MATH 137 Assignments

Arnav Patri

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Assignment 1

Claim 1. *If $b > 2$, then a solution to (1) exists.*
Proof. Let $b > 2$ and $x = 1$. Then

$$|x+1| + 2|x-1| = 1 + 2(0) = 1 < 2 < b$$

Therefore for all $b > 2$, then $x = 1$ is a solution to (1).
Claim 2. *If $b \leq 2$, then a solution to (1) does not exist.*
Proof. Let $b \leq 2$ and $x \in \mathbb{R}$.

Case 1: Let $x \leq -1$. Then

$$|x+1| \geq 0 \qquad \text{and} \qquad |x-1| \geq 2$$

so

$$|x+1| + 2|x-1| \geq 0 + 2(2) = 4 > 2 \geq b$$

Case 2: Let $-1 \leq x < 1$. Then

$$|x+1| = x+1 \qquad \text{and} \qquad |x-1| = 1-x$$

so

$$|x+1| + 2|x-1| = x + 1 + 2 - 2x = 3 - x \geq 2 \geq b$$

Case 3: Let $x \geq 1$. Then

$$|x+1| \geq 2 \qquad \text{and} \qquad |x-1| \geq 0$$

so

$$|x+1| + 2|x-1| = 2 + 2(0) = 2 \geq b$$

Therefore (1) is not true for all $b \leq 2$ for all $x \in \mathbb{R}$, meaning that (1) is false if and only if $b \leq 2$.
□

Assignment 2

a) **Claim.** $L \geq 0$.

Proof. Assume that $L < 0$. This means that there is some $N \in \mathbb{N}$ for which $n \geq N$ means that for any $\varepsilon > 0$,

$$|a_n - L| < \varepsilon$$

As $a_n > 0$ and $L < 0$,

$$|a_n - L| = a_n - L$$

Letting $\varepsilon = L$,

$$\begin{aligned} a_n - L &\leq L \\ a_n &\leq 2L < 0 \end{aligned}$$

which is a contradiction, so $L \geq 0$. $\square$.

b) **Claim.**

$$\lim_{n\to\infty} \frac{2}{a_n + 5} = \frac{2}{L+5}$$

Proof. Let $a_n \to L \geq 0$; that is, for any $\varepsilon > 0$, there is some cutoff $N \in \mathbb{N}$ for which $n \geq N$ implies

$$|a_n - L| < \varepsilon$$

Letting $n > N$,

$$\begin{aligned} \left|\frac{2}{a_n+5} - \frac{2}{L+5}\right| &= \left|\frac{2(L+5) - 2(a_n+5)}{(a_n+5)(L+5)}\right| \\ &\leq \frac{2|L - a_0|}{5(L+5)} \\ &< \frac{2\varepsilon}{5L+25} \end{aligned}$$

Defining

$$\varepsilon_1 = \frac{2\varepsilon}{5L+25}$$

it is clear that

$$\lim_{n\to\infty} \frac{2}{a_n+5} = \frac{2}{L+5} \qquad \square$$