Calculus III

Arnav Patri

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Chapter 12

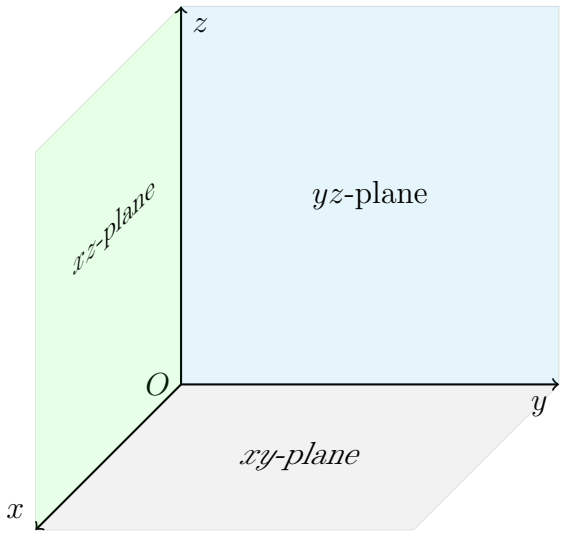
Vectors and the Geometry of Space

12.1 Three-Dimensional Coordinate Systems

Any point in a plane can be represented as an ordered pair of real numbers. Because this uses two numbers, a plane is called two-dimensional. To locate a point in space, a triplet of real-numbers is required.

3D Space

Before points can be represented in 3D space, a fixed point O (the origin) and three perpendicular lines that pass through it, called the **coordinate axes**. These axes are labeled the x-, y-, and z-axes. In general, the former two are horizontal while the third is vertical. The direction of the z-axis is determined by the **right-hand rule**. Curling the fingers of the right hand from the positive x-axis to the positive y-axis, the thumb will point in the direction of the positive z-axis.
The three coordinate axes determine the three **coordinate planes**.



Three planes divided space into eight **octants**. Illustrated above are the positive xz-, yz-, and xy-planes, constituting the **first octant**.
A point's **coordinates** are an ordered triple of real numbers. A point's **projection** onto a plane is the point with two of the same coordinates, the third becoming 0.

Plane	xy	yz	xz
(a, b, c)	$(a, b, 0)$	$(0, b, c)$	$(a, 0, c)$

The set of all ordered triples is the cartesian product of three sets of all real numbers, denoted appropriately by $\mathbb{R}^3$ and defined as

$$\mathbb{R}^3 = \mathbb{R} \times \mathbb{R} \times \mathbb{R} = \{(x, y, z) \mid x, y, z \in \mathbb{R}\}$$

A one-to-one correspondence between points in space and ordered triples in $\mathbb{R}^3$ is a **three-dimensional coordinate system**. It should be noted that the first octant can be described as the set of points for which all coordinates are positive.

Distances and Spheres

Distance Formula in Three Dimensions The distance $|P_1P_2|$ between points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is

$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Equation of a Sphere The equation of a sphere with center (h, k, l) and radius r is

$$(x - h)^2 + (y - k)^2 + (z - l)^2 = r^2$$

12.2 Vectors