AP Calc BC Project: Derivatives of a^u and e^u

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1st Hour

Problems

1. $\dfrac{\mathrm{d}}{\mathrm{d}x}e^{3x^2}$
2. $\dfrac{\mathrm{d}}{\mathrm{d}x}e^{x^3+2x}$
3. $\dfrac{\mathrm{d}}{\mathrm{d}x}e^{\ln(3x)}$
4. $\dfrac{\mathrm{d}}{\mathrm{d}x}e^{3\sin(2x)}$
5. $\dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{e^{5x^2}}{e^{2x^4-1}}$
6. $\dfrac{\mathrm{d}}{\mathrm{d}x}2^{\ln x}$
7. $\dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{e^x-e^{-x}}{e^x+e^{-x}}$
8. $\dfrac{\mathrm{d}}{\mathrm{d}x}5^{\sin\left(x^3\right)}$
9. $\dfrac{\mathrm{d}}{\mathrm{d}x}2^{e^{4x}}$
10. $\dfrac{\mathrm{d}}{\mathrm{d}x}e^x(x^2+3)(x^3+4)$
11. $\dfrac{\mathrm{d}}{\mathrm{d}x}3^{e^{2x}}$
12. $\dfrac{\mathrm{d}}{\mathrm{d}x}e^{\sin\left(x^2\right)\cos^2(x)}$
13. $\dfrac{\mathrm{d}}{\mathrm{d}x}6^{14xe^{3x^2}}$
14. $e^{\sqrt{\tan x}}$
15. $e^{\ln\left(6\sqrt{x}\right)}+3^{x^2+4}-\sin^2 x$

Solutions

1.

$$\frac{\mathrm{d}}{\mathrm{d}x}e^{3x^2} \implies u = 3x^2 \implies \frac{\mathrm{d}u}{\mathrm{d}x} = 6x$$
$$= e^u \frac{\mathrm{d}u}{\mathrm{d}x} = 6xe^{3x^2}$$

2.

$$\frac{\mathrm{d}}{\mathrm{d}x}e^{x^3+2x} \implies u = x^3 + 2x \implies \frac{\mathrm{d}u}{\mathrm{d}x} = 3x^2 + 2d$$
$$= e^u \frac{\mathrm{d}u}{\mathrm{d}x} = (3x^2 + 2)e^{x^3+2x}$$

3.

$$\frac{\mathrm{d}}{\mathrm{d}x}e^{\ln(3x)} = \frac{\mathrm{d}}{\mathrm{d}x}3x = 3$$

4.

$$\frac{\mathrm{d}}{\mathrm{d}x}e^{3\sin(2x)} \implies u = 3\sin(2x) \implies \frac{\mathrm{d}u}{\mathrm{d}x} = 6\cos(2x)$$
$$= e^u \frac{\mathrm{d}u}{\mathrm{d}x} = 6e^{3\sin(2x)}\cos(2x)$$

5.

$$\frac{\mathrm{d}}{\mathrm{d}x}\frac{e^{5x^2}}{e^{2x^4-1}} = \frac{\mathrm{d}}{\mathrm{d}x}e^{-2x^4+5x^2+1} \implies u = -2x^4 + 5x^2 + 1 \implies \frac{\mathrm{d}u}{\mathrm{d}x} = -8x^3 + 10x$$
$$= e^u \frac{\mathrm{d}u}{\mathrm{d}x} = e^{-2x^4+5x^2+1}(-8x^3 + 10x)$$

6.

$$\frac{\mathrm{d}}{\mathrm{d}x}2^{\ln x} \implies a = 2, u = \ln x \implies \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{x}$$
$$= a^u \ln(a)\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{2^{\ln x}\ln(2)}{x}$$

7.

$$\frac{\mathrm{d}}{\mathrm{d}x}\frac{e^x - e^{-x}}{e^x + e^{-x}} \implies u_1 = x, u_2 = -x \implies \frac{\mathrm{d}u_1}{\mathrm{d}x} = 1, \frac{\mathrm{d}u_2}{\mathrm{d}x} = -1$$
$$= \frac{e^{u_1}\frac{\mathrm{d}u_1}{\mathrm{d}x} - e^{u_2}\frac{\mathrm{d}u_2}{\mathrm{d}x}}{e^{u_1}\frac{\mathrm{d}u_1}{\mathrm{d}x} + e^{u_2}\frac{\mathrm{d}u_2}{\mathrm{d}x}} = \frac{e^x + e^{-x}}{e^x - e^x}$$

8.

$$\frac{\mathrm{d}}{\mathrm{d}x}5^{\sin\left(x^3\right)} \implies a = 5, u = \sin\left(x^3\right) \implies \frac{\mathrm{d}u}{\mathrm{d}x} = 3x^2\sin\left(x^3\right)$$
$$= a^u \ln(a)\frac{\mathrm{d}u}{\mathrm{d}x} = 5^{\sin\left(x^3\right)}\left(3x^2\sin\left(x^3\right)\ln(5)\right)$$

9.

$$\frac{\mathrm{d}}{\mathrm{d}x}2^{e^{4x}} \implies u_1 = e^{4x} \implies \frac{\mathrm{d}u_1}{\mathrm{d}x} \implies u_2 = 4x \implies \frac{\mathrm{d}u_2}{\mathrm{d}x} = 4$$
$$\frac{\mathrm{d}u_1}{\mathrm{d}x} = e^{u_2}\frac{\mathrm{d}u_2}{\mathrm{d}x} = 4e^{4x}$$
$$\frac{\mathrm{d}}{\mathrm{d}x}2^{e^{4x}} = a^{u_1}\ln(a)\frac{\mathrm{d}u_1}{\mathrm{d}x} = 2^{e^{4x}}\left(4e^{4x}\ln(2)\right) = 2^{e^{4x}+2}e^{4x}\ln(2)$$

10.

$$\frac{\mathrm{d}}{\mathrm{d}x}e^x(x^2+3)(x^3+4) \implies u =$$