15.6.13. (p. 1093)

Arnav Patri

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Evaluate the triple integral

$$\iiint_E y \, \mathrm{d}V$$

where

$$E = \{(x, y, z) \mid 0 \leq x \leq 3, 0 \leq y \leq x, x - y \leq z \leq x + y$$

$$\iiint_E y \, \mathrm{d}V = \int_0^3 \int_0^x \int_{x-y}^{x+y} y \, \mathrm{d}z \, \mathrm{d}y \, \mathrm{d}x = \int_0^3 \int_0^x [yz]_{x-y}^{x+y} \, \mathrm{d}y \, \mathrm{d}x = \int_0^3 \int_0^x [y(x + y - (x - y)] \, \mathrm{d}y \, \mathrm{d}x$$

$$= \int_0^3 \int_0^x \left[2y^2\right] \mathrm{d}y \, \mathrm{d}x = \int_0^3 \left[\frac{2y^3}{3}\right]_0^x \mathrm{d}x = \int_0^3 \left[\frac{2x^3}{3}\right] \mathrm{d}x = \left[\frac{x^4}{6}\right]_0^3 = \frac{27}{2}$$