Assignment 1

Claim 1. textitIf $b > 2$, then a solution to (1) exists.
Proof. Let $b > 2$ and $x = 1$. Then

$$|x+1| + 2|x-1| = 1 + 2(0) = 1 < 2 < b$$

Therefore for all $b > 2$, then $x = 1$ is a solution to (1).
Claim 2. *If $b \leq 2$, then a solution to (1) does not exist.*
Proof. Let $b \leq 2$ and $x \in \mathbb{R}$.

Case 1: Let $x \leq -1$. Then

$$|x+1| \geq 0 \qquad \text{and} \qquad |x-1| \geq 2$$

so

$$|x+1| + 2|x-1| \geq 0 + 2(2) = 4 > 2 \geq b$$

Case 2: Let $-1 \leq x < 1$. Then

$$|x+1| = x+1 \qquad \text{and} \qquad |x-1| = 1-x$$

so

$$|x+1| + 2|x-1| = x + 1 + 2 - 2x = 3 - x \geq 2 \geq b$$

Case 3: Let $x \geq 1$. Then

$$|x+1| \geq 2 \qquad \text{and} \qquad |x-1| \geq 0$$

so

$$|x+1| + 2|x-1| = 2 + 2(0) = 2 \geq b$$

Therefore (1) is not true for all $b \leq 2$ for all $x \in \mathbb{R}$, meaning that (1) is false if and only if $b \leq 2$. $\square$