Assignment 5

Arnav Patri

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1. Find the Directional derivative of the function $f(x,y) = x^3y - 2x^2y^2$ in the direction of the vector $\vec{v} = \hat{\imath} + 2\hat{\jmath}$ at the point $P(1,1)$.

   Solution

$$
\begin{aligned}
\nabla f(x,y) &= \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle = \langle 3x^2y - 4xy^2, x^3 - 4x^2y \rangle \\
\nabla f(1,1) &= \langle -1, -3 \rangle \\
\vec{u} &= \frac{\vec{v}}{|\vec{v}|} = \frac{\vec{v}}{\sqrt{1^2 + 2^2}} = \frac{\vec{v}}{\sqrt{5}} \\
D_{\vec{u}} f(1,1) &= \nabla f(1,1) \cdot \vec{u} = \frac{(-1)(1) + (-3)(2)}{\sqrt{5}} = -\frac{7}{\sqrt{5}}
\end{aligned}
$$

2. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for $x^2 - y^2 + z^2 - 2z = 4$

   Solution
   Let $F(x,y,z) = x^2 - y^2 + z^2 - 2z$. If $z \neq 1$,

$$
\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{2x}{2z-2} = -\frac{x}{z-1} \qquad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{-2y}{2z-1} = \frac{y}{z-1}
$$

3. Find the local maximum and local minimum values and saddle points of $f(x,y) = xy - 2x - 2y - x^2 - y^2$.

   Solution

$$
\begin{aligned}
f_x(x,y) &= y - 2 - 2x = 0 & f_y(x,y) &= x - 2 - 2y = 0 \\
y &= 2x + 2 & y &= \frac{x}{2} - 1 \\
2x + 2 &= \frac{x}{2} - 1 \\
4x + 4 &= x - 2 \\
3x &= -6 \\
x &= -2 & y &= -2 \\
f(-2,2) &= 4 \\
f_{xx}(x,y) &= -2 & f_{yy}(x,y) &= -2 \\
f_{xx}(-2,-2) &= -2 & f_{yy}(-2,-2) &= -2 \\
f_{xy}(x,y) &= 1 \\
f_{xy}(-2,-2) &= 1 \\
D(-2,-2) &= (-2)(-2) - 1^2 \\
&= 3
\end{aligned}
$$

   As $D(-2,-2) > 0$ and $f_{xx}(-2,-2) < 0$, the point $(-2,-2,4)$ is a relative maximum of f. As f lacks any other critical values, it lacks relative minima and saddle points.