Arnav Patri

Chapter 4

Divisibility Rules

3. $\sum d_i \mid 3 \implies n \mid 3$

5. $d_{\text{final}} = 0, 5 \implies n \mid 5$

7. $0.2(n - d_{\text{final}}) \mid 7 \implies n \mid 7$

11. (odds from right) $-$ (evens from right) $\mid 11 \implies n \mid 11$

13. $4d_{\text{final}} + \text{remaining} \mid 13 \implies n \mid 13$

Chapter 6

Summation Formulas

k	$\sum_{i=1}^{n} i^k$
0	n
1	$\frac{n(n+1)}{2}$
2	$\frac{n(n+1)(2n+1)}{6}$
k	$\left(\sum_{i=1}^{n} i^{k-2}\right)^2$

Counting Rules

- **Product Rule:** If a procedure can be decomposed into a sequence of two tasks, one with n_1 possible ways of being completed and another with n_2 ways, there are $n_1 n_2$ total ways to carry out the procedure.

- **Sum Rule:** If a task can be completed either in one of n_1 ways or in one of $n_1 2$ ways, where there is no overlap between the sets of n_1 and n_2 ways, then there are $n_1 + n_2$ ways to complete the task.

- **Subtraction Rule:** If a task can be completed in either n_1 or n_2 ways, then the number of ways to do the task is $n_1 + n_2$ minus the number of ways that are shared between both.

$$|A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2|$$

- **Division Rule:** If a task can be done using a procedure that can be carried out n ways and exactly d of n ways correspond to every way, there are n/d ways to complete the task.

Permutations and Combinations

$n, r \in \mathbb{Z}^+, r \leq n$

$$P(n,r) = \frac{n!}{(n-r)!} \quad C(n,r) = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Type	Repetition?	Formula
r-permutations	N	$\frac{n!}{(n-r)!}$
r-combinations	N	$\frac{n!}{r!(n-r)!}$
r-permutations	Y	n^r
r-combinations	Y	$\frac{(n+r-1)!}{r!(n-1)!}$

Permutations of Indistinguishable Objects (n total, n_i of category i, k categories)

$$\frac{n!}{n_1! n_2! \cdots n_k!} = \frac{n!}{\prod_{i=1}^{k} n_i!}$$

Boxes

- **Distinguishable Objects, Distinguishable Boxes** (n total, n_i in box i, k boxes)

$$\frac{n!}{n_1! n_2! \cdots n_k!} = \frac{n!}{\prod_{i=1}^{k} n_i!}$$

- **Indistinguishable Objects, Distinguishable Boxes** (r indistinguishable objects, n distinguishable boxes)

$$C(n+r-1,r) = \frac{(n+r-1)!}{r!(n-1)!}$$

- **Distinguishable Objects, Indistinguishable Boxes**

$$\sum_{j=1}^{k} \frac{1}{j!} \sum_{i=0}^{j-1} (-1)^i \binom{j}{i} (j-i)^n$$

Binomials

Binomial Theorem $n \in \mathbb{N}$

$$(x+y)^n = \sum_{i=0}^{n} \binom{n}{i} x^{n-i} y^i$$

Pascal's Identity $n, k \in \mathbb{Z}^+, k \leq n$

$$\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}$$

Vandermonde's Identity $m, n, r \in \mathbb{N}, r \leq m, n$

$$\binom{m+n}{r} = \sum_{k=0}^{r} \binom{m}{r-k} \binom{n}{k}$$

Chapter 5

Induction

Principle of Mathematical Induction ($\mathbb{Z}^+$)

$$(P(1) \wedge \forall k (P(k) \Rightarrow P(k+1))) \Rightarrow \forall n P(n)$$

Proofs

1. Express the statement to be proven as "for all $n \geq b$, $P(n)$" for fixed integer b.
2. Show $P(b)$ is true (basis).
3. Identify inductive hypothesis as "Assume that $P(k)$ is true for an arbitrary fixed integer $k \geq b$".
4. State what must be proven under the assumption to prove the hypothesis' validity.
5. Prove that $P(k+1)$ is true under the assumption (inductive).
6. Identify the conclusion of the inductive step.
7. State the conclusion that "by mathematical induction, $P(n)$ is true for all integers n with $n > b$".