15.2.9. (pg. 1059)

Arnav Patri

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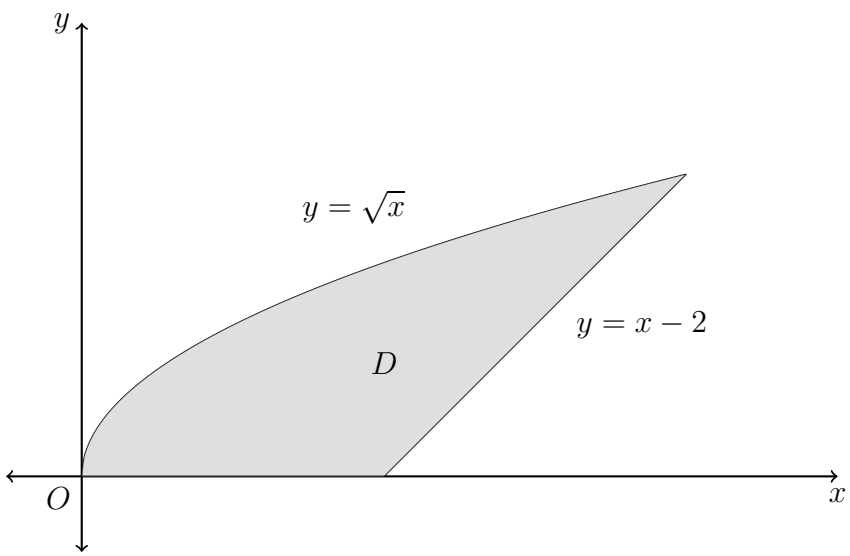
$$f(x, y) = xy$$



(a) Express the double integral $\iint_D f(x, y)\,\mathrm{d}A$ as an iterated integral for the given function f and region D.

$$\begin{aligned} y &= \sqrt{x} \\ x &= y^2 \end{aligned} \qquad\qquad \begin{aligned} y &= x - 2 \\ x &= y + 2 \end{aligned}$$

$$\begin{aligned} y^2 &= y + 2 \\ 0 &= y^2 - y - 2 \\ &= (y - 2)(y + 1) \\ y &= -1, 2 \end{aligned}$$

$$\iint_D f(x, y)\,\mathrm{d}A = \int_0^2 \int_{y^2}^{y+2} xy\,\mathrm{d}x\,\mathrm{d}y$$

(b) Evaluate the iterated integral.

$$\begin{aligned}
\int_0^2 \int_{y^2}^{y+2} xy \, \mathrm{d}x \, \mathrm{d}y &= \int_0^2 \left[\frac{x^2 y}{2} \right]_{y^2}^{y+2} \mathrm{d}y \\
&= \int_0^2 \left[\frac{(y+2)^2 y}{2} - \left(\frac{y^5}{2} \right) \right] \mathrm{d}y \\
&= \int_0^2 \left[\frac{y^3 + 4y^2 + 4y - y^5}{2} \right] \mathrm{d}y \\
&= \left[-\frac{y^6}{12} + \frac{y^4}{8} + \frac{2y^3}{3} + y^2 \right]_0^2 \\
&= -\frac{64}{12} + \frac{16}{8} + \frac{16}{3} + 4 \\
&= 6
\end{aligned}$$