error}) = 1 - \text{power} = 1 - 0.75 = 0.25$$

c. Power can be increased by increasing α or the sample size.

67. Error probabilities and power

a. The power of the test is one minus the probability of making a Type II, making it 0.86.

b. The probability of making a Type I error is equal to the significance level, so it is 0.05.

70.

$$z = \frac{\frac{64}{100} - 0.5}{\sqrt{\frac{\frac{64}{100}\left(1-\frac{64}{100}\right)}{100}}} = \frac{0.64 - 0.5}{\sqrt{\frac{0.64(1-0.64)}{0.36}}100}$$

The answer is therefore **a**.

71. A significance test requires randomness, Large Counts (for approximate Normality), and the 10% condition (for independence), so the answer is **e**.

72.

$$P\text{-value} = 1 - \texttt{normalcdf}\,(\text{lower} : -2.43, \text{upper} : 2.43, \mu : 0, \sigma : 1) \approx 0.015$$

As the P-value is between 0.01 and 0.05, the answer is **b**.

73. The only interval that doesn't contain 0.3 is $(0, 19, 0.27)$, so the answer is **a**.

74.

$$P(\text{Type II Error}) = 1 - \text{power} = 1 - 0.9 = 0.1$$

The probability of a Type II error is 0.1, so the answer is **b**.

9.3 Tests About a Difference in Proportions

85. Bag lunch?

a. The survey is states to be an SRS.

$$n_1 \leq 0.1N_1, n_2 \leq 0.1N_2 \hat{p}_1 - \hat{p}_2 = \frac{52}{80} - \frac{78}{104} = -0.1$$

$$\hat{p}_C = \frac{52+78}{80+104} \approx 0.707$$

$$n_1\hat{p}_C, n_1(1-\hat{p}_C), n_2\hat{p}_C, n_2(1-\hat{p}_C) \geq 10$$

$$s_{\hat{p}_1-\hat{p}_2} = \sqrt{\hat{p}_C\left(1-\hat{p}_C\right)\left(\frac{1}{n_1}-\frac{1}{n_2}\right)} \approx \sqrt{0.707\left(1-0.707\right)\left(\frac{1}{80}-\frac{1}{104}\right)} \approx 0.068$$

$$z = \frac{\hat{p}_1-\hat{p}_2-\mu_{\hat{p}_1-\hat{p}_2}}{s_{\hat{p}_1-\hat{p}_2}} \approx \frac{-0.1-0}{0.707} \approx -1.471$$

$$P\text{-value} = \texttt{normalcdf}\left(\text{lower}: -\infty, \text{upper}: z \approx -1.471, \mu: 0, \sigma: 1\right) \approx 0.071$$

As 0.071 is greater than $\alpha = 0.05$, H_0 cannot be rejected, and the data does not provide convincing evidence for H_a.

b. If there is no difference between the proportions of sophomores and seniors at Phoebe's school that bring a bag lunch, there is a 7.1% chance of the sample difference being at most -0.1 in a random sample.

87. Preventing peanut allergies

a. The study is random and the 10% condition is met.

$$\hat{p}_1 - \hat{p}_2 = \frac{10}{307} - \frac{55}{321} \approx -0.139$$

$$\hat{p}_C = \frac{10+55}{307+321} \approx 0.104$$

$$s_{\hat{p}_1-\hat{p}_2} = \sqrt{\hat{p}_C\left(1-\hat{p}_C\right)\left(\frac{1}{n_1}-\frac{1}{n_2}\right)} = \sqrt{0.104\left(1-0.104\right)\left(\frac{1}{307}-\frac{1}{321}\right)} \approx 0.024$$

$$z = \frac{\hat{p}_1-\hat{p}_2-\mu_{\hat{p}_1-\hat{p}_2}}{s_{\hat{p}_1-\hat{p}_2}} \approx \frac{-0.139-0}{0.024} \approx -4.256$$

$$P\text{-value} = \texttt{normalcdf}\left(\text{lower}: -\infty, \text{upper}: z \approx -4.256, \mu: 0, \sigma: 1\right) \approx 0$$

As the P-value is less than the significance level $\alpha = 0.05$, H_0 can be rejected and the data convincingly supports H_a.

b. As H_0 was rejected, only a Type II error could have been made.

c. The study was an experiment, so the results are not generalizable.

d. This confidence interval states with 95% certainty that the difference is between -0.185 and -0.93 while the test simply stated that it was not 0.

89. Preventing peanut allergies

a. The sample size increasing decreases the standard error and therefore the standardized test statistic, making it more likely for the P-value to fall below α, making H_0 more likely to be rejected, decreasing the probability of a Type II error, increasing the power. This would, however, increase the cost of the study considerably.

b. Increasing the α increases the chances of the P-value falling below α, making H_0 more likely to be rejected, decreasing the probability of a Type II error, increasing the power. This would make a Type I error more likely, though.

c. This would eliminate a source of variability, making it easier to reject H_0, increasing the probability of a Type II error and therefore increasing the power. This would limit the scope of inference, though.

93. Texting and driving

a. p_A = proportion of people that received version A that answered "Yes"; p_B = proportion of people that received version B that answered "Yes"; $H_0 : p_A - p_B = 0$; $H_a : p_A - p_B > 0$.

b.

$$\hat{p}_C = \frac{x_A + x_B}{n_A + n_B} = \frac{18 + 14}{25 + 25} = 0.64$$

$$n_A(1 - \hat{p}_C) = 9 < 10$$

The Large Counts condition is not met, so standard error cannot be calculated, so the P-value cannot be calculated.

c.

$$\hat{p}_A - \hat{p}_b = \frac{18}{25} - \frac{14}{25} = 0.16$$

$$P\text{-value} = P(\hat{p}_A - \hat{p}_B \geq 0.16) = \frac{14}{100} = 0.14$$

d. The probability of getting a difference between the proportions of people that received each version that answered "Yes" of at least 0.16 in a random sample is about 0.14. As this value is greater than the significance level $\alpha = 0.05$, H_0 cannot be rejected and the data does not provide convincing evidence for H_a.

95.

$$H_0 : p_M - p_F = 0 \qquad\qquad H_a : p_M - p_F \neq 0$$

The answer is therefore **a**.

96. The P-value was greater than $\alpha = 0.1$, so H_0 cannot be rejected, and there is not convincing evidence supporting H_a, so the answer is **b**.

97.

$$
\begin{aligned}
s_{\hat{p}_1-\hat{p}_2} &= \sqrt{\hat{p}_C\left(1-\hat{p}_C\right)\left(\frac{1}{n_1}-\frac{1}{n_2}\right)} = \sqrt{\frac{500+410}{550+484}\left(1-\frac{500+410}{550+484}\right)\left(\frac{1}{550}-\frac{1}{484}\right)} \\
&\approx \sqrt{0.88\left(0.12\right)\left(\frac{1}{550}-\frac{1}{484}\right)}
\end{aligned}
$$

The answer is therefore **c**.

98. The total number of rats in each group is 10, so it is not possible for the Large Counts condition to be met, making the answer **e**.