Transform from a function of t to a function of s

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1)

$$\begin{aligned}\mathscr{L}\{\mathrm{e}^{3t}\} &= \int_0^\infty \mathrm{e}^{-st}\mathrm{e}^{3t}\,\mathrm{d}t = \int_0^\infty \mathrm{e}^{-t(s-3)}\,\mathrm{d}t = \left[-\frac{\mathrm{e}^{-t(s-3)}}{(s-3)}\right]_0^\infty \\ &= 0 + \frac{1}{s-3} = \frac{1}{s-3}\end{aligned}$$

Using theorem 7.1.1,

$$\mathscr{L}\{e^{3t}\} = \frac{1}{s-3}$$

2)

$$\mathscr{L}\{f(t)\} = \int_0^\infty \mathrm{e}^{-st} f(t)\,\mathrm{d}t = F(s)$$

3) Using theorem 7.1.1,

$$\mathscr{L}\{\sin(3t)\} = \frac{3}{s^2+3^2} = \frac{3}{s^2+9}$$

4) Using theorem 7.1.1,

$$\mathscr{L}\{\cos^2(2t)\} = \mathscr{L}\left\{\frac{1+\cos(4x)}{2}\right\} = \frac{1}{2}\frac{1}{s} + \frac{1}{2}\frac{s}{s^2+4^2} = \frac{1}{2s} + \frac{s}{2s^2+32} = \frac{s^2+8}{s^3+16s}$$