

## Assignment 2

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1. Find the angle between the vectors  $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$  and  $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$ .

**Solution:**

$$\begin{aligned} |\vec{a}||\vec{b}| \cos \theta &= \vec{a} \cdot \vec{b} \\ \sqrt{1^2 + (-2)^2 + (1)^2} \sqrt{2^2 + 1^2 + (-1)^2} \cos \theta &= (1)(2) + (-2)(1) + (1)(-1) \\ \sqrt{6}\sqrt{6} \cos \theta &= -1 \\ \theta &= \arccos\left(-\frac{1}{6}\right) \approx 100^\circ \end{aligned}$$

2. Find the cross product of the vectors  $\vec{a} = \langle 2, -1, 4 \rangle$  and  $\vec{b} = \langle 3, 0, -2 \rangle$ .

**Solution:**

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 4 \\ 3 & 0 & -2 \end{vmatrix} = \langle (-1)(-2) - (4)(0), -1((2)(-2) - (4)(3)), (2)(0) - (-1)(3) \rangle = \langle 2, 16, 3 \rangle$$

3. Find the equation of the plane  $Ax + By + Cz = D$  through the points  $P(1, -1, 2)$ ,  $Q(-1, 0, 1)$ , and  $R(1, -1, 1)$ .

**Solution**

$$\overrightarrow{PQ} = \langle -1 - 1, 0 - (-1), 1 - 2 \rangle = \langle -2, 1, -1 \rangle$$

$$\overrightarrow{PR} = \langle 1 - 1, -1 - (-1), 1 - 2 \rangle = \langle 0, 0, -1 \rangle$$

$$\vec{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 1 & -1 \\ 0 & 0 & -1 \end{vmatrix} = \langle (1)(-1) - (-1)(0), -1((-2)(-1) - (-1)(0)), (-2)(0) - (-1)(0) \rangle = \langle -1, -2, 0 \rangle$$

$$D = n_x x + n_y y + n_z z = -1(1) + (-2)(-1) + 0(2) = -1 + 2 + 0 = 1$$

$$1 = -x - 2y$$