Prep 1.

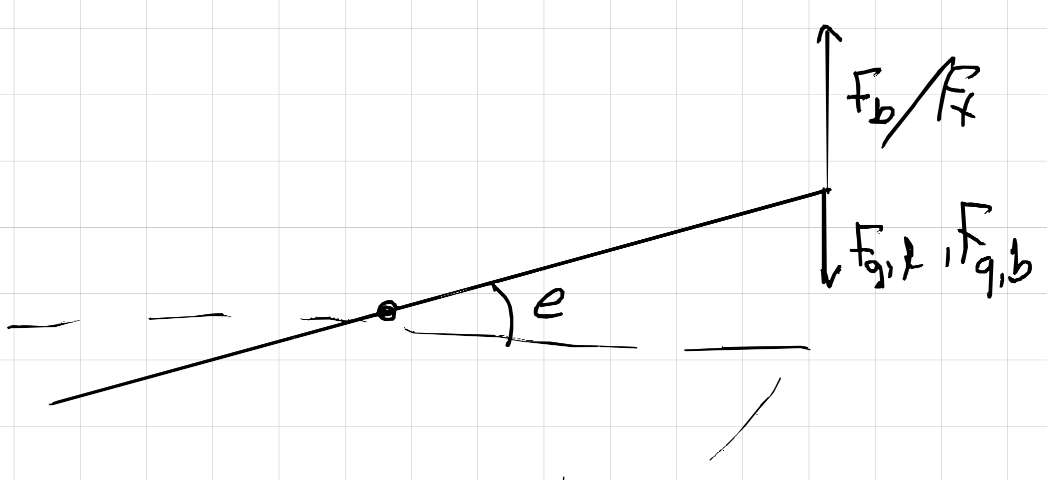
P1

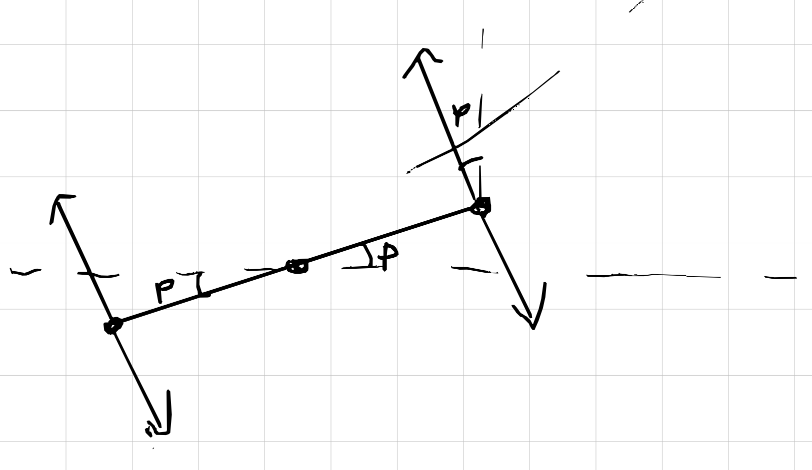
$$J_p \ddot{p} = F_b l_p - F_f l_r$$

$$J_p \ddot{p} = l_p (K_f V_b - K_f V_f)$$

$$J_p \ddot{p} = l_p \, K_f V_d \qquad (L_1 = l_p K_f)$$

$$\Rightarrow J_p \ddot{p} = L_1 V_d$$





$$J_e \ddot{e} = l_h \cdot F_s \cdot \cos p + m_c l_c g \cos e - 2 m_p l_h \cos e$$

$$J_e \ddot{e} = \underbrace{l_h K_f V_s}_{L_3} \cos p + \underbrace{(m_c l_c g - 2 m_p l_h)}_{L_2} \cos e$$

$$\Rightarrow J_e \ddot{e} = L_2 \cos e + L_3 \cos p$$

$$J_\lambda \ddot{\lambda} = -F_s l_h \sin p \cdot \cos e$$

$$J_\lambda \ddot{\lambda} = L_4 \cos e \sin$$

P2

$$\bar{V_s} = V_s - V_{s,0}$$

Small angle approx $\Rightarrow \cos\theta \approx 1$ & $\sin\theta \approx \theta$

$$J_p \ddot{p} = L_1 V_d \Rightarrow \underline{\underline{\ddot{p} = \frac{L_1}{J_p} V_d = K_1 V_d}}$$

$$J_e \ddot{\bar{e}} = L_2 \cos(\bar{e}) + L_3 (\bar{V_s} + V_{s,0}) \cos(\bar{p})$$

$V_{s,0} = V_s$ at eq

$$J_e \ddot{e} = 0 = L_2 + L_3 V_{s,0} \Rightarrow V_s = -\frac{L_2}{L_3}$$

$$J_e \ddot{\bar{e}} = L_2 \cos(\bar{e}) + L_3 \left(\bar{V_s} - \frac{L_2}{L_3}\right) \cos(\bar{p})$$

$$\Rightarrow J_e \ddot{\bar{e}} = L_3 \bar{V_s} \Rightarrow \underline{\underline{\ddot{\bar{e}} = \frac{L_3}{J_e} \bar{V_s} = K_2 \bar{V_s}}}$$

$$J_\lambda \ddot{\bar{\lambda}} = L_4 (\bar{V_s} + V_{s,0}) \cos(\bar{e}) \sin(p)$$

$$\ddot{\bar{\lambda}} = \frac{L_4}{J_\lambda} (\bar{V_s} + V_{s,0}) \cdot 1 \cdot p$$

$$\ddot{\bar{\lambda}} = K_3 p$$

P3

$$\ddot{p} = K_1 K_{pp} (p_c - p) - K_1 K_{pd} \dot{p}$$

$$s^2 p = K_1 K_{pp} p_c - K_1 K_{pp} p - K_1 K_{pd} s p$$

$$p (s^2 + s K_1 K_{pd} + K_1 K_{pp}) = K_1 K_{pp}$$

$$\Rightarrow G(s) = \frac{p(s)}{p_c(s)} = \frac{K_1 K_{pp}}{s^2 + s K_1 K_{pd} + K_1 K_{pp}}$$

$$s^2 + s K_1 K_{pd} + K_1 K_{pp} = 0$$

$$= s^2 + 2\zeta \omega_0 s + \omega_0^2$$

$$\omega_0 = \sqrt{K_1 K_{pd}}$$

$$2\zeta\omega_0 = K_1 K_{pp} \Rightarrow \zeta = \frac{K_1 K_{pp}}{2\omega_0} = \frac{K_1 K_{pp}}{2\sqrt{K_1 K_{pd}}}$$

$$= \frac{\sqrt{K_1} K_{pp}}{2\sqrt{K_{pd}}}$$

$\zeta > 1$, damped

$\boxed{\zeta = 1, \text{ critically damped}}$ if possible

$\zeta < 1$, underdamped

ω_0 not too close to possible values

K_{pp} and K_{pd} below saturation

$$(s - \lambda_1)(s - \lambda_2) = s^2 - (\lambda_1 + \lambda_2) + \lambda_1 \lambda_2$$

$$\Rightarrow K_{pp} = \frac{\lambda_1 \lambda_2}{K_1} \qquad K_{pd} = \frac{-\lambda_1 + \lambda_2}{K_1}$$

$\zeta = 1$

Two complex roots

$0 < \zeta < 1$:

Første lab

$$K_1 = \frac{l_p K_f}{J_p} \qquad l_p = 0{,}175$$

$$V_{s,0} = 8 \qquad K_1 = \frac{l_p K_f}{2 m_p l_p^2} = 31{,}746$$

$$V_s = 8$$

Kritisk

$K_{pd} = 3{,}15$ $\quad K_{pp} = 0{,}63$

$K_{pd} = 10$ $\quad K_{pp} = 1{,}123$ Metning

$K_{pd} = 5$ $\quad K_{pp} = 0{,}794$

$$\xi = \frac{\sqrt{K_1} K_{pp}}{2\sqrt{K_{pd}}} \qquad \omega_0 = \sqrt{K_1 K_{pd}}$$

Overdempet

$K_{pd} = 5$ $\quad K_{pp} = 3{,}969$

$$s^2 + s K_1 K_{pd} + K_1 K_{pp} = 0$$

Underdempet

$K_{pd} = 5$ $\quad K_{pp} = 0{,}397$

$$s^2 + s K_1 K_{pd} + K_1 K_{pp} = 0$$

$K_{pd} = 0{,}2205$ $\quad K_{pp} = 0{,}315$

Kræsjet i bordet

Andre lab prep

P1

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p \\ \dot{p} \\ \dot{e} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & K_1 \\ K_2 & 0 \end{bmatrix} \begin{bmatrix} \bar{V}_s \\ V_d \end{bmatrix}$$

$$\ddot{p} = K_1 V_d = \frac{L_p K_f}{J_p} V_d$$

$$\ddot{e} = K_2 \bar{V}_s = \frac{L_3}{J_e} V_d = \frac{L_h K_f V_s}{J_e} V_d$$

P2 $\mathcal{C} = [B \quad AB \quad A^2B]$

$$AB = \begin{bmatrix} 0 & K_1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad A^2B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\mathcal{C} = \begin{bmatrix} 0 & 0 & 0 & K_1 & 0 & 0 \\ 0 & K_1 & 0 & 0 & 0 & 0 \\ K_2 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$\underline{\operatorname{rank}(\mathcal{C}) = 3}$

P3 $r = \begin{bmatrix} p_c \\ \dot{e}_c \end{bmatrix}$ $\qquad u = Fr - Kx$

$$\dot{x} = Ax + B(Fr - Kx) = (A - BK)x + BFr = 0$$

$$(A - BK)x_\infty = -BFr \Rightarrow x_\infty = -BFr(A - BK)^{-1}$$

$$\cancel{y_\infty = C[\underbrace{BFr(BK - A)^{-1}}_{x_\infty}] = r}$$

$$\Rightarrow F = [C(BK - A)^{-1}B]^{-1}$$

P4 Tuning: $\qquad Y = CX$

Bryson's rule $\qquad C \cdot \begin{bmatrix} p \\ \dot{p} \\ \dot{e} \end{bmatrix}$

$$Q_{ii} = \frac{1}{\text{maximum acceptable value } x_i^2}$$

$$R_{jj} = \frac{1}{\text{—"— } u_j^2}$$

$$Q = \begin{bmatrix} \frac{1}{p_{max}^2} & & 0 \\ & \frac{1}{\dot{p}_{max}^2} & \\ 0 & & \frac{1}{\dot{e}_{max}^2} \end{bmatrix} = \begin{bmatrix} 1 & & 0 \\ & 1 & \\ 0 & & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} \frac{1}{V_{s_{max}}^2} & 0 \\ 0 & \frac{1}{V_{d_{max}}^2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} 2.$$

P5

$$\begin{bmatrix} \dot{x} \\ \dot{x}_a \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x \\ x_a \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ I \end{bmatrix} r$$

$$\bar{A} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \qquad \bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix} \qquad \bar{G} = \begin{bmatrix} 0 \\ I \end{bmatrix}$$

$$u = \bar{F}r - \bar{K}\begin{bmatrix} x \\ x_a \end{bmatrix} = \bar{F}r - [K \quad K_a]\begin{bmatrix} x \\ x_a \end{bmatrix}$$

$$\begin{bmatrix} \dot{x} \\ \dot{x}_a \end{bmatrix} = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix}\begin{bmatrix} x \\ x_a \end{bmatrix} - \begin{bmatrix} B \\ 0 \end{bmatrix}[K \quad K_a]\begin{bmatrix} x \\ x_a \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix}\bar{F}r + \begin{bmatrix} 0 \\ -I \end{bmatrix} r$$

$$\begin{bmatrix} \dot{x} \\ \dot{x}_a \end{bmatrix} = \begin{bmatrix} A - BK & BK_a \\ C & 0 \end{bmatrix}\begin{bmatrix} x \\ x_a \end{bmatrix} + \begin{bmatrix} BF \\ -I \end{bmatrix} r$$

$$\bar{A} = \left[\begin{array}{ccc|cc} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \hline -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \end{array}\right] \qquad \bar{G} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\bar{B} = \begin{bmatrix} 0 & 0 \\ 0 & K_1 \\ K_2 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad \bar{C} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \dot{x} \\ \dot{x}_a \end{bmatrix} = \begin{bmatrix} A - BK & BK_a \\ C & 0 \end{bmatrix}\begin{bmatrix} x \\ x_a \end{bmatrix} + \begin{bmatrix} BF \\ -I \end{bmatrix} r$$

$$\dot{x} = (A - BK)x_\infty + BK_a x_{a\infty} + BFr = 0$$

$$x_\infty = -(A - BK)^{-1}B(K_a x_{a,\infty} + BFr)$$

$$Cx_\infty = -C(A - BK)^{-1}B(K_a x_{a,\infty} + Fr)$$

$$r_0 - Cx_\infty = C(A - BK^{-1})B(K_a x_{a\infty} + Fr) = 0$$

$$x = \begin{bmatrix} p \\ \dot{p} \\ e \\ \int \\ \int \end{bmatrix}$$

1

Labdag 2

1. $Q = I^{3\times 3}$

   $R = I^{2\times 2}$

2. $Q = \begin{bmatrix} 10 & & \\ & 1 & \\ & & 1 \end{bmatrix}$ $\quad R = I^{2\times 2}$

3. $Q = \begin{bmatrix} 100 & & 0 \\ & 1 & \\ 0 & & 1 \end{bmatrix}$

4: $\begin{pmatrix} 50 & 0 \\ 0 & 1 \end{pmatrix}$

5. $Q = \begin{bmatrix} 1 & & 0 \\ & 10 & \\ 0 & & 1 \end{bmatrix}$

6. $Q = \begin{bmatrix} 10 & & 0 \\ & 10 & \\ 0 & & 1 \end{bmatrix}$

$Q = \begin{bmatrix} 10 & & \\ & 1 & \\ & & 5 \end{bmatrix}$ Beste

$R = \begin{pmatrix} 0{,}5 & 0 \\ 0 & 2 \end{pmatrix}$

$K_f \approx \frac{J_p}{l_p V_b} \ddot{p}$

Prep 3 - Luenberger observer

$\dot{p} = K_1 V_d$

$\dot{e} = K_2 \tilde{V}_s$

$\dot{\lambda} = K_3 p$

$$\dot{x} = \begin{bmatrix} \dot{\tilde{p}} \\ \ddot{\tilde{p}} \\ \dot{\tilde{e}} \\ \ddot{\tilde{e}} \\ \dot{\tilde{\lambda}} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ k_3 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{p} \\ \dot{\tilde{p}} \\ \tilde{e} \\ \dot{\tilde{e}} \\ \tilde{\lambda} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & K_1 \\ 0 & 0 \\ K_2 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{V}_s \\ \tilde{V}_d \end{bmatrix}$$

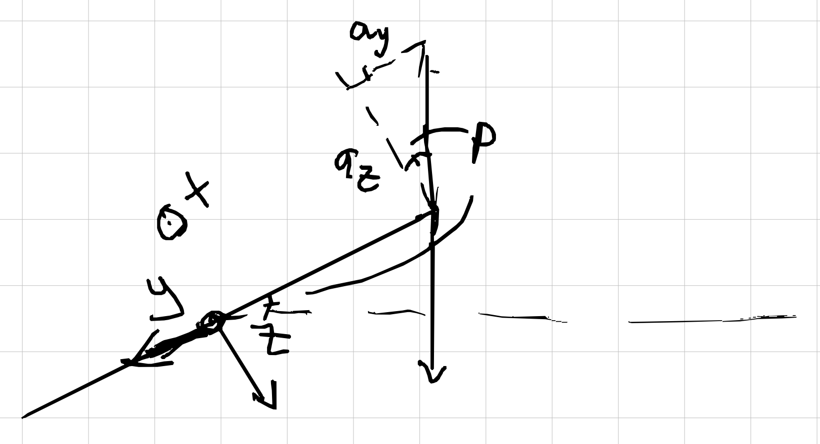
$$\tilde{y} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{p} \\ \dot{\tilde{p}} \\ \tilde{e} \\ \dot{\tilde{e}} \\ \tilde{\lambda} \end{bmatrix} = \begin{bmatrix} \tilde{p} \\ \tilde{e} \end{bmatrix}$$

2x1

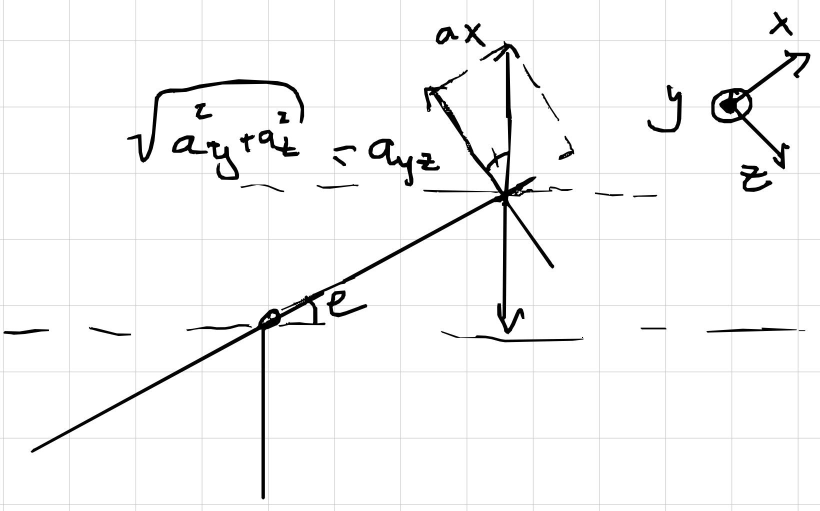
$$C_{obs} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Closed-loop system uses output to regulate.

B



$$p = \text{atan}\left(\frac{a_y}{a_z}\right)$$



$$\tan^{-1}\left(\frac{a_x}{\sqrt{a_y^2+a_z^2}}\right) = e$$

2.3.4

$\lambda = -\frac{1}{T_d}$ → very negative poles lead to fast convergence

- Too fast leads to saturation
- More noise requires a higher gain

$$\tilde{y}_{obs} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{p} \\ \dot{\tilde{p}} \\ \tilde{e} \\ \dot{\tilde{e}} \\ \tilde{\lambda} \end{bmatrix} = \begin{bmatrix} \tilde{p} \\ \tilde{e} \end{bmatrix}$$

2x1

$\tilde{p} = \tilde{e}$ $\qquad$ $\tilde{\lambda} = \tilde{e}$ affected by noise

Lab 3

Data_4.1.1

poler = [-1 -1 -2 -2 -3] Treg derivation

Data_4.1.2

[-1 -1 -2 -20 -3] Bedre

Data_4.1.3

[-10 -1 -2 -20 -2] Litt bedre

Data 4.1.4

[-10 -10 -20 -20 -30] Bedre, men litt whind-up

Data 4.1.5

[-100 -100 -200 200 -300] Veldig treg - overdempet

Data 4.1.6

[1 1 2 2 3]

Ustabil, fullt pådrag.

Prep Kalman

$$\dot{x} = \begin{bmatrix} \dot{\tilde{p}} \\ \ddot{\tilde{p}} \\ \dot{\tilde{e}} \\ \ddot{\tilde{e}} \\ \dot{\lambda} \\ \ddot{\tilde{\lambda}} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ k_3 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{p} \\ \dot{\tilde{p}} \\ \tilde{e} \\ \dot{\tilde{e}} \\ \tilde{\lambda} \\ \dot{\tilde{\lambda}} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & k_1 \\ 0 & 0 \\ k_2 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{V}_s \\ \tilde{V}_d \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & 1 & \\ & & & & & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Q_D & R_d relation is the weight of measurements (Q_D) to model (R_D)

Q_D low - slow regulation

$Q_D \to \infty$ Follows noise.

Find balance with good response, but not reacting too much to noise

$$\hat{x}_{k|k-1} = A\hat{x}_{k-1} + Bu_{k-1} \quad \text{prior}$$

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + L_k(y_k - C\hat{x}_{k|k-1}) \quad \text{posterior}$$

$$\hat{x}_{0|0} = E[x] = \mu_{x_0} \leftarrow \text{mean}()$$

$$P_{0|0} = E[e_0 e_0^T] = E[(x_0 - \mu_{x_0})(x_0 - \mu_{x_0})^T] = C_{x_0}$$

$\uparrow$ cov()

$$\hat{x}_{k|k-1} = A\hat{x}_{k-1|k-1} + Bu_{k-1}$$

$$P_{k|k-1} = AP_{k-1|k-1}A^T + Q$$

Update estimate:

$$L_k = P_{k|k-1}C^T(CP_{k|k-1}C^T + R)^{-1}$$

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + L_k(y_k - C\hat{x}_{k|k-1})$$

$$P_{k|k} = (I - L_kC)P_{k|k-1}(I - L_kC)^T + L_kR_kL_k^T$$

Hva er R_K og R_I i diskretisering