Discussion 12: Transform y, y', y'', y''', y''''

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1. By definition,

$$\mathscr{L}\{y\} = \int_0^\infty \mathrm{e}^{-st} y \, \mathrm{d}t = Y(s)$$

2.

$$\begin{aligned}
\mathscr{L}\{y'\} &= \int_0^\infty \mathrm{e}^{-st} y' \, \mathrm{d}t && \text{(definition of } \mathscr{L}\text{)} \\
&= \left[\mathrm{e}^{-st} y\right]_0^\infty + \int_0^\infty s\mathrm{e}^{-st} y \, \mathrm{d}t && \text{(integration by parts)} \\
&= -y_0 + s\mathscr{L}\{y\} && \text{(definition of } \mathscr{L}\text{)} \\
&= sY(s) - y_0
\end{aligned}$$

3.

$$\begin{aligned}
\mathscr{L}\{y''\} &= \int_0^\infty \mathrm{e}^{-st} y'' \, \mathrm{d}t && \text{(definition of } \mathscr{L}\text{)} \\
&= \left[\mathrm{e}^{-st} y'\right]_0^\infty + \int_0^\infty s\mathrm{e}^{-st} y' \, \mathrm{d}t && \text{(integration by parts)} \\
&= -y_0' + s\mathscr{L}\{y'\} && \text{(definition of } \mathscr{L}\text{)} \\
&= -y_0' + s^2 Y(s) - sy_0 && \text{(substituting } \mathscr{L}\{y'\}\text{)} \\
&= s^2 Y(s) - sy_0 - y_0'
\end{aligned}$$

4.

$$\begin{aligned}
\mathscr{L}\{y'''\} &= \int_0^\infty \mathrm{e}^{-st} y'' \, \mathrm{d}t && \text{(definition of } \mathscr{L}\text{)} \\
&= \left[\mathrm{e}^{-st} y''\right]_0^\infty + \int_0^\infty s\mathrm{e}^{-st} y'' \, \mathrm{d}t && \text{(integration by parts)} \\
&= -y_0'' + s\mathscr{L}\{y''\} && \text{(definition of } \mathscr{L}\text{)} \\
&= -y_0'' + s^3 Y(s) - s^2 y_0 - sy_0' && \text{(substituting } \mathscr{L}\{y''\}\text{)} \\
&= s^3 Y(s) - s^2 y_0 - sy_0' - y_0''
\end{aligned}$$

5.

$$
\begin{aligned}
\mathscr{L}\{y''''\} &= \int_0^\infty \mathrm{e}^{-st} y''' \,\mathrm{d}t && \text{(definition of } \mathscr{L}) \\
&= \left[\mathrm{e}^{-st} y'''\right]_0^\infty + \int_0^\infty s\mathrm{e}^{-st} y''' \,\mathrm{d}t && \text{(integration by parts)} \\
&= -y_0''' + s\mathscr{L}\{y'''\} && \text{(definition of } \mathscr{L}) \\
&= -y_0''' + s^4 Y(s) - s^3 y_0 - s^2 y_0' - sy''0 && \text{(substituting } \mathscr{L}\{y'''\}) \\
&= s^4 Y(s) - s^3 y_0 - s^2 y_0' - sy_0'' - y_0'''
\end{aligned}
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