

Assignment 5

Arnav Patri

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1. Find the Directional derivative of the function $f(x, y) = x^3y - 2x^2y^2$ in the direction of the vector $\vec{v} = \hat{i} + 2\hat{j}$ at the point $P(1, 1)$.

Solution

$$\begin{aligned}\nabla f(x, y) &= \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle = \langle 3x^2y - 4xy^2, x^3 - 4x^2y \rangle \\ D_{\vec{v}}f(x, y) &= \nabla f(x, y) \cdot \vec{v} = (3x^2y - 4xy^2)(1) + (x^3 - 4x^2y)(2) \\ &= 3x^2y - 4xy^2 + 2x^3 - 8x^2y = 2x^3 - 5x^2y - 4xy^2\end{aligned}$$

2. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for $x^2 - y^2 + z^2 - 2z = 4$

Solution

Let $F(x, y, z) = x^2 - y^2 + z^2 - 2z$. If $z \neq 1$,

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{2x}{2z - 2} = -\frac{x}{z - 1} \qquad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{-2y}{2z - 2} = \frac{y}{z - 1}$$

3. Find the local maximum and local minimum values and saddle points of $f(x, y) = xy - 2x - 2y - x^2 - y^2$.

Solution

$$\begin{aligned}f_x(x, y) &= y - 2 - 2x = 0 & f_y(x, y) &= x - 2 - 2y = 0 \\ y &= 2x + 2 & y &= \frac{x}{2} - 1 \\ 2x + 2 &= \frac{x}{2} - 1 \\ 4x + 4 &= x - 2 \\ 3x &= -6 \\ x &= -2 & y &= -2 \\ f(-2, 2) &= 4 \\ f_{xx}(x, y) &= -2 & f_{yy}(x, y) &= -2 \\ f_{xx}(-2, -2) &= -2 & f_{yy}(-2, -2) &= -2 \\ f_{xy}(x, y) &= 1 \\ f_{xy}(-2, -2) &= 1 \\ D(-2, -2) &= (-2)(-2) - 1^2 \\ &= 3\end{aligned}$$

As $D(-2, -2) > 0$ and $f_{xx}(-2, -2) < 0$, the point $(-2, -2, 4)$ is a relative maximum of f . As f lacks any other critical values, it lacks relative minima and saddle points.