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10 Graphs

A graph $G = (E, V)$ is comprised of edge set E and vertex set V.

Graph Terminology			
Type	Directed?	Multiple Edges?	Loops?
Simple	N	N	N
Multi-	N	Y	N
Psuedo-	N	Y	Y
Simple Directed	Y	N	N
Directed Multi-	Y	Y	Y
Mixed	Y/N	Y	Y

Two vertices are *adjecent/neighbors* if there is an edge connected them. Such an edge is *incident* with both vertices.
The set of all neighbors of a vertex v, denoted $N(v)$, is the *neighborhood* of v. The neighborhood of $A \subset V$, denoted $N(A)$, is the set of all vertices in G that are adjectent to at least one vertex in A.
A vertex v's *degree*, denoted $\deg v$, in an undirected graph is the number of edges incident with it, with loops being counted twice.
The sum of the degrees of every vertex of an undirected graph is $2|E|$.

The *initial vertex* of a *directed edge* or *arc* (u, v) in a digraph is u while the *terminal/end vertex* is u. (u, v) is *adjacent from* u and *adjacent to* v.
A vertex v's *in-degree*, denoted $\deg^- v$, is the number of edges that terminate at v, while its *out-degree*, denoted $\deg^+ v$, is the number of edges that start at v.

A *complete graph on n vertices*, denoted K_n, is the simple graph that contains exactly one edge between each pair of distinct vertices. Its outline can be drawn as a regular polygon with n vertices. Each pair of nodes can then be connected. It has $\binom{n}{2}$ edges.
A *cycle* C_n for $n \geq 3$ consists of n vertices and edges connecting each vertex to exactly two other nodes. It can be drawn as a regular polygon with n vertices.
A *wheel* W_n is obtained by adding an additional vertex to C_n that all other vertices connect to. This can be drawn as a regular polygon with n vertices with an additional node in the center that connects to all other vertices.
An *n-dimensional hypercube* or *n-cube* Q_n is a graph with 2^n vertices representing all bit strings of length n with edges connecting vertices differing in exactly one bit position. Q_1 is a line, Q_2 a square, Q_3 a cube, and so on.
The sum of every vertex's in-degrees is equal to that of their out-degrees, both of which $|E|$.

A simple graph is *bipartite* if V can be partitioned into two disjoint subsets V_1 and V_2 such that every edge connects a vertex in V_1 to one in V_2; that is to say, no two edges in the same subset are connected.
A *complete bipartite graph* $K_{m,n}$ is a bipartite graph with $|V_1| = m$ and $|V_2| = n$ such that there is an edge from every vertex in V_1 to every vertex in V_2.

The union of 2 simple graphs $G_1 = (V_1, E_1$ and $G_2 = (V_2, E_2)$ is the simple graph $G_1 \cup G_2 = (V_1 \cup V_2, E_1 \cup E_2)$.

A graph's *adjacency matrix* is the $|V| \times |V|$ matrix $\mathbf{A}_G = [a_{i,j}]$ where $a_{i,j}$ is equal to the number of edges connecting v_i and v_j. The ordering may be arbitrary.
For a digraph's adjacency matrix, $a_{i,j}$ is equal to the number of arcs connecting starting at v_i and ending at v_j.
A graph's *incidence matrix* is the $|V| \times |E|$ matrix $\mathbf{M}_G = [m_{i,j}]$ where $m_{i,j}$ is 1 if e_j is incident to m_i and 0 otherwise.

Two simple graphs are *isomorphic* if there is a one-to-one and onto function f between the vertex sets with the property that a and b are adjacent in the first graph if and only if $f(a)$ and $f(b)$ are in the other. Such a function is called an *isomorphism*. Two simple graphs that are not isomorphic are *nonisomorphic*.

A *path* is a sequence of connected edges. It is denoted by the sequence of edges. It *passes through* nodes while *traversing* edges.

A path is a *circuit* if it begins and ends at the same node.

A path is *simple* if it does not contain the same edge more than once.

An undirected graph is *connected* if there is a path between every pair of vertices. One that is not connected is *disconnected*. To *disconnect* a graph is to remove vertices and/or edges to produce a disconnected subgraph.

A *connected component* of a graph G is a connected subgraph of it that is not a proper subgraph of another connected subgraph of G.

A digraph is *strongly connected* if there is a path from u to v and from v to u for any pair of vertices in the graph. It is *weakly connected* if there is a path between every pair of nodes in the underlying undirected graph.

An *Euler circuit* is a simple circuit containing every edge. An *Euler path* is a simple path containing every edge.

Every vertex of graph with an Euler circuit must be of even degree. All but 2 nodes of a graph with an Euler path must be of even degree. These conditions are necessary and sufficient.