AP Physics C: Electricity and Magnetism

Arnav Patri

November 10, 2022

Chapter 1

Electrostatics and Gauss' Law

1.1 Electrostatics

1.1.1 Electric Charge

An electron is negatively charged.
Two particles of the same polarity repel each other while those of opposite polarity are attracted.
Conductors are materials in which electrons are able to move relatively freely. **Nonconductors/Insulators** are the opposite, limiting electron movement.
Semiconductors are materials that are between conductors and insulators in terms of conductivity.
Superconductors are perfect conductors.
Atoms are comprised of positively charged protons, negatively charged electrons, and neutral (though very slightly negatively charged) neutrons. In conductors, the outermost electrons are able to move relatively freely. These mobile electrons are called **conduction electrons**. **Induction** describes the phenomenon of neutral conductors being attracted to charged ones.
Coulomb's law describes the electrostatic force between two particles of charges q_1 and q_2 as

$$F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{r^2} \qquad \text{(Coulomb's Law)}$$

where $\varepsilon_0 \approx 8.85 \times 10^{-12}\,\mathrm{C^2/N^2\,m^2}$, the **vacuum permittivity constant**. This is often rewritten as

$$F = k\frac{|q_1||q_2|}{r^2} \qquad \text{(Coulomb's Law)}$$

where $k = \frac{1}{4\pi\varepsilon_0} \approx 8.99 \times 10^9\,\mathrm{N\,m^2/C^2}$ is the **electrostatic constant** or the **Coulomb constant**.
The electrostatic force is pointed either directly towards or away from the other particle. If multiple are acting on the same particle, the net force is the *vector* sum.
Particles that interact through the electrostatic force form a *third-law pair*.
One **shell theories** hypothesizes that a shell with uniform charge density acts like a single particle at its center from the perspective of a particle outside the shell while another claims that it cancels out, providing no net force to a particle within the shell.
Electric charge is **quantized**, meaning that it can only take on certain (*discrete*) values.
A particle's charge q can be written as ne, where n is a nonzero integer and $e \approx 1.609 \times 10^{-19}\,\mathrm{C}$ is the elementary charge

$$q = ne$$

The proton, neutron, and electron, denoted p, n, and e (or e-) respectively, have the corresponding charges e, 0, and $-e$.
The net charge of an isolated system is always conserved.

1.1.2 Electric Fields

The **electric field $\vec{E}$** is the vector field of the electric charge on every point in a region surrounding a charged object. It is measured in N/C To measure it, a **positive** charge q_0, called a *test charge* is placed at a point. The electrostatic force F is then measured on the test charge. The electric field at this point is defined to be

$$\vec{E} = \frac{\vec{F}}{q_0}$$

The magnitude of electric field due to a point charge q at any point of distance r from said point charge is

$$E = \frac{F}{q_0} = k\frac{|q|}{r^2}$$

The direction vector $\vec{d}$ of a dipole typically goes from the negative end to the positive.
The **dipole moment $\vec{p}$** is defined as

$$\vec{p} = q\vec{d}$$

The dipole moment always attempts to align with the direction of the field, making it simple to see the direction of rotation of the dipole. The torque $\vec{\tau}$ on a dipole in an electric field is

$$\vec{\tau} = \vec{p} \times \vec{E}$$

Linear charge density is denoted as λ as is found as

$$\mathrm{d}q = \lambda\,\mathrm{d}s$$

for a curved rod of charge Q and length s. It is generally useful to use θ, where

$$\theta = \frac{s}{r} \implies \mathrm{d}s = r\,\mathrm{d}\theta$$

A vector into the paper is denoted on one end by $\otimes$ while one pointing out is denoted by $\odot$.
Work W is the integral of force with respect to displacement, making it

$$W = \int_C \vec{F} \cdot \mathrm{d}\vec{r}$$

The work done by a conservative force is denoted by W_c, a change in potential energy by ΔU, and the gravitational force by F_g. It should be noted that gravity is a conservative force and that

$$W_c = -\Delta U$$

As such,

$$\Delta U_g = mg(\Delta h)$$

1.2 Gauss' Law

The **area vector $\mathbf{d}\vec{A}$** for an area element on a surface is a vector with magnitude equal to area $\mathrm{d}A$ of the element that is perpendicular to the surface pointing outwards.
The **electric flux $\mathbf{d}\Phi_E$** is given by

$$\mathrm{d}\Phi_E = \vec{E} \cdot \mathrm{d}\vec{A} \tag{electric flux}$$

with units $\mathrm{N/C\,m^2}$. The **total flux** through a surface is found by the surface integral

$$\Phi_E = \iint_S \vec{E} \cdot \mathrm{d}\vec{A} \tag{total flux}$$

Through a **closed surface** (as used in Gauss' law),

$$\Phi_E = \oiint_S \vec{E} \cdot \mathrm{d}\vec{A} = \oiint_S E\,\mathrm{d}A\cos\varphi \tag{net flux}$$

where φ is the angle between the electric field and the surface.
For a uniform electric field,

$$\Phi_E = \oiint_S E\,\mathrm{d}A\cos\varphi = E\cos\varphi \oiint_S \mathrm{d}A = EA\cos\varphi$$

The relationship between the surface and the field can be described by flux as

Φ_E	< 0	0	> 0
φ	$< 90°$	$90°$	$> 90°$

Gauss' law relates the net flux Φ_E of an electric field through a closed (Gaussian) surface to the net charge q_{enc} enclosed by the surface as

$$\varepsilon_0 \Phi_E = q_{\text{enc}} \tag{Gauss' law}$$

If excess charge is placed on a conductor, the charge will move to the surface.
Everywhere inside a conductor, $E_{\text{net}} = 0$.
A **(uniform) surface charge density $\boldsymbol{\sigma}$** is equal to

$$\sigma = \frac{q}{A} \tag{uniform surface charge density}$$

The magnitude of the electric field outside of a conductor with uniform surface charge density σ is

$$E = \frac{\sigma}{\varepsilon_0} \tag{conducting surface}$$

That outside of an insulator is

$$E = \frac{\sigma}{2\varepsilon_0} \tag{insulator}$$

The magnitude of the electric field produced by a uniform spherical shell of radius R is

$$E = \begin{cases} k\dfrac{q}{r^2} & r \geq R \\ 0 & r < R \end{cases}$$

A **(uniform) volume charge density $\boldsymbol{\rho}$** is equal to

$$\rho = \frac{q}{V} \tag{uniform volume charge density}$$

Within a sphere of radius R with uniform volume charge density, the magnitude of the field is radial:

$$E = \left(k\frac{Q}{R^3}\right) r \tag{uniform charge, field at $r \leq R$}$$

Chapter 2

Conductors, Capacitors, and Dielectrics

2.1 Electric Potential

Electrostatic forces are **conservative**, so $W_C = -\Delta U$. It can then be seen that

$$\Delta U = -\int_C \vec{F} \cdot \mathrm{d}\vec{s} = -q \int_C \vec{E} \cdot \mathrm{d}\vec{s}$$

The change in **electric potential $\boldsymbol{V}$** (measured in volts (V)) is found as

$$\Delta V = \frac{\Delta U}{q} = -\frac{q \int_C \vec{E} \cdot \mathrm{d}\vec{s}}{q} = -\int_C \vec{E} \cdot \mathrm{d}\vec{s}$$

If the initial potential is set to 0, then

$$V = -\int_C \vec{E} \cdot \mathrm{d}\vec{s}$$

Adjacent points with the same electric potential form an **equipotential surface**, which can be imaginary or real.
The electric potential from a point charge can be found as

$$V_f - V_i = -\int_C \vec{E} \cdot \mathrm{d}s = -\int_r^\infty E \, \mathrm{d}r = k\frac{q}{r}$$

Setting V_f to 0 (at ∞),

$$V_i = V = k\frac{q}{r}$$

The potential due to a collection of n charged particles is simply the sum of the individual potentials:

$$V = \sum_{i=1}^{n} V_i = \sum_{i=1}^{n} k\frac{q_i}{r_i} \qquad (n \text{ charged particles})$$

Note that direction is not considered.
As a convention, positively charged particles produce positive potentials while negative ones produce negative potentials.
The potential energy of a system of particles is the sum of the potential energies of every pair of

particles in the system. It is equal to the work required to assemble the system with particles that are initially at rest and infinitely far apart. For two particles of distance r,

$$U = k\frac{q_1 q_2}{r} \qquad \text{(2-particle system)}$$

The x component of an electric field can be found from potential as

$$E_x = -\frac{\mathrm{d}V_x}{\mathrm{d}x}$$

For a continuous charge distribution over an extended object, the net potential can be found as

$$V = \int_C \mathrm{d}V = k\int_C \frac{\mathrm{d}q}{r}$$

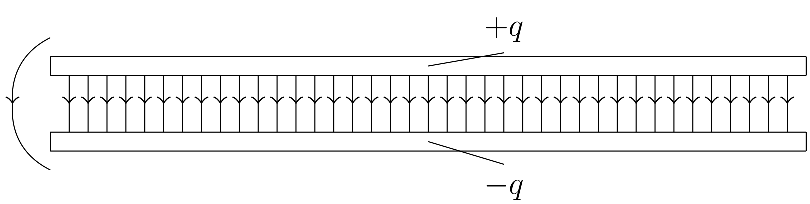
A substitution can then be made using the appropriate charge density.

2.2 Capacitance

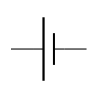
A capacitor is comprised of 2 isolated conductors with charges $+q$ and $-a$. Its **capacitance *C*** is defined as

$$C = \frac{|q|}{V}$$

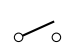
where V is the potential difference between the plates. It is measured in C/V or **Farads (F)**. By its definition, capacitance is always positive.
A capacitor's capacitance is a constant inherent to its physical attributes.
A parallel-plate capacitor is comprised of 2 parallel plates of area A separated by a distance d. The charges on the faces of the plates facing each other are of magnitude q and opposite signs.
The electric field due to a parallel-plate capacitor is uniform only between the plates.



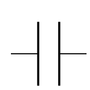
A battery is denoted by



where the larger side is positive and the shorter negative.
An open switch is denoted by



A capacitor is denoted by



When a circuit with a battery, an open switch, and an uncharged capacitor is completed by closing the switch, conduction electrons shift, resulting in the capacitor plates being of opposite charges.

Gauss' law can be used to relate the electric field between a capacitor's plates to the charge q on either plate as

$$V = -\int_C \vec{E} \cdot \mathrm{d}\vec{s} = -\int_{-}^{+} E \,\mathrm{d}s$$

It is assumed that the plates of the capacitor are large and close enough for fringing to be negligible, making $\vec{E}$ constant between the plates. Using a Gaussian surface that encloses just the charge q on the positive plate,

$$\oiint \vec{E} \cdot \mathrm{d}A = EA = \frac{q_{\mathrm{enc}}}{\varepsilon_0} = \frac{q}{\varepsilon_0}$$

so

$$E = \frac{q}{A\varepsilon_0} \qquad \text{and} \qquad q = EA\varepsilon_0$$

Where A is the plate's area. Therefore

$$V = \int_{-}^{+} E \,\mathrm{d}s = E \int_0^d \mathrm{d}s = Ed$$

Substituting CV for q yields

$$C = \frac{q}{V} = \frac{EA\varepsilon_0}{Ed} = \frac{A\varepsilon_0}{d} \qquad \text{(parallel-plate capacitor)}$$

Consider a cylindrical capacitor of length L formed by 2 coaxial cylinders of radii a and b. Assume that $L \gg b$ so that fringing may be neglected. Each plate has charge q, so

$$q = EA\varepsilon_0 = E\varepsilon_0(2\pi rL)$$

Using Gauss' law,

$$\oiint \vec{E} \cdot \mathrm{d}A = E(2\pi rL) = \frac{q_{\mathrm{enc}}}{\varepsilon_0} = \frac{q}{\varepsilon_0}$$

so

$$E = \frac{1}{2\pi\varepsilon_0}\frac{q}{rL}$$

and

$$V = \frac{q}{2L\pi\varepsilon_0}\int_a^b \frac{\mathrm{d}r}{r} = \frac{q}{2L\pi\varepsilon_0}\ln\left(\frac{b}{a}\right)$$

The capacitance is then

$$C = \frac{q}{V} = 2\pi\varepsilon_0 \frac{L}{\ln(b/a)}$$

Capacitors connected in parallel can be replaced by a single capacitor with the same total charge q_{eq} and potential difference V as the original capacitors.
When a potential difference V is applied across several parallel capacitors, that potential difference V is applied to each capacitor. The total charge q_{eq} is the the sum of the charges of each individual capacitor.

$$q_{\mathrm{eq}} = \sum q_i$$

The equivalent capacitance C_{eq} is then simply

$$C_{\mathrm{eq}} = \sum C_i \qquad \text{(capacitors in parallel)}$$

Capacitors connected in series can be replaced by a single capacitor with the same total charge and potential difference.
When a potential difference V is applied across several series capacitors, the capacitors all have the same charge q. The sum of the potential differences across all capacitors is equal to the applied potential difference V.

$$V_{\text{eq}} = \sum q \left(\frac{1}{C_i}\right)$$

The reciprocal of the equivalent capacitance C_{eq} is then

$$\frac{1}{C_{\text{eq}}} = \sum \frac{1}{C_i} \qquad \text{(capacitors in series)}$$

The **electric potential energy $\boldsymbol{U_c}$** of a charged capacitor is

$$U_c = \frac{q^2}{2C} = \frac{1}{2}CV^2 \qquad \text{(potential energy)}$$

This is equal to the work required to charge the capacitor. This energy can be viewed as being stored in the electric field between the plates.
Every electric field has an associated stored energy. In a vacuum, the **energy density $\boldsymbol{u}$** in a field of magnitude E is

$$u = \frac{1}{2}\varepsilon_0 E^2 \qquad \text{(energy density)}$$

A **dielectric** is an insulating material placed between the plates of a capacitor. This increases the structural integrity of the capacitor while increasing its capacitance.
The **dielectric constant $\boldsymbol{\kappa}$** is a unitless constant that is the ratio of the final capacitance to the initial capacitance.

$$C = \kappa C_0$$