Homework Set 1

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4 Number Theory and Cryptography

4.2 Integer Representations and Algorithms

4.2.1 1–11 odd, 21, 23

1. a) $231 = (1110\,0111)_2$
   b) $4532 = (1\,0001\,1011\,0100)_2$
   c) $97644 = (1\,0111\,1101\,0110\,1100)_2$

3. a) $(1\,1111)_2 = 37$
   b) $(10\,0000\,0001)_2 = 513$
   c) $(1\,0101\,0101)_2 = 215$
   d) $(110\,1001\,0001\,0000)_2 = 26896$

5. a) $(572)_8 = 378$
   b) $(1604)_8 = 900$
   c) $(432)_8 = 275$
   d) $(2417)_8 = 1295$

7. a) $(80\text{E})_{16} = (1000\,0000\,1110)_2$
   b) $(135\text{AB})_{16} = (0001\,0011\,0101\,1010\,1011)_2$
   c) $(\text{ABBA})_{16} = (1010\,1011\,1011\,1010)_2$
   d) $(\text{DEFACED})_{16} = (1101\,1110\,1111\,1010\,1100\,1110\,1101)_2$

9. $(\text{ABCDEF})_{16} = (1010\,1011\,1100\,1101\,1110\,1111)_2$

11. $(1011\,0111\,1011)_2 = (\text{B7B})_{16}$

21. a)
$$\begin{array}{r} {\scriptstyle 1}\quad\;\; {\scriptstyle 1\,1\,1} \\ 1\,0\,0\,0\,1\,1\,1 \\ +\,1\,1\,1\,0\,1\,1\,1 \\ \hline 1\,0\,1\,1\,1\,1\,1\,0 \end{array}$$
$(100\,0111)_2 = 71$, $(111\,0111)_2 = 119$, $71 \times 119 = 8449 = (10\,0001\,0000\,0001)_2$

   b)
$$\begin{array}{r} {\scriptstyle 1}\;\; {\scriptstyle 1\,1}\quad {\scriptstyle 1}\quad {\scriptstyle 1} \\ 1\,1\,1\,0\,1\,1\,1\,1 \\ +\,1\,0\,1\,1\,1\,1\,0\,1 \\ \hline 1\,1\,0\,0\,1\,1\,0\,1\,0 \end{array}$$
$(1110\,1111)_2 = 239$, $(1011\,1101)_2 = 189$, $239 \times 189 = 45171 = (1011\,0000\,0111\,0011)_2$

   c)

23.

a)
$$\begin{array}{r} {\scriptstyle 1\,1}\\ {\scriptstyle 1}763\\ +147\\ \hline 1132 \end{array}$$

$$\begin{array}{r} 763\\ \times 147\\ \hline {\scriptstyle 1\,1}\\ {\scriptstyle 2}6645\\ {\scriptstyle 1}3714\\ +763\\ \hline 144305 \end{array}$$

b)
$$\begin{array}{r} 6001\\ +\ 272\\ \hline 6273 \end{array}$$

$$\begin{array}{r} 6001\\ \times\ 272\\ \hline {\scriptstyle 1}\ 14002\\ 52007\\ +14002\\ \hline 2134272 \end{array}$$

c)
$$\begin{array}{r} {\scriptstyle 1\,1\,1}\\ 1111\\ +\ 777\\ \hline 2110 \end{array}$$

$$\begin{array}{r} {\scriptstyle 1\,1\,1}\\ 1111\\ \times\ 777\\ \hline 7777\\ 7777\\ +7777\\ \hline 1107667 \end{array}$$

d)
$$\begin{array}{r} 54321\\ +3456\\ \hline 57777 \end{array}$$

$$\begin{array}{r} 54321\\ \times 3456\\ \hline {\scriptstyle 2\quad 1\,1}\\ {\scriptstyle 1\,1}4123\,46\\ 336025\\ 261504\\ +205163\\ \hline 237326216 \end{array}$$

4.3 Primes and Greatest Common Divisors

4.3.1 1, 3, 5, 15, 17, (19 extra credit)