Assignment 4

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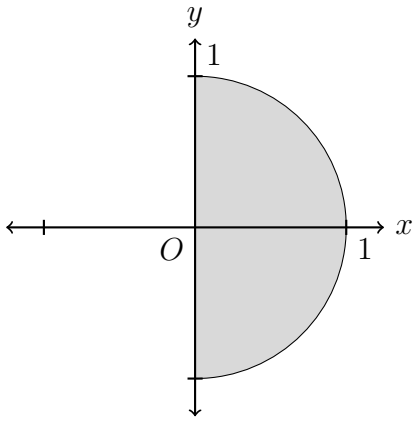
1) Find and sketch the domain of the function $f(x,y) = \sqrt{x} + \sqrt{1-x^2-y^2}$.

Solution

$$\begin{aligned} \sqrt{x} &\implies x \geq 0 \\ \sqrt{1-x^2-y^2} &\implies x^2 + y^2 \leq 1 \end{aligned}$$

The latter equation is that of a circle, so the region domain is simply the region bounded by the positive half of the circle $x^2 + y^2 = 1$.



2) Let $f(x,y) = 4 - x^2 - 5y^2$. Find $f_x(1,1)$ and $f_y(1,1)$ and interpret these numbers as slopes.

Solution

$$\begin{aligned} f_x(x,y) &= -2x & \qquad f_y(x,y) &= -10y \\ f_x(1,1) &= -2(1) = -2 & \qquad f_y(1,1) &= -10(1) = -10 \end{aligned}$$

At the point where $x = 1$ and $y = 1$, the slopes of the lines tangent to $f(x,y) = 4 - x^2 - 5y^2$ parallel to the x- and y-axes respectively are -2 and -10.

3) Let $f(x,y) = x^3 + xy^2 - 3y^2$. Find f_x, f_y, f_{xx}, f_{yy}, and f_{xy}.

Solution

$$\begin{aligned} f_x(x,y) &= 3x^2 + y^2 & \qquad f_y(x,y) &= 2xy - 6y \\ f_{xx}(x,y) &= 6x & \qquad f_{yy}(x,y) &= 2x - 6 \\ f_{xy}(x,y) &= 2y \end{aligned}$$

4) Find the equation of the tangent plane for $z = 3x^2 + y^2$ at $P(1, 1, z_0)$.

Solution

$$
\begin{aligned}
z_0 &= 3(1)^2 + (1)^2 = 4 \\
\left.\frac{\partial z}{\partial x}\right|_{(x,y)=(1,1)} &= 6x|_{(x,y)=(1,1)} = 6(1) = 6 \\
\left.\frac{\partial z}{\partial y}\right|_{(x,y)=(1,1)} &= 2y|_{(x,y)=(1,1)} = 2(1) = 2 \\
z &= 6(x-1) + 2(y-1) + 4 \\
&= 6x - 6 + 2y - 2 + 4 \\
&= 6x + 2y - 4
\end{aligned}
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