Problems

1.

Solutions

1.

$$\lim_{n\to\infty}\left[\frac{n+5}{3n+5}\right]=\frac{1}{3}\neq 0 \implies \text{diverges by alternating series test}$$

2.

$$\lim_{n\to\infty}\left[\frac{n^{2/5}}{n^5}\right]=0 \qquad \frac{1}{(n+1)^{6/2}}<\frac{1}{n^{6/2}} \implies \text{converges by alternating series test}$$

$$\sum_{n=1}^{\infty}\left|\frac{(-1)^n n^{3/2}}{n^5}\right|=\sum_{n=1}^{\infty}\left[\frac{1}{n^{7/2}}\right] \implies p=\frac{7}{2}>1 \implies \text{converges by } p\text{-series}$$

$$\implies \text{converges absolutely}$$

3.

$$\lim_{n\to\infty}\left[\frac{(-1)^{2n}}{5^n}\right]=\lim_{n\to\infty}\left[\frac{1}{5^n}\right]=0=0$$

$$\frac{1}{5^{n+1}}<\frac{1}{5^n} \implies \text{converges by alternating series test}$$

$$\sum_{n=1}^{\infty}\left|\frac{(-1)^n(-1)^{2n}}{5^n}\right|=\sum_{n=1}^{\infty}\left[\frac{1}{5^n}\right]=\sum_{n=1}^{\infty}\left(\frac{1}{5}\right)^n \implies r=\frac{1}{5}<1$$

$$\implies \text{converges by geometric series}$$

$$\implies \text{converges absolutely}$$

4.

5.

$$\lim_{n\to\infty}\left[\frac{n}{n^2+4}\right]=0 \text{ by comparative growth rates}$$

$$\frac{n+1}{(n+1)^2+4}<\frac{n}{n^2+4} \implies \text{converges by alternating series test}$$

$$\sum_{n=0}^{\infty}\left|\frac{(-1)^{n=1}n}{n^2+4}\right|=\sum_{n=0}^{\infty}\left[\frac{n}{n^2+4}\right]$$

$$\sum_{n=0}^{\infty}\left[\frac{n}{n^2}\right]=\sum_{n=0}^{\infty}\left[\frac{1}{n}\right] \implies p=1\geq 1 \implies \text{diverges by } p\text{-series}$$

$$\lim_{n\to\infty}\left|\frac{n}{n^2+4}\times\frac{n}{1}\right|=\lim_{n\to\infty}\left[\frac{n^2}{n^2+4}\right]=1 \implies \text{diverges by limit comparison test}$$

$$\implies \text{converges conditionally}$$

6. $$\lim_{n\to\infty}\left[\frac{n!}{e^{2n+1}}\right] = \infty \text{ by comparative growth rates} \neq 0 \implies \text{diverges by } n^{\text{th}} \text{ term test}$$

7.